Designing and Calculating Bounded Areas

STAGE 2 Mathematical Methods

Purpose

In this task, you will use your knowledge of functions and calculus to model the outline of a visual design, such as a stylised logo, architectural silhouette, natural formation, or a cultural/geometric pattern.

Your final design must be created using multiple functions, joined to form a continuous, coherent outline. This task requires a combination of mathematical modelling, domain restrictions, function transformations, and calculus-based reasoning.

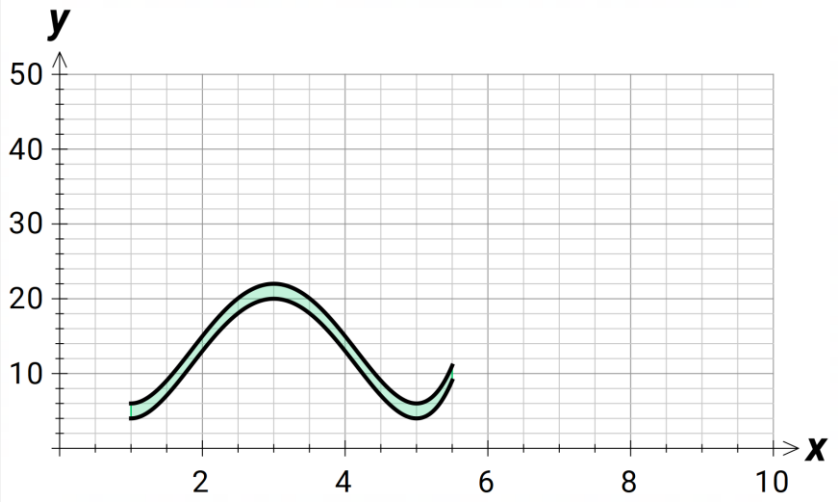
Assessment Type:	Mathematical Investigation	**Task Weighting**	20%

Task Summary

Part A – Working with a Given Function and Designing with Functions

You are given the quartic function $f_1(x) = (x-1)^2(x-5)^2 + 6$, for $1 \leq x \leq 5.5$.

A second curve, labelled as $y = f_2(x)$, has been created by vertically translating down $y = f_1(x)$ by 2 units. The green area between the two curves is shown on the diagram below.



Ensure that these functions are integrated into your final design, potentially in the form of a dome, wave, or arch, depending on your creative intent.

- State the second function, $y = f_2(x)$.
- Show that the green area is exactly 9 units².

Consider what this area might represent in a real-world context or design scenario. On the given functions, you can apply transformations such as reflection, translation, or dilations. This can be incorporated at various stages of your design in Part B, at the beginning, middle, or end of your design.

Extend your work from the given functions, $y = f_1(x)$ and $y = f_2(x)$ by creating a complete design made up of **pairs of curves** that form enclosed areas.

Requirements:

1. In addition to the given pair of functions, you must Include at least five different functions to complete your design.
2. Use at least three different types of function (e.g., polynomial, root, trigonometric, exponential, etc.).
3. Restrict the domain of each function to control which part of the graph is shown.
4. Use pairs of functions to form enclosed areas throughout your design.
5. Ensure at least two joins between curves are smooth (same gradient at the connection). Prove this algebraically.
6. Produce a clear, labelled graph of your full design using *FX Graph*, *Desmos*, *Geogebra* or similar software.

Part B – Calculations of the Area

Apply calculus and mathematical reasoning to analyse your full design.

1. Calculate the area between at least three pairs of curves in your design using algebraic integration.
2. Justify any assumptions you made in your modelling process.
3. Discuss the limitations of your model and how it simplifies real-world structures.

Assessment Guide

In constructing your investigation, you need to use the following structure:

Introduction

- Define the problem that has been presented to you (the 'what')
- Describe clearly and concisely how this task is applicable to your learning and to the real world (the 'why'- the significance of the task)
- List all methods you are going to demonstrate to solve this problem, including any use of technology (the 'how')

Mathematical Calculations & Analysis

- Follow all the instructions provided with your problem. Provide answers to all questions. Use headings where appropriate.
- Provide relevant diagrams or graphs that could help illustrate any concepts you are explaining.
- Explain each step in your mathematical reasoning with notes or short sentences. Justify all choices you make when developing your design.
- Discuss the reasonableness and limitations of the mathematical results in your design.
    - Limitations are situations where the mathematics you have used is a simplification of what happens in the real world or where the mathematics is restricted by certain rules or numbers.

Conclusion

Summarise your findings by:

- Outlining any observations or findings you have made.
- Clearly describe and illustrate your results.
- Suggest any alterations you could make to improve your design.

Appendices

Includes any additional information you would like to include that strengthens your justification or documents process that is not directly essential in your analysis section.

You are expected to provide a bibliography for any resources you have used outside of the Mathematical Methods course during this task.

Submission Requirements		Modified
Format (including text type)	Written report	☐
Word Count / Length	Maximum 12 A4 pages (not including the *Appendix*) Minimum font size 10	

Deadlines	Date	Submission Requirements
Checkpoint #1		
Due	TBC	

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Assessment Design Criteria *What you will be assessed on*	**Task Specific Clarification** *To meet the level descriptors make sure you:*
Concepts and Techniques	
• Knowledge and understanding of concepts and relationships.	• Accurate solutions using differential calculus method.
• Selection and application of mathematical techniques and algorithms to find solutions to problems in a variety of contexts.	• Appropriate selection of techniques to investigate, formulate and develop your design.
• Application of mathematical models.	• Sufficient steps in the modelling process
• Use of electronic technology to find solutions to mathematical problems.	• Efficient use of technology (***FX Graph, Desmos, Geogebra*** or other graphing packages, and graphics calculators) to find various functions for your design.
Reasoning and Communication	
• Interpretation of mathematical results. • Drawing conclusions from mathematical results, with an understanding of their reasonableness and limitations.	• Clear, efficient, and precise interpretation of the mathematical results. All choices of functions used clearly justified. • In-depth discussion of any limitations to the design developed or techniques used to determine the joint of the functions and the reasonableness of the results.
• Use of appropriate mathematical notation, representations, and terminology.	• Report constructed in the appropriate format. All graphs clearly labelled. Correct notation proficiently used for all calculations.
• Communication of mathematical ideas and reasoning to develop logical arguments.	• Mathematical arguments clear and easy to follow. Steps in developing, testing, and proving your conjectures, must be clearly outlined.

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	Concepts and Techniques	Reasoning and Communication
A	Comprehensive knowledge and understanding of concepts and relationships. Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts. Successful development and application of mathematical models to find concise and accurate solutions. Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems.	Comprehensive interpretation of mathematical results in the context of the problem. Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations. Proficient and accurate use of appropriate mathematical notation, representations, and terminology. Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments.
B	Some depth of knowledge and understanding of concepts and relationships. Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts. Some development and successful application of mathematical models to find mostly accurate solutions. Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems.	Mostly appropriate interpretation of mathematical results in the context of the problem. Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations. Mostly accurate use of appropriate mathematical notation, representations, and terminology. Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments.
C	Generally competent knowledge and understanding of concepts and relationships. Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts. Successful application of mathematical models to find generally accurate solutions. Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally appropriate interpretation of mathematical results in the context of the problem. Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations. Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy. Generally effective communication of mathematical ideas and reasoning to develop some logical arguments.
D	Basic knowledge and some understanding of concepts and relationships. Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation of mathematical results. Drawing some conclusions from mathematical results, with some awareness of their reasonableness or limitations. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology, with limited accuracy in solving routine problems.	Limited interpretation of mathematical results. Limited understanding of the meaning of mathematical results, and their reasonableness or limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning.