$= \frac{1}{2} F(x^2)$.

PS7 6th dan 2015.

Lec 27

$x = a\sin\theta \quad \theta = \sin^{-1}\frac{x}{a}$

5D-1 $\int \frac{dx}{(a^2-x^2)^{\frac{3}{2}}} = \int \frac{a\cos\theta\, d\theta}{(a^2-a^2\sin^2\theta)^{\frac{3}{2}}} = \int \frac{a\cos\theta\, d\theta}{a^3\cos^3\theta} \quad dx = a\cos\theta\, d\theta$

$= \frac{1}{a^2}\int \frac{d\theta}{\cos^2\theta} = \frac{1}{a^2}\tan\theta + c = \frac{x}{a^2\sqrt{a^2-x^2}} + c$

2 $\int \frac{x^3\,dx}{\sqrt{a^2-x^2}} = \int \frac{a^3\sin^3\theta\, a\cos\theta\, d\theta}{a\cos\theta} = a^3\int \sin^3\theta\, d\theta = a^3\int (1-\cos^2\theta)(-du)$

$x = a\sin\theta \qquad \cos\theta = \frac{\sqrt{a^2-x^2}}{a}$

$= a^3\int (u^2-1)\,du$

$= a^3\left(\frac{1}{3}u^3 - u + c\right)$

$= \frac{1}{3}a^3\cos^3\theta + c$

$= \frac{1}{3}(a^2-x^2)^{\frac{3}{2}} + c$

7 $\int \frac{\sqrt{x^2-a^2}\,dx}{x^2} =$

$x = a\sec\theta$

$\int \frac{a\tan\theta\; a\sec\theta\tan\theta\, d\theta}{a^2\sec^2\theta}$

$= \int \tan^2\theta\cos\theta\, d\theta = \int \frac{\sin^2\theta}{\cos\theta}\, d\theta$

$\sec\theta = \frac{x}{a} \qquad \cos\theta = \frac{a}{x}$

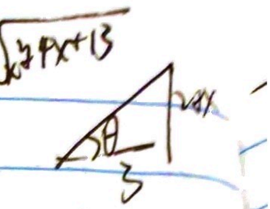
$= \int \frac{1-\cos^2\theta}{\cos\theta}\, d\theta$

$= \int (\sec\theta - \cos\theta)\, d\theta$

$= -\sin\theta + \ln(\tan\theta + \sec\theta) + c$

$= \ln\left(x + \sqrt{x^2-a^2}\right) - \frac{\sqrt{x^2-a^2}}{x} + c$

10 $\displaystyle\int \frac{dx}{(x^2+4x+13)^{\frac{3}{2}}} = \int \frac{dx}{((x+2)^2+3^2)^{\frac{3}{2}}}$ let $2+x=u$



$\displaystyle = \int \frac{du}{(u^2+3^2)^{\frac{3}{2}}} = \int \frac{1}{27}\frac{du}{\left(\left(\frac{u}{3}\right)^2+1\right)^{\frac{3}{2}}}$ let $\frac{u}{3} = \tan\theta$, $du = 3\sec^2\theta\, d\theta$

$\displaystyle = \frac{1}{27}\int \frac{du}{(\tan^2\theta+1)^{\frac{3}{2}}} = \frac{1}{27}\int \frac{3\sec^2\theta\, d\theta}{\sec^3\theta} = \frac{1}{27}\int \cos\theta\, d\theta$

$\displaystyle = \frac{\sin\theta}{9} = \frac{(2+x)}{9\sqrt{x^2+4x+13}} + C$

Lec 28 ~~...~~ 21st Jan 2025

5B-2 $\displaystyle\int \frac{x\,dx}{(x-2)(x+3)}$ $\displaystyle\frac{x}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$ $\displaystyle\frac{x}{x-2} = B + \frac{A(x+3)}{x-2}$

$\displaystyle = \int\left(\frac{1}{5x-10} + \frac{3}{5x+15}\right)dx$ $\displaystyle\frac{x}{x+3} = A + \frac{B(x-2)}{x+3}$

$A = \frac{2}{5}$, $B = \frac{3}{5}$

$= \ln|5x-10| + \ln|5x+15| + C$

3 $\cancel{\displaystyle\frac{x}{(x-4)(x+3)} = \frac{Ax+B}{x-4} + \frac{C}{x+3}}$ $\displaystyle\frac{x}{(x-2)(x+2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x+3}$

$\displaystyle\frac{x}{(x+2)(x+3)} = A + \frac{B(x-2)}{x+2} + \frac{C(x-2)}{x+3}$ $A = \frac{2}{4\times 5} = \frac{1}{10}$

$\displaystyle\frac{x}{(x-2)(x+3)} = \frac{A(x+2)}{x-2} + \frac{C(x+2)}{x+3} + B$ $B = \frac{-2}{-4\times 1} = \frac{1}{2}$

$\displaystyle\frac{x}{(x-2)(x+2)} = \frac{A(x+3)}{x-2} + \frac{B(x+3)}{x+2} + C$ $C = \frac{-3}{-5\times(-1)} = -\frac{3}{5}$

$\displaystyle\int \frac{x\,dx}{(x-4)(x+3)} = \ln|10x-20| + \ln\left|\frac{1}{2x+4}\right| - 3\ln|5x+15| + C$

5 $\displaystyle\frac{3x+2}{(x+1)^2 x} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x}$

$\displaystyle\frac{3x+2}{x} = A(x+1) + B + \frac{C(x+1)^2}{x}$ $B = 1$

$\displaystyle\frac{3x+2}{(x+1)^2} = \frac{Ax}{x+1} + \frac{Bx}{(x+1)^2} + C$ $C = 2$

$\displaystyle\frac{3x+2}{(x+1)^2 x} = \frac{A}{x+1} + \frac{1}{(x+1)^2} + \frac{2}{x}$ $\displaystyle\frac{5}{4} = \frac{A}{2} + \frac{1}{4} + 2$

$A = -3$

$\int \frac{3x+2}{(x+1)^2 x}dx = 2\ln\left|x+1\right| + 2\ln|x| - \frac{1}{x+1} + C$

6 $\frac{2x-9}{(x^2+9)(x+2)} = \frac{Ax+B}{x^2+9} + \frac{C}{x+2}$

$\frac{2x-9}{(x^2+9)} = \frac{(Ax+B)(x+2)}{x^2+9} + C$

$\frac{-4-9}{13} = C \quad C = -1$

$\frac{2x-9}{(x^2+9)(x+2)} = \frac{Ax+B}{x^2+9} - \frac{1}{2}$

$\frac{1}{2} + \frac{-9}{18} = \frac{B}{9} \quad B = 0$

$\frac{2 \cdot 9}{10 \times 3} = \frac{A}{10} - \frac{1}{3} \quad A - \frac{9}{10} \cdot \frac{7}{3}$

$A = \frac{1}{10} + 17 = 1$

$\int \frac{-1}{x+2}dx = -\ln|x+2| + C$

$\int \frac{x\,dx}{x^2+9} = \frac{1}{9}\int \frac{x\,dx}{(\frac{x}{3})^2+1}$ let $\frac{x}{3} = \tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$= \frac{1}{9}\int \frac{3\tan\theta \cdot 3\sec^2\theta\, d\theta}{\sec^2\theta} = \int \tan\theta\, d\theta$

$= -\ln|\cos\theta| + C = \ln\left|\frac{\sqrt{x^2+9}}{3}\right| + C = \frac{1}{2}\ln|x^2+9| + C$

$\int \frac{2x-9}{(x^2+9)(x+2)} = -\ln|x+2| + \frac{1}{2}\ln(x^2+9) + C$

10 4) $\int \frac{(x^2+1)dx}{x^2+2x+2} = \int \frac{(x^2+1)dx}{(x+1)^2+1}$ let $x+1 = \tan\theta$

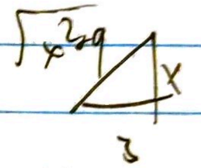
$dx = \sec^2\theta\, d\theta$

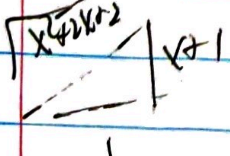
$\int(\sec^2\theta - 1)d\theta$

$= \int \frac{(\tan^2\theta - 2\tan\theta + 2)\sec^2\theta\, d\theta}{\sec^2\theta} = 2\ln|\cos\theta| + 2\theta + \int \frac{1}{\cos^2\theta}d\theta$

$= 2\ln|\cos\theta| + 2\theta + \tan\theta - \theta + C$

$= 2\ln\frac{1}{\sqrt{x^2+2x+2}} + \tan^{-1}(x+1) + x + C$





Lec 29 21/1/2015

5F-1a) $\int x^a \ln x\,dx$ $\quad u' = x^a \quad u = \frac{1}{a+1}x^{a+1}$

$= \frac{1}{a+1}x^{a+1}\ln x + \int \frac{1}{a+1}x^{a}\,dx \quad v = \ln x \quad v' = \frac{1}{x}$

$= \frac{1}{a+1}x^{a+1}\ln x + \int \frac{1}{a+1}x\,dx = \frac{1}{a+1}x^{a+1}\ln x + \frac{1}{(a+1)^2}x^{a+1} + C$

2d) $\int x^n e^{ax}\,dx \quad u' = e^{ax} \quad u = \frac{1}{a}e^{ax}$

$= \int \frac{n}{a}x^{n-1}e^{ax}\,dx + \frac{1}{a}x^n e^{ax} + C \quad v = x^n \quad v' = nx^{n-1}$

2b) $\int x^2 e^x\,dx = x^2e^x - \int e^x \cdot 2x\,dx \quad u' = e^x \quad u = e^x$

$\int e^x x\,dx = xe^x - \int e^x = xe^x - e^x \quad v = x^2 \quad v' = 2x$

$\int x^2 e^x\,dx = x^2e^x - 2xe^x + 2e^x \quad v = x \quad v' = 1$

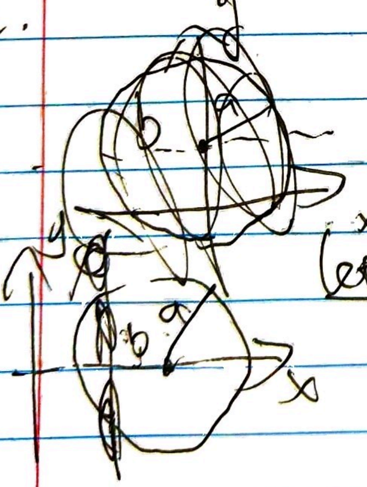
3 $\int \sin^{-1}(4x)\,dx = x\sin^{-1}(4x) - \int \frac{4x\,dx}{\sqrt{1-16x^2}} \quad u' = 1 \quad u = x$

$= x\sin^{-1}4x - \int \frac{du}{2\sqrt{1-u}} = x\sin^{-1}(4x) - \sqrt{1-u} + C \quad v = \sin^{-1}(4x) \quad v' = \frac{1}{\sqrt{1-x^2}}$

$= x\sin^{-1}(4x) - \sqrt{1-16x^2} + C$

PART II 5/2/2015

2.



$y = \sqrt{a^2-(x-b)^2}$

$V = \int_{b-a}^{b+a} 2\sqrt{a^2-(x-b)^2} \cdot 2\pi x\,dx$

let $x-b = u$

$V = \int_{-b}^{a} \sqrt{a^2-u^2} \cdot (u+b)\,du$

let $(x-b)^2 = u$

$\int_{-b}^{a-b}$

$V = 2\int_{b-a}^{b+a} \sqrt{a^2-(x-b)^2} \cdot 2\pi x\,dx$

$= 2\cdot 2\pi \int_{-a}^{a} \sqrt{a^2-u^2} \cdot (u+b)\,du$

$V = 4\pi \int_{-a}^{a} u\sqrt{a^2 - u^2}\, du + 4\pi b \int_{-a}^{a} \sqrt{a^2 - u^2}\, du$

let $a^2 - u^2 = v$ $\quad$ $dv = 2u\,du$

$V = 2\pi \int_0^{a^2} \sqrt{v}\, dv + 4\pi b \int_{-a}^{a} \sqrt{a^2-u^2}\, du$

$= 2\pi \left(\frac{2}{3}(v)^{\frac{3}{2}}\right)\Big|_0^{a^2} + \frac{b}{2}\pi a^2$

$= 2\pi^2 a^2 b$.

3 a) $\int \tan^{n+2} x\,dx = \int \tan^n x \tan^2 x\, dx \qquad \int uv' = uv - \int u'v$

$= \int \tan^n x \sec^2 x\, dx - \int \tan^n x\, dx$

$= \frac{1}{n+1} \tan^{n+1} x + C - \int \tan^n x\, dx$

b) $\int \tan^4 x\, dx = \frac{1}{n+1} \tan^3 x - \int \tan^2 x\, dx$

$= \frac{1}{n+1} \tan^3 x - \int (\sec^2 x - 1)\, dx$

$= \frac{1}{n+1} \tan^3 x + x - \tan x + C$

4 $\sec x = \frac{1}{\cos x} = \frac{\cos x}{\cos^2 x} = \frac{\cos x}{1 - \sin^2 x}$

$\int \sec x\, dx = \int \frac{\cos x}{1-\sin^2 x} dx = \int \frac{d\sin x}{1 - \sin^2 x} = \int \frac{du}{1-u^2} = \int \frac{du}{(1-u)(1+u)}$

$= \ln(1-\sin x) \ln(1+\sin x) + C = \frac{1}{2} \ln\left(\frac{1+\sin x}{1-\sin x}\right) + C$

b) $\int \sec x\,dx = \cancel{\frac{1}{2}\ln\left(1-\sin^2 x\right)} + C$

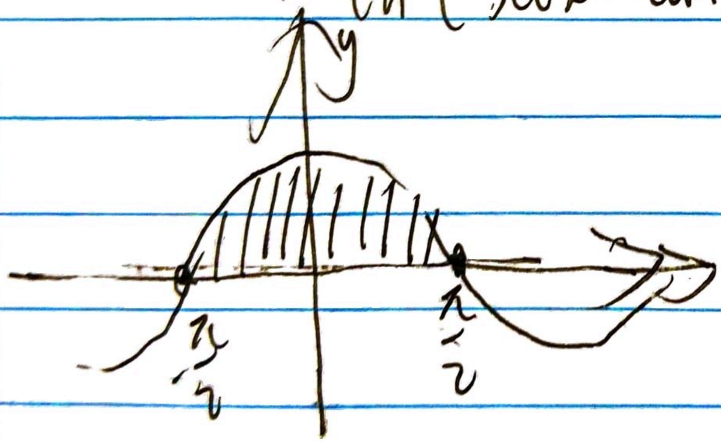
$= \cancel{\ln(\cos x) + C}$

$= \frac{1}{2}\ln\left(\frac{1+\sin^2 x+2\sin x}{1-\sin^2 x}\right) + C$

$= \frac{1}{2}\ln(\sec^2 x + \tan^2 x + 2\tan x\sec x) + C$

$= \ln(\sec x + \tan x) + C$

5.



$V = \int_0^{\frac{\pi}{2}} \cos x \cdot 2\pi x\,dx$

$= 2\pi \int_0^{\frac{\pi}{2}} x\cos x\,dx$

$= 2\pi\left(x\sin x\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x\,dx\right)$

$= 2\pi\left(\frac{\pi}{2} - \left(-\cos x\Big|_0^{\frac{\pi}{2}}\right)\right) = 2\pi\left(\frac{\pi}{2} - 1\right) = \pi^2 - 2\pi$