Lecture 1 1 Jul

1C 3 a) $f(x) = 1/(2x+1)$

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{2x+2\Delta x+1} - \frac{1}{2x+1}}{\Delta x} = \frac{\frac{2x+1-2x-2\Delta x-1}{4x^2+4\Delta x x+2x+2x+2\Delta x+1}}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{-2}{4x^2+4\Delta x \cdot x+4x+2\Delta x+1} = \frac{-2}{4x^2+4x+1}$

b) $f(x) = 2x^2+5x+4$

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2(x^2+2\Delta x \cdot x+\Delta x^2)+5x+5\Delta x+4-2x^2-5x-4}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{4\Delta x \cdot x+2\Delta x^2+5\Delta x}{\Delta x} = \lim_{\Delta x \to 0} (4x+2\Delta x+5) = 4x+5$

c) $f(x) = \frac{1}{x^2+1}$

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{x^2+\Delta x^2+2\Delta x \cdot x+1} - \frac{1}{x^2+1}}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\frac{x^2+1-x^2-\Delta x^2-2\Delta x \cdot x-1}{x^4+2x^3\Delta x+x^2\Delta x^2+x^2+x^2+\Delta x^2+2x\Delta x+1}}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\frac{-\Delta x^2-2\Delta x \cdot x}{x^4+2x^3 \cdot \Delta x+x^2\Delta x+2x^2+\Delta x^2+2x\Delta x+1}}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{-\Delta x-2x}{x^4+2x^3 \cdot \Delta x+x^2\Delta x+2x^2+\Delta x^2+2x\Delta x+1}$

$= \frac{-2x}{x^4+2x^2+1}$

e) a) $f'(x) = \frac{-2}{4x^2+4x+1} = +1$

$-2 = 4x^2+4x+1$

$4x^2+4x+3=0$

$(2x+1)^2 = -2$

Do Not Exist

$f'(x) = 0$

Do Not Exist

$f'(x) = -1$

$(2x+1) = \pm\sqrt{2}$

$x = \frac{\pm\sqrt{2}-1}{2}$

$f\left(\frac{\sqrt{2}-1}{2}\right) = \frac{\sqrt{2}}{2}$ $f\left(\frac{-\sqrt{2}-1}{2}\right) = -\frac{\sqrt{2}}{2}$

$\therefore \left(\frac{\sqrt{2}-1}{2}, \frac{\sqrt{2}}{2}\right)$ and $\left(\frac{-\sqrt{2}-1}{2}, -\frac{\sqrt{2}}{2}\right)$

3) b. $f'(x) = 1$

$4x+5=1$

$x=-1$

$f(-1) = 1$

$(-1, 1)$

$f'(x) = -1$

$4x+5=-1$

$x = -\frac{3}{2}$

$f(-\frac{3}{2}) = \frac{9}{2} - \frac{15}{2} + \frac{8}{2}$

$= 1$

$(-\frac{3}{2}, 1)$

$f'(x) = 0$

$4x+5=0$

$x = -\frac{5}{4}$

$f(-\frac{5}{4}) = \frac{25}{8} - \frac{25}{4} + \frac{16}{4}$

$= \frac{25}{8} - \frac{18}{8} = \frac{7}{8}$

4a) $f(1) = \frac{1}{3}$

$(y - \frac{1}{3}) = f'(1)(x-1)$

$y - \frac{1}{3} = \frac{-2}{1+2+1}(x-1)$

$y - \frac{1}{3} = -\frac{1}{2}x + \frac{1}{2}$

$y = -\frac{1}{2}x + \frac{5}{6}$

$\frac{(-1)x+5}{9}$

b) $f(a) = 2a^2+5a+4$

$y - 2a^2 - 5a - 4 = f'(a)(x-a)$

$y - 2a^2 - 5a - 4 = (4a+5)(x-a)$

$y - 2a^2 - 5a - 4 = (4a+5)x - 4a^2 - 5a$

$y = (4a+5)x - 2a^2 + 4$

5. $y - f(a) = f'(a)(x-a)$

$y' = \lim_{\Delta x \to 0} \frac{(x+\Delta x+1)^2 - (x+1)^2}{\Delta x} = \frac{(2x+\Delta x+2)(\Delta x)}{\Delta x} = 2x+\Delta x+2$

$= 2x+2$

$2a+2$

$y - 1 - (a+1)^2 = (2a+2)(x-a)$

$y - 1 - (a^2+2a+1) = 2ax + 2x - 2a^2 - 2a$

$y - 1 - a^2 + 2a - 1 = (2a+2)x - 2a^2 - 2a$

$y = (2a+2)x - a^2 - 4a + 2$

$0 = (2a+2)\,0 - a^2 - 4a + 2$

$a^2 + 4a - 2 = 0$

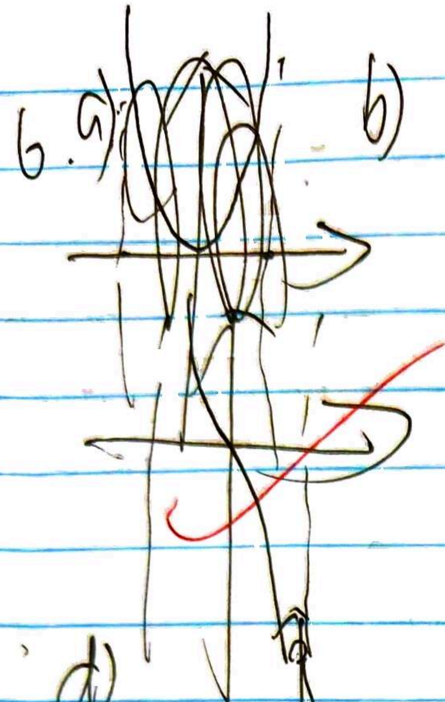
$a = \frac{-4 \pm \sqrt{16+8}}{2} = \frac{-4 \pm 2\sqrt{6}}{2} = -2 \pm \sqrt{6}$

$\therefore y_1 = (-2+2\sqrt{6})x$

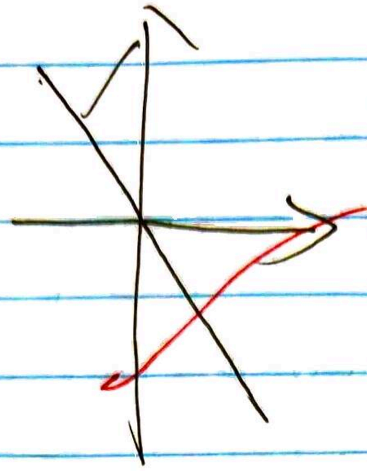
$y_2 = (-2-2\sqrt{6})x$

$y = (2\sqrt{2}-2)x$ and $y = (-2\sqrt{2}-2)x$.

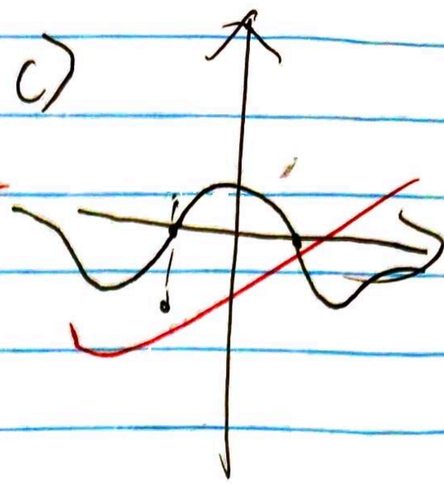
6. a)



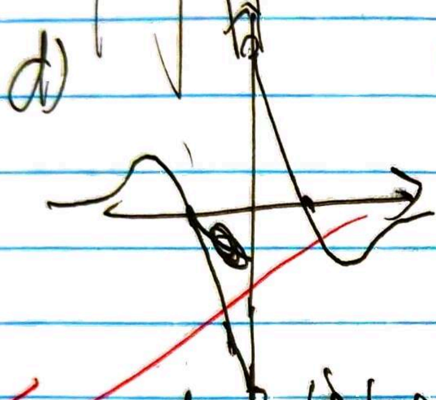
b)



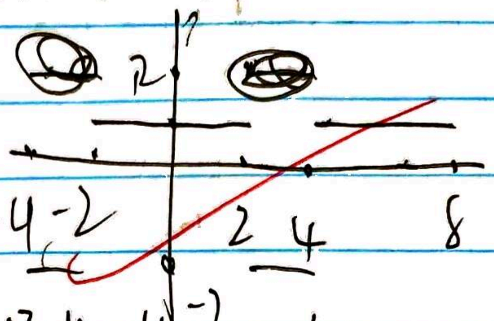
c)



d)



e) period: 6



B-2 a) $v = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{6t+6\Delta t-16(t+\Delta t)^2-6t+16t^2}{\Delta t} = \lim_{\Delta t \to 0} \frac{6\Delta t+32t\Delta t-16\Delta t^2}{\Delta t} = \lim (b+32t-16\Delta t)$

$= b+32t$

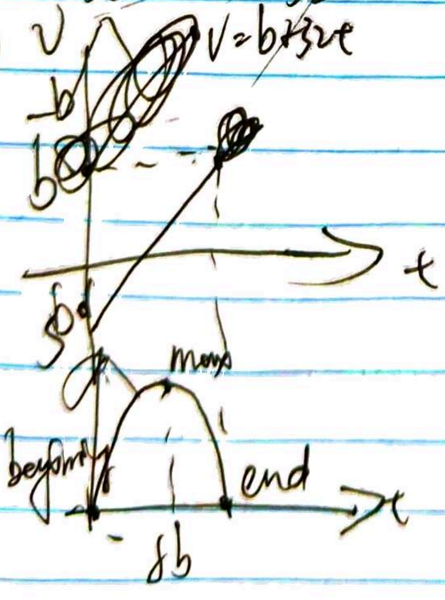
b) $v=0$

$b+32t=0$

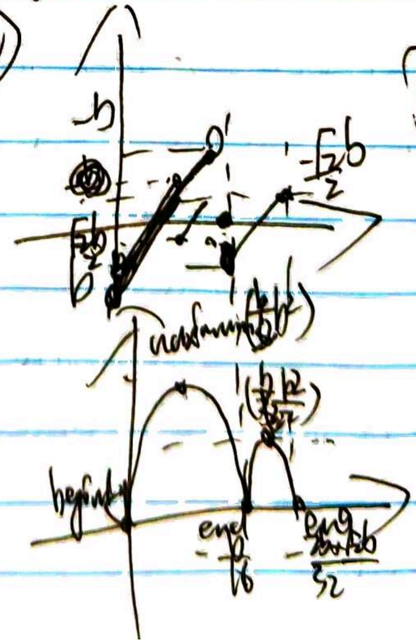
$t=-\frac{b}{32}$

c) $s\left(-\frac{b}{32}\right) = -\frac{b^2}{32} - 16 \cdot \frac{b^2}{32\times 32} = -\frac{b^2}{2\times 32} - \frac{b^2}{2\times 32} = -\frac{3b^2}{64}$

d)

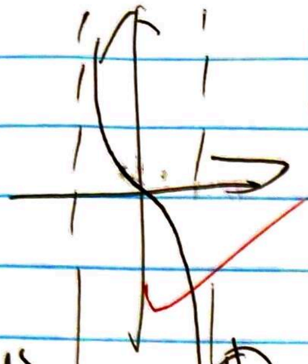


e)

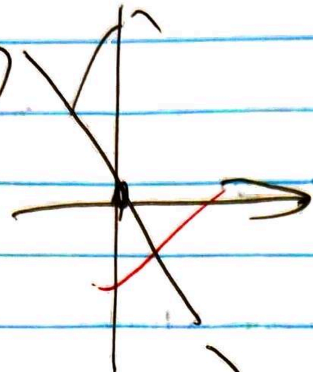


f)

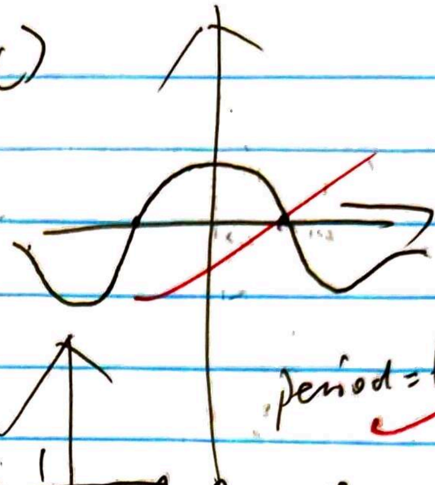
6 a)



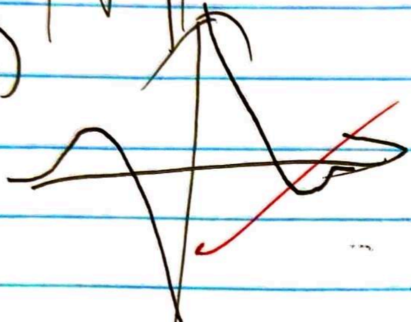
b)



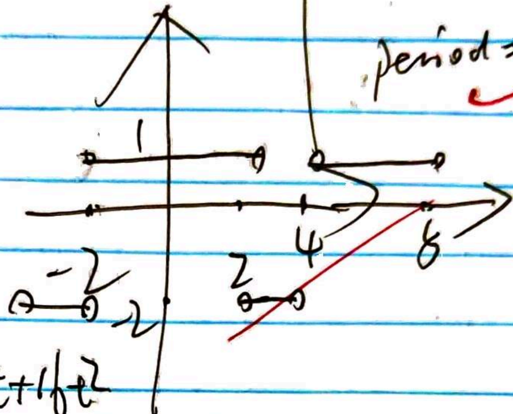
c)



d)



e)



period = 6

1B-2 a) $v = \lim_{\Delta t \to 0} \frac{bt + b\Delta t - 16(t+\Delta t)^2 - bt + 16t^2}{\Delta t}$

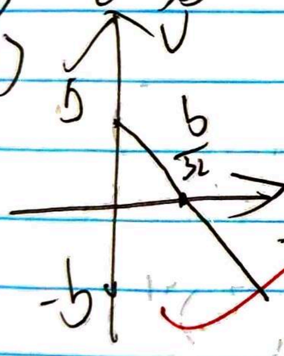
$= b - 32t$

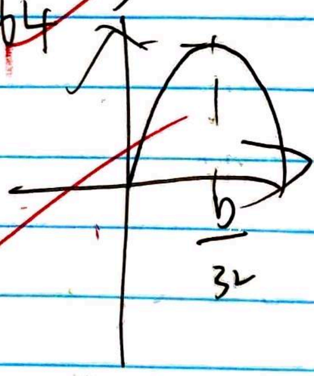
b) $v = 0$

$t = \frac{b}{32}$

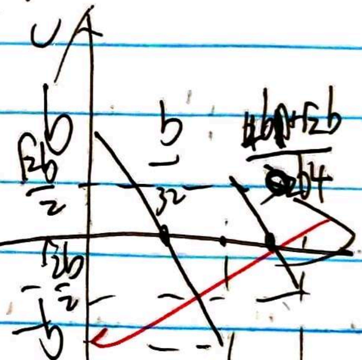
c) $s\left(\frac{b}{32}\right) = \frac{b^2}{64}$

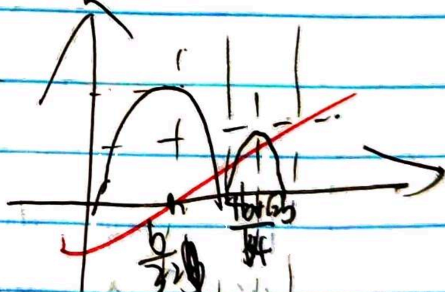
d)





e)





f) $\frac{b_1}{16} + \frac{b_2}{16} + \frac{b_3}{16} + \dots = \frac{b}{16}\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}^2} + \dots\right)$

$= \frac{\frac{b}{16}}{1 - \frac{1}{\sqrt{2}}} \approx 3.4$

1C-1 a) $A = \pi r^2$

$$A' = \lim_{\Delta r \to 0} \frac{\pi(r+\Delta r)^2 - \pi r^2}{\Delta r} = \lim_{\Delta r \to 0} \frac{\pi r^2 + 2\pi r \Delta r + \Delta r^2 - \pi r^2}{\Delta r} = 2\pi r$$