* 1. **Selection of the area calculation methods based on the algebraic integration**

For regions formed by different types of curves, a suitable selection of area calculation methods is crucial. Generally, in , area of Region is evaluated by a double integral

Regions bounded by 2 functions and bounded by an enclosed parametric curve have different applications of this equation. For regions like , its area is

which is exactly the Eq. 1 used in Part A when evaluating the area of . For parametric curves, Green’s theorem is introduced:

where means integrate on an enclosed curve, and means the boundary of . By letting , there is

Note that this area has direction. If integrate along the curve clockwise, it gives a negative value.

Additionally, for an affine transformation , there is

where is the determinant of a matrix , indicating the area scaling factor. For a non-linear transformation , the Jacobian reflects the linear transformation behaviour in each infinitesimal element in the region.Summing these behaviours yields the scaling of the area:

|  |  |  |  |
| --- | --- | --- | --- |
| **Summary: Select the area calculation method in different situations** | | | |
| Region bounded by functions | Region enclosed by parametric curve | Affine transformation | Non-linear transformation |
| Eq. 2 | Eq. 3 | Eq. 4 | Eq. 5 |