**Design and Calculating Bounded Area**

**Introduction**

**1 Part A - Working with a Given Function and Designing with Functions**

**1.1 Calculate the area of the given region**

Given a function . The second function is created by vertically translating down by 2 units. The region bounded by the two curves is

To calculate its area, there are 2 methods.

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| An intuitive way: the region between 2 curves is formed by translation, hence its area is equal to the area of a 4.5 by 2 rectangle (you can see that as you put all the infinitesimal rectangles of the Riemann summation together). |  |
| This area can also be evaluated by the formula:  This is a general formula to calculate the region bounded by a pair of functions in a given domain. To use this formula, the equation of is required. By the definition of translation, . |  |

**1.2 Affine transformation and parametric equation**

The task asks to integrate the region between the given function and into the final design, and it is encouraged to use transformations including reflection, translation, or dilations, which are **affine transformations**.

A simple definition of an affine transformation is to perform a linear transformation on a vector space, and additionally, a translation. Generally, an affine transformation on vector in can be written as

where is a 2 by 2 matrix representing a linear transformation, and represents a translation. The transformation of a function is more complicated. For this specific purpose, since what is transformed is the whole shape of a function , which means the actual subject of the transformation is every point on the plane, namely

This equation naturally satisfies the restriction on the **domain.** But note that after the transformation, the relation between and will probably no longer obey the restriction of a function (one-to-one correspondence). To better describe this kind of curve, **parametric equations** are introduced. Hence, a curve can be written as

where vector P records the position of a point on the curve at time t. Consequently, the whole curve is the trajectory of the point. According to Eq. 1.2, a general affine transformation on a parametric equation is

Here are some examples on .

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| **Operation and comment** | **Graph** | **Working out** |
| **Parameterise**  To parameterise a function is all about picking the most accessible parameter to make life easier. Here for , the best choice is to let . | N/A | Let , |
| **Translate to get**  Since it is only a translation, the linear transformation matrix should be identical, namely  Translating down by 2 units gives the translation vector |  |  |
| **Scale transform**  To compress vertically to 1/5 of the original size and stretch horizontally to four times the original size requires the matrix  By the way, if the graph is sclaed by a negative factor, the graph will be flipped.  Since there is no translation, b is the zero vector. |  |  |
| **Shear transform**  A shear transformation is a type of linear transformation that slants the shape of an object. To shear the region in the y-direction by 1/2 requires the matrix  Since there is no translation, b is the zero vector. | C:\Users\Administrator\AppData\Local\Microsoft\Windows\INetCache\Content.Word\transformation_shear.png |  |
| **Rotation transform**  A general rotation transformation can be denoted as  where positive represents a counter-clockwise rotation. Here put , then | C:\Users\Administrator\AppData\Local\Microsoft\Windows\INetCache\Content.Word\transformation_rot.png |  |

There are two very good things about affine transformations. First, they are composable. For example, the transformation to first scale the given curves and , copy a pair of curves and , flip this pair of curves around the y-axis, translate them to align with the original curves, rotate them all by 180 degrees, and finally translate them to 10 units above the origin, can be written as

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Another advantage of affine transformations is that the amount of the area changed in the transform can be easily calculated by the determinant of the transformation matrix. This will be further discussed in Part B.

**1.3 Non-linear transformation**