18.01 Exam 2 25 Oct. 2024. 75

1.

a. $\sin(\pi+\frac{1}{100}) \approx -\sin(\frac{1}{100}) \approx -\frac{1}{100}$

b. ~~$\sqrt{101} = \sqrt{100+1} = 100+0.5 = 100.5$~~

$\sqrt{101} \approx 10 + \frac{1}{2\sqrt{100}} + \frac{1}{2}(-\frac{1}{4})100^{-\frac{3}{2}}\cdot 1$

$= 10 + \frac{1}{20} + (-\frac{1}{8})\frac{1}{\sqrt{100}^3}$

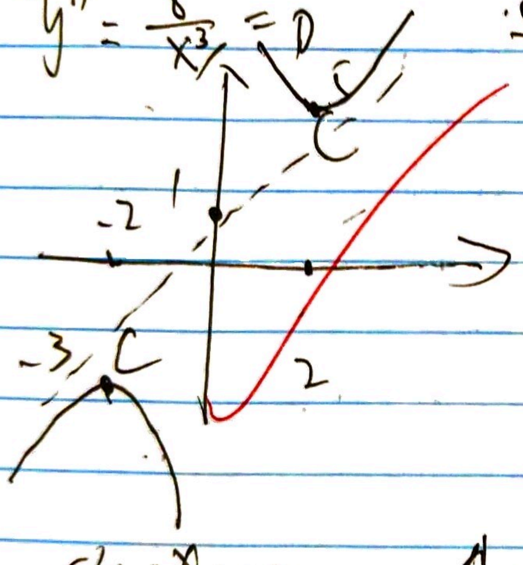
$= 10 + \frac{1}{20} - \frac{1}{8000} = 10.049875 \approx 10.05$

2. $y' = -\frac{4}{x^2}+1 = 0$; $x^2=4$ $x=\pm 2$

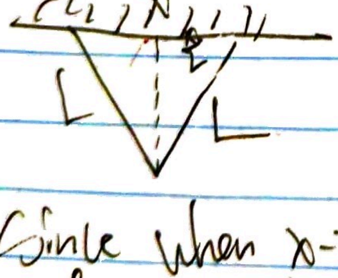
$y''=\frac{8}{x^3} = 0$

$\frac{4}{x}+x+1=0$

$4+x^2+x=0$ <0.



3.



$A(x) = \frac{x}{2}\cdot\sqrt{L^2-\frac{x^2}{4}}$

$A'(x) = \frac{1}{2}\sqrt{L^2-\frac{x^2}{4}} + \frac{x}{2}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{L^2-\frac{x^2}{4}}}(0-\frac{x}{2})$

$= \frac{\sqrt{L^2-\frac{x^2}{4}}}{2} - \frac{x^2}{8\sqrt{L^2-\frac{x^2}{4}}}$

$= \frac{4L^2-x^2-x^2}{8\sqrt{L^2-\frac{x^2}{4}}}$

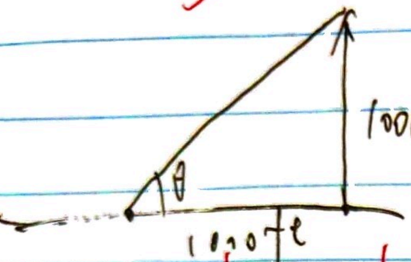
$= \frac{2L^2-x^2}{4\sqrt{L^2-\frac{x^2}{4}}} = 0.$ $x=\sqrt{2}\cdot L$

Since when $x\to 0$ and $x\to\infty$, $A(x)\to 0$, the A_{max} must occur when x is the only critical point $x=\sqrt{2}L$

describe the differen...

P4. -13

$h|_{t=10} = 1000 \qquad \theta = 45°$



$\sin\theta = \frac{h}{\sqrt{1000^2+h^2}}$

$\frac{d}{dt}\sin\theta = \frac{d}{dt}\frac{h}{\sqrt{1000^2+h^2}} \qquad \frac{h}{2\sqrt{1000^2+h^2}} \cdot 2h\cdot h'$

$\tan\theta = \frac{h}{1000} = \frac{1}{100}t^2 \qquad \cos\theta\cdot\frac{d\theta}{dt} = \frac{}{10^6+h^2}$

$\sec^2\theta\cdot\frac{d\theta}{dt} = \frac{1}{50}t$

$h'|_{t=10} = 20t|_{t=10} = 200$

$\frac{d\theta}{dt} = \frac{\cos^2\theta}{50}\cdot t$

$\frac{d\theta}{dt}\Big|_{t=10} = \frac{200\sqrt{2\times10^6} - \frac{1000}{2\sqrt{2}\times10^3}\cdot 400\times1000}{2\times10^6\cdot\cos\frac{\pi}{4}}$

$\frac{d\theta}{dt} = \frac{h'\sqrt{10^6+h^2} - \frac{2h^2h'}{2\sqrt{10^6+h^2}}}{(10^6+h^2)\cos\theta}$

$\frac{d\theta}{dt}\Big|_{t=10} = \frac{t}{100} = \frac{200\sqrt{2}\cdot1000 - 100\sqrt{2}\times1000}{\sqrt{2}\times10^6} = \frac{1}{10}$

$= \frac{2h'(10^6+h^2) - 2h^2h'}{2(10^6+h^2)^{\frac{3}{2}}\cdot\cos\theta} = \frac{200\sqrt{2}\times1000}{\sqrt{2}\times1000\times1000}$

$\frac{1}{50}(t+1)\sin^2\theta + 1$

$= \frac{1}{50}(\sin^2\theta\, t + 1 - \sin^2\theta)$

$= \frac{h'(10^6)}{(10^6+h^2)^{\frac{3}{2}}\cdot\cos\theta}$

$\frac{d^2\theta}{dt^2} = \frac{1}{50}(\sin^2\theta\cdot t+1) = \frac{3}{14}$

$\frac{d\theta}{dt} = \frac{1}{50}\left(\frac{(t-1)(1000^2)}{1000^2+t^2}+1\right), \quad t\in[0,+\infty), \ \theta\in(0,\frac{\pi}{2})$

$= \frac{2\times10^7 t}{(10^6+100t^4)^{\frac{3}{2}}\cdot\cos\theta} \qquad \frac{d\theta}{dt} = \frac{}{(10^6+100t^4)^{\frac{1}{2}}\cdot\tan\theta} \geq 0$

$\cancel{\frac{d^2\theta}{dt^2} = \frac{2\times10^7(10^6+100t^4)^{\frac{3}{2}}\cos\theta - 2\times10^7t\left(\frac{3}{2}\right)(10^6+100t^4)^{\frac{1}{2}}(400t^3)\cos\theta}{(10^6+100t^4)^{3}\cdot\cos\theta}}$

$= \frac{1}{50}\left(\frac{100t^3 - 100t^2 + 100^2}{1000^2+100t^2}\right)$

Increase on $\mathbb{R}$

decrease on ϕ

P5 i. $\int \cos(3x)\,dx = \frac{\sin 3x}{3} + C$

ii. $\int x e^{(x^2)}\,dx = \int \frac{e^{(x^2)}}{2}\,d(x^2) = \frac{e^{x^2}}{2} + C$

b. $\cancel{\int \frac{1}{y^3}\,dy}$ $\quad \frac{dy}{dx} = \frac{1}{y^3}$

$dy = \frac{1}{y^3}\,dx$

$\cancel{y = \int \frac{1}{y^3}\,dx}$

$dy \cdot y^3 = dx$ $\qquad y(0) = (-C)^{\frac{1}{4}} = 1$

$x = \int y^3\,dy$ $\qquad -C = 1$

$C = -1$

$x = \frac{y^4}{4} + C$ $\qquad \therefore y = (4x+1)^{\frac{1}{4}}$

$y = (4x)^{\frac{1}{4}}$

P6 $f'(x) = e^{(x^2)}$, $f(0) = 10$. $\quad -10$ $\quad \frac{e^{x^2} \cdot 2x \cdot 2x - 2e^{x^2}}{4x^2}$ $\quad 0 < C < 1$

$f(x) = \int e^{(x^2)}\,dx + C = \cancel{\ldots} = \frac{e^{x^2}}{2x}$ $\qquad f(1) = f(0) + f'(c) = 10 + e^{c^2}$

$\cancel{f(x)}$

$\cancel{fe}$ let $u = x^2$

$du = 2x\,dx$ $\qquad dx = \frac{du}{2x} = \frac{du}{2\sqrt{u}}$ $\qquad \therefore \cancel{10 < e^{(c^2)} 10 + e}$

$f(x) = \int \frac{e^u}{2\sqrt{u}}\,du = \frac{1}{2}\int \frac{e^u}{\sqrt{u}}\,du = \cancel{\ldots}$ $\qquad 1 < e^{(c^2)} < e$

$f'(1) = e$ $\quad f'(0) =$ $\qquad 11 < f(1) < 10 + e$

$f(x) = f(0) + f'(1)(x) = ex + 10$

$f'(x) = e^{(x^2)} \geq 0$ $\qquad \therefore f(1) > f(0) = 10$