Exam 1 1st Jul 2024. 98

P1 $y' = \frac{2}{3}x$

$y'|_{x=1} = \frac{2}{3}$

$\frac{2}{3}(x-1) = y - \frac{1}{3}$

$y = \frac{2}{3}x - \frac{1}{3}$

P2 a. $\frac{d}{dx}\left(\frac{x}{\sqrt{1-x}}\right) = \frac{\sqrt{1-x} - x\cdot\frac{1}{2}(1-x)^{-\frac{1}{2}}\cdot(-1)}{1-x} = \frac{\sqrt{1-x} + x^{\frac{1}{2}}\cdot\frac{1}{\sqrt{1-x}}}{1-x} = \frac{\sqrt{1-x} + \frac{x}{2\sqrt{1-x}}}{1-x}$

b. $\frac{d}{dx}\left(\frac{\cos 2x}{x}\right) = \frac{-2\sin 2x \cdot x - \cos 2x}{x^2}$

c. $y'(x) = e^{2f(x)}\cdot 2f'(x)$

d. $\frac{d}{dx}(\ln(\sin x)) = \frac{1}{\sin x}\cdot\cos x = \cot x.$

P3 $\frac{dy}{dx}(y^4 + xy) = 0$

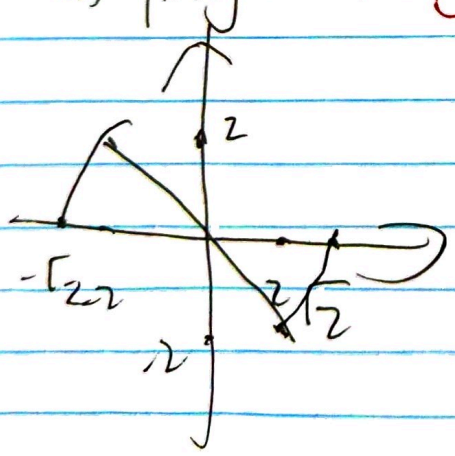
$4y^3y' + y + xy' = 0$

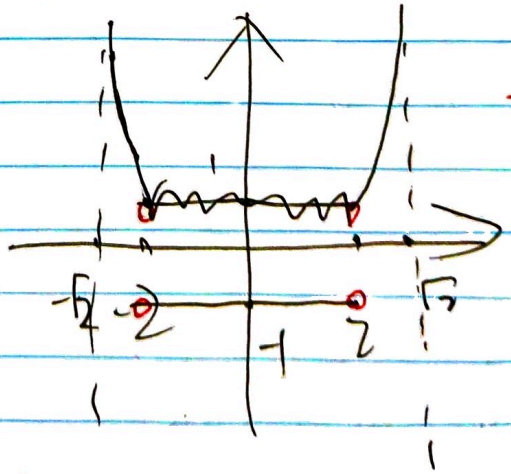
$y'(4y^3 + x) = -y$

$y' = \frac{-y}{4y^3 + x}$

$\frac{dy}{dx}\Big|_{x=3,y=1} = \frac{-1}{4+3} = -\frac{1}{7}$

P4





P5 $f(x)=\begin{cases} a & x<1 \\ ax^3+1 & x\geq 1 \end{cases}$

$a=4+1=5$

$a+b=1^{10}+1$

$b=-2$

$\therefore \begin{cases} a=5 \\ b=-2. \end{cases}$

P6 a. $\lim_{x\to 0}\frac{(1+2x)^{10}-1}{x}=\frac{d}{dx}(2x)^{10}\Big|_{x=\frac{1}{2}}=20x\cdot 2\Big|_{x=\frac{1}{2}}=20$

b. $\lim_{x\to 0}\frac{\sqrt{\cos x}-1}{x}=\frac{d}{dx}(\cos x)\Big|_{x=0}=\left(\frac{1}{2}\frac{1}{\sqrt{\cos x}}\cdot(-\sin x)\right)\Big|_{x=0}=\frac{1}{2}\cdot 0=0$

P7 $\frac{d}{dx}a^x=\lim_{\Delta x\to 0}\frac{a^{x+\Delta x}-a^x}{\Delta x}=\lim_{\Delta x\to 0}a^x\frac{a^{\Delta x}-1}{\Delta x}$

pf: Let $M(a)=\lim_{\Delta x\to 0}\frac{a^{\Delta x}-1}{\Delta x}$

$\therefore \frac{d}{dx}a^x=M(a)a^x.$

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