Lecture 4 Chain rule 26th Jul

1F-1 a) Method 1: $y=(x^2+2)^2 = x^4+4x^2+4$

$y' = 4x^3+8x$

Method 2: Let $t = x^2+2$

$y = t^2$

$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = 2t\cdot 2x = (2x^2+4)(2x)$

$= 4x^3+8x.$

b) $t = x^2+2$

$y = t^{100}$

$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = 100t^{99}\cdot 2x = 100(x^2+2)^{99}\cdot 2x$

7. $y' = 10x^9(x^2+1)^{10} + x^{10}\cdot 10(x^2+1)^9\cdot 2x$

$= 10x^9(x^2+1)^{10} + 20x^{11}(x^2+1)^9$

b. $f(x) = f(-x)$

$f'(x) = f'(x)\cdot(-x)' = -f'(x)$

$f(x) = -f(-x)$

$f'(x) = -f'(x)\cdot(-x)' = f'(x).$

7\. b) $\frac{dm}{dv} = -\frac{1}{2}\frac{m_0}{(\sqrt{1-\frac{v^2}{c^2}})^3} \cdot \left(-\frac{2v}{c^2}\right) = \frac{m_0 v}{c^2(\sqrt{1-\frac{v^2}{c^2}})^3}$

d) $\frac{dQ}{dt} = \frac{a(1+bt^2)^3 - at \cdot 3(1+bt^2)^2(2bt)}{(1+bt^2)^6}$

$= \frac{a(1+bt^2)^3 - 6abt^2(1+bt^2)^2}{(1+bt^2)^6}$

U-1 a) $y' = \cos(5x^2)\cdot 10x = 10x\cos(5x^2)$

b) $y' = 2\cos(3x)\cdot 3 = 6\cos(3x)$ $6\sin(3x)\cos(3x)$

m) $\frac{d}{dx}(\cos(2x)) = -2\sin(2x)$

$\frac{d}{dx}(\cos^2 x - \sin^2 x) = 2\cos x\cdot(-\sin x) - 2\sin x\cdot\cos x = -4\sin x\cos x$

$\frac{d}{dx}(2\cos^2 x) = 4\cos x\cdot(-\sin x) = -4\sin x\cos x$

~~Yes they are equal~~ No

$\cos(2x) = \cos(x+x) = \cos x\cdot\cos x - \sin x\cdot\sin x = \cos^2 x - \sin^2 x$

$\cos^2 x - \sin^2 x = 1 - \cos^2 x - 1 + \cos^2 x = 2\cos^2 x - 1$

$\therefore \cos(2x) = \cos^2 x - \sin^2 x \neq 2\cos^2 x$

10-1 b) $y' = \frac{x+5-x}{x^2+10x+25} = \frac{5}{x^2+10x+25}$

$y'' = \frac{-(5)(2x+10)}{(x+5)^4} = \frac{-10x-10}{(x+5)^4} = \frac{-10(x+5)}{(x+5)^4} = \frac{-10}{(x+5)^3}$

5a) $y' = u'(x)v(x) + u(x)v'(x)$

$y'' = u''(x)v(x) + u'(x)v'(x) + u'(x)v'(x) + u(x)v''(x)$

$= u''(x)v(x) + u(x)v''(x) + 2u'(x)v'(x)$

$y''' = u'''(x)v(x) + u''(x)v'(x) + u'(x)v''(x) + u(x)v'''(x) + 2u''(x)v'(x)$

$= u'''(x)v(x) + 3u''(x)v'(x) + 3u'(x)v''(x) + u(x)v'''(x)$ $+2u'(x)v''(x)$

$$y''' = u^{(3)}v + \binom{3}{1}u^{(2)}v^{(1)} + \binom{3}{2}u^{(1)}v^{(2)} + uv^{(3)}$$

$$= u'''v + 3u''v' + 3u'v'' + uv'''$$

$$y = x^p(1+x)^q$$

$$y^{(p+q)} = (x^p)^{(p+q)} \cdot (1+x)^q + \binom{p+q}{1}(x^p)^{(p+q-1)} \cdot ((1+x)^q)^{(1)} + \dots + (x^p) ((1+x)^q)^{(p+q)}$$

$$= \binom{p+q}{q} p! \cdot q! = \frac{(p+q)!}{p!(q!)} \cdot p! \cdot q! = (p+q)!$$