Part II

0. None

1. $y = \frac{(x-1)}{x+1}$ $\quad y' = \frac{(x+1)-(x-1)}{(x+1)^2} = \frac{2}{x^2+2x+1}$

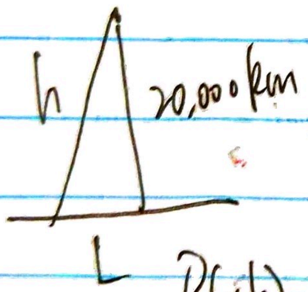
$\frac{(x+1)^2}{(x+1)^2}$

$\frac{-2x}{(x+1)(x-1)}$ $\quad \frac{x^2+1}{(x+1)(x-1)}$

$y_1 = \frac{x^2+1}{(x+1)(x-1)}$ $\quad y_2 = \frac{-2x}{(x+1)(x-1)}$

$= \frac{x^2+1}{x^2-1}$ $\quad = \frac{-2x}{x^2-1}$

2.

(i) 

$L = \sqrt{h^2 - 4\times10^6}$

$L_0 = \sqrt{h_0^2 - 4\times10^6}$

$P(\Delta h) = \frac{\Delta L}{\Delta h} = \frac{L - L_0}{\Delta h}$

$P(1) = \frac{\sqrt{(25001)^2 - 4\times10^6} - \sqrt{(25000)^2 - 4\times10^6}}{1} \approx 1.0032153$

$P(10^{-1}) = 1.0032154294658$

$P(10^{-2}) = 1.0032154408691$

$C = 1.0032154[illegible]5$

(b) $p(\Delta h) = \frac{\sqrt{h^2-20000^2}-\sqrt{h_0^2-20000^2}}{\Delta h}$

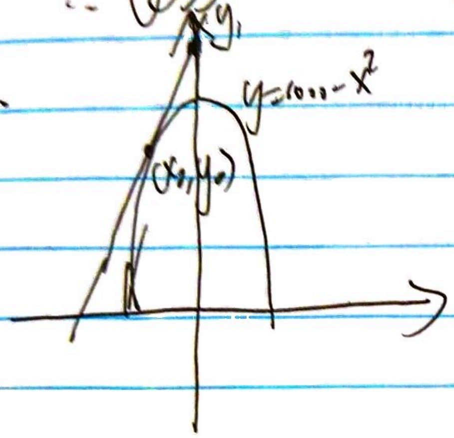
$p(1) = \frac{\sqrt{20002^2-20000^2}-\sqrt{20001^2-20000^2}}{1} = 1.0050370539575$

$p(10^{-1}) = 1.005037823025$

$p(10^{-2}) = 1.00503730500035$.

∴ less accurately.

3.



$y' = -2x$

~~$y_0 = 1000 - x$~~

~~$2x_0(x-x_0) = y_0 - x_0$~~

$y_1 = -2x_0 \cdot x + 1100$

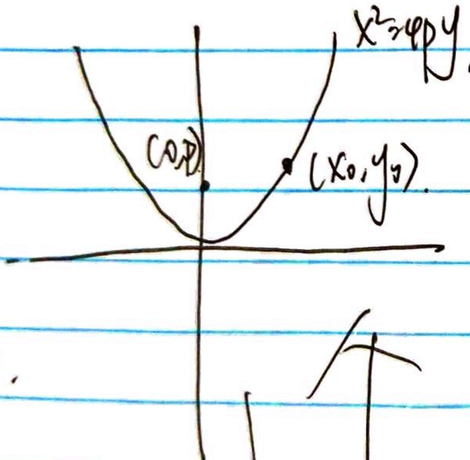
$-2x_0 \cdot x_0 + 1100 = 1000 - x_0^2$

$x_0^2 = 100$

$x_0 = \pm 10$

$y = y_0 = 1000 - x_0^2 = 900$ m.

4. Textbook 3.1/21.



a) $y = \frac{x^2}{4p}$

$y' = \frac{1}{4p} \cdot 2x = \frac{x}{2p}$

$y'|_{x=x_0} \cdot (x-x_0) = y - y_0$

$\frac{x_0}{2p}(x-x_0) = y - \frac{x_0^2}{4p}$

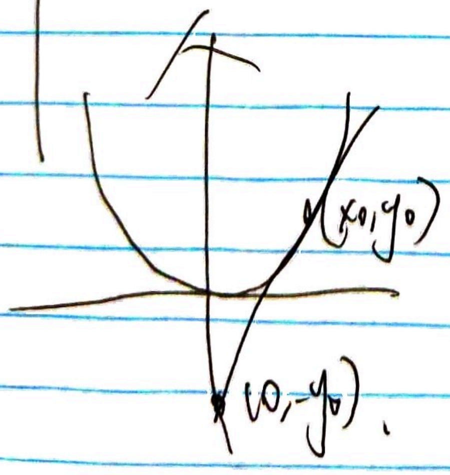
$y = \frac{x_0 x}{2p} - \frac{x_0^2}{2p} + \frac{x_0^2}{4p}$

$= -\frac{x_0^2}{4p} + \frac{x_0}{2p} \cdot x$

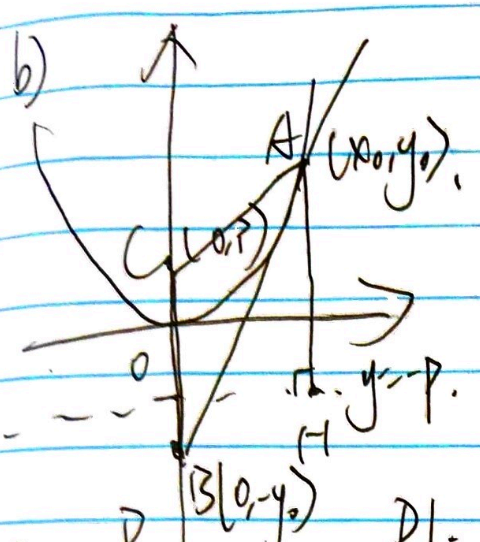
$y|_{x=0} = -\frac{x_0^2}{4p} = -y_0$

∴ the tangent at (x_0,y_0) has a y-int $-y_0$.

(b).



b)

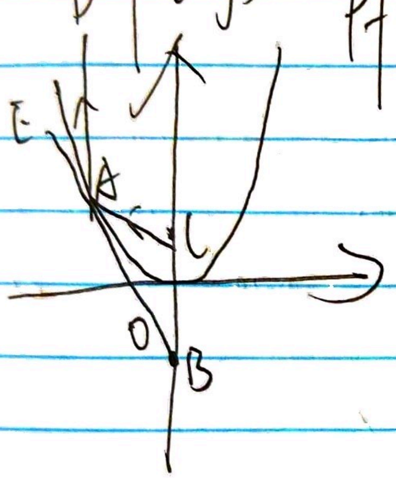


on a parabola, $AC = AH = y_0+p$

$BC = y_0+p = AH = AC$.

$\therefore \triangle CAB$ is a isosceles $\triangle$

c)



Pf: As what is shown in picture A is any point on a parabola.

From b) we know that $\angle CAB = \angle CBA$.

$\because \angle CAB = \angle DAE$

$\therefore \angle DAE = \angle CAB$

$\therefore DA \parallel BC$.

$\square$

5. a) $V(t) = (10-t)^2/5$

$V(5) = 5 \quad V(0) = 20$

$\text{rate} = \dfrac{V(0)-V(5)}{5} = 3$ L/min

b) Water ~~dar~~ drains out as $t=10$.

the rate $r(t) = \cancel{\frac{dV}{dt}}\ \dfrac{dV}{dt} = \dfrac{2(10-t)\cdot(-1)}{5} = \dfrac{-20+2t}{5} = \frac{2}{5}t - 4$

$\therefore r(t) = \frac{2}{5}t - 4$ where $t \in [5, 10]$.

6. 2.5/19.

(d) $\lim_{x\to\infty} x \cdot \sin\frac{1}{x} = x \cdot \frac{1}{x} = 1$

(f) $\lim_{x\to 0} \dfrac{\sin^2 x}{3x^2} = \dfrac{x^2}{3x^2} = \dfrac{1}{3}$

(g) $\lim_{x\to 0} \dfrac{\sin 2x}{\sin 3x} = \dfrac{2x}{3x} = \dfrac{2}{3}$

20(c) $\lim_{x\to 0} \frac{x^2}{1-\cos x} = \lim_{x\to 0} \frac{}{\sin^2 x} = \frac{}{x^2} = 1$

(g). $\lim_{x\to 0} \frac{3x^2+4x}{\sin 2x} = \lim_{x\to 0} \frac{3x^2+4x}{2x} = \lim_{x\to 0}(\frac{3}{2}x+2) = 2$

22(a) $f(\theta) = \frac{1-\cos\theta}{\theta^2}$

θ	$f(\theta)$
1	0.459697694
0.1	0.499583472
0.01	0.499995833
0.001	0.499999960

Conjecture:

$\lim_{\theta\to 0} f(\theta) = 0.5$

(b) pf: $\lim_{\theta\to 0} \frac{\cos\theta(1-\cos\theta)}{\theta^2} = \lim_{\theta\to 0} \frac{1-\cos^2\theta}{\theta^2} = \lim_{\theta\to 0} \frac{\sin^2\theta}{\theta^2}$

$\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta^2} = \lim_{\theta\to 0} \frac{1-(1-2\sin^2\frac{\theta}{2})}{\theta^2} = \lim_{\theta\to 0} \frac{2\sin^2\frac{\theta}{2}}{\theta^2} = 2\,\frac{\frac{\theta^2}{4}}{\theta^2} = \frac{1}{2}$

□

7. a) $D(uvw) = u(vw)' + u'(vw)$
$= u(v'w + vw') + u'(vw)$
$= uv'w + uvw' + u'vw$

b) $D(u_1u_2\cdots u_n) = u_1'u_2\cdots u_n + u_1u_2'\cdots u_n + \cdots + u_1u_2\cdots u_n'$

proof: as $n=3$, $D(u_1u_2u_3) = u_1u_2u_3' + u_1u_2'u_3 + u_1'u_2u_3$

assume there the proposition is true when $n=k$.

as $n=k+1$

$D(u_1\cdots u_{k+1}) = D(u_1\cdots u_k)u_{k+1} + (u_1\cdots u_k)u_{k+1}'$
$= (u_1'u_2\cdots u_k + \cdots + u_1u_2\cdots u_k')u_{k+1} + u_1\cdots u_k u_{k+1}'$
$= u_1'u_2\cdots u_k u_{k+1} + \cdots + u_1u_2\cdots u_k' + u_1u_2\cdots u_k u_{k+1}'$

$\therefore D(u_1u_2\cdots u_n) = u_1'u_2\cdots u_n + \cdots + u_1u_2\cdots u_n'$

□.