Homework 2

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Problem 1

1.Closed form function

Q: Implement a function closed_form_1 that computes this closed form solution given the features $\mathbf{X}$, labels Y (using Python or Matlab).

In [51]:

```
import numpy as np
import pandas as pd
from numpy.linalg import inv
import matplotlib.pyplot as plt

climate_change_1 = pd.read_csv('climate_change_1.csv')
climate_change_1
```

Out[51]:

	Year	Month	MEI	CO2	CH4	N2O	CFC-11	CFC-12	TSI	Aerosols	Ten
0	1983	5	2.556	345.96	1638.59	303.677	191.324	350.113	1366.1024	0.0863	0.1
1	1983	6	2.167	345.52	1633.71	303.746	192.057	351.848	1366.1208	0.0794	0.1
2	1983	7	1.741	344.15	1633.22	303.795	192.818	353.725	1366.2850	0.0731	0.1
3	1983	8	1.130	342.25	1631.35	303.839	193.602	355.633	1366.4202	0.0673	0.1
4	1983	9	0.428	340.17	1648.40	303.901	194.392	357.465	1366.2335	0.0619	0.1
5	1983	10	0.002	340.30	1663.79	303.970	195.171	359.174	1366.0589	0.0569	0.0
6	1983	11	-0.176	341.53	1658.23	304.032	195.921	360.758	1366.1072	0.0524	0.2
7	1983	12	-0.176	343.07	1654.31	304.082	196.609	362.174	1366.0607	0.0486	0.0
8	1984	1	-0.339	344.05	1658.98	304.130	197.219	363.359	1365.4261	0.0451	0.0
9	1984	2	-0.565	344.77	1656.48	304.194	197.759	364.296	1365.6618	0.0416	0.0
10	1984	3	0.131	345.46	1655.77	304.285	198.249	365.044	1366.1697	0.0383	0.0
11	1984	4	0.331	346.77	1657.68	304.389	198.723	365.692	1365.5660	0.0352	-0.0
12	1984	5	0.121	347.55	1649.33	304.489	199.233	366.317	1365.7783	0.0324	0.0
13	1984	6	-0.142	346.98	1634.13	304.593	199.858	367.029	1366.0956	0.0302	-0.0
14	1984	7	-0.138	345.55	1629.89	304.722	200.671	367.893	1366.1145	0.0282	-0.0
15	1984	8	-0.179	343.20	1643.67	304.871	201.710	368.843	1365.9781	0.0260	0.0
16	1984	9	-0.082	341.35	1663.60	305.021	202.972	369.800	1365.8669	0.0239	0.0
17	1984	10	0.016	341.68	1674.65	305.158	204.407	370.782	1365.7869	0.0220	-0.0
18	1984	11	-0.351	343.06	1677.10	305.263	205.893	371.770	1365.6802	0.0202	-0.1
19	1984	12	-0.611	344.54	1672.15	305.313	207.308	372.701	1365.7617	0.0188	-0.2
20	1985	1	-0.561	345.25	1663.42	305.301	208.537	373.623	1365.6082	0.0164	-0.0
21	1985	2	-0.602	346.06	1666.21	305.243	209.543	374.681	1365.7085	0.0160	-0.1
22	1985	3	-0.737	347.66	1678.34	305.165	210.368	376.004	1365.6570	0.0141	-0.0
23	1985	4	-0.484	348.20	1675.24	305.093	211.111	377.635	1365.5120	0.0138	-0.0
24	1985	5	-0.731	348.92	1666.83	305.045	211.823	379.539	1365.6366	0.0128	0.0
25	1985	6	-0.086	348.40	1659.40	305.027	212.512	381.642	1365.6964	0.0126	-0.0
26	1985	7	-0.156	346.66	1654.25	305.049	213.165	383.905	1365.6509	0.0121	-0.0
27	1985	8	-0.392	344.85	1654.41	305.126	213.803	386.223	1365.7499	0.0116	0.0
28	1985	9	-0.541	343.20	1668.31	305.250	214.501	388.500	1365.6653	0.0102	-0.0

	Year	Month	MEI	CO2	CH4	N2O	CFC-11	CFC-12	TSI	Aerosols	Ten
29	1985	10	-0.140	343.08	1681.56	305.395	215.327	390.676	1365.5269	0.0101	-0.0
...	...	...	...	...	...	...	...	...	...	...	
278	2006	7	0.628	382.38	1765.95	319.872	249.247	539.725	1365.8212	0.0038	0.4
279	2006	8	0.759	380.45	1762.66	319.930	248.981	539.682	1365.7067	0.0041	0.4
280	2006	9	0.793	378.92	1776.04	320.010	248.775	539.566	1365.8419	0.0043	0.4
281	2006	10	0.892	379.16	1789.02	320.125	248.666	539.488	1365.8270	0.0044	0.4
282	2006	11	1.292	380.18	1791.91	320.321	248.605	539.500	1365.7039	0.0049	0.4
283	2006	12	0.951	381.79	1795.04	320.451	248.480	539.377	1365.7087	0.0054	0.5
284	2007	1	0.974	382.93	1799.66	320.561	248.372	539.206	1365.7173	0.0054	0.6
285	2007	2	0.510	383.81	1803.08	320.571	248.264	538.973	1365.7145	0.0051	0.4
286	2007	3	0.074	384.56	1803.10	320.548	247.997	538.811	1365.7544	0.0045	0.4
287	2007	4	-0.049	386.40	1802.11	320.518	247.574	538.586	1365.7228	0.0045	0.4
288	2007	5	0.183	386.58	1795.65	320.445	247.224	538.130	1365.6932	0.0041	0.3
289	2007	6	-0.358	386.05	1781.81	320.332	246.881	537.376	1365.7616	0.0040	0.3
290	2007	7	-0.290	384.49	1771.89	320.349	246.497	537.113	1365.7506	0.0040	0.3
291	2007	8	-0.440	382.00	1779.38	320.471	246.307	537.125	1365.7566	0.0041	0.3
292	2007	9	-1.162	380.90	1794.21	320.618	246.214	537.281	1365.7159	0.0042	0.4
293	2007	10	-1.142	381.14	1802.38	320.855	246.189	537.380	1365.7388	0.0041	0.3
294	2007	11	-1.177	382.42	1803.79	321.062	246.178	537.319	1365.6680	0.0042	0.2
295	2007	12	-1.168	383.89	1805.58	321.217	246.261	537.052	1365.6927	0.0040	0.2
296	2008	1	-1.011	385.44	1809.92	321.328	246.183	536.876	1365.7163	0.0038	0.0
297	2008	2	-1.402	385.73	1803.45	321.345	245.898	536.484	1365.7366	0.0036	0.1
298	2008	3	-1.635	385.97	1792.84	321.295	245.430	535.979	1365.6726	0.0034	0.4
299	2008	4	-0.942	387.16	1792.57	321.354	245.086	535.648	1365.7146	0.0033	0.2
300	2008	5	-0.355	388.50	1796.43	321.420	244.914	535.399	1365.7175	0.0031	0.2
301	2008	6	0.128	387.88	1791.80	321.447	244.676	535.128	1365.6730	0.0031	0.3
302	2008	7	0.003	386.42	1782.93	321.372	244.434	535.026	1365.6720	0.0033	0.4
303	2008	8	-0.266	384.15	1779.88	321.405	244.200	535.072	1365.6570	0.0036	0.4
304	2008	9	-0.643	383.09	1795.08	321.529	244.083	535.048	1365.6647	0.0043	0.3
305	2008	10	-0.780	382.99	1814.18	321.796	244.080	534.927	1365.6759	0.0046	0.4
306	2008	11	-0.621	384.13	1812.37	322.013	244.225	534.906	1365.7065	0.0048	0.3
307	2008	12	-0.666	385.56	1812.88	322.182	244.204	535.005	1365.6926	0.0046	0.3

308 rows × 11 columns

In [52]:

```
climate_change_1_train=climate_change_1.iloc[0:284]
#climate_change_1_train
climate_change_1_test=climate_change_1.iloc[284:308]
#climate_change_1_test
```

In [53]:

```
def closed_form_1(df: pd.core.frame.DataFrame, column: int = 10)-> np.ndarray:
    X = df.drop(df.columns[column],axis=1).to_numpy()
    X = np.concatenate([np.ones((len(X),1)),X],axis = 1)
    # X: the features
    Y = df.iloc[:,[column]].to_numpy()
    Y = Y.reshape((len(Y)))
    # Y: the results
    theta = inv(X.T @ X)@ X.T @ Y
    return theta

def closed_form_2(X:np.ndarray, Y:np.ndarray)-> np.ndarray:
    theta = inv(X.T @ X)@ X.T @ Y
    return theta

closed_form_1(climate_change_1_train)[1:]
```

Out[53]:

```
array([ 8.23965077e-03, -3.61230802e-03,  6.44651665e-02,  2.51139083e
-03,
        1.87173456e-04, -1.63078511e-02, -6.27428702e-03,  3.42652458e
-03,
        9.51816184e-02, -1.54295992e+00])
```

In [54]:

```
# Using scipy to check:
from sklearn.linear_model import LinearRegression as lm
X=climate_change_1_train.drop(climate_change_1_train.columns[10],axis=1).to_numpy()
Y=climate_change_1_train.iloc[:,[10]].to_numpy()
l=lm().fit(X,Y)
l.coef_
```

Out[54]:

```
array([[ 8.23964892e-03, -3.61230811e-03,  6.44651665e-02,
         2.51139097e-03,  1.87173443e-04, -1.63078496e-02,
        -6.27428715e-03,  3.42652470e-03,  9.51816165e-02,
        -1.54295993e+00]])
```

2.R2

Q: Write down the mathematical formula for the linear model and evaluate the model R2 on the training set and the testing set.

In [55]:

```
r_sq = l.score(X, Y)
r_sq
```

Out[55]:

```
0.7549422940386257
```

3.Significant Variables

Q: Which variables are significant in the model?

In [56]:

```
import statsmodels.api as sm
mod = sm.OLS(Y,X)
fit = mod.fit()
p_values = fit.summary2().tables[1]['P>|t|']
p_values
```

Out[56]:

```
x1      6.334282e-08
x2      1.798940e-03
x3      5.820911e-20
x4      2.569273e-02
x5      7.630079e-01
x6      2.974904e-02
x7      4.331089e-08
x8      1.284179e-08
x9      7.060829e-08
x10     6.742765e-13
Name: P>|t|, dtype: float64
```

```
MEI,CO2,N2O,CFC-11,CFC-12,TSI,Aerosols are significant in the model(0.05
significant level).
We can ignore the influence of the year and month later.
```

4. For climate_change_2.csv

Q: Write down the necessary conditions for using the closed form solution. And you can apply it to the dataset climate_change_2.csv, explain the solution is unreasonable.

In [24]:

```
climate_change_2 = pd.read_csv('climate_change_2.csv')
#climate_change_2
```

In [25]:

```
climate_change_2_train=climate_change_2.iloc[0:284]
climate_change_2_test=climate_change_2.iloc[284:308]
closed_form_1(climate_change_2_train)[1:]
```

Out[25]:

```
array([ 2.01083594e-12, -5.36251599e-12,  5.10582687e-11, -1.28112521e
-11,
        1.00000000e-03,  5.87157545e-11,  4.17712392e-12, -3.05891423e
-12,
        2.25990338e-11,  1.08553680e-10, -1.01428310e-11])
```

In [29]:

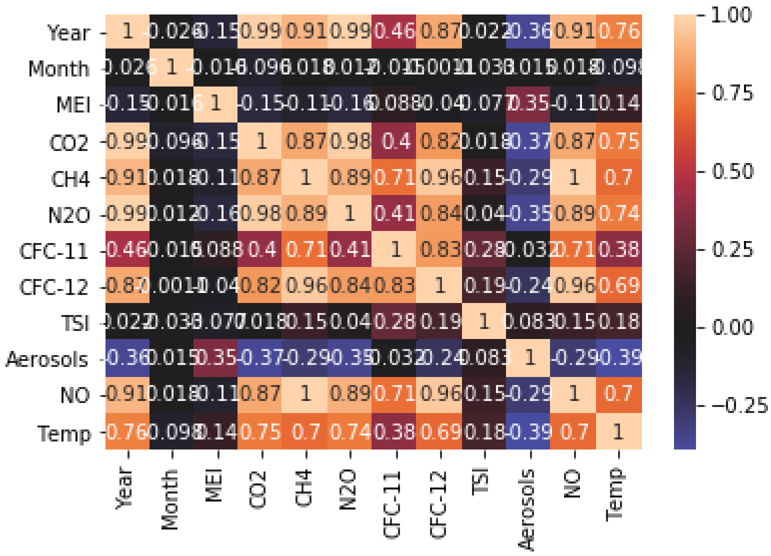
```
from sklearn.linear_model import LinearRegression as lm
X=climate_change_2_train.drop(climate_change_2_train.columns[10],axis=1).to_numpy()
Y=climate_change_2_train.iloc[:,[10]].to_numpy()
l=lm().fit(X,Y)
l.coef_
```

Out[29]:

```
array([[ 1.12051329e-17,  6.66402228e-18,  1.04772243e-17,
         7.73061207e-18,  1.00000000e-03, -1.08634025e-17,
         7.13888333e-18, -6.95792118e-18, -1.76950359e-17,
         1.12119206e-16,  1.91583910e-16]])
```

In [28]:

```
import pandas as pd
climate_change_2_corr = climate_change_2.corr()
# Visualization
import matplotlib.pyplot as mp, seaborn
seaborn.heatmap(climate_change_2_corr, center=0, annot=True)
mp.show()
```



It can be concluded from the correlation matrix that NO and CH4 are completely linearly correlated, so there is no inverse matrix, and the formula is invalid. So the solution is unreasonable.

Problem 2----Regularization

1.Loss Function

Q: Please write down the loss function for linear model with L1 regularization, L2 regularization, respectively.

In [34]:

```
#L1 regularization
def L1Norm(l, theta):
    return  np.dot(np.abs(theta), np.ones(theta.size)) * l

def L1NormPartial(l, theta):
    return np.sign(theta) * l

# For linear regression, the derivative of J function is:
def __Jfunction(self):
    sum = 0
    for i in range(0, self.m):
        err = self.__error_dist(self.x[i], self.y[i])
        sum += np.dot(err, err)
        sum += Regularization.L2Norm(0.8, self.theta)
        return 1/(2 * self.m) * sum
```

In [37]:

```
#L2 regularization
def L2Norm(l, theta):
    return  np.dot(theta, theta) * l

def L2NormPartial(l, theta):
    return theta * l

# For linear regression, the derivative of J function is:
def __partialderiv_J_func(self):
        sum = 0
        for i in range(0, self.m):
            err = self.__error_dist(self.x[i], self.y[i])
            sum += np.dot(self.x[i], err)
            sum += Regularization.L2NormPartial(0.8, self.theta)
            return 1/self.m * sum
```

2.Closed Form Solution

Q: The closed form solution for linear model with L2 regularization: $\boldsymbol{\theta} = (\mathbf{XTX} + \boldsymbol{\lambda}\mathbf{I}){-}\mathbf{1XTY}$ where I is the identity matrix. Write a function closed_form_2 that computes this closed form solution given the features X, labels Y and the regularization parameter λ.

We can answer questions 2 and 4 together.

In [48]:

```
def closed_form_2():

    dataset = pd.read_csv("climate_change_1.csv")
    X = dataset.get(["MEI","CO2","CH4","N2O","CFC-11","CFC-12","TSI","Aerosols"])

    y = dataset.get("Temp")

    X = np.column_stack((X,np.ones(len(X))))

    for lambda1 in [10,1,0.1,0.01,0.001]:
        X_train = X[:284]
        X_test = X[284:]
        y_train = y[:284]
        y_test = y[284:]

        X_train=np.mat(X_train)
        y_train = np.mat(y_train).T
        xTx = X_train.T*X_train
        w = 0
        print("="*25+"L2 Regularization (lambda is "+str(lambda1)+")"+"="*25)
        I_m= np.eye(X_train.shape[1])
        if np.linalg.det(xTx+lambda1*I_m)==0.0:
            print("xTx is invertible")
        else:
            print(np.linalg.det(xTx+lambda1*I_m))
            w= (xTx+lambda1*I_m).I*(X_train.T*y_train)
        wights = np.ravel(w)
        y_train_pred = np.ravel(np.mat(X_train)*np.mat(w))
        y_test_pred = np.ravel(np.mat(X_test)*np.mat(w))
        coef_=wights[:-1]
        intercept_=wights[-1]

        X_train = np.ravel(X_train).reshape(-1,9)
        y_train = np.ravel(y_train)

        print("Coefficient: ",coef_)
        print("Intercept: ",intercept_)
        print("the model is: y = ",coef_,"* X +(",intercept_,")")
        y_train_avg = np.average(y_train)

        R2_train = 1-(np.average((y_train-y_train_pred)**2))/(np.average((y_train-y_
        print("R2 in Train : ",R2_train)

        y_test_avg = np.average(y_test)
        R2_test = 1-(np.average((y_test-y_test_pred)**2))/(np.average((y_test-y_test
        print("R2 in Test : ",R2_test)

closed_form_2()
```

```
=========================L2 Regularization (lambda is 10)=============
============
4.052005289688253e+33
Coefficient:  [ 0.04054315  0.00814554  0.00020508 -0.01608137 -0.0063
6145  0.003689
  0.00126458 -0.02443305]
Intercept:  -0.0002202205828863274
the model is:  y =  [ 0.04054315  0.00814554  0.00020508 -0.01608137 -
0.00636145  0.003689

  0.00126458 -0.02443305] * X +( -0.0002202205828863274 )
```

```
R2 in Train :   0.6803719394071281
R2 in Test :   -0.7061640575416965
========================L2 Regularization (lambda is 1)==============
===========
4.182558175861993e+31
Coefficient:  [ 0.04395558  0.00804313  0.00021395 -0.01693027 -0.0064
6627  0.00376881
  0.00146759 -0.21177258]
Intercept:  -0.0022945422838525635
the model is: y =  [ 0.04395558  0.00804313  0.00021395 -0.01693027 -
0.00646627  0.00376881
  0.00146759 -0.21177258] * X +( -0.0022945422838525635 )
R2 in Train :   0.6897571586198687
R2 in Test :   -0.5861726468586046
========================L2 Regularization (lambda is 0.1)============
=============
1.0051083854786037e+30
Coefficient:  [ 5.06851277e-02  6.98925378e-03  1.30761990e-04 -1.4815
6599e-02
 -6.07864608e-03  3.66100278e-03  1.36118274e-03 -8.71332452e-01]
Intercept:  -0.025045661913281534
the model is: y =  [ 5.06851277e-02  6.98925378e-03  1.30761990e-04 -
1.48156599e-02
 -6.07864608e-03  3.66100278e-03  1.36118274e-03 -8.71332452e-01] * X
+( -0.025045661913281534 )
R2 in Train :   0.7110310866063567
R2 in Test :   -0.36213522139292387
========================L2 Regularization (lambda is 0.01)===========
==============
6.930175866500259e+28
Coefficient:  [ 5.46344723e-02  6.35012916e-03  7.94610956e-05 -1.3479
4077e-02
 -5.83699154e-03  3.59093203e-03  1.44947810e-03 -1.26505174e+00]
Intercept:  -0.26232414556713424
the model is: y =  [ 5.46344723e-02  6.35012916e-03  7.94610956e-05 -
1.34794077e-02
 -5.83699154e-03  3.59093203e-03  1.44947810e-03 -1.26505174e+00] * X
+( -0.26232414556713424 )
R2 in Train :   0.7153953027375966
R2 in Test :   -0.24446585990645597
========================L2 Regularization (lambda is 0.001)==========
===============
6.742979655964518e+27
Coefficient:  [ 5.53981612e-02  6.25686043e-03  7.26293229e-05 -1.3335
9358e-02
 -5.81554289e-03  3.58444627e-03  3.15485922e-03 -1.32868779e+00]
Intercept:  -2.5924696366355677
the model is: y =  [ 5.53981612e-02  6.25686043e-03  7.26293229e-05 -
1.33359358e-02
 -5.81554289e-03  3.58444627e-03  3.15485922e-03 -1.32868779e+00] * X
+( -2.5924696366355677 )
R2 in Train :   0.7168000467674187
R2 in Test :   -0.21604536980238898
```

3.Comparasion

Q: Compare the two solutions in problem 1 and problem 2 and explain the reason why linear model with L2 regularization is robust. (using climate_change_1.csv)

It will reduce the coefficient of unimportant prediction factors close to 0 and avoid overfitting. In L2 model, it is less sensitive to single variable, so it is more robust.

4.Change the regularization parameter λ

Q: You can change the regularization parameter λ to get different solutions for this problem. Suppose we set λ = 10, 1, 0.1, 0.01, 0.001, and please evaluate the model R2 on the training set and the testing set. Finally, please decide the best regularization parameter λ. (Note that: As a qualified data analyst, you must know how to choose model parameters, please learn about cross validation methods.)

The anwser can see the above(in Q2).

Problem 3 — Feature Selection

1.Workflow

Q: From Problem 1, you can know which variables are significant, therefore you can use less variables to train model. For example, remove highly correlated and redundant features. You can propose a workflow to select feature.

Solution: For m features, from k=1 to k = m: We can choose k features from m features, and establish C (m, K) models, then choose the best one (MSE minimum or R2 maximum); Then select an optimal model from the m optimal models.

2.Better Model

Train a better model than the model in Problem 2.

In [41]:

```
import numpy as np
import pandas as pd
from statsmodels.stats.outliers_influence import variance_inflation_factor
from sklearn import linear_model

#Variance Inflation Factor
def vif(X, thres=10.0):
    col = list(range(X.shape[1]))
    dropped = True
    while dropped:
        dropped = False
        vif = [variance_inflation_factor(X.iloc[:,col].values, ix) for ix in range(X
        maxvif = max(vif)
        maxix = vif.index(maxvif)
        if maxvif > thres:
            del col[maxix]
            print('delete=',X.columns[col[maxix]],'   ', 'vif=',maxvif )
            dropped = True
    print('Remain Variables:', list(X.columns[col]))
    print('VIF:', vif)
    return list(X.columns[col])

dataset = pd.read_csv("climate_change_1.csv")
X = dataset.get(["MEI","CO2","CH4","N2O","CFC-11","CFC-12","TSI","Aerosols"])

y = dataset.get("Temp")

X_train = X[:284]
X_test = X[284:]
y_train = y[:284]
y_test = y[284:]
d = vif(X_train)
print(d)

X = dataset.get( ['MEI', 'CFC-12', 'Aerosols'])
y = dataset.get("Temp")
X_train = X[:284]
X_test = X[284:]
y_train = y[:284]
y_test = y[284:]

regr = linear_model.LinearRegression()
regr.fit(X_train,y_train)
print('coefficients(b1,b2...):',regr.coef_)
print('intercept(b0):',regr.intercept_)
y_train_pred = regr.predict(X_train)

R2_1 = regr.score(X_train, y_train)
print(R2_1)
R2_2 = regr.score(X_test, y_test)
print(R2_2)
```

```
delete= CFC-11    vif= 239743.2424704495
delete= Aerosols    vif= 29867.18540477364
delete= CFC-11    vif= 11884.79599294173
delete= CFC-12    vif= 502.06957361985695
delete= CFC-12    vif= 122.31236225671839
Remain Variables: ['MEI', 'CFC-12', 'Aerosols']
VIF: [1.2888871669460935, 1.33868239281389, 1.48103752609454]
```

```
['MEI', 'CFC-12', 'Aerosols']
coefficients(b1,b2...): [ 5.54993375e-02  1.86365387e-03 -2.08242114e+
00]
intercept(b0): -0.6553255026654846
0.5996443150479794
0.004717686204946725
```

Problem 4 — Gradient Descent

Gradient descent algorithm is an iterative process that takes us to the minimum of a function. Please write down the iterative expression for updating the solution of linear model and implement it using Python or Matlab in gradientDescent function.

In [7]:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

def costFunc(X,Y,theta):
    #cost func
    inner=np.power((X*theta.T)-Y,2)
    return np.sum(inner)/(2*len(X))

def gradientDescent(X,Y,theta,alpha,iters):
    temp = np.mat(np.zeros(theta.shape))
    cost = np.zeros(iters)
    thetaNums = int(theta.shape[1])

    for i in range(iters):
        error = (X*theta.T-Y)
        for j in range(thetaNums):
            derivativeInner = np.multiply(error,X[:,j])
            temp[0,j] = theta[0,j]-(alpha*np.sum(derivativeInner)/len(X))
        theta = temp
        cost[i]=costFunc(X,Y,theta)
    return theta,cost


dataset = pd.read_csv("climate_change_1.csv")
X = dataset.get(["MEI","CO2","CH4","N2O","CFC-11","CFC-12","TSI","Aerosols"])

y = dataset.get("Temp")
X = np.column_stack((np.ones(len(X)),X))
X_train = X[:284]
X_test = X[284:]
y_train = y[:284]
y_test = y[284:]

X_train = np.mat(X_train)
Y_train = np.mat(y_train).T

for i in range(1,9):
    X_train[:,i] = (X_train[:,i] - min(X_train[:,i])) / (max(X_train[:,i]) - min(X_t

theta_n = (X_train.T*X_train).I*X_train.T*Y_train
print("theta =",theta_n)
theta = np.mat([0,0,0,0,0,0,0,0,0])
iters = 100000
alpha = 0.001

finalTheta,cost = gradientDescent(X_train,Y_train,theta,alpha,iters)
print("final theta ",finalTheta)
print("cost ",cost)

fig, bx = plt.subplots(figsize=(8,6))
bx.plot(np.arange(iters), cost, 'r')
bx.set_xlabel('Iterations')
bx.set_ylabel('Cost')
bx.set_title('Error vs. Training Epoch')
plt.show()
```

```
theta = [[-0.07698894]

 [ 0.29450977]
```

```
 [ 0.28935427]
 [ 0.02211171]
 [-0.27724073]
 [-0.53156629]
 [ 0.7376296 ]
 [ 0.17604596]
 [-0.22725924]]
final theta  [[-0.09315388  0.26327692  0.20584575  0.05590722  0.1773
908  -0.10907193
   0.09177624  0.13999486 -0.2096555 ]]
cost  [0.04678781 0.04652792 0.04626986 ... 0.00428416 0.00428416 0.00
428416]
```