PS 5. 10 Dec

Lecture 19. 10 Dec

3I-1 $\int_3^6 \frac{1}{\sqrt{x-2}}\,dx = \int_{\sqrt{3-2}}^{\sqrt{6-2}} \frac{1}{u}\,d(u^2+2) = \int_1^2 \frac{1}{u}\cdot 2u\,du = 2u\Big|_1^2 = 2$

2a). $\int_0^2 \sqrt{3x+5}\,dx = \int_{\sqrt{5}}^{\sqrt{11}} u\,d\frac{u^2-5}{3} = \int_{\sqrt{5}}^{\sqrt{11}} u\cdot\frac{1}{3}\cdot 2u\,du = \int_{\sqrt{5}}^{\sqrt{11}} \frac{2}{3}u^2\,du$

$= \frac{2}{3}\cdot\frac{1}{3}u^3\Big|_{\sqrt{5}}^{\sqrt{11}} = \frac{2}{9}\left(11\sqrt{11} - 5\sqrt{5}\right) = \frac{22\sqrt{11}-10\sqrt{5}}{9}$

3a) $\int_1^2 \frac{x\,dx}{x^2+1} = \int_2^5 \frac{\sqrt{u-1}\,2u\,du}{u} = \int_2^5 2\sqrt{u-1}\,du = 2\cdot\frac{2}{3}(u-1)^{\frac{3}{2}}\Big|_2^5$

$= \frac{4}{3}(8-1) = \frac{28}{3}$

5a)  $\int_0^{\pi} \sin x = -\cos x\Big|_0^{\pi} = -(-1-1) = 2$

3E-6b) $\int_0^{\pi} \sin^2 x\,dx$ ~~$= \int_0^{\pi}(\sin^2 x+\cos^2 x)\,dx = \int_0^{\pi} 1\,dx = \pi$~~

$< \int_0^{\pi} \sin x\,dx = -\cos x\Big|_0^{\pi} = 2$

c) $\int_{10}^{20} \sqrt{x^2+1}\,dx > \int_{10}^{20} x\,dx = \frac{1}{2}x^2\Big|_{10}^{20} = \frac{1}{2}(400-100) = 150$

4J-2 $R = \int_0^{60} x_0 e^{-kt}\cdot r\,dt = x_0 r\cdot\left(-\frac{1}{k}e^{-kt}\right)\Big|_0^{60} = \left(\frac{x_0 r}{k} - \frac{x_0 r e^{-60k}}{k}\right)$

Lec 20. 12 Dec

3E-1. $L\left(\frac{1}{x}\right) = \int_1^{\frac{1}{x}} \frac{dt}{t} = \int_{1}^{x} u\,d\frac{1}{u} = \int_1^x -u\cdot\frac{1}{u^2}\,du = -\int_1^x \frac{1}{u}\,du = L(x)$

3a) $\int_0^1 \frac{\sqrt{u}}{e^u}\,de^u = \int_0^1 \sqrt{u}\,du = \frac{2}{3}u^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3}(1-0) = \frac{2}{3}$

3D-1 a) $\left(\ln(x+\sqrt{x^2+a^2}) - \ln a\right)'$

~~$= [\ln(\sqrt{x^2+a^2}) + \ln(\ldots)]' = [\ln(\ldots) + \ln(\ldots)]$~~

$= \frac{1+\frac{1}{2\sqrt{x^2+a^2}}\cdot 2x}{x+\sqrt{x^2+a^2}} = \frac{\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}}}{x+\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$

$\therefore f(x) = \int_0^x \frac{dt}{\sqrt{t^2+a^2}} = \ln(x+\sqrt{x^2+a^2}) - \ln a + C$

$f(0) = \ln a - \ln a + C = \int_0^0 \frac{dt}{\sqrt{t^2+a^2}} = 0.\quad \therefore C = 0.$

$\therefore LHS = RHS.$

21 An...

6). $\int_c^c \frac{dt}{\sqrt{t^2+a^2}} = 0 = \ln(c+\sqrt{c^2+a^2})$

$c+\sqrt{c^2+a^2}=1$

$\sqrt{c^2+a^2}=1-c$

$c^2+a^2=(1-c)^2=c^2-2c+1$

$2c=1-a^2$

$c=\frac{1-a^2}{2}$

4b) ~~$\int \sin x^3 dx = \ldots -\cos x^3 \ln(3x^2)$~~

$f(x)=\int_0^x \sin(t^3)\, dt+2$

c) $f(x)=\int_1^x \sin(t^3)\, dt-1$

5. a) $\lim_{\delta x\to 0}\int_1^{1+\delta x} \frac{t\,dt}{\sqrt{1+t^4}}$ = ~~$\delta x \cdot \left.\frac{t}{\sqrt{1+t^4}}\right|_{t=1}$~~ = ~~$\delta x \int \ldots$~~

$\frac{1}{\delta x}\int_1^{1+\delta x}\frac{t\,dt}{\sqrt{1+t^4}}$ $=\delta x \cdot \frac{1}{\sqrt{2}}$ ~~$=2\delta x \int \ldots du$~~ ~~$=2\delta x\int \frac{1}{2}(u^2-1)\,du \ldots$~~

$=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

b) $F(x)=\int_0^x \frac{t}{\sqrt{1+t^4}}\,dt$

$F'(x)=\frac{t}{\sqrt{1+t^4}}$

$F'(1)=\frac{1}{\sqrt{2}}$

$\int_1^{1+\delta x}\frac{t}{\sqrt{1+t^4}} = F(1+\delta x)-F(1)$

$\lim_{\delta x\to 0}\frac{1}{\delta x}\int_1^{1+\delta x}\frac{t\,dt}{\sqrt{1+t^4}}$ $\lim_{\delta x\to 0}\frac{F(1+\delta x)-F(1)}{\delta x}=F'(1)=\frac{1}{\sqrt{2}}$

8 a) $f(x)=[2x(\sin x+1)]'=2(\sin x+1)+2x(\cos x)=2x\cos x+2\sin x+2$

$f\left(\frac{\pi}{2}\right)=\pi\cdot 0+2+2=4$

~~b) $F\left(\frac{x}{2}\right)=2x(\sin x+1)$~~

~~$f(x)=4x(\sin 2x+1)$~~

$f(x)=$ ~~$f'(x)=4(\sin 2x+1)+4x(2\cos 2x)=4\sin 2x+8x\cos 2x+4$~~

Yes No Maybe

Uni

(a) $\int_0^{\sqrt{2}x} e^{-\frac{u^2}{2}}\, d\frac{\sqrt{2}}{2}u = \frac{\sqrt{2}}{2}\int_0^{\sqrt{2}x} e^{-\frac{u^2}{2}}\, du$

$F(x) = \frac{\sqrt{2}}{2}\int_0^{\sqrt{2}x} e^{-\frac{u^2}{2}}\, du$

$F(\frac{\sqrt{2}}{2}x) = \frac{\sqrt{2}}{2}\int_0^{x} e^{-\frac{u^2}{2}}\, du$

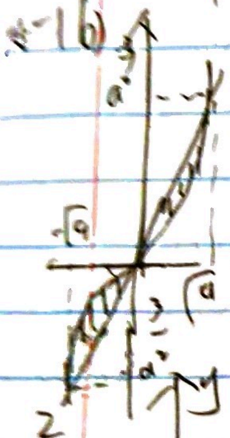
$E(x) = \frac{\sqrt{2}}{\sqrt{2\pi}} F(\frac{\sqrt{2}}{2}x) = \frac{1}{\sqrt{\pi}} F(\frac{\sqrt{2}}{2}x)$

$\lim_{x\to\infty} E(x) = \frac{\sqrt{2}}{2}\cdot\frac{1}{\sqrt{2\pi}} = \frac{1}{2}$

c) $E(b) - E(a)$

Lec 24 15 Dec

1(b)

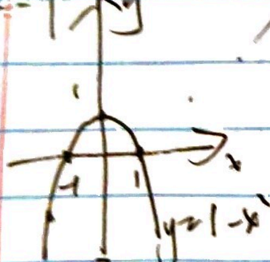


$x^3 = ax \quad x=0$

$x^2 = a$

$x = \pm\sqrt{a} \quad A = 2\int_0^{\sqrt{a}} (ax - x^3)\,dx = 2\left(\frac{a}{2}x^2 - \frac{1}{4}x^4 \Big|_0^{\sqrt{a}}\right)$

$= 2\left(\frac{a^2}{2} - \frac{a^2}{4} - 0\right) = \frac{a^2}{2}$



Method 1 $A = \int_{-1}^{1} (1-x^2)\,dx = x - \frac{x^3}{3}\Big|_{-1}^{1}$

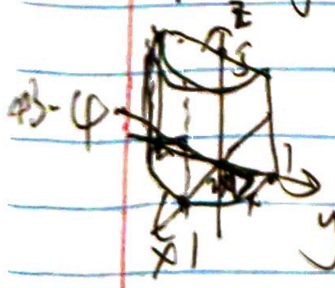
$= \left(1 - \frac{1}{3} + 1 - \frac{1}{3}\right) = \frac{4}{3}$

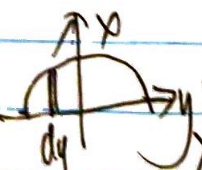
Method 2 $x = \pm\sqrt{1-y}$

$A = 2\int_0^1 \sqrt{1-y}\,dy = 2\left(-(1-y)^{\frac{3}{2}}\cdot\frac{2}{3}\Big|_0^1\right)$

$= 2\left(0 + \frac{2}{3}\right) = \frac{4}{3}$

#3-(4)





$x^2+y^2=1$

$x = \sqrt{1-y^2}$

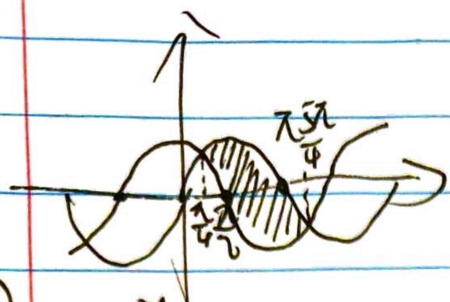
$z = 2x = 2\sqrt{1-y^2}$

$V = \int_{-1}^{1} (1-y^2)\,dy$

$= \left(y - \frac{y^3}{3}\Big|_{-1}^{1}\right)$

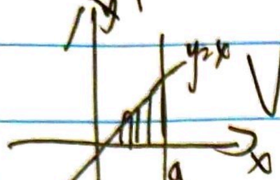
$= \left(1 - \frac{1}{3} - \left(-1 + \frac{1}{3}\right)\right) = \frac{4}{3}$

-4



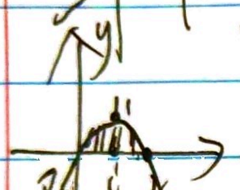
$$A=\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\sin x-\cos x)\,dx = \left(-\cos x-\sin x\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\right)$$
$$=\left(\frac{\sqrt2}{2}+\frac{\sqrt2}{2}\right)-\left(-\frac{\sqrt2}{2}-\frac{\sqrt2}{2}\right)=\sqrt2+\sqrt2=2\sqrt2$$

-1 d)



$$V=\int_0^a \pi x^2dx = \frac{\pi}{3}x^3\Big|_0^a=\frac{\pi a^3}{3}$$

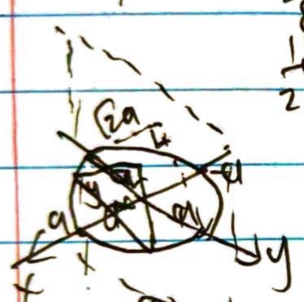
e)



$$V=\int_0^2 \pi y^2dx=\int_0^2\pi(2x-x^2)^2dx=\int_0^2\pi(4x^2+x^4-4x^3)dx$$
$$=\pi\left(\frac{4}{3}x^3+\frac{1}{5}x^5-x^4\Big|_0^2\right)=\pi\left(\frac{32}{3}+\frac{32}{5}-16\right)$$
$$=\pi\left(\frac{160+96-240}{15}\right)=\frac{16\pi}{15}$$

$16 \times 15 = 80,\ 16,\ 240$

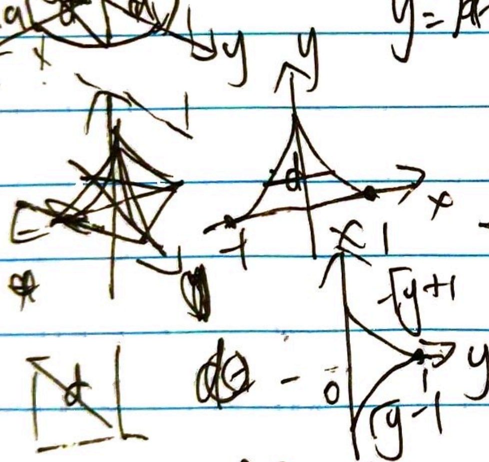
6



$x^2+y^2=a^2 \quad A=(\sqrt2 y)^2\cdot\frac12=y^2=a^2-x^2$

$y=\sqrt{a^2-x^2} \quad V=\int_{-a}^{a}(a^2-x^2)dx=a^2x-\frac{x^3}{3}\Big|_{-a}^{a}$
$$=a^3-\frac{a^3}{3}+a^3+\left(\frac{-a^3}{3}\right)=\frac{4}{3}a^3$$

7



$~~A=\int_{-1}^{0}(x+1)^2dx+\int_0^1(x-1)^2dx~~$

$y=(x\pm1)^2$

$x=\pm\sqrt{y}\pm1$

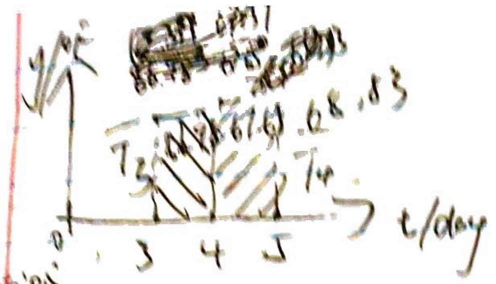
$d=-\sqrt{y}+1-\sqrt{y}+1=2-2\sqrt{y}$

$$A=\left(\frac{d}{\sqrt2}\right)^2=\frac{d^2}{2}=\frac{4(1-\sqrt{y})^2}{2}=2(y+1-2\sqrt{y})=2y-4\sqrt{y}+2$$
$$V=\int_0^1(2y-4\sqrt{y}+2)dy=2\int_0^1(y-2\sqrt{y}+1)dy=2\left(\frac{y^2}{2}-2\cdot\frac{2}{3}y^{\frac32}+y\Big|_0^1\right)$$
$$=2\left(\frac{y^2}{2}-\frac{4}{3}y^{\frac32}+y\right)=2\left(\frac12-\frac43+1\right)=2\left(\frac{3-8+6}{6}\right)=\frac13$$

PS5 PART II

1\. a) $\frac{dy}{dt}=\frac{y}{100} \quad \int\frac{dy}{y}=\int\frac{dt}{100} \quad \ln y=\frac{t}{100}+C \quad y=Ae^{\frac{t}{100}} \quad y(0)=A=65$

$y(3)=65e^{0.03}=66.98 \quad y(4)=65e^{0.04}=67.65 \quad y(5)=65e^{0.05}=68.33$

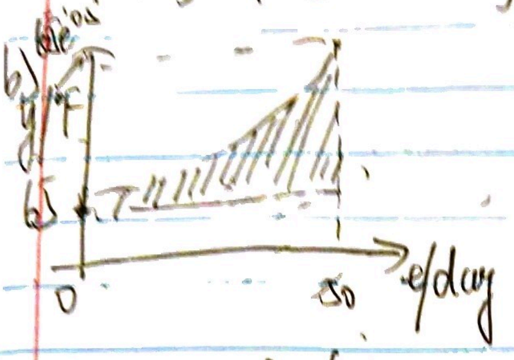


$T_3 = \int_3^4 65e^{0.01t}\,dt = 6500e^{0.01t}\Big|_3^4 = 67.32°F$

$= 67.003$ ~~...~~ $= 67°F$

$T_4 = \int_4^5 65e^{0.01t}\,dt = 6500e^{0.01t}\Big|_4^5 = 67.997$

b)



$\int_0^{50} (65e^{0.01t} - 65)\,dt = 6500e^{0.01t} - 65t\Big|_0^{50}$

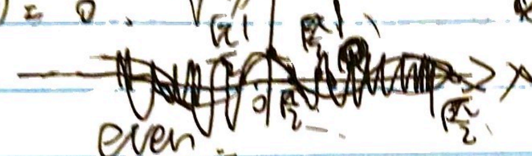
$= 966.69°F$

2. $f'(x) = \cos x^2$ ... $f'(x)$ is even. $f(x)$ is odd.

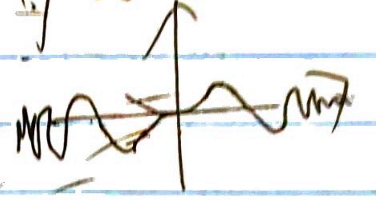
$f(0) = 0$.

$f''(x) = -\sin x^2 \cdot 2x$

a)



even

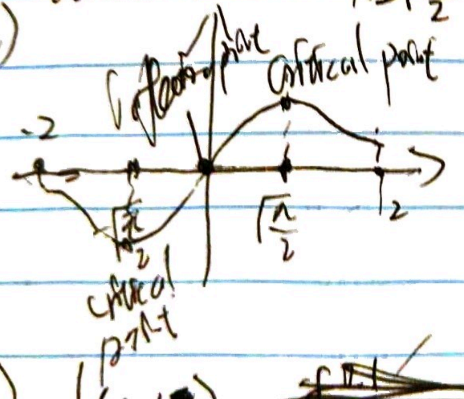


b)

local maxima $x = \pm\sqrt{\frac{\pi}{2}+2k\pi}$, $k \in \mathbb{Z}^+$

local minima $x = \pm\sqrt{\frac{3\pi}{2}+2k\pi}$, $k \in \mathbb{Z}^+$

c)



$\int_0^x \cos t^2\,dt \approx \sum_{k=0}^{n-1} \cos\left(\frac{kx}{n}\right)^2 \cdot \frac{x}{n}$

d) $f(0.1) =$

$f'(x) = \cos x^2$ $\quad f''(x) = -\sin x^2 \cdot 2x$ $\quad f''(0) = 0$

$f^{(3)}(x) = -\cos x^2 \cdot 4x^2 - \sin x^2 \cdot 2$ $\quad f^{(3)}(0) = 0$

$f^{(4)}(x) = \sin x^2 \cdot 8x^3 - 12\cos x^2 \cdot x$

$f^{(5)}(x) = -\cos x^2 \cdot 2x \cdot 8x^3 + \sin x^2 \cdot 24x^2 - 12\cos x^2 + 12x\sin x^2 \cdot 2x$

$f^{(5)}(0) = -12$

$$f(0.1) \approx f(0) + 0.1 f'(0) + \frac{(0.1)^2}{2} f^{(2)}(0) + \frac{(0.1)^3}{6} f^{(3)}(0) + \frac{(0.1)^4}{24} f^{(4)}(0) + \frac{(0.1)^5}{120} f^{(5)}(0)$$

$$= 0 + 0.1 + \frac{10^{-5}}{120} \times 2 = 0.1 + 10^{-6} = \cancel{0.1000001} \quad 0.099999$$

e) i) $g(c_2 x) = \int_0^{c_2 x} \cos\left(\frac{\pi}{2} u^2\right) du$

let $u = \frac{t}{\sqrt{\frac{\pi}{2}}}$ $\quad du = \frac{1}{\sqrt{\frac{\pi}{2}}} dt$

$g(c_2 x) = \frac{1}{\sqrt{\frac{\pi}{2}}} \int_0^{\sqrt{\frac{\pi}{2}} c_2 x} \cos t^2 \, dt$ $\quad$ turn the critical points

$c_1 g(c_2 x) = c_1 \sqrt{\frac{\pi}{2}} \int_0^{\sqrt{\frac{\pi}{2}} c_2 x} \cos t^2 \, dt.$ $\quad$ into integers.

put $c_1 = \sqrt{\frac{\pi}{2}}$, $c_2 = \frac{1}{\sqrt{\frac{\pi}{2}}} = \sqrt{\frac{2}{\pi}}$

$c_1 g(c_2 x) = \int_0^x \cos t^2 \, dt.$

ii) $h(x^2) = \int_0^{x^2} \frac{\cos v}{\sqrt{v}} dv$. $\quad$ let $v = t^2$. $\quad dv = 2t\,dt$.

$h(x^2) = \int_0^x \frac{\cos t^2}{t} \cdot 2t \, dt = 2\int_0^x \cos t^2 \, dt$

$f(x) = \frac{1}{2} h(x^2).$

iii) let $z = xt^2$. $\quad dz = 2xt \, dt$

$p(x) = \sqrt{x} \int_0^1 \cos(xt^2) dt = \sqrt{x} \int_0^x \cos z \cdot \cancel{2xt} \, dz = \sqrt{x} \int_0^x \cos z \cdot \frac{1}{2\sqrt{z}\sqrt{x}} dz$

$= \frac{1}{2} \int_0^x \frac{\cos z}{\sqrt{z}} dz = \int_0^{\sqrt{x}} \cos t^2 \, dt$

$h(x^2) = \int_0^x \cos t^2 \, dt = f(x).$

c) 7.3/22.

a) $dV = -cA(h)\, dt$

$\int dV = cA(h)\,dt$ $\quad \cancel{V = -c \frac{dA}{dt}}$ $\quad = \frac{dh}{dt}\frac{dV}{dt} = -cA(h)\,dh$

$V(h) = \int_0^h A(x)\, dx$ $\quad \int \frac{dh}{dt} dV = -c \int A(h)\, dh$

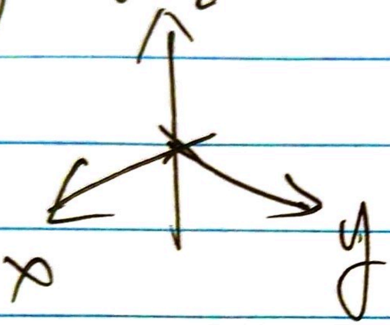
$\frac{dh}{dt} V = -c \, V(h)$

$\frac{dh}{dt} = -c.$

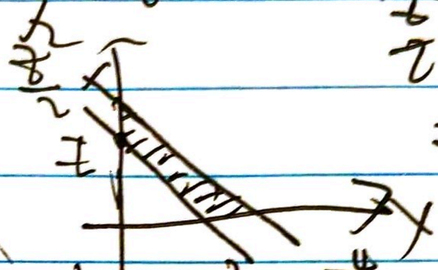
b) $\cancel{V(h) \cdot \int A(h) \cdot c \, A(h) \, dh = -c \int A(h) \, dt}$

$\frac{h_0}{c}$

b) $y > \frac{z^2}{2} - \frac{x}{8}$



$y < z - x$



$z^2 < z$ & $z < 1$

$\frac{z^2}{2} < z$ $\quad \frac{z}{2} > z1$

$z < 2$ $\quad z > 2$

$\frac{z^2}{2} - z < 0$ $\quad dz$

$z(z-2) < 0$

$$A = \begin{cases} \frac{z^2}{2} - \frac{z^4}{8}, & z < 2 \\ 0 & z = 2 \\ \frac{z^4}{8} - \frac{z^2}{2}, & z > 2 \end{cases}$$

$$V = \int_0^1 \left(\frac{z^2}{2} - \frac{z^4}{8}\right) dz = \frac{z^3}{6} - \frac{z^5}{40}\Big|_0^1 = \frac{1}{6} - \frac{1}{40} = \frac{20}{120} - \frac{3}{120} = \frac{17}{120}$$