**Design and Calculating Bounded Area**

**Introduction**

**1 Part A - Working with a Given Function and Designing with Functions**

**1.1 Calculate the area of the given region**

Given a function . The second function is created by vertically translating down by 2 units. To calculate the area between the two curves, there are 2 methods.

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| An intuitive way: the region between 2 curves is formed by translation, hence its area is equal to the area of a 4.5 by 2 rectangle (you can see that as you put all the infinitesimal rectangles of the Riemann summation together). |  |
| This area can also be evaluated by the formula:  This is a general formula to calculate the region bounded by a pair of functions in a given domain. To use this formula, the equation of is required. By the definition of translation, . |  |

**1.2 Affine transformation and parametric equation**

The task asks to integrate the region between the given function and into the final design, and it is encouraged to use transformations including reflection, translation, or dilations, which are **affine transformations**.

A simple definition of an affine transformation is to perform a linear transformation on a vector space, and additionally, a translation. Generally, an affine transformation on vector in can be written as

where is a 2 by 2 matrix representing a linear transformation, and represents a translation. The transformation of a function is more complicated. For this specific purpose, since what is transformed is the whole shape of a function , which means the actual subject of the transformation is every point on the plane, namely

This equation naturally satisfies the restriction on the **domain.** But note that after the transformation, the relation between and will probably no longer obey the restriction of a function (one-to-one correspondence). To better describe this kind of curve, **parametric equations** are introduced. Hence, a curve can be written as

where vector P records the position of a point on the curve at time t. Consequently, the whole curve is the trajectory of the point. According to Eq. 1.2, a general affine transformation on a parametric equation is

Here are some examples on .

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| **Operation and comment** | **Graph** | **Working out** |
| **Parameterise**  To parameterise a function is all about picking the most accessible parameter to make life easier. Here for , the best choice is to let . | N/A | Let , |
| **Translate to get**  Since it is only a translation, the linear transformation matrix should be identical, namely  Translating down by 2 units gives the translation vector  Note that the actual transformation is imposed on the whole vector space The transformed base vectors are intended to explicitly show the transformation on the whole coordinate frame |  |  |