* 1. **Design of region *D3***
     1. **Introduction to Bézier Curves**

Bézier Curves are a type of interpolation method intended to approximate a real-world shape that otherwise has no mathematical representation. The general equation of a Bezier curve of order n is

Where are the control points**.** The curves are expected to be rather wavy when designing the pattern using Bezier curves. It can be achieved by using a high-order Bezier curve, but there are problems. First, when there are many control points, it is hard to imagine the shape of the curve. Secondly, one cannot modify only a certain part of the curve, because if one control point is moved, the whole curve will change. In addition, applying a higher-order curve can cause a problem of oscillation at the edges of an interval. It is called **Runge's phenomenon**. A viable solution is a composite Bezier curve, namely composing a curve of multiple shorter Bezier curves. To do that, the smoothness of their joints must be guaranteed. The derivative of Eq. 1 is

(for the full derivation, see Appendix 2) Put and , the conclusion can be obtained that: the tangent vector at the start point is and at the terminal point is . Thus, to ensure that the two curves have continuity at the junction is to put the four control points near the junction collinearly, namely

* + 1. **Design based on Bézier Curves**

|  |  |  |  |
| --- | --- | --- | --- |
| Figure 1. D3 | Figure 2. Continuous Joints between B1 and B2 | Figure 3 Continuous Joints between B6 and B7 | Region D3 is enclosed by 8 Bezier curves with control points *.* The curves are connected end to end and form an enclosed shape (see Figure 1). Note that Bezier curves naturally have a domain of . As shown in Figure 2 & Figure 3, there are 2 pairs of curves that has continuous joints. They are taken as samples to show calculations (for full equations see Appendix 3). |

|  |  |
| --- | --- |
| Figure 4. B2    Figure 5. B1 | (see Figure 4) has control points  Substitute into Eq. 1, there is  **To ensure G1 continuity, it must be**  That is to say,  Under this constraint, the control points of B1 (see Figure *5*) are designed to be  where the k is set to be 3.90. Hence the equation is |
| Figure 6. B4    Figure 7. B5 | (see Figure 6) has control points  Note there is only one turning point is required for this curve. So it only needs 3 control points, namely it is a quadratic Bezier curve. Substitute into Eq. 1, there is  **To ensure G1 continuity, it must be**  **That is to say,**  Under this constraint, the control points of B5 (see Figure 7) are designed to be |

**Appendix 3. Equations.**

The control points are,

According to Eq. 1,