

# ***FUNKCJA***

$$f(x) = \frac{x^2-16}{9-x^2}$$

## **1. Dziedzina**

$$9 - x^2 = 0$$

$$x \in \mathbb{R} \setminus \{3, -3\}$$

## **2. Miejsca zerowe**

$$f(x) = 0$$

$$\frac{x^2-16}{9-x^2} = 0$$

...

$$x^2 - 16 = 0$$

$$x_0 = -4 \vee x_0 = 4$$

## **3. Przecięcie z osią OY**

$$f(0) = y_0$$

$$\frac{0^2-16}{9-0^2} = y_0$$

$$y_0 = -\frac{16}{9}$$

## **4. Parzystość funkcji**

$$\frac{x^2-16}{9-x^2} = \frac{(-x)^2-16}{9-(-x)^2}$$

...

$$\frac{x^2-16}{9-x^2} = \frac{x^2-16}{9-x^2}$$

$$\bigvee_{x \in X} f(x) = f(-x)$$

## 5. Granice

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 - \frac{16}{x^2}\right)}{x^2 \left(\frac{9}{x^2} - 1\right)} = -1$$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x^2 \left(1 - \frac{16}{x^2}\right)}{x^2 \left(\frac{9}{x^2} - 1\right)} = -1$$

$$\lim_{x \rightarrow -3^+} f(x) = \left[ \begin{array}{c} - \\ + \end{array} \right] = -\infty$$

$$\lim_{x \rightarrow -3^-} f(x) = \left[ \begin{array}{c} - \\ - \end{array} \right] = \infty$$

$$\lim_{x \rightarrow 3^+} f(x) = \left[ \begin{array}{c} - \\ - \end{array} \right] = \infty$$

$$\lim_{x \rightarrow 3^-} f(x) = \left[ \begin{array}{c} - \\ + \end{array} \right] = -\infty$$

## 6. Asymptoty

a) pozioma

-  $\{-1\}$

b) pionowa

-  $\{-3, 3\}$

c) ukośna

- brak

## 7. Miejsce zerowe pochodnej

$$f'(x) = -\frac{14x}{(9-x^2)^2}$$

$$f'(x) = 0$$

$$-\frac{14x}{(9-x^2)^2} = 0$$

$$x = 0$$

### 7.1 Znak pochodnej

$$f'(x) > 0, x \in (-\infty, -3) \cup (-3, 0)$$

$$f'(x) = 0, x \in \{-3, 3\}$$

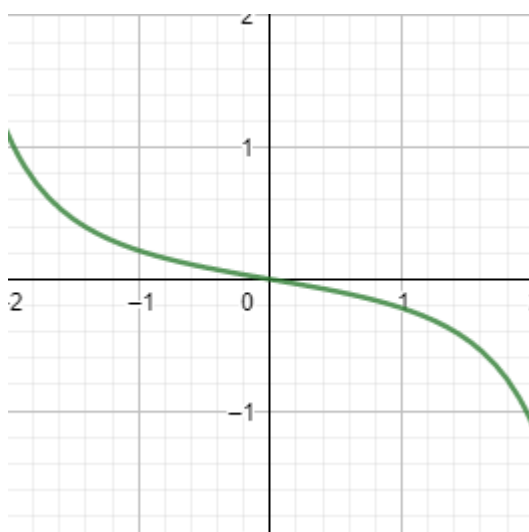
$$f'(x) < 0, x \in (0, 3) \cup (3, \infty)$$

### 7.2 Monotoniczność

$$f \nearrow, x \in (-\infty, -3), (-3, 0)$$

$$f \searrow, x \in (0, 3), (3, \infty)$$

## 8. Minimum i Maximum



$$f(0) = -\frac{16}{9} \approx -1,78 \Rightarrow \text{Max}$$

*Brak Min*

9. Tabela

|         |                 |      |            |      |            |                          |
|---------|-----------------|------|------------|------|------------|--------------------------|
|         | $(-\infty, -4)$ | $-4$ | $(-4, -3)$ | $-3$ | $(-3, 0)$  | $0$                      |
| $f'(x)$ | $+$             | $+$  | $+$        | $X$  | $+$        | $0$                      |
| $f(x)$  | $\nearrow$      | $0$  | $\nearrow$ | $X$  | $\nearrow$ | $Max$<br>$-\frac{16}{9}$ |

|         |            |     |            |     |               |
|---------|------------|-----|------------|-----|---------------|
|         | $(0, 3)$   | $3$ | $(3, 4)$   | $4$ | $(4, \infty)$ |
| $f'(x)$ | $-$        | $X$ | $-$        | $-$ | $+$           |
| $f(x)$  | $\searrow$ | $X$ | $\searrow$ | $0$ | $\searrow$    |

10. Wykres

