$\frac{y}{r} = 1$

Practise Final exam. 22/2/2015 $250-25-13 = 212/250 = 85\%$

P1 (a) $\frac{d}{dx}\frac{\ln x}{x^2} = \frac{\frac{1}{x}\cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{1-2\ln x}{x^3}$

b) $\frac{d}{du}\sqrt{3\sin^2 u+2} = \frac{1}{2\sqrt{3\sin^2 u+2}} \cdot 6\cos u \sin u = \frac{3\sin u\cos u}{\sqrt{3\sin^2 u+2}}$

c) $\frac{d^n}{dx^n} e^{kx}\Big|_{x=0} = k^n e^{kx}\Big|_{x=0} = k^n$

P2 $\cancel{2xy^2+x^2yy'+3y^2y'=2}$

$\cancel{y' = \frac{2-2xy^2}{3y^2+2x^2y}}$

$2xy^2+2x^2yy'+3y^2y'=0$

$y' = \frac{-2xy^2}{3y^2+2x^2y} = \frac{-2}{3+2} = -\frac{2}{5}$

$\cancel{y'\big|_{x=1} = \frac{-2}{3+2}}$

$\therefore y=1$ $\quad y = -\frac{2}{5}x + \frac{7}{5}$

P3 $y = \cos^{-1}x$ $\quad x = \cos y$ $\quad Dx = D\cos y$ $\quad \sin^{-1} = \frac{1}{y}$

$y' = -\frac{1}{\sqrt{1-x^2}}$ $\quad 1 = -D\sin y \cdot y'$ $\quad D\cos^{-1}x = D\sin^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$\cancel{Dx = \sqrt{1-x^2}}$

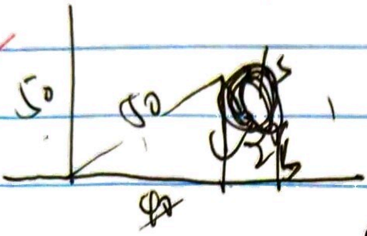
P4 $0+0+a = 0+2$

$a=2$.

$f'(x) = \begin{cases} 2x+1, & x\le 0 \\ b, & x>0 \end{cases}$ $\quad 1=b$

$\therefore \begin{cases} a=2 \\ b=1 \end{cases}$

P5



$v_x^2+v_y^2 = 2^2$

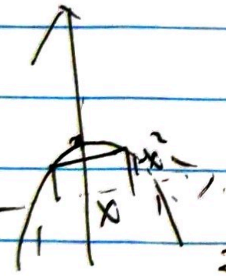
$50^2 = x^2+y^2$

$0 = 2xv_x + 2yv_y$

$40\times 3 \cdot v_y = 80 v_x$

$v_x = \frac{120}{80} = \frac{3}{2}$ m/s

P6



$A(x) = 2x(1-x^2)$

$= 2x - 2x^3$

$A'(x) = 0 \quad 2-6x^2 = 0$

$\cancel{2x(1-x)(1+x)=0}$ $\quad 2(1-\sqrt{3}x)(1+\sqrt{3}x) = 0$

$\cancel{x=0,1,-1}$. $\quad x = \pm\frac{\sqrt{3}}{3}$

$A\left(\frac{\sqrt{3}}{3}\right) = \frac{2\sqrt{3}}{3}\left(\frac{2}{3}\right) = \frac{4\sqrt{3}}{9}$

P7 a) $\frac{y}{x-1} \cdot y' = -1$

$y' = \frac{1-x}{y}$

b) $\frac{dy}{dx} = \frac{1-x}{y}$

$\int y\,dy = \int (1-x)\,dx$

$\frac{1}{2}y^2 = x - \frac{x^2}{2} + c$

$\therefore y^2 = -x^2+2x+c$

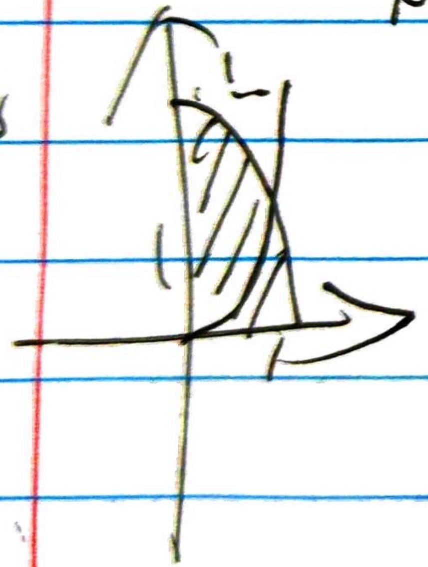
c) $y^2 = -x^2 + 2x + c$

$y^2 + x^2 - 2x + 1 = c$

$y^2 + (x-1)^2 = c$

A circle with the centre at $(1, 0)$

P8



$V = \int_0^1 (1-x^2) \cdot 2\pi x \, dx$

$2\pi\left(\frac{x^2}{2} - \frac{1}{4}x^4\right)\Big|_0^1$

$= \frac{\pi}{2}$

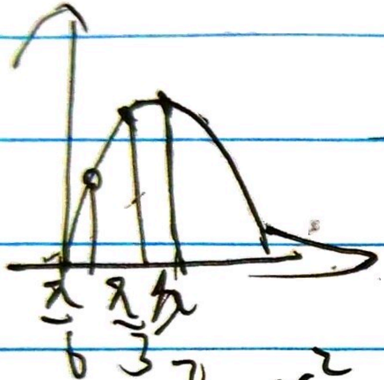
P9

~~$= 2\ln\frac{2}{2} - 2\ln 2 \quad 2\ln 2(\ln 2 - 1) + \frac{\pi}{4}$~~

P9 $\int_0^{\frac{\pi}{2}} \sin^2 x \approx \frac{\frac{\pi}{2}}{3}\cdot\left(\cancel{\frac{y_0+y_1+y_1+y_2+y_2+y_3}{2}}\right) = \frac{\pi}{6}\left(\frac{1}{2}+\frac{1}{4}+\frac{3}{4}\right)$

$= \frac{\pi}{6}\cdot\frac{3}{2}$

$= \frac{\pi}{4}$



$y_0 = 0$

$y_1 = \sin^2\frac{\pi}{6} = \frac{1}{4}$

$y_2 = \sin^2\frac{\pi}{3} = \frac{3}{4}$

$y_3 = \sin^2\frac{\pi}{2} = 1$

P10 $F'(x) = e^{-x^2} + C$

$F'(0) = e^0 + C = 0$

$C = -1$

$F'(1) = e^{-1} - 1 = \frac{1}{e} - 1$

$F''(x) = -2x e^{-x^2}$

$F''(1) = -2\cdot\frac{1}{e}$

$= -\frac{2}{e}$

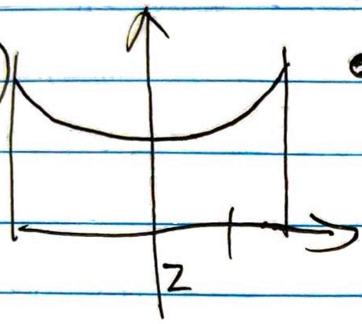
b) let $t = \frac{y}{2}$ $\quad dt = \frac{dy}{2}$

$$\int_{\frac12}^{1} e^{-t^2}\cdot 2dt = 2\int_{\frac12}^{1} e^{-t^2}\,dt$$

$$= 2(F(x))\Big|_{\frac12}^{1}$$

$$= 2F(1) - 2F(\tfrac12)$$

P11 a)



$y = \frac{1}{10}x^2$

$\frac{dy}{dx} = \frac{1}{5}x$

$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$= \sqrt{1+\frac{1}{25}x^2}\,dx$

$S = 2\int_0^1 \sqrt{1+\frac{1}{25}x^2}\,dx$

b) $\int_{-1}^{1} \frac{1}{10}x^2\,dx$ $\quad -8$

$= \frac{1}{30}x^3\Big|_{-1}^{1}$

$= \frac{1}{30} + \frac{1}{30} = \frac{2}{30} = \frac{1}{15}$ m.

$\int_{-10}^{10} \frac{1}{10}x^2\,dx = \frac{1}{30}$

let $\frac{x}{5} = \tan\theta$

$\theta = \arctan$

$d\frac{x}{5} = d\tan\theta$

$= \sec^2 x\, d\theta$

P12 a) $\int_0^1 \frac{dx}{x^2+3x+2} = \int_0^1 \frac{dx}{(x+1)(x+2)} = \int_0^1 \frac{1}{x+1} - \int_0^1 \frac{1}{x+2}$

Let $\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$ $\quad = \ln(x+1)\Big|_0^1 - \ln(x+2)\Big|_0^1$

$\frac{1}{x+2} = A + \frac{(x+1)B}{x+2}$ let $x=-1$ $\quad A=1$ $\quad = \ln 2 - \ln 3 + \ln 2$

$\frac{1}{x+1} = \frac{(x+2)A}{x+1} + B$ let $x=-2$ $\quad B=-1$ $\quad = 2\ln 2 - \ln 3$

$\therefore \frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}$

b) $\int x^2 \ln x \, dx$ $\quad u = \ln x \quad u' = \frac{1}{x}$; $v' = x^2 \quad v = \frac{x^3}{3}$

$= \frac{x^3 \ln x}{3} - \int \frac{x^2 dx}{3}$

$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$

$= \frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + C$

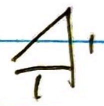
P13 $\int_0^{\arctan 1} \frac{\sec^2 u \, du}{(\tan^2 u + 1)^2}$ $\quad dx = d\tan u = \sec^2 u \, du$

$= \int_0^{\arctan 1} \frac{\sec^2 u \, du}{(\sec^2 u)^2} = \int_0^{\arctan 1} \cos^2 u \, du = \frac{1}{2}\int_0^{\arctan 1} 2\cos^2 u - 1 + 1 \, du$

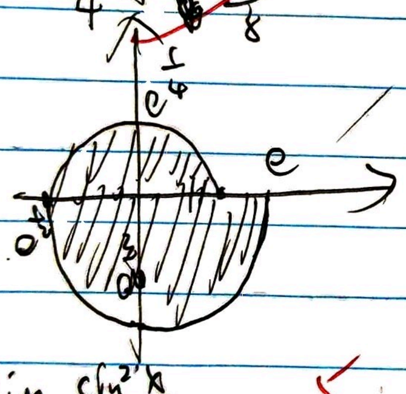
$= \frac{1}{2}\int_0^{\arctan 1} \cos 2u + 1 \, \frac{d2u}{2} = \frac{1}{4}(\sin 2u + 2u)\Big|_0^{\arctan 1}$

$= \frac{1}{4}\sin 2\arctan 1 + \frac{1}{2}\arctan 1$

$= \frac{1}{4} + \frac{\pi}{8}$



P14



$A = \int_0^{2\pi} \frac{1}{2} r^2 \, d\theta$

$= \int_0^{2\pi} \frac{1}{2} e^{\frac{\theta}{\pi}} \, d\theta$

$= \frac{1}{2}\pi e^{\frac{\theta}{\pi}}\Big|_0^{2\pi}$

$= \frac{1}{2}\pi(e^2 - 1)$

P15 a) $\lim_{x\to 0} \frac{\sin^2 x}{1-\cos x}$ $\quad -5$

$= \lim_{x\to 0} \cancel{\frac{2\sin x \cos x}{1+\sin x}} = 0$ $\quad \lim_{x\to 0} \frac{2\sin x \cos x}{\sin x} = 2\cos x = 2.$

b) $\lim_{x\to 1} \frac{(\ln x)^2}{x-1}$

$= \lim_{x\to 1} \frac{2\ln x \cdot \frac{1}{x}}{1} = 0$

c) $\lim_{x\to\infty} x^2 e^{-x}$

$= \lim_{x\to\infty} \frac{2}{e^x} = 0$

P16 $\int_1^\infty \frac{dx}{x^{3/2}} = -2x^{-\frac{1}{2}}\Big|_1^\infty = -2(0-1) = 2$

1A

P17 1° $p \le 0$ $\sum_{n=1}^{\infty} \frac{n}{\sqrt{5}} = \infty$ ~~diverges~~ diverges

2° $p > 0$ $\sum_{1}^{\infty} \frac{n}{\sqrt{4+n^p}} \sim \int_1^\infty \frac{x\,dx}{\sqrt{x^p}} = \int_1^\infty \frac{dx}{x^{\frac{p}{2}-1}}$

I. $\frac{p}{2}-1 > 1$ $p > 4$ $\int_1^\infty \frac{dx}{x^{\frac{p}{2}-1}} = \frac{1}{-\frac{p}{2}+2} x^{-\frac{p}{2}+2}\Big|_1^\infty$ converges

II. $\frac{p}{2}-1 \le 1$ $\int_1^\infty \frac{dx}{x^{\frac{p}{2}-1}} \ge \int_1^\infty \frac{dx}{x} = \ln x\Big|_1^\infty$ diverges

$\therefore$ For $x \in$ $(4, +\infty)$, convergent.

P18 a) $f(x) = \sqrt{1+x} = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n$

$f'(0) = 1$

$f'(x) = \frac{1}{2\sqrt{1+x}}$ $f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(1+x)^{-\frac{3}{2}}$ $f''(0) = -\frac{1}{4}$

$f'''(x) = \frac{3}{8}(1+x)^{-\frac{5}{2}}$ $f'''(0) = \frac{3}{8}$

-1 $= 1 + \frac{x}{2} + \cancel{\frac{1}{2} \cdot \frac{1}{4} x^2}$ $-\frac{1}{2} \cdot \frac{1}{4} x^2$ $+ \frac{1}{2\cdot 3} \cdot \frac{3}{8} \cdot x^3$

$= 1 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{16}$

b) $\sqrt{1.2} = f(0.2) = 1 + 0.1 + \frac{0.04}{8} + \frac{0.008}{16}$

$= 1 + 0.1 ~~+0.005~~ + 0.0005$

$= 1.1055$ 1.0955

P19 $\tan^{-1} x = \int \frac{1}{1+x^2}$ -10 $\frac{1}{1+x^2} = 1 - x^2 + x^4 + \ldots$

$f(x) = \frac{1}{1+x^2}$ $f(0) = 1$ $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots$

$f'(x) = -1 \cdot (1+x^2)^{-2} \cdot 2x$ $f'(0) = 0$

$f''(x) = + 2(1+x^2)^{-3} \cdot 2x \cdot 2x - ((1+x^2)^{-2} \cdot 2)$ $f''(0) = 2$

$f'''(x) = \cancel{-(\ldots)(1+x^2)^{-2}}$ $24(1+x^2)^{-2}x$

$-6(1+x^2)^{-4} \cdot 8x^3 \cancel{+ 2(1+x^2)^{-2} \cdot 8x + 2(1+x^2) \cdot 4x}$ $f'''(0) = 0$

$f''''(x) = 24(1+x^2)^{-5} \cdot 16x^4 - 6(1+x^2)^{-4} \cdot 24x^2 - 48(1+x^2)^{-3}x + 24(1+x^2)^{-2}$

$f''''(0) = 24.$