Model Appendix: Time Variation in the News-Returns Relationship

Paul Glasserman* Fulin Li† Harry Mamaysky‡

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1 One-period model

Assume a one period model, where an agent (indexed by i) solves the following mean-variance portfolio problem with benchmarking penalty

$$\max_w w^\top (\mu_i - P) - \frac{\gamma_i}{2} w^\top \Sigma w - \frac{1}{2}(w - x)^\top \Lambda_i (w - x) \tag{1}$$

for $w, x, \mu_i, P \in \mathbb{R}^N$. P is the vector of security prices, w is the agent's portfolio holdings, μ_i is agent i's expectations about end-of-period security values, and Σ is the covariance matrix of end-of-period security values conditional on the investor's information set. The vector x captures the benchmark target, and $\Lambda_i \in \mathbb{R}^{N \times N}$ is a symmetric matrix which represents the deviation penalty. $\gamma_i \geq 0$ is the investor's risk aversion.

The first-order condition for the problem is

$$\mu_i - P - \gamma_i \Sigma w - \Lambda_i (w - x) = 0.$$

Rearranging we find

$$w_i = (\gamma_i \Sigma + \Lambda_i)^{-1} (\mu_i - P + \Lambda_i x). \tag{2}$$

The price elasticity of demand is

$$\frac{\partial w_i}{\partial P} = -(\gamma_i \Sigma + \Lambda_i)^{-1}, \tag{3}$$

so higher benchmarking penalty *or* higher risk aversion play a similar role of decreasing price elasticity. Note (3) is also the elasticity of demand with respect to an investor's beliefs about future returns μ_i.

*Columbia Business School, pg20@columbia.edu.
†University of Chicago Booth School of Business, fli3@chicagobooth.edu.
‡Columbia Business School, hm2646@columbia.edu.

Market clearing requires

$$\sum_i \phi_i w_i = S,$$

where ϕ_i is the fraction of the population represented by the i-th investor with $\sum_i \phi_i = 1$, and $S \in \mathbb{R}^N$ is the supply of shares. Using (2) we get

$$\sum_i \phi_i \left(\gamma_i \Sigma + \Lambda_i\right)^{-1} \left(\mu_i - P + \Lambda_i x\right) = S. \tag{4}$$

After rearranging

$$\sum_i \phi_i \left(\gamma_i \Sigma + \Lambda_i\right)^{-1} \left(\mu_i + \Lambda_i x\right) - S = \sum_i \phi_i (\gamma_i \Sigma + \Lambda_i)^{-1} P.$$

Assume that Σ is a diagonal matrix with all entries equal to σ^2. And assume that the benchmarking penalty $\Lambda_i = \lambda_i I$ for $\lambda_i \in \mathbb{R}$ and I an $N \times N$ identity matrix. The market clearing condition for stock $j \in \{1, \ldots, N\}$ is therefore

$$\sum_i \frac{\phi_i}{\gamma_i \sigma^2 + \lambda_i} \left(\mu_{ij} + \lambda_i x_j\right) - S_j = P_j \sum_i \frac{\phi_i}{\gamma_i \sigma^2 + \lambda_i}.$$

The risk premium in the price is not central to the present analysis, so we set $S_j = x_j = 0, \forall j$. With this we get the following equation for the equilibrium price of security j

$$\sum_i \frac{\phi_i}{\gamma_i \sigma^2 + \lambda_i} \mu_{ij} = P_j \sum_i \frac{\phi_i}{\gamma_i \sigma^2 + \lambda_i},$$

from which we get

$$P_j = \left(\sum_i \frac{\phi_i}{\gamma_i \sigma^2 + \lambda_i} \right)^{-1} \sum_i \frac{\phi_i}{\gamma_i \sigma^2 + \lambda_i} \mu_{ij}. \tag{5}$$

Since the supply of shares and the benchmark target x_j are zero, there is no risk discount in the price and P_j is simply the weighted average investor belief about future payoff of stock j.

We now specialize the equilibrium to three types of investors. A fraction ϕ_1 represents non-institutional investors. These investors have $\gamma_1 = 1$ (without loss of generality) and face no benchmarking constraints on portfolio holdings so $\lambda_1 = 0$. A fraction ϕ_2 of investors are financial intermediaries. They are less risk-averse than non-institutional investors, so $\gamma_2 = \gamma < 1$ and face no benchmarking restrictions, so $\lambda_2 = 0$. Shin (2009) shows that a VaR constraint leads to an equivalent optimization problem to (1) where γ represents the shadow cost of the VaR constraint. We can therefore interpret γ for financial intermediaries as a measure of the degree to which they are constrained in their risk-taking activities. Finally, passive institutional investors, or indexers, have zero risk-aversion but face a benchmarking restriction $\lambda_3 > 1$, because the i-th indexer is required by mandate to not deviate too far away from its benchmark index x (assumed to be zero). The degree to which the i-th investor is constrained is given by $\gamma_i \sigma^2 + \lambda_i$, so assuming $\sigma^2 \approx 1$, we

Investor	Weight	Risk Aversion	Benchmarking	Constrained
Non-institutional	ϕ_1	$\gamma_1 = 1$	$\lambda_1 = 0$	Medium
Intermediaries	ϕ_2	$\gamma_2 < 1$	$\lambda_2 = 0$	Least
Passive	ϕ_3	$\gamma_3 = 0$	$\lambda_3 > 1$	Most

can say that intermediaries are less constrained than non-institutional investors, and non-institutional investors are less constrained than indexers. The key features of the three types of investors can be summarized as follows:

We now specify how each investor group updates beliefs based on news. Assume that I_j represents good news about security j, and that $\mu_{ij} = f_i(I_j, \dots)$ with $\partial\mu_{ij}/\partial I_j > 0$, where the $\dots$ indicate that beliefs can depend on other factors besides news. We assume that non-institutional investors and intermediaries update their beliefs in proportion to the information content τ of news, so

$$\frac{\partial \mu_{ij}}{\partial I_j} = f(\tau) \qquad \text{for } i \in \{1, 2\},$$

where $f(\tau) \in (0, 1)$ and is increasing in τ. Furthermore, we assume that $\sigma^2 = \sigma^2(\tau) \in (0, 1)$ for $i = 1, 2$ and that $\sigma^2(\tau)$ is decreasing in the information content τ of news.[1] Indexers don't update beliefs in response to news, so $\partial\mu_{3j}/\partial I_j = 0$. As in (1), this reflects institutional constraints on the behavior of indexers.

Using the price from (5) we find that the sensitivity of P_j to news is

$$\frac{\partial P_j}{\partial I_j} = \frac{\frac{\phi_1}{\sigma^2(\tau)} + \frac{\phi_2}{\gamma_2 \sigma^2(\tau)}}{\frac{\phi_1}{\sigma^2(\tau)} + \frac{\phi_2}{\gamma_2 \sigma^2(\tau)} + \frac{\phi_3}{\lambda_3}} f(\tau).$$

Note that the denominator is positive, and that $\phi_1 = 1 - \phi_2 - \phi_3$. Making this substitution and multiplying by σ^2 we find:

$$\begin{aligned} \frac{\partial P_j}{\partial I_j} &= \frac{1 - \phi_2 - \phi_3 + \frac{\phi_2}{\gamma_2}}{1 - \phi_2 - \phi_3 + \frac{\phi_2}{\gamma_2} + \frac{\phi_3}{\lambda_3}\sigma^2(\tau)} f(\tau), \\ &= \frac{1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 - \phi_3}{1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 + \frac{\sigma^2(\tau) - \lambda_3}{\lambda_3}\phi_3} f(\tau). \end{aligned} \tag{6}$$

The following properties hold in equilibrium:

Proposition 1. *Price sensitivity to news increases with more intermediaries and decreases with more passive investors.*

Proposition 2. *Price sensitivity to news increases when intermediaries are less constrained, i.e., have lower γ.*

[1] This would happen under normality if news I_j consisted of the dividend plus noise, and τ was the precision of the noise term – though this argument ignores the equilibrium effect of the impact of precision on the informativeness of prices. But I think it would still go through even in equilibrium.

Proposition 3. *Price sensitivity to news increases as the information content of news grows.*

To check Proposition 3, observe that higher τ increases $f(\tau)$ and decreases $\sigma^2(\tau)$ and both effects tend to increase $\partial P_j/\partial I_j$.

To check Proposition 2, note that in the top expression in (6) both the numerator and denominator increase by the same amount when γ falls; since both are positive and the numerator is smaller than the denominator, this increases the price sensitivity, i.e., for $x, y > 0$, $\frac{d}{dx}(\frac{x}{x+y}) = \frac{1}{x+y} - \frac{x}{(x+y)^2} = \frac{1}{x+y}(1 - \frac{x}{x+y}) > 0$.

In what follows, we drop $f(\tau)$ from (6) since it's positive and does not affect the sign of the derivatives. To check Proposition 1, note that:

$$\frac{\partial}{\partial \phi_2}\left(\frac{\partial P_j}{\partial I_j}\right) = \frac{\frac{1-\gamma_2}{\gamma_2}}{1 + \frac{1-\gamma}{\gamma}\phi_2 + \frac{\sigma^2 - \lambda}{\lambda}\phi_3} - \frac{1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 - \phi_3}{(1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 + \frac{\sigma^2(\tau) - \lambda_3}{\lambda_3}\phi_3)^2}\frac{1-\gamma_2}{\gamma_2}.$$

Since the denominator is positive, the sign of this is the same as the sign of

$$\left(1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 + \frac{\sigma^2(\tau) - \lambda_3}{\lambda_3}\phi_3\right) - \left(1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 - \phi_3\right) = \sigma^2(\tau)\phi_3 > 0.$$

To check that more indexers decrease the price sensitivity to news, note that

$$\frac{\partial}{\partial \phi_3}\left(\frac{\partial P_j}{\partial I_j}\right) = -\frac{1}{1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 + \frac{\sigma^2(\tau) - \lambda_3}{\lambda_3}\phi_3} - \frac{1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 - \phi_3}{(1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 + \frac{\sigma^2(\tau) - \lambda_3}{\lambda_3}\phi_3)^2}\frac{\sigma^2(\tau) - \lambda_3}{\lambda_3}.$$

Since the denominator is positive, the sign of this is the same as the sign of

$$-\left(1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 + \frac{\sigma^2(\tau) - \lambda_3}{\lambda_3}\phi_3\right) - \left(1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 - \phi_3\right)\frac{\sigma^2(\tau) - \lambda_3}{\lambda_3}.$$

Since $\lambda_3 > 1$ and $\sigma^2(\tau) < 1$ the sign of the second term above is positive and the entire expression is therefore less than

$$-\left(1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 + \frac{\sigma^2(\tau) - \lambda_3}{\lambda_3}\phi_3\right) + \left(1 + \frac{1-\gamma_2}{\gamma_2}\phi_2 - \phi_3\right) = -\sigma^2(\tau)\phi_3 < 0.$$

2 Two-period model

We extend the model to two periods, which allows us to make predictions on price *under-reaction* to news, conditional on investor composition, intermediary constraints, and news informativeness. The key feature of the two-period model is based on Hong and Stein (1999, HS): we assume that non-institutional investors and intermediaries behave like *newswatchers* in HS: they "formulate their asset demands based on the static-optimization notion that they buy and hold until the liquidating dividend" and "they do not condition on current or past prices." HS refer to this as a "Walrasian equilibrium with private valuations," as opposed to a rational expectations equilibrium. HS motivate this behavior as a simple form of bounded rationality (see their discussion on page 2149). This assumption can be thought of as a reduced form version Sims' (2011) rational inattention. Since we are focused on price responses to public news over a short time interval, we are not concerned about overreaction, and so do not introduce HS's momentum traders.

We assume that a fraction $\theta \in [0, 1]$ of the non-institutional and intermediary sector pays attention to stock j in period 0, and the remaining fraction $1 - \theta$ pays attention to the stock in period 1. As in HS, time 0 investors remain in the market in period 1. Investors who pay attention to stock j receive the same public signal I_j. We interpret θ as the technological capacity constraint faced by investors. As technology improves, investors are able to follow more stocks, and with respect to stock j, a greater fraction of investors is able to follow the stock in period 0. Stock j pays a liquidating dividend in period 2. Our newswatchers optimize objective function (1) using their information I_j with respect to the liquidating dividend. As in HS, they do not condition on prices. The $1 - \theta$ fraction of investors who do not follow stock j in period 0 simply stay out of the market for j until period 1 – they do not have capacity to devote to following stock j in period 0. The indexers behave as before. We assume $\mu_{1j} = \mu_{2j} = \mu$ and $\mu_{3j} = 0$ for simplicity.

Given our assumptions, the period 1 price is the same as the price in the one-period model, and is given by (5), i.e.,

$$P_{1j} = \frac{\frac{\phi_1}{\sigma^2}\mu + \frac{\phi_2}{\gamma_2\sigma^2}\mu}{\frac{\phi_1}{\sigma^2} + \frac{\phi_2}{\gamma_2\sigma^2} + \frac{\phi_3}{\lambda_3}} = \frac{\phi_1\mu + \frac{\phi_2}{\gamma_2}\mu}{\phi_1 + \frac{\phi_2}{\gamma_2} + \frac{\phi_3}{\lambda_3}\sigma^2}.$$

The period 1 price reflects *all* information. Given the HS assumptions, the period 0 equilibrium is identical to the period 1 equilibrium, except the fraction of non-institutional and intermediary investors is given by $\theta\phi_1$ and $\theta\phi_2$. The period 0 price is therefore

$$P_{0j} = \frac{\phi_1\mu + \frac{\phi_2}{\gamma_2}\mu}{\phi_1 + \frac{\phi_2}{\gamma_2} + \frac{\phi_3}{\lambda_3}\frac{\sigma^2}{\theta}}. \tag{7}$$

The HS newswatcher assumption leads to a very tractable equilibrium, and, as in their paper, there is price underreaction. To see this note that

$$P_{0j} = \alpha(\theta)P_{1j} \qquad \text{and} \qquad P_{1j} - P_{0j} = (1 - \alpha(\theta))P_{1j} \tag{8}$$

where

$$\alpha(\theta) = \frac{\phi_1 + \frac{\phi_2}{\gamma_2} + \frac{\phi_3}{\lambda_3}\sigma^2}{\phi_1 + \frac{\phi_2}{\gamma_2} + \frac{\phi_3}{\lambda_3}\frac{\sigma^2}{\theta}} \qquad \text{and} \qquad \alpha(\theta) \in [0, 1].$$

Note that $\alpha(\theta)$ is increasing in θ. Therefore, when news I_j arrives, the period 0 price reaction will be smaller than the full (period 1) price reaction, and the price change from period 0 to period 1, $P_{1j} - P_{0j}$, will be nonzero and will go in the same direction as the period 0 price response:

$$\frac{\partial}{\partial I_j}(P_{1j} - P_{0j}) > 0.$$

In light of (8) the following proposition is immediate:

Proposition 4. *Propositions 1, 2, and 3 all apply to the period 0 price response to news and to the period 1 return $P_{1j} - P_{0j}$ in response to news.*

Finally, if technology improves, i.e., as θ increases, the model makes an unambiguous prediction:

Proposition 5. *As θ increases, the period 0 price response to news $\partial P_{0j}/\partial I_j$ increases, and the period 1 price change in response to news $\partial(P_{1j} - P_{0j})/\partial I_j$ decreases.*

This follows from (8) and the fact that $\alpha(\theta)$ is increasing in θ.

Of course, allowing the period 1 investors to participate in period 0 trading while conditioning on the period 0 price of j, or allowing for arbitrageurs who can profit from understanding the dynamics of the model, would make our results less stark. But the main intuition of price underreaction and its dependence on technological constraints would remain.

2.1 Period 0 and 1 holdings by intermediaries

From (2), the demands of the three investor types for stock j are

$$\begin{aligned}
w_{1j} &= \frac{1}{\sigma^2}(\mu_j - P_j) \\
w_{2j} &= \frac{1}{\gamma_2\sigma^2}(\mu_j - P_j) \\
w_{3j} &= -\frac{1}{\lambda_3}P_j,
\end{aligned}$$

where we assume that $\mu_{1j} = \mu_{2j} = \mu_j > 0$ and $\mu_{3j} = 0$. Market clearing thus requires that

$$\theta\phi_1 w_{1j} + \theta\phi_2 w_{2j} + \phi_3 w_{3j} = S_j > 0.$$

Plugging in the above demands, P_j must satisfy

$$\theta\frac{\phi_1}{\sigma^2}(\mu_j - P_j) + \theta\frac{\phi_2}{\gamma_2\sigma^2}(\mu_j - P_j) - \frac{\phi_3}{\lambda_3}P_j = S_j.$$

Rearraging we find that

$$\begin{aligned}
P_j &= \left(\theta\frac{\phi_1}{\sigma^2} + \theta\frac{\phi_2}{\gamma_2\sigma^2}\right)\left(\theta\frac{\phi_1}{\sigma^2} + \theta\frac{\phi_2}{\gamma_2\sigma^2} + \frac{\phi_3}{\lambda_3}\right)^{-1}\mu_j - \left(\theta\frac{\phi_1}{\sigma^2} + \theta\frac{\phi_2}{\gamma_2\sigma^2} + \frac{\phi_3}{\lambda_3}\right)^{-1}S_j \\
&= \frac{\theta\phi_1\gamma_2\lambda_3 + \theta\phi_2\lambda_3}{\theta\phi_1\gamma_2\lambda_3 + \theta\phi_2\lambda_3 + \phi_3\gamma_2\sigma^2}\mu_j - \frac{\gamma_2\sigma^2\lambda_3}{\theta\phi_1\gamma_2\lambda_3 + \theta\phi_2\lambda_3 + \phi_3\gamma_2\sigma^2}S_j.
\end{aligned}$$

The period 0 demand of the intermediary sector $X_2(\theta) = \theta\phi_2 w_2$ is therefore given by

$$\begin{aligned}
X_{2j}(\theta) &= \theta\phi_2\frac{1}{\gamma_2\sigma^2}(\mu_j - P_j) \\
&= \theta\phi_2\frac{1}{\gamma_2\sigma^2}\frac{\phi_3\gamma_2\sigma^2}{\theta\phi_1\gamma_2\lambda_3 + \theta\phi_2\lambda_3 + \phi_3\gamma_2\sigma^2}\mu_j + \frac{\theta\phi_2\lambda_3}{\theta\phi_1\gamma_2\lambda_3 + \theta\phi_2\lambda_3 + \phi_3\gamma_2\sigma^2}S_j \\
&= \frac{\phi_2\phi_3}{\phi_1\gamma_2\lambda_3 + \phi_2\lambda_3 + \phi_3\gamma_2\sigma^2/\theta}\mu_j + \frac{\phi_2\lambda_3}{\phi_1\gamma_2\lambda_3 + \phi_2\lambda_3 + \phi_3\gamma_2\sigma^2/\theta}S_j.
\end{aligned}$$

Given that all quantities in $X_{2j}(\theta)$ are positive, it is easy to see that $\partial X_{2j}(\theta)/\partial\theta > 0$, and therefore $X_{2j}(1) - X_{2j}(\theta) > 0$ for $\theta < 1$. So the intermediary sector adds to its holdings of stock j in period 1. Furthermore, since $\partial\mu_j/\partial I_j > 0$, we will have that $\partial(X_{2j}(1) - X_{2j}(\theta))/\partial I_j > 0$, meaning that with good news, intermediaries increase their period 1 buying by even more.

References

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