

## homework-3

```
library(bis557)
```

### QUESTION 1

```
set.seed(1234)
X <- matrix(rnorm(9), nrow = 3, ncol = 3)
XX <- t(X) %*% X
svd(XX)$d
#> [1] 8.2407073 1.2242344 0.1353224

p <- c(0.9999, 0.1, 1e-15)
D <- p * (1-p)
H <- t(X) %*% diag(D) %*% X
svd(H)$d
#> [1] 5.058342e-02 5.383925e-04 4.540159e-16
```

From the above results we can know the linear Hessian  $X^T X$  is well conditioned, as the ratio of the largest singular value to the smallest value is approximately 61. However the logistic variation has the ratio around  $10^{14}$ , which means it is ill-conditioned.

### QUESTION 2

`glm_new()` implements a first-order solution for the GLM maximum likelihood problem using only gradient information. In the following examples, a constant step size and a Momentum adaptive step size are used to fit the GLM model.

```
library(bis557)

#devtools::build_vignettes()
n <- 5000
p <- 3
beta <- c(-1, 0.2, 0.1)
X <- cbind(1, matrix(rnorm(n * (p-1)), ncol = p - 1))
eta <- X %*% beta
lambda <- exp(eta)
y <- rpois(n, lambda = lambda)

#Example 1
fit_linear_model <- glm_new(X, y, mu_fun = function(eta) exp(eta), var_fun = identity,
                             step = 0.0001, method = "constant")
fit_linear_model$coefficients
#> [1,]
#> [1,] -0.95730122
#> [2,] 0.19535022
#> [3,] 0.03065324

#Example 2
fit_linear_model2 <- glm_new(X, y, mu_fun = function(eta) exp(eta), var_fun = identity, step = 0.0001,
```

```

friction = 0.9, method = "momentum")
fit_linear_model2$coefficients
#>           [,1]
#> [1,] -0.94106260
#> [2,]  0.22389656
#> [3,]  0.05124123

```

### QUESTION 3

The function `multi_class()` is built to classify multi-categorical variables based on logistic regression. It returns the class with the highest probability and the corresponding coefficients.

```

library(bis557)
data(iris)
X <- cbind(1, as.matrix(iris[, -5]))
y <- ifelse(iris$Species==levels(iris$Species)[1], 1, ifelse(iris$Species==levels(iris$Species)[2], 2, 3))

fit_linear_model <- multi_class(X, y, itr = 50, tolerance = 1e-10)
fit_linear_model$class
#> [1] "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1"
#> [19] "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1"
#> [37] "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "1" "2" "2" "2"
#> [55] "2" "2" "2" "2" "2" "2" "2" "2" "2" "2" "2" "2" "2" "2" "2" "3" "2"
#> [73] "2" "2" "2" "2" "2" "2" "2" "2" "2" "2" "3" "2" "2" "2" "2" "2" "2"
#> [91] "2" "2" "2" "2" "2" "2" "2" "2" "2" "3" "3" "3" "3" "3" "3" "3" "3"
#> [109] "3" "3" "3" "3" "3" "3" "3" "3" "3" "3" "3" "3" "3" "3" "3" "3" "3"
#> [127] "3" "3" "3" "3" "3" "3" "3" "2" "3" "3" "3" "3" "3" "3" "3" "3" "3"
#> [145] "3" "3" "3" "3" "3" "3"
fit_linear_model$coefficients
#>           [,1]           [,2]           [,3]           [,4]           [,5]
#> [1,] -22.349934 18.5270872 14.826355 -34.661353 -46.625047
#> [2,]  7.378487 -0.2453567 -2.796568  1.313643 -2.778344
#> [3,] -42.637804 -2.4652202 -6.680887  9.429385 18.286137

```

### REFERENCE

Textbook CASL P124 - 138.