TRIGONOMÉTRIE : FORMULAIRE

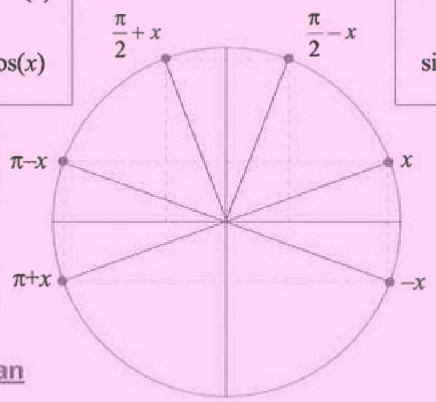
Angles associés

Une lecture efficace du cercle trigonométrique permet de retrouver les relations suivantes :

$$\cos\left(\frac{\pi}{2}+x\right) = -\sin(x) \qquad \sin\left(\frac{\pi}{2}+x\right) = \cos(x)$$

$$\cos\left(\frac{\pi}{2}-x\right) = \sin(x) \qquad \sin\left(\frac{\pi}{2}-x\right) = \cos(x)$$

$$\cos(\pi - x) = -\cos(x) \qquad \sin(\pi - x) = \sin(x)$$



$$\cos(\pi + x) = -\cos(x) \qquad \sin(\pi + x) = -\sin(x)$$

$$\cos(-x) = \cos(x) \qquad \sin(-x) = -\sin(x)$$

Relations entre cos, sin et tan

$$\cos^2(x) + \sin^2(x) = 1 \qquad 1 + \tan^2(x) = \frac{1}{\cos^2(x)}$$

Formules d'addition

$$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b) \qquad \cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b) \qquad \sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\tan(a - b) = \frac{\tan(a) - \tan(b)}{1 + \tan(a)\tan(b)} \qquad \tan(a + b) = \frac{\tan(a) + \tan(b)}{1 - \tan(a)\tan(b)}$$

Formules de duplication

$$\cos(2a) = \cos^2(a) - \sin^2(a) = 2\cos^2(a) - 1 = 1 - 2\sin^2(a) \qquad \sin(2a) = 2\sin(a)\cos(a) \qquad \tan(2a) = \frac{2\tan(a)}{1-\tan^2(a)}$$

Extensions : $\cos(3a) = 4\cos^3(a) - 3\cos(a)$ $\qquad$ $\sin(3a) = 3\sin(a) - 4\sin^3(a)$ $\qquad$ $\tan(3a) = \frac{3\tan(a)-\tan^3(a)}{1-3\tan^2(a)}$

Au delà, utiliser la formule de Moivre.

Formules de linéarisation

$$\cos^2(a) = \frac{1+\cos(2a)}{2} \qquad \sin^2(a) = \frac{1-\cos(2a)}{2} \qquad \tan^2(a) = \frac{1-\cos(2a)}{1+\cos(2a)}$$

Extensions : $\cos^3(a) = \frac{\cos(3a) + 3\cos(a)}{4}$ $\qquad$ $\sin^3(a) = \frac{-\sin(3a) + 3\sin(a)}{4}$ $\qquad$ $\tan^3(a) = \frac{-\sin(3a) + 3\sin(a)}{\cos(3a) + 3\cos(a)}$

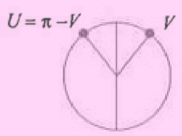
Au delà, utiliser les formules d'Euler. Les formules d'Euler permettent également de montrer que :

$$\cos(a)\cos(b) = \frac{1}{2}[\cos(a-b) + \cos(a+b)] \qquad \cos(a)\sin(b) = \frac{1}{2}[\sin(a+b) - \sin(a-b)] \qquad \sin(a)\sin(b) = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$

$$\cos(p) + \cos(q) = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right) \qquad \cos(p) - \cos(q) = -2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$

$$\sin(p) + \sin(q) = 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right) \qquad \sin(p) - \sin(q) = 2\sin\left(\frac{p-q}{2}\right)\cos\left(\frac{p+q}{2}\right)$$

Résolution d'équations trigonométriques



$$\cos(U) = \cos(V) \iff (U = V\ [2\pi] \text{ ou } U = -V\ [2\pi])$$

$$\sin(U) = \sin(V) \iff (U = V\ [2\pi] \text{ ou } U = \pi - V\ [2\pi])$$

$$\tan(U) = \tan(V) \iff U = V\ [\pi]$$

Expression du cosinus, sinus et tangente en fonction de la tangente de l'angle moitié

Si $t = \tan\left(\frac{a}{2}\right)$, on a : $\qquad \cos(a) = \frac{1-t^2}{1+t^2}$; $\sin(a) = \frac{2t}{1+t^2}$; $\tan(a) = \frac{2t}{1-t^2}$