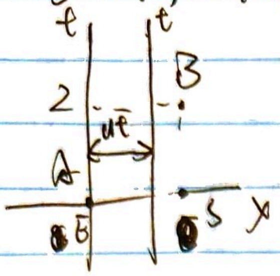
PS VII 18/4/2025

1. 

~~A(0,0) B(0,2)~~

i) $\Delta t' = \frac{\Delta t - u\Delta x/c^2}{\sqrt{1-u^2/c^2}}$

$= \frac{2 - 0.8c \cdot 8.3\, c/c^2}{\sqrt{1-0.8c^2/c^2}}$

$= \frac{2-6.64}{\sqrt{0.2}} \approx -7.78$ min

ii) $\Delta t' = \frac{\Delta t + u\Delta x/c^2}{\sqrt{1-u^2/c^2}} = 14.4$ min

2 a) $\Delta x = \frac{0 - v\Delta t'}{\sqrt{1-v^2/c^2}}$ $\quad v = \frac{\Delta x}{\Delta t'}\sqrt{1-v^2/c^2}$

$v^2 = \left(\frac{\Delta x}{\Delta t'}\right)^2 - \left(\left(\frac{v}{c}\right)\left(\frac{\Delta x}{\Delta t'}\right)\right)^2$

$v^2\left(1+\left(\frac{1}{c}\cdot\frac{\Delta x}{\Delta t'}\right)^2\right) = \left(\frac{\Delta x}{\Delta t'}\right)^2$

$v = \sqrt{\frac{(\Delta x/\Delta t')^2}{1+\left(\frac{1}{c}\cdot\Delta x/\Delta t'\right)^2}} = \sqrt{\frac{c^2}{\left(\frac{\Delta t' c}{\Delta x}\right)^2+1}} = \sqrt{\frac{3\times10^8 m/s}{\left(\frac{5 min}{8.3 min}\right)^2+1}}$

~~$= 2.6\times10^8$ m/s~~ $= 0.86c$

b) $\Delta t = \frac{\Delta t' - v\Delta x'/c^2}{\sqrt{1-v^2/c^2}} = \frac{5}{\sqrt{1-v^2/c^2}} = \frac{5}{\sqrt{1-(0.86)^2}} = 9.8$ min

c) $\Delta s^2 = (c\Delta t')^2 - \Delta x'^2 = (5\times 60\, c)^2 =$ 90000 light-second

3. $\Delta x = \frac{\Delta x' - u\Delta t'}{\sqrt{1-u^2/c^2}} = \frac{-2.97\times10^8 m/s \times 2\times10^{-6} s}{\sqrt{1-(2.97\times10^8/3\times10^8)^2}} = -4210$ m $= -4.21$ km

$h = h_0 + \Delta x = 95.79$ km

$\Delta x' = 0$

4. i) $\Delta x' = \frac{\Delta x - u\Delta t}{\sqrt{1-u^2/c^2}} = 0$

$$u = \frac{\Delta x}{\Delta t} = \frac{730\,m}{4.96\times10^{-6}s} = 1.47\times10^{8}\,m/s$$

ii) for S', R is before B

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 = \cancel{(3\times10^8 \times 4.96\times10^{-6})^2 + (750)^2 = 2.75\times10^6}\ m$$

$$= (c\Delta t')^2 - (\Delta x')^2$$

$$\cancel{(c\Delta t')^2 =}$$

$$\Delta t' = \sqrt{(c\Delta t)^2 - (\Delta x)^2}\cdot\frac{1}{c} = 4.32\times10^{-6}\,s$$

5. $\cancel{L = \frac{L_0}{\sqrt{1-u^2/c^2}} = \frac{5}{3}L_0}$

$$u = \frac{\frac{c}{2}+\frac{c}{2}}{1+\left(\frac{c}{2}\right)^2/c^2} = \frac{c}{\frac{5}{4}} = \frac{4}{5}c$$

$$\cancel{L = \frac{L_0}{\sqrt{1-u^2/c^2}} = \frac{L_0}{\sqrt{1-\frac{16}{25}}} = \frac{5}{3}L_0}$$

$$L = \sqrt{1-u^2/c^2}\,L_0 = \frac{3}{5}L_0$$

7. $P = \left(\frac{mc}{\sqrt{1-u^2/c^2}}, \frac{mu}{\sqrt{1-u^2/c^2}}\right)$

$$p = \frac{mu}{\sqrt{1-u^2/c^2}} \qquad p' = \frac{2mu}{\sqrt{1-4u^2/c^2}} = \frac{4mu}{\sqrt{1-u^2/c^2}} = 4p$$

$$\sqrt{1-u^2/c^2} = 2\sqrt{1-4u^2/c^2}$$

$$1-u^2/c^2 = 4-16u^2/c^2$$

$$15u^2/c^2 = 3 \qquad \frac{u}{c} = \sqrt{\frac{3}{15}} = \frac{1}{\sqrt{5}}$$

7. $P_1 + P_0 = P_2$

$\left(\frac{m_0 c}{\sqrt{1-v^2/c^2}}, \frac{m_0 v}{\sqrt{1-v^2/c^2}}\right) \quad (m_0 c, 0)$

$$P_2^2 = M^2c^2 = (P_1+P_0)^2 = P_1^2 + P_0^2 + 2P_1 \cdot P_0$$

$$= m_0^2c^2 + m_0^2c^2 + \frac{2m_0^2c^2}{\sqrt{1-v^2/c^2}}$$

$$M^2 = 2m_0^2 + \frac{2m_0^2}{\sqrt{1-v^2/c^2}}$$

$$M = m_0\sqrt{2\left(1+\frac{1}{\sqrt{1-v^2/c^2}}\right)}$$

8. i) $p = p_1 + p_2 = m_0 v$

$M = 2m_0 \quad V = \frac{p}{M} = \frac{v}{2}$

ii) $P_1' = \left(\frac{m_0 c}{\sqrt{1-V^2/c^2}}, \frac{m_0 V}{\sqrt{1-V^2/c^2}}\right) \quad P_2' = \left(\frac{m_0 c}{\sqrt{1-V^2/c^2}}, -\frac{m_0 V}{\sqrt{1-V^2/c^2}}\right)$

$v = \frac{2V}{1+V^2/c^2}$ $V < c$

$v = \frac{2V}{1+V^2/c^2}$

$v + \frac{v}{c^2}V^2 = 2V$

$vV^2 - 2c^2V + vc^2 = 0$

$vV^2 - 2c^2V + vc^2 = 0$

$V = \frac{2c^2 \pm \sqrt{4c^4 - 4v^2c^2}}{2v}$

$V = \frac{c^2 \pm \sqrt{c^2(1-v^2)}}{v}$

$P_1 = \left(\frac{m_0 c}{\sqrt{1-v^2/c^2}}, \frac{m_0 v}{\sqrt{1-v^2/c^2}}\right), \quad P_0 = (m_0 c, 0)$

$= \frac{c^2}{v}\left(1 \pm \sqrt{1-v^2/c^2}\right)$

$= \frac{c^2}{v}\left(1-\frac{1}{\gamma}\right)$

$P_t = P_1 + P_0 = (m_0 c(1+\gamma), m_0 v\gamma)$

$= v\frac{1-\frac{1}{\gamma}}{\frac{v^2}{c^2}}$

$P_t' = (Mc, 0) = P_1' + P_2'$

$= v\frac{\gamma}{1+\gamma}$

$$\cancel{\frac{2m_0c}{\sqrt{1-v^2/c^2}} = m_0c(1+\gamma)}$$

$$\cancel{c^2 - v^2 = \frac{2c^2}{1+\gamma}}$$

$$\cancel{v^2 = c^2 - \frac{2c^2}{1+\gamma}} = \frac{c^2((1+\gamma)^2-4)}{(1+\gamma)^2}$$

$$\cancel{v = \frac{c^2(\gamma^2+1+2\gamma-4)}{1+\gamma}}$$

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

$$0 = \frac{m_0 v\gamma}{\sqrt{1-\beta^2}} - \beta m_0 c(1+\gamma)$$

$$m_0 v\gamma = \beta m_0 c(1+\gamma)$$

$$\cancel{0 = m_0 v_1 - \beta m_0 c - \frac{\beta}{\gamma} m_0 c}$$

$$V = \frac{v\gamma}{1+\gamma}$$

$$\cancel{\left(1+\frac{1}{\gamma}\right)\beta = v}$$

$$k^2 = \omega^2 - \vec{k}^2 = 0$$

$$(P_1+P_2)^2 = P_1^2 + P_2^2 + 2P_1 \cdot P_2 = 2m_0^2$$

$$= m_0^2\left(\cancel{(\gamma_1^2+\gamma_2^2) - (\gamma_1\vec{v}_1)^2 - (\gamma_2\vec{v}_2)^2}\right) + 2m_0^2(\gamma_1\gamma_2 - \gamma_1\gamma_2\vec{v}_1\cdot\vec{v}_2)$$

$$\cancel{m_0^2\left((\gamma_1+\gamma_2)^2 - (\gamma_1\vec{v}_1+\gamma_2\vec{v}_2)^2\right)}$$

$$= 2m_0^2(1+\gamma_1\gamma_2 - \gamma_1\gamma_2\vec{v}_1\cdot\vec{v}_2)$$

If $k^2 = (P_1+P_2)^2$

$$0 = 2m_0^2 + E_1E_2 - \vec{p}_1\cdot\vec{p}_2 \qquad \vec{p}_1\cdot\vec{p}_2 = 2m_0^2 + E_1E_2$$

But $p^2 = E^2 - m^2c^2$ $\therefore p < E$ $\therefore \vec{p}_1\cdot\vec{p}_2 < |\vec{p}_1||\vec{p}_2| < E_1E_2$ $\therefore$ Impossible

10. $K = (\nu, \vec{k}) \quad P = (E, \vec{p}) \quad P' = (E', \vec{p}\,')$

$$P = K + P' = (E, 0)$$

$$\cancel{m_0^2 c^2 = m_0'^2 c^2 + \delta^2}$$

$$\cancel{m_0^2 = m_0'^2 + \delta^2}$$

$$\cancel{P^2 = K^2 + P'^2 + 2K \cdot P} \qquad m_0' = m_0 - \delta$$

$$\cancel{m_0^2 = m_0'^2 + 2(\nu m_0'^2 - \vec{k} \cdot \vec{p}\,')}$$

$$P' = K - P$$

$$P'^2 = K^2 + P^2 - 2K \cdot P$$

$$m_0'^2 = m_0^2 - 2(\nu m_0)$$

$$\cancel{m_0^2} - 2\delta m_0 + \delta^2 = \cancel{m_0^2} - 2\nu m_0$$

$$2\nu m_0 = 2\delta m_0 + \delta^2$$

$$\nu = \delta + \frac{\delta^2}{2m_0}$$

11. $\cancel{P = \left(\frac{m_0 c}{\sqrt{1-u^2/c^2}}\right)}$ $P = \left(\frac{m_0 c}{\sqrt{1-v^2/c^2}}, \frac{m_0 v}{\sqrt{1-v^2/c^2}}\right) = (\gamma_v m_0 c, \gamma_v m_0 v)$

$$P' = (P_0', P_1') \qquad P_0' = \frac{P_0 - \beta P_1}{\sqrt{1-\beta^2}}, \; P_1' = \frac{P_1 - \beta P_0}{\sqrt{1-\beta^2}}, \; \left(\beta = \frac{u}{c}\right)$$

$$w = \frac{v-u}{1-\frac{uv}{c^2}} \qquad P' = \frac{1}{\sqrt{1-w^2/c^2}}(m_0 c, m_0 w) = (\gamma_w m_0 c, \gamma_w m_0 w)$$

$\gamma_v = \frac{1}{\sqrt{1-v^2/c^2}}$ $\quad \gamma_u = \frac{1}{\sqrt{1-u^2/c^2}}$

$\gamma_w = \frac{1}{\sqrt{1-w^2/c^2}}$ $\quad w = \frac{v-u}{1-uv/c^2}$

$\gamma_w^2(1-w^2/c^2) = 1$

$$\cancel{\gamma_w^2(c^2-w^2) = c^2}$$

$$\cancel{\gamma_w^2\left(c^2-\left(\frac{v-u}{1-uv}\right)^2\right) = c^2}$$

$$\cancel{\gamma_w^2\left((c^2-uv)^2c^2-(v-u)^2\right) = c^2(c^2-uv)^2}$$

$$\gamma_w^2\left(1-\frac{1}{c^2}\left(\frac{v-u}{1-\frac{uv}{c^2}}\right)^2\right) = 1$$

$$\gamma_w^2\left(1-\left(\frac{v-u}{c^2-uv}\right)^2c^2\right) = 1$$

$$\gamma_w^2\left((c^2-uv)^2-c^2(v-u)^2\right) = (c^2-uv)^2$$

$$\gamma_w^2\left((c^4-2c^2uv+u^2v^2)-c^2(v^2-2uv+u^2)\right) = (c^2-uv)^2$$

$$\gamma_w^2(c^4-c^2v^2-c^2u^2+u^2v^2) = (c^2-uv)^2$$

$$\gamma_w^2(c^2-u^2)(c^2-v^2) = (c^2-uv)^2$$

$$\gamma_w^2 = \frac{(c^2-uv)^2/c^4}{\left(\sqrt{1-u^2/c^2}\sqrt{1-v^2/c^2}\right)^2} \quad \gamma_u\gamma_v$$

$$\gamma_w = \gamma_u\gamma_v\left(1-\frac{uv}{c^2}\right)$$

$$p_0' = \gamma_w m_0 c$$

$$= \gamma_u\gamma_v\left(1-\frac{uv}{c^2}\right)m_0 c$$

$$= \gamma_u\left(\gamma_v m_0 c - \gamma_v m_0 v \cdot \frac{u}{c}\right)$$

$$= \gamma_u(p_0 - \beta p_1)$$

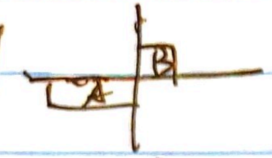
$$p_1' = \gamma_u(p_1 - \beta p_0)$$

$$\beta = \frac{u}{c}.$$

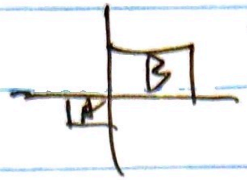
12. Events.

A | B

1.

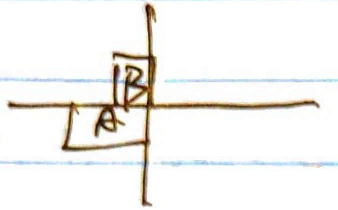


$(0, 0)$

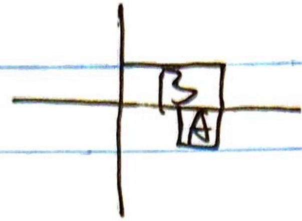


$(0, 0)$

2.

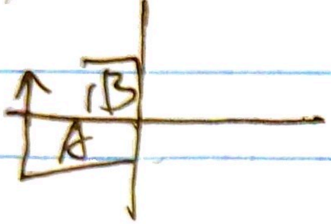


$(t_2, 0)$

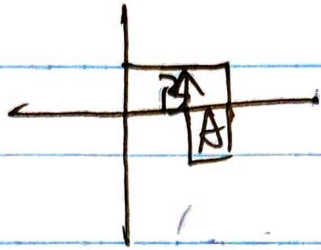


~~(L_0, t_2')~~ (t_2', L_0)

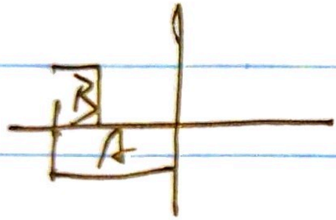
3.



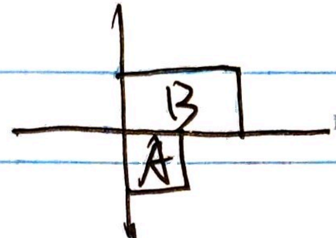
$(t_3, -L_0)$



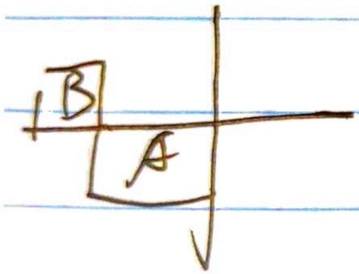
~~(L_0, t)~~ $(t_3',$ A_3' $)$



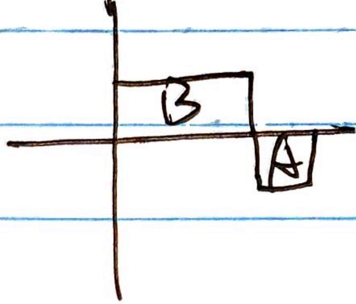
$(t_4, -L_0)$ ~~$(-L_0, t_4)$~~



$(t_4', 0)$ ~~$(0, t_4)$~~



$(t_5, -L_0)$ ~~$(-L_0, t_5)$~~



(t_5', L_0)

$\Delta t_{32} = 0 \quad \Delta x_{32} = -L_0 = -1$

$\Delta t'_{32} = t'_3 - t'_2 \quad \Delta x'_{32} = x'_3 - x'_2$

$\Delta x'_{32} = (\Delta x_{32} + u\Delta t_{32})\gamma = -\gamma L_0 = -\gamma$

$x'_3 = \Delta x'_{32} + x'_2 = L_0 - \gamma L_0 = 1 - \gamma L_0$

in the missile mission.

$\Delta t'_{32} = (t_{32} + u x_{32}/c^2)\gamma = \frac{u}{c^2}(-L_0)\gamma = -\gamma u/c^2$

$(c\Delta t'_{32})^2 + (\Delta x'_{32})^2 = \gamma^2 u^2/c^2 + \gamma^2 = \gamma^2(1 + u^2/c^2) = \gamma^2/\gamma^2 = 1$

$\Rightarrow (c\Delta t_{32})^2 + (\Delta x_{32})^2 = 1$