PS XII

1. $\Delta S = \int_{T_1}^{T_2} \frac{cm\,dT}{T} \cancel{= cm\int_{T_1}^{T_2}\frac{dT}{T} = cm\ln\frac{T_2}{T_1}} + \frac{\Delta Q_{melt}}{T_1}$

$= cm\ln\frac{T_2}{T_1} + \cancel{\Delta Q_{melt}}\ \frac{mL_f}{T_1}$

$= m\left(c\ln\frac{T_2}{T_1} + \frac{L}{T_1}\right)$

$= 1\left(4.2\times\ln\frac{273+50}{273} + \frac{334}{273}\right)$

$= 1.93\ J/K$

2. $\Delta S = \int_{T_1}^{T^*}\frac{cm_1\,dT}{T} + \int_{T_2}^{T^*}\frac{cm_2\,dT}{T}$

$= c\left(m_1\ln\frac{T^*}{T_1} + m_2\ln\frac{T^*}{T_2}\right)$

$= 4.2\times\left(500\ln\frac{57.5+273}{80+273} + 300\ln\frac{57.5+273}{20+273}\right)$

$= 13.4\ J/K$

$m_1\Delta T_1 = m_2\Delta T_2$

$m_1(T_1 - T^*) = m_2(T^* - T_2)$

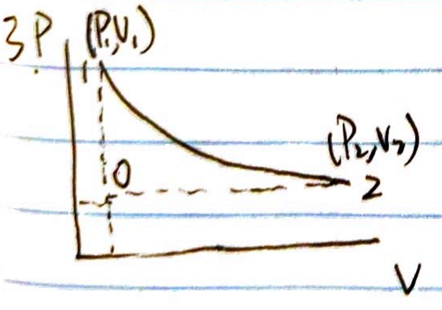
$(m_1+m_2)T^* = m_1T_1 + m_2T_2$

$T^* = \frac{m_1T_1+m_2T_2}{m_1+m_2}$

$= \frac{500\times80+300\times20}{800}$

$= 57.5^\circ C$

3.



$S_2 - S_1 = \int_{12} dS$

$= \int_{10} dS + \int_{02} dS$

$= C_v\int_{T_1}^{T_0}\frac{dT}{T} + C_p\int_{T_0}^{T_2}\frac{dT}{T}$

$= C_p\ln\frac{T_2}{T_0} + (C_p - R)\ln\frac{T_0}{T_1}$

$= C_p\left(\ln\frac{T_0}{T_1} + \ln\frac{T_2}{T_0}\right) - R\ln\frac{T_0}{T_1}$

$= C_p\ln\frac{T_2}{T_1} - R\ln\frac{P_0V}{P_1V} = C_p\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$

if 1→2 is an adiabatic curve.

$\Delta Q = 0$. $dS = 0$

$$\Delta S = \int_{T_1}^{T_2} dS = 0$$

$$\delta Q = dU + PdV = 0$$

$$\frac{3}{2} nR\,dT + \frac{nRT}{V} dV = 0$$

$$nC_v\,dT + \frac{nRT}{V} dV = 0$$

$$\frac{C_v}{R} \cdot \frac{dT}{T} + \frac{dV}{V} = 0$$

$$\int_{12} \frac{C_v}{R} \cdot \frac{dT}{T} + \frac{dV}{V} = \frac{C_v}{R} \ln \frac{T_2}{T_1} + \ln \frac{V_2}{V_1} = 0$$

$$\ln\left[\left(\frac{T_2}{T_1}\right)^{C_v/R} \cdot \frac{V_2}{V_1}\right] = 0$$

$$\left(\frac{T_2}{T_1}\right)^{C_v/R} \cdot \frac{V_2}{V_1} = 1$$

$$\left(\frac{T_2}{T_1}\right)^{C_v/R} = \frac{V_1}{V_2}$$

$$T^{C_v/R} \cdot V = C$$

$$\left(\frac{PV}{R}\right)^{C_v/R} \cdot V = C'$$

$$P^{C_v/R}\, V^{(1 + C_v/R)} = C$$

$$PV^{\frac{C_v + R}{C_v}} = PV^{\frac{C_p}{C_v}} = C$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{C_p/C_v}$$

$$S_2 - S_1 = C_p \ln\left(\frac{V_1}{V_2}\right)^{R/C_v} - R \ln\left(\frac{V_1}{V_2}\right)^{C_p/C_v} = \frac{C_p R}{C_v}\left(\ln\left(\frac{V_1}{V_2} \cdot \frac{V_2}{V_1}\right)\right) = 0$$