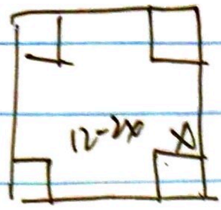
Lecture 11 2 Sep

2C-1

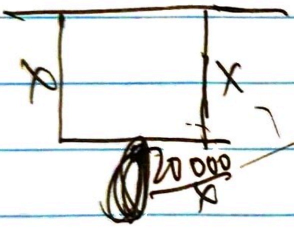


$V(x) = (12-2x)^2 x$
$= 4(x-6)^2 x$
$= 4x(x^2-12x+36)$
$= 4x^3 - 48x^2 + 144x \quad x \in [0, 6]$

$V'(x) = 12x^2 - 96x + 144 = 0$
$x^2 - 8x + 12 = 0$
$(x-2)(x-6) = 0$
$x = 2, 6.$

$V_{max} = \max\{V(0), V(2), V(6)\}$
$= \max\{0, 128, 0\}$
$= V(2) = 128$. inch^3

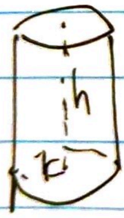
2C-2.



$f(x) = 2x + \frac{20000}{x} \geq 2\sqrt{40000} = 400$ if and only if $x=100$

$f_{min}(x) = f(100) = 400$ feets.

2C-5



$A = 2\pi r \cdot h + \pi r^2 \quad h = \frac{A - \pi r^2}{2\pi r} = \frac{A}{2\pi r} - \frac{r}{2}$

$V = \cancel{2\pi r}\, \pi r^2 h$

$V(r) = \frac{\pi r^2 A}{2\pi r} - \frac{\pi r^3}{2}$
$= \frac{A}{2} r - \frac{\pi}{2} r^3$

$V'(r) = -\frac{3\pi}{2} r^2 + \frac{A}{2} = 0$

$3\pi r^2 - A = 0$
$r = \frac{\pm\sqrt{12A\pi}}{6\pi}$
$= \frac{\sqrt{3A\pi}}{3\pi} = \sqrt{\frac{A}{3\pi}}$

$\cancel{2\pi r^2} = \cancel{2\pi r} h + \cancel{\pi r^2} \quad \frac{h}{r} = 1$

6-11 ⑥ $S = cxy^3$

$x^2+y^2=4r^2$

$x=\sqrt{4r^2-y^2}$

$S = c\sqrt{4r^2-y^2}\,y^3$

$S^2 = c^2y^6(4r^2-y^2)$

$S^2 = 4c^2r^2y^6 - y^8$

$2SS' = -8y^7 + 24c^2r^2y^5$

$S' = \frac{-8y^4+24c^2r^2y^2}{2c\sqrt{4r^2-y^2}} = \frac{-4y^4+12c^2r^2y^2}{c\sqrt{4r^2-y^2}} = \frac{-4y^2(y^2-3c^2r^2)}{c\sqrt{4r^2-y^2}}$

$S'=0$ as $y=\sqrt{3}\,cr$

$\cancel{x=\sqrt{4r^2-y^2}}$

$\frac{x^2}{y^2} = \frac{4r^2}{3cr^2}-1$

$\frac{x}{y} = \sqrt{\frac{4}{3c}-1}$

(3a) $f(x) =$

$(100-x)(200+\frac{5x}{2})$

$= 20000+50x-\frac{5}{2}x^2$

$225 \times 90 = 20250$

$f'(x) = -5x+50 = 0$

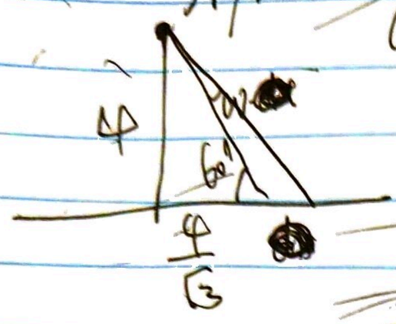
$x=10$

$f_{max}(x) = f(10) = 90\times 225 = \$20250$

price: $200+\frac{5x}{2} = \$225$.

Lecture 12 2 Sep

1.5-2



$\omega = \frac{6\pi}{60s} = \frac{\pi}{10}$ rad/s

$4\tan\theta = x$

$\frac{d}{dt}(4\tan\theta) = x'$

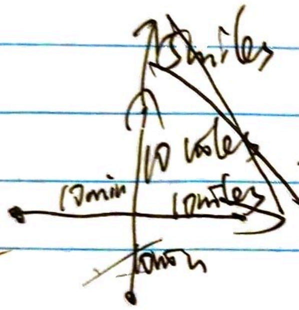
$4 \cdot \frac{1}{\tan\theta} \cdot \theta' = x'$

$x' = \frac{\pi}{5\cdot 10} \cdot \frac{\sqrt{3}}{3} \cdot \frac{-2}{(\times 4)}$

$= \frac{-2\sqrt{3}\pi}{15}$

$V = \frac{-2\sqrt{3}\pi}{15}$ miles/s

3



$a^2 + b^2$

$x^2 = 15^2 + 10^2$

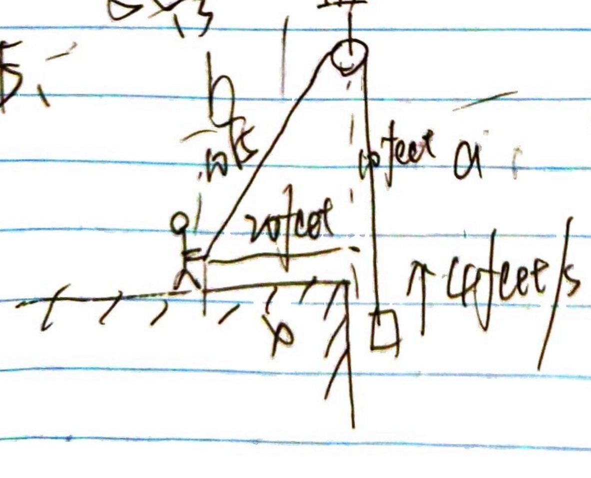
$\frac{d}{dt}(x^2) = 2aa' + 2bb'$

$2x \cdot x' = 30 \times 30 + 20 \times 30$

$x' = \frac{1500}{2\sqrt{225+100}} = \frac{150}{\sqrt{13}} = \frac{150\sqrt{13}}{13}$ miles/s

$\frac{6\sqrt{325}}{\frac{6\cdot 5}{13}}$

5.

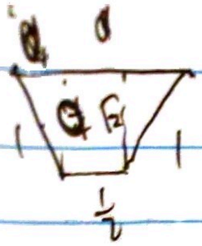


$x^2 = b^2 - a^2$

$2xx' = 2bb' - 2aa'$

$40x' = 20\sqrt{5} \cdot 4 - 20 \cdot 4$

$x' = 2(\sqrt{5}-1)$ feet/sec



$$\frac{1}{4} = \frac{1}{4}\ m^2/s$$

$$\left(\frac{1}{2} + 2h + \frac{1}{2}\right) h \cdot \frac{1}{2} = S$$

$$\frac{1}{2}h + h^2 = S$$

$$\frac{d}{dt}\left(\frac{1}{2}h + h^2\right) = \frac{d}{dt} S$$

$$\frac{1}{2}h' + 2h \cdot h' = S'$$

$$\frac{3}{2}h' = S'$$

$$h' = \frac{2}{3}S' = \frac{1}{16}\ m/s$$

Lecture 13 9 Sep 2024.

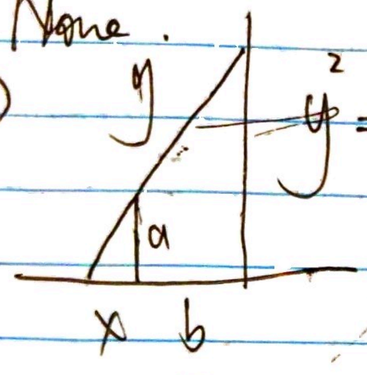
None

~~Lecture 14 9 Sep 2024~~

3. Part II.

None.

(1)



$$y^2 = \left((x+b)\frac{a}{x}\right)^2 + (x+b)^2 = \left(a + \frac{ab}{x}\right)^2 + (x+b)^2$$

$$\cancel{= (x+b)^2\left(\left(\frac{a}{x}\right)^2 + 1\right)}$$

$$\cancel{2yy' = 2(x+b)\left(\left(\frac{a}{x}\right)^2 + 1\right) + (x+b)^2\left(\frac{2a}{x} \cdot \frac{-a}{x^2}\right)}$$

$$\cancel{2yy' = (x+b)\left(2\left(\frac{a}{x}\right)^2 + 2 + (x+b)\left(\frac{-2a^2}{x^3}\right)\right)}$$

$$\cancel{2yy' = (x+b)\left(\frac{2a^2}{x^2} + \frac{2a^2}{x^2} + \frac{-2a^2b}{x^3} + 2\right)}$$

$$2yy' = \left(2a + \frac{2ab}{x}\right)\left(\frac{-ab}{x^2}\right) + (2x + 2b)$$

$$= 2x + 2b - \frac{2a^2b}{x^2} - \frac{2a^2b^2}{x^3}$$

$$y' = \frac{x^4 + bx^3 - a^2bx - a^2b^2}{x^3 y} = \frac{(x+b)(x^3 - a^2b)}{x^3 y}$$

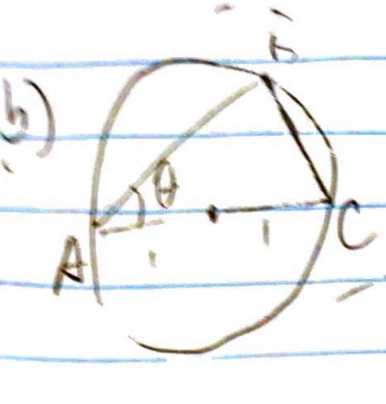
$$x = \sqrt[3]{a^2 b}$$

$$\cancel{\left(a + a^{\frac{1}{3}}b^{\frac{2}{3}}\right)^2 + \left(a^{\frac{2}{3}}b^{\frac{1}{3}} + b\right)^2}$$

$$= a^2 + 2a^{\frac{4}{3}}b^{\frac{2}{3}} + a^{\frac{2}{3}}b^{\frac{4}{3}} + a^{\frac{4}{3}}b^{\frac{2}{3}} + 2a^{\frac{2}{3}}b^{\frac{4}{3}} + b^2$$

$$\cancel{= a^2 + 3a^{\frac{4}{3}}b^{\frac{2}{3}} + 3a^{\frac{2}{3}}b^{\frac{4}{3}} + b^2}$$

b)



$|\overline{AB}| = 2\cos\theta$

$|\overline{BC}| = 2\theta$

$$T(\theta) = \frac{2\cos\theta}{3} + \frac{2\theta}{6}$$

$$= \frac{2\cos\theta + \theta}{3}$$

$$\frac{d}{d\theta}T(\theta) = \frac{-2\sin\theta + 1}{3} = 0$$

$$-2\sin\theta + 1 = 0$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

7. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$

$$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}y' = 0$$

$$y' = \frac{-\frac{2}{3}x^{-\frac{1}{3}}}{\frac{2}{3}y^{-\frac{1}{3}}} = -\sqrt[3]{\frac{x}{y}}$$

$$y - y_0 = y'|_{x=x_0}(x - x_0)$$

$$y_0 = (1 - x^{\frac{2}{3}})^{\frac{3}{2}} = \sqrt{1 - 3x^{\frac{2}{3}} + 3x^{\frac{4}{3}} - x^{\frac{8}{3}}}$$

put: $y_1 = y|_{x=0} = y_0 - x_0 y'|_{x=x_0} = (1 - x_0^{\frac{2}{3}})^{\frac{3}{2}} + x_0^{\frac{4}{3}}(1 - x_0^{\frac{2}{3}})^{-\frac{1}{2}}$

$$x_1 = x|_{y=0} = \frac{-y_0}{y'|_{x=x_0}} + x_0 = \frac{(1 - x_0^{\frac{2}{3}})}{x_0^{-\frac{1}{3}}} + x$$

$$y_1^2 + x_1^2 = \frac{(1 - x_0^{\frac{2}{3}})^4 + x_0^{\frac{8}{3}} + 2(1 - x_0^{\frac{2}{3}})^2 x_0^{\frac{4}{3}}}{(1 - x_0^{\frac{2}{3}})} + (1 - x_0^{\frac{2}{3}})^2 x_0^{-\frac{2}{3}} + x_0^2 + 2x_0^{\frac{2}{3}}(1 - x_0^{\frac{2}{3}})$$

let $t = x^{\frac{3}{2}}$

$$\frac{(1 - 2t + t^2)^2 + t^4 + 2(1-t)^2 t^2}{1-t} + (1 - 2t + t^2)t^{-1} + t^3 + 2t(1$$

$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ $\qquad y = \left(1 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$

$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}y' = 0$

$x^{-\frac{1}{3}} + y^{-\frac{1}{3}}y' = 0$

$\frac{1}{\sqrt[3]{y}}y' = -\frac{1}{\sqrt[3]{x}}$

$y' = -\sqrt[3]{\frac{y}{x}} = -y^{\frac{1}{3}} \cdot x^{-\frac{1}{3}} = -\left(1 - x^{\frac{2}{3}}\right)^{\frac{1}{2}} x^{-\frac{1}{3}}$

$= -\sqrt{\left(1 - x^{\frac{2}{3}}\right)x^{-\frac{2}{3}}} = -\sqrt{x^{-\frac{2}{3}} - 1}$

$\frac{1}{y}y'^3 = -\frac{1}{x}$

$y'^3 = -x$

$y - y_0 = y'(x - x_0)$ $\quad -y_0^{\frac{1}{3}}x_0^{-\frac{1}{3}}x + y_0^{\frac{1}{3}}x_0^{\frac{2}{3}} + \left(1 - x_0^{\frac{2}{3}}\right)^{\frac{3}{2}}$

$y - \left(1 - x_0^{\frac{2}{3}}\right)^{\frac{3}{2}} = y'(x - x_0)$

$x =$

$$x = x_0 - \frac{(1-x_0^{\frac{2}{3}})^{\frac{3}{2}}}{y'}$$

$$= x_0 + (x_0^{\frac{2}{3}})^{\frac{3}{2}}(x_0^{-\frac{2}{3}}-1)$$

$$= x_0^{\frac{1}{3}}$$

$$y = \cancel{(1-x_0^{\frac{2}{3}})^{\frac{1}{2}}(-2x_0^{\frac{2}{3}}+1)}$$

$$y_0 - y'x_0 = (1-x_0^{\frac{2}{3}})^{\frac{3}{2}} + \sqrt{x^{-\frac{2}{3}}-1}\; x_0$$

$$= (1-x_0^{\frac{2}{3}})^{\frac{3}{2}} + \sqrt{1-x^{\frac{2}{3}}}\; x_0^{\frac{2}{3}}$$

$$= \sqrt{1-x_0^{\frac{2}{3}}}\left[(1-x_0^{\frac{2}{3}}) + x_0^{\frac{2}{3}}\right]$$

$$= \sqrt{1-x_0^{\frac{2}{3}}}$$

$$\sqrt{x^2+y^2} = \sqrt{x_0^{\frac{2}{3}}+1-x_0^{\frac{2}{3}}} = \sqrt{1} = 1$$

3. a) $\frac{d}{dh}(L^2+20000^2) = \frac{d}{dh}h^2$

$$2L\cdot L' + 0 = 2h$$

$$\frac{dL}{dh} = \frac{2h}{2L} = \frac{h}{L} = \frac{h}{\sqrt{h^2-4\times10^8}} = \sqrt{\frac{h^2}{h^2-4\times10^8}} = \sqrt{\frac{1}{1-\frac{4\times10^8}{h^2}}}$$

$\cancel{f'(x) = 2x+2.}$

$$\frac{dL}{dh} = \frac{h}{L} = \frac{h}{\sqrt{h^2 - 4\times10^8}}$$

$$\frac{d\left(\frac{h}{L}\right)}{dh} = \frac{\sqrt{h^2-4\times10^8} - h\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{h^2-4\times10^8}}\cdot 2h}{h^2-4\times10^8}$$

$$= \frac{h^2-4\times10^8-h^2}{(h^2-4\times10^8)^{\frac{3}{2}}} = \frac{-4\times10^8}{(h^2-4\times10^8)^{\frac{3}{2}}} < 0$$

$$\lim_{h\to20000^+} \frac{h}{L} = +\infty \quad \therefore \frac{h}{L} > 1.$$

$$\lim_{h\to+\infty} \frac{h}{L} = 1$$

$$\frac{\Delta L}{\Delta h} = \frac{2h_0+\Delta h}{2L+\Delta L} = \frac{2h+\Delta h}{L_0+L} = \frac{2h+\Delta h}{\sqrt{h_0^2-4\times10^8}+\sqrt{h^2-4\times10^8}}$$

as $\Delta h \to 0$. $\frac{\Delta L}{\Delta h} = \frac{h}{L} = \frac{dL}{dh}$

$\cancel{\frac{d}{d\Delta h}\left(\frac{\Delta L}{\Delta h}\right)}$

$\cancel{\Delta L\left(\sqrt{(h+\Delta h)^2-4\times10^8}+\sqrt{h^2-4\times10^8}\right) = 2h+\Delta h \cdot \Delta h}$

$$\frac{d}{d\Delta h}\frac{\Delta L}{\Delta h} = \frac{(L_0+L) - (2h+\Delta h)\left(\frac{1}{2\sqrt{(h+\Delta h)^2-4\times10^8}}\, 2(h+\Delta h)\right)}{\left(\sqrt{h_0^2-4\times10^8}+\sqrt{h^2-4\times10^8}\right)^2}$$

$$= \frac{L_0+L+(h+h_0)\left(-\frac{h_0}{L_0}\right)}{(L_0+L)^2} > 0.$$

$\therefore \frac{\Delta L}{\Delta h} > \frac{dL}{dL}$

$\cancel{\frac{\Delta L}{\Delta h} \le 2}$ $\quad 2 ? \ \frac{\Delta L}{\Delta h} \ge \frac{dL}{dh}$

$\cancel{\frac{2h+\Delta h}{2L+\Delta L} \le 2}$ $\quad \frac{dL}{dh} \le 2$

$\frac{h}{L} \le 2$

$\cancel{2h+\Delta h \le 4L+2\Delta L}$

$h \le 2L = 2\sqrt{h^2-4\times10^8}$

$h^2 \le 4h^2-16\times10^8$

$3h^2 \ge 16\times10^8$

$h \ge \sqrt{\frac{16}{3}\times10^8} \approx 23094.01$ when $\Delta h \to 0$.

when $\Delta h < 0$.

$\frac{\Delta L}{\Delta h} \le 2$. $\frac{h+h_0}{L+L_0} \le 2$. $h+h_0 \le 2L+2L_0 = 2\left(\sqrt{h^2-4\times10^8}+\sqrt{h_0^2\ldots}\right.$

$L^2+20000^2=h^2$

put $h_0=20000$

$L^2+20000^2=(h_0-\Delta h)^2$

$\Delta h=-h+20000$

Δh

$\Delta L=-L$

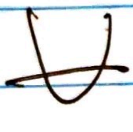
$\frac{\Delta L}{\Delta h}=\frac{L}{h-20000}\le 2$

$2h-40000\ge L$

$2h-40000\ge\sqrt{h^2-4\times10^8}$

$4h^2-16\times10^4 h+16\times10^8\ge h^2-4\times10^8$

$3h^2-16\times10^4 h+20\times10^8\ge 0$



$h\le\frac{16\times10^4-\sqrt{\ldots\times10^8}}{6}=\frac{16\times10^4-4\times10^4}{6}=2\times10^4$

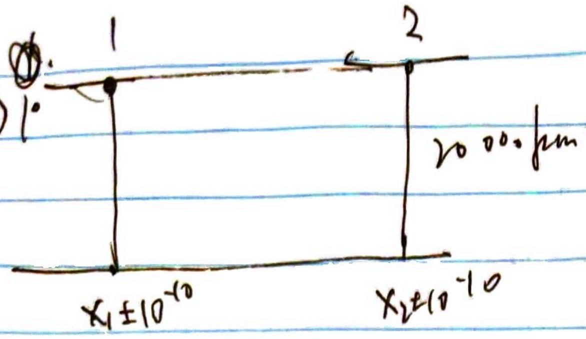
$h\ge 33333\frac{1}{3}$

$\therefore h\ge 2\times10^4$

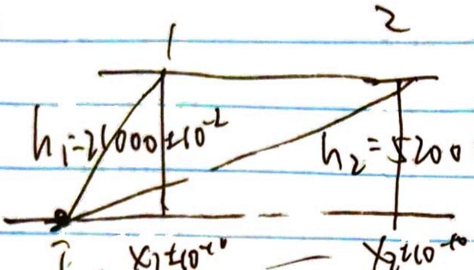
$\therefore h\ge 33333\frac{1}{3}$

ribe the diff...

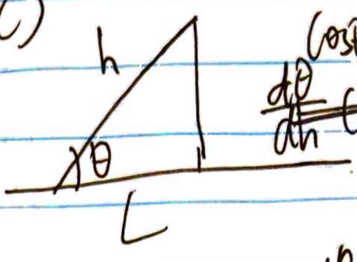
b) $1°$



$1.a°$



c)



$\cos\theta = \frac{L}{h}$

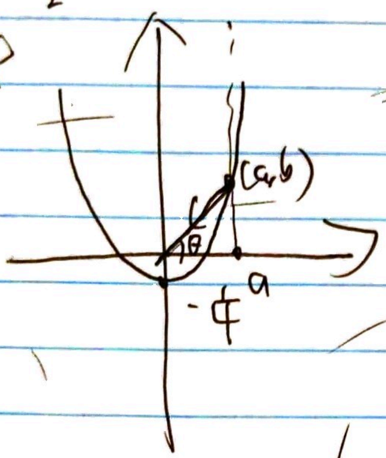
$\cancel{\frac{d\theta}{dh}(\sin\theta) = \frac{dL}{dh}\left(\frac{1}{h^2}\right)}$

$$-\sin\theta \cdot \frac{d\theta}{dh} = \frac{\frac{dL}{dh}\cdot h - L}{h^2}$$

$$-\sin\theta \cdot \frac{d\theta}{dh} = \frac{1}{h}\frac{dL}{dh} - \frac{L}{h^2}$$

$$\frac{dL}{dh} = -h\sin\theta\frac{d\theta}{dh} + \frac{L}{h}$$

4



$$\tan\theta = \frac{b}{a} = \frac{a^2 - \frac{1}{4}}{a}$$

$$\tan\theta = a - \frac{1}{4a}$$

$$\cancel{\frac{d}{d\theta}(\tan\theta) = \frac{d}{d\theta}\left(a - \frac{1}{4a}\right)}$$

$$\frac{d}{da}\left(\frac{\sin\theta}{\cos\theta}\right) = \frac{d}{da}\left(a - \frac{1}{4a}\right)$$

$$\sec^2\theta \cdot \frac{d\theta}{da} = 1 - \frac{1}{4}(-1)\left(\frac{1}{a^2}\right)$$

$$= 1 + \frac{1}{4a^2}$$

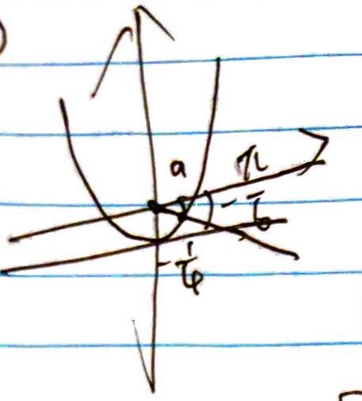
$$\frac{d\theta}{da} = \left(1 + \frac{1}{4a^2}\right)\cos^2\theta$$

$$= \cos^2\theta \cdot \frac{\cos^2}{4a}$$

$$\cancel{\frac{4a^2+1}{4a^2} \cdot \frac{a^2 - \frac{1}{4}}{a}}$$

$$= -\cancel{\frac{4a^4 - \frac{1}{4}}{4a^3}} - a + \frac{1}{16a^3}$$

a)



$$da = \frac{\sec^2\theta}{1+\frac{1}{4a^2}} \cdot d\theta$$

$$= \frac{4a^2\sec^2\theta}{4a^2+1} \cdot d\theta$$

$\tan\theta = a - \frac{1}{4a}$

$\theta = \frac{\pi}{6}$

$a^2 - \frac{1}{4} + \frac{\sqrt{3}}{3}a = 0$

$$da = \frac{4a^2}{\frac{3}{4}(4a^2+1)} \cdot d\theta$$

$1^\circ \quad a = -\frac{\sqrt{3}}{2}$

$a = \frac{-\frac{\sqrt{3}}{3} \pm \sqrt{\frac{1}{3}+1}}{2}$

$$da = \frac{3}{3} d\theta = d\theta = 10^{-3}$$

$a = \frac{\frac{(-1 \pm 2)\sqrt{3}}{3}}{2} = -\frac{\sqrt{3}}{2}$ or $\frac{\sqrt{3}}{6}$

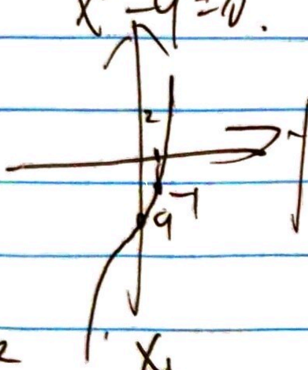
$2^\circ \quad a = \frac{\sqrt{3}}{6}$

$$da = \frac{\frac{1}{3}}{1} = \frac{1}{3} d\theta = \frac{1}{3} \cdot 10^{-3}.$$

b). $x = \sqrt[3]{9}$

$x^3 - 9 = 0$. $\quad f(x) = x^3 - 9$

$f'(x) = 3x^2$



$\Rightarrow f'(x_0)(x_1 - x_0) = 0 - f(x_0)$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 + \frac{1}{12} = \frac{25}{12}$

k	x_k	$x_n - \sqrt[3]{9}$
0	2	0.08008438
1	$\frac{25}{12}$	~~0.003250~~ 3.24951×10^{-3}
2	2.080089	~~0.000005~~ 5.06584×10^{-6}
3	2.080084	~~0.00000~~ 1.23372×10^{-11}

the fx-CG50AU Casio only show $\sqrt[3]{9}$ to 9 d.p.

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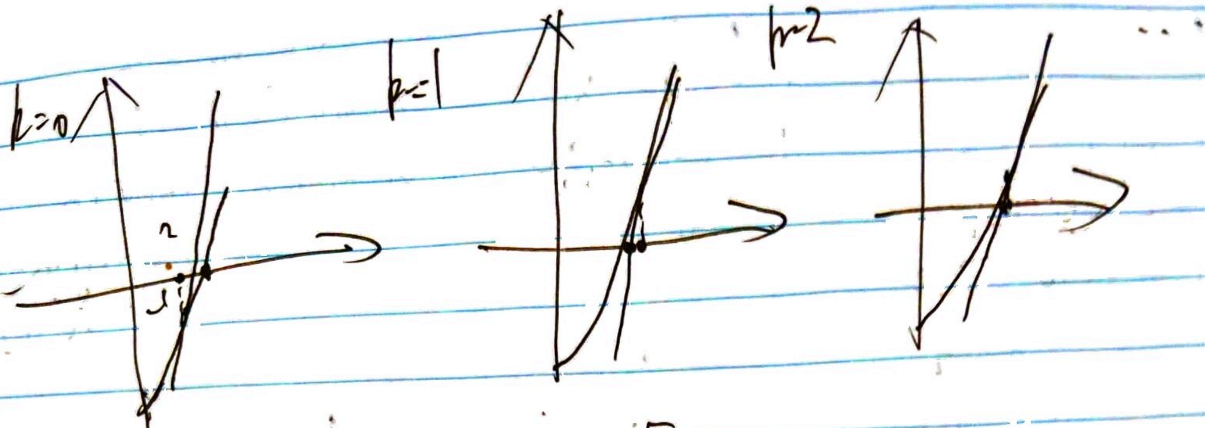
...ct. There is a certain ran...

...t Objectives

Units

b)



$\therefore x_0 < \sqrt[3]{9}$, $\quad x_1, x_2 \cdots x_{k>0} > \sqrt[3]{9}$. and $x_{k+1} < x_k$

c) $9^{\frac{1}{3}} = \left(8\left(1+\frac{1}{8}\right)\right)^{\frac{1}{3}} = 2\left(1+\frac{1}{8}\right)^{\frac{1}{3}}$

Let $f(x) = (1+x)^{\frac{1}{3}}$

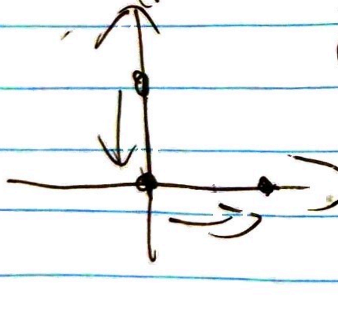
$= 1 + \frac{1}{3}x + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2}x^2$

$9^{\frac{1}{3}} \approx 2f\left(\frac{1}{8}\right) = 2\left(1 + \frac{1}{24} - \frac{2}{9\times 2}\cdot\frac{1}{64}\right)$

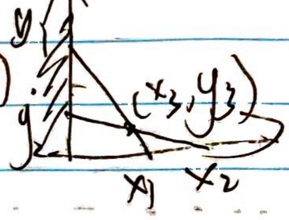
$= 2\left(1 + \frac{1}{24} - \frac{1}{18\times 64}\right) = \frac{599}{576}$ $= \frac{599}{288} \approx 2.0798611$

(d) $\left|9^{\frac{1}{3}} - \frac{599}{288}\right| \approx 2.22712 \times 10^{-4}$.

6.



(ii)



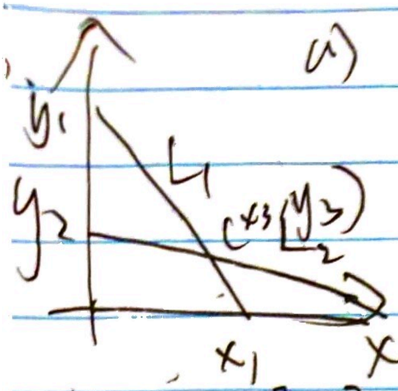
$$\begin{cases} \frac{x_3}{x_1} + \frac{y_3}{y_1} = 1 \\ \frac{x_3}{x_2} + \frac{y_3}{y_2} = 1 \end{cases}$$

$y_3 = \left(\frac{1}{y_1} - \frac{1}{y_2}\right) \cdot x_3 \left(\frac{x_1 - x_2}{x_1 x_2}\right)$

$\frac{y_1 y_2 x_3 (x_1 + x_2)}{(y_2 - y_1) x_1 x_2}$

$$\frac{x_3}{x_1} + \frac{y_3}{y_1} = \frac{x_3}{x_2} + \frac{y_3}{y_2}$$

$y_1 x_2 y_2 x_3 + x_1 x_2 y_2 y_3 = x_1 y_1 y_2 x_3 + x_1 y_1 x_2 y_3$



a)

$$\begin{cases} \dfrac{y_3}{y_1} + \dfrac{x_3}{x_1} = 1 \\ \dfrac{y_3}{y_2} + \dfrac{x_3}{x_2} = 1 \end{cases}$$

$$\frac{x_3}{x_1} - \frac{x_3}{x_2} = \frac{y_3}{y_1} - \frac{y_3}{y_2}$$

$$x_3\left(\frac{x_2 - x_1}{x_1 x_2}\right) = y_3\left(\frac{y_2 - y_1}{y_1 y_2}\right)$$

$$x_3 = y_3\left(\frac{x_1 x_2}{x_2 - x_1}\right)\left(\frac{y_2 - y_1}{y_1 y_2}\right)$$

b)

$$x_2^2 + y_2^2 = 1$$
$$x_1^2 + y_1^2 = 1$$

$$x^2 + y^2 = 1$$

$$\frac{d}{dx}\left(x^2 + y^2\right) = 0$$

$$2x + 2yy' = 0$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = y' = -\frac{x}{y}$$

c)

$$x_2 = x_1 + \Delta x$$
$$y_2 = y_1 + \Delta y \qquad = y_1 - \frac{\Delta x \cdot x_1}{y_1}$$

$$y_1 - \frac{y_1}{x_1} x_3 = y_2 - \frac{y_2}{x_2} x_3$$

$$\left(\frac{y_1}{x_1} - \frac{y_2}{x_2}\right) x_3 = y_1 - y_2$$

$$x_3 = \left(\frac{x_1 x_2}{y_1 x_2 - y_2 x_1}\right)(y_1 - y_2)$$

describe the differenc...

$$x_3 = \frac{(x_1 x_2)(y_1 - y_2)}{y_1 x_2 - y_2 x_1}$$

$$= \cancel{\frac{(x_1^2 + \Delta x x_1)\left(\frac{\Delta x \cdot x_1}{y_1}\right)}{x_1 y_1 + \Delta x y_1 - x_1 y_1 + \frac{\Delta x x_1^2}{y_1}}}$$

$$= \cancel{\frac{\Delta x x_1^3 + \Delta x^2 x_1^2}{x_1 y_1^2 + \Delta x y_1^2 - x_1 y_1^2 + \Delta x x_1^2}}$$

$$= \frac{(x_1^2 + x_1 \Delta x)(\Delta y)}{(y_1 + \Delta y) x_1 - y_1 (x_1 + \Delta x)} = \frac{\Delta y (x_1^2 + x_1 \Delta x)}{\cancel{x_1 y_1} + \Delta y x_1 - \cancel{x_1 y_1} - y_1 \Delta x}$$

$$= \frac{x_1^2 + x_1 \Delta x}{x_1 - y_1 \frac{\Delta x}{\Delta y}}$$

(1) $\Delta x \to 0$, $x_3 = \frac{x_1^2}{x_1 + y_1 \cdot \frac{y_1}{x_1}} = \frac{x_1^3}{x_1^2 + y_1^2} = x_1^3$

the same, $\lim_{\Delta x \to 0} y_3 = y_1^3$

d) $x_3^{\frac{2}{3}} + y_3^{\frac{2}{3}} = (x_1^3)^{\frac{2}{3}} + (y_1^3)^{\frac{2}{3}} = x_1^2 + y_1^2 = 1.$