This project examined the analysis of geometric regions bounded by curves, incorporating a variety of functions, including polynomial, trigonometric, logarithmic and exponential functions, to model boundaries with differing growth and saturation behaviours. Observations indicate that these functional choices allowed for flexible and accurate modelling of curve shapes.

Mathematical methods, including Bezier curves, piecewise Hermite interpolation and the discrete Fourier transformation, proved effective in simplifying design tasks and ensuring continuity and smoothness where necessary. A pattern is observed that the design methods based on adjustment, like constructing functions and Bezier curves, are more flexible in design, but sometimes hard to control and imagine the shape, while methods based on interpolation have a more automatic process but require more calculation. Patterns on methods using sample points like Bezier curves and polynomial interpolation indicate are also observed that with more sample points, the output design is more accurate, but at the same time tends to have Runge’s phenomenon, which can be solved by using piecewise and composite curves.

Differentiation techniques are applied in this investigation mainly to ensure the smooth connection between curves. Observations indicate that curves and functions like (Bezier curves and Piecewise Cubic Hermite Interpolating Polynomials (PCHIP)) that have pre-defined slope properties at joints can better ensure smooth connections and easy to make adjustments without interfering original shape.

The use of definite integrals was central to quantifying areas between the curves. Analytical integration of piecewise-defined functions, combined with the curve integration of parametric equations, provided reliable results. The effects of affine transformations and non-linear transformations on the area are also investigated. It is observed that for some special regions, calculating and applying the scaling factor of a transformation on the original area is easier than doing the integration directly. By using various integration techniques, the overall area of the final design is calculated to be 106.90 unit2 (2 d.p.).

Potential improvements include using Monte Carlo methods to numerically validate the area, increasing the density of sample points for designs formed by interpolations in regions where the curves change rapidly, or experimenting with alternative functional forms to better capture extreme variations. These adjustments would further reduce approximation errors and enhance the precision of integral-based area calculations.