$= 4\pi \left(\Big|_0 \int \sin x \, dx \right)$

$= 2\pi \left(\frac{\pi}{2} - \left(-\cos x \Big|_0^{\frac{\pi}{2}} \right) \right) = 2\pi \left(\frac{\pi}{2} - 1 \right) = \pi^2 - 2\pi$

PS 8A 6/2/~~2004~~ 2025

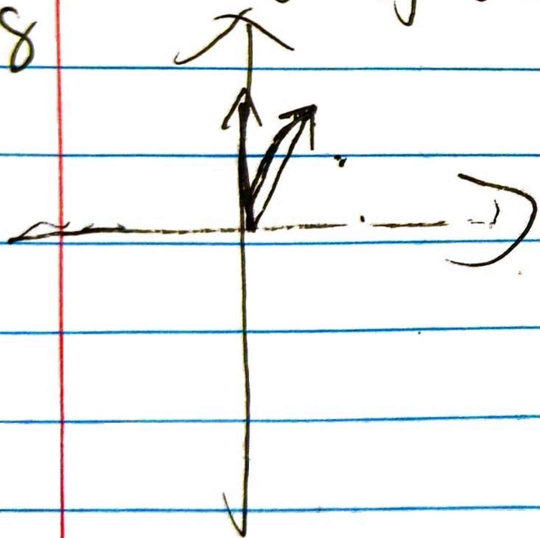
Lec 37.

4.5-2 $x^2 = t^2 + \frac{1}{t^2} + 2 \quad y^2 = t^2 + \frac{1}{t^2} - 2$

$x^2 - y^2 = 4$

3 $(x-1)^2 + (y-4)^2 = 1$

8



$\begin{cases} x = t\cos\left(\frac{\pi}{2} - \frac{\pi t}{3}\right) \\ y = t\sin\left(\frac{\pi}{2} - \frac{\pi t}{6}\right) \end{cases}$

4P - 1d) $s=\int_1^2\sqrt{(dx)^2+(dy)^2}=\int_1^2\sqrt{(dx)^2+\left(\frac{1}{3}\cdot\frac{3}{2}(2+x^2)^{\frac{1}{2}}\cdot 2x\,dx\right)^2}$

$=\int_1^2\sqrt{(dx)^2+\left(\sqrt{2+x^2}\,x\,dx\right)^2}=\sqrt{(dx)^2+(2+x^2)x^2(dx)^2}$

$=\int_1^2\sqrt{1+2x^2+x^4}\,dx=\int_1^2(x^2+1)\,dx$

$=\frac{1}{3}x^3+x\Big|_1^2=\left(\frac{8}{3}+2-\frac{1}{3}-1\right)=\frac{7}{3}+1=\frac{10}{3}$

4. $\begin{cases}x=t^2\\ y=t^3\end{cases}$ $ds=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

$=\sqrt{(2t)^2+(3t^2)^2}\,dt$

$=\sqrt{4t^2+9t^4}\,dt$

$=t\sqrt{4+9t^2}\,dt$

$=\frac{1}{2}\sqrt{4+9t^2}\,dt^2$

$s\Big|_0^2=\int_0^2 ds=\frac{1}{2}\int_0^2\sqrt{4+9t^2}\,dt^2$

$=\frac{1}{2}\cdot\frac{2}{3}(4+9t^2)^{\frac{3}{2}}\cdot\frac{1}{9}\Big|_0^2$

$=\frac{1}{27}(4+9t^2)^{\frac{3}{2}}\Big|_0^2$

$=\frac{(40)^{\frac{3}{2}}}{27}-\frac{8}{27}$

$=\frac{(2\sqrt{10})^3}{27}-\frac{8}{27}=\frac{80\sqrt{10}}{27}-\frac{8}{27}$

$=\frac{80\sqrt{10}-8}{27}$

5 $y = (e^{x} + e^{-x})/2$

$ds = \sqrt{(dx)^2 + (\frac{1}{2}(e^{x} - e^{-x})dx)^2}$

$\sqrt{(\frac{1}{4}(e^{2x} + e^{-2x} - 2) + 1)(dx)^2}$

$= \frac{1}{2}\sqrt{(e^{2x} \cancel{+ e^{-2x}} - 1)} \quad dx$ $\int_1^2 \sqrt{2t + \frac{2}{t^4}}\, dt$

$s = \int ds = \frac{1}{2}\int \sqrt{e^{2x} + e^{-2x} - 1}\, dx$

8 $\begin{cases} x = e^{t}\cos t \\ y = e^{t}\sin t \end{cases}$ $ds = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt$

$t \in [0, 10]$

$ds = \sqrt{(e^{t}\cos t - e^{t}\sin t)^2 + (e^{t}\sin t + e^{t}\cos t)^2}\, dt$

$= \sqrt{2e^{2t}(\cos^2 t + \sin^2 t)}\, dt$

$\cancel{= \sqrt{2}\, e^{t}(\sin(t + \frac{\pi}{4}))\, dt}$

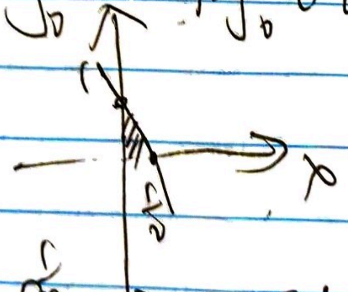
$\cancel{s = \int_0^{10} ds = \sqrt{2}\int_0^{10} e^{t}\sin(t + \frac{\pi}{4})\, dt}$

$= \sqrt{2}\, e^{t}\, dt$

$s = \int_0^{10} ds = \sqrt{2}\int_0^{10} e^{t}\, dt = \sqrt{2}\, e^{t}\Big|_0^{10} = \sqrt{2}(e^{10} - 1)$

CP4-2

$y = 1 - 2x$



$ds = \sqrt{(dx)^2 + (dy)^2}$

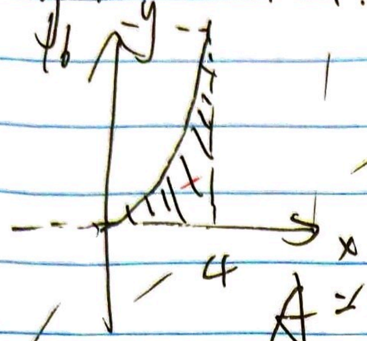
$= \sqrt{(dx)^2 + (2dx)^2}$

$A = \int_0^{\frac{1}{2}} (1 - 2x) \cdot 2\pi \cdot \sqrt{5}\, dx$ $= \sqrt{5}\, dx$

$= 2\sqrt{5}\pi \int_0^{\frac{1}{2}} (1 - 2x)\, dx$

$= 2\sqrt{5}\pi\, (x - x^2)\Big|_0^{\frac{1}{2}} = \frac{\sqrt{5}\pi}{2}.$

44-5 $y=x^2$ $0\le x\le 4$.



$ds=\sqrt{(dx)^2+(dy)^2}$

$=\sqrt{(dx)^2+4x^2(dx)^2}$

$=\sqrt{(4x^2+1)}\,dx$

$A=\int_0^4 2x\pi\cdot ds$

$=\int_0^4 2\pi x\sqrt{4x^2+1}\,dx$

$=\pi\int_0^4\sqrt{4x^2+1}\,dx^2$

$=\pi\left(\frac{2}{3}(4x^2+1)^{\frac{3}{2}}\cdot\frac{1}{4}\right)\Big|_0^4$

$=\pi\left(\frac{2}{3}(64+1)^{\frac{3}{2}}\cdot\frac{1}{4}-\frac{1}{6}\right)\Big|_0^4$

$=\pi\left(\frac{65}{6}\sqrt{65}-\frac{1}{6}\right)$

$=\frac{65\sqrt{65}-1}{6}\pi$

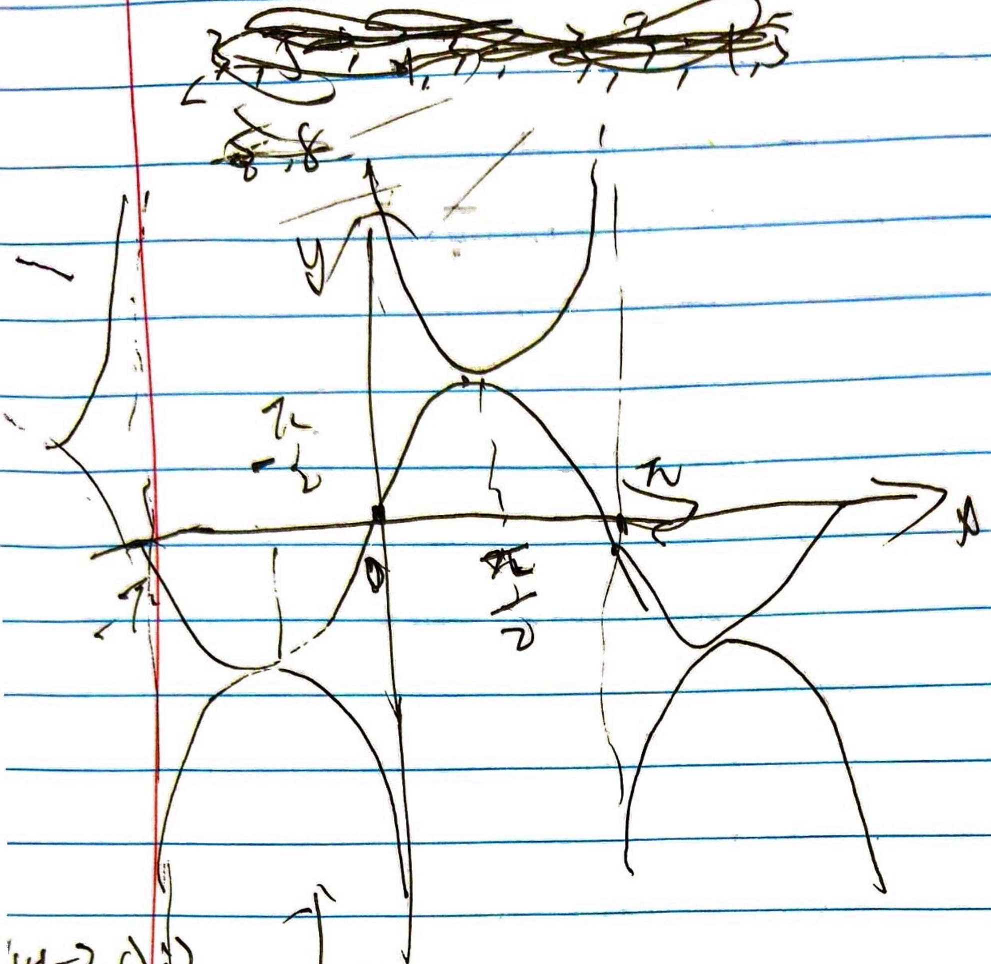
44-b) $(-2,0)$ $-2=r\cos\theta$ $0=r\sin\theta$.

$r=2$, $\theta=\pi$.

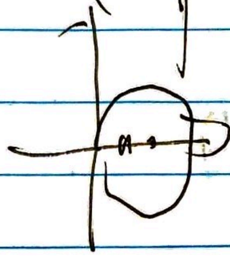
f) $(0,-2)$. $\begin{cases}0=r\cos\theta\\-2=r\sin\theta\end{cases}\Rightarrow\begin{cases}r=2\\\theta=\frac{3\pi}{2}\end{cases}$

g) $(\sqrt{3},-1)$ $\begin{cases}\sqrt{3}=r\cos\theta\\-1=r\sin\theta\end{cases}$ $r^2=3+1=4$ $r=2$

$\begin{cases}\cos\theta=\frac{\sqrt{3}}{2}\\-\frac{1}{2}=\sin\theta\end{cases}$ $\theta=\frac{3\pi}{2}+\frac{\pi}{3}=\frac{11\pi}{6}$



(a) i)



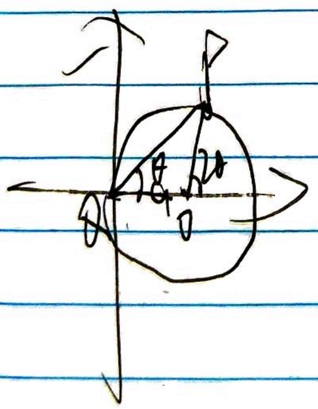
$(x-a)^2+y^2=a^2$

$r^2(\sin^2\theta+\cos^2\theta)-2ar\cos\theta+\cancel{a^2}=\cancel{a^2}$

$r^2=2ar\cos\theta$

$r=2a\cos\theta$

ii)



$P=(a+a\cos 2\theta,\ a\sin 2\theta)$

$a(1+\cos 2\theta)=r\cos\theta$

$2a\cos^2\theta=r\cancel{\cos\theta}$

$r=2a\cos\theta$

$a\sin 2\theta=r\sin\theta$

$a\,2\cos\theta\cancel{\sin\theta}=r\cancel{\sin\theta}$

$r=2a\cos\theta$

~~4(1-3,f) $r = a\cos(2\theta)$ $x = r\cos\theta = (2\cos^2\theta - 1)\cos\theta$ $= 2\cos^2\theta - 1$ $y = r\sin\theta$ $= 1 - 2\sin^2\theta$ $x^2+y^2 = r^2\cos(2\theta) = r^3$~~

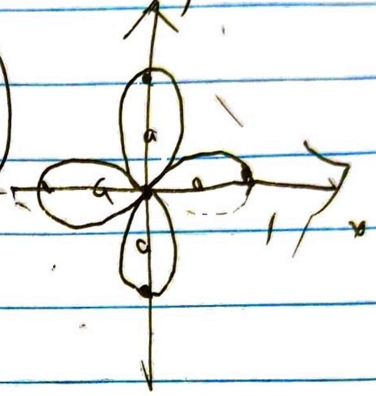
4(1-3,f) $r = a\cos(2\theta)$

$= a(2\cos^2\theta - 1) = a\left(2\frac{x^2}{x^2+y^2} - 1\right)$

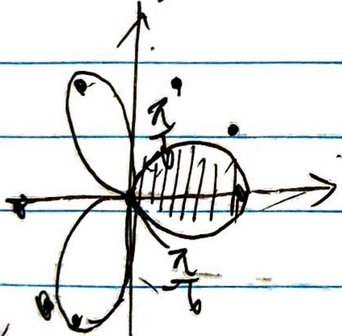
$\frac{1}{a}\sqrt{x^2+y^2} + 1 = \frac{2x^2}{x^2+y^2}$

$\frac{1}{a}(x^2+y^2)^{\frac{3}{2}} = x^2 - y^2$

$\frac{(x^2+y^2)^{\frac{3}{2}}}{x^2-y^2} = a$



4I-2 $r = a\cos 3\theta$



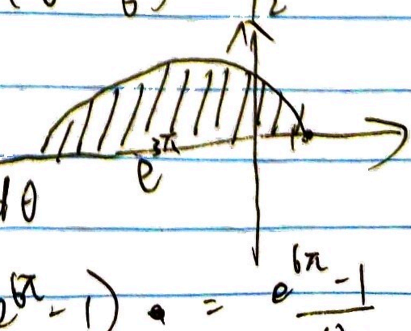
$A = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2}(a\cos 3\theta)^2 \, d\theta$

$= \frac{a^2}{4}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos 6\theta + 1 \, d\theta$

$= \frac{a^2}{4}\left(\frac{1}{3}\sin 6\theta + \theta\right)\Big|_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$

$= \frac{a^2}{4}\left(\frac{\pi}{6} + \frac{\pi}{6}\right) = \frac{\pi a^2}{12}$

3 $0 \le r \le e^{3\theta}$, $\theta \in [0, \pi]$



$A = \int_0^{\pi} \frac{1}{2}(e^{3\theta})^2 \, d\theta = \frac{1}{2}\int_0^{\pi} e^{6\theta} \, d\theta$

$= \frac{1}{2}\left(\frac{1}{6}e^{6\theta}\right)\Big|_0^{\pi} = \frac{1}{12}(e^{6\pi} - 1) = \frac{e^{6\pi} - 1}{12}$

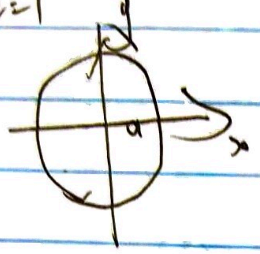
PART II 11/2/2015

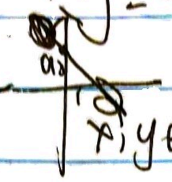
1(a) $\sqrt[k]{\frac{x}{a}} = \cos t \quad \sqrt[k]{\frac{y}{a}} = \sin t$

$\left(\frac{x}{a}\right)^{\frac{2}{k}} + \left(\frac{y}{a}\right)^{\frac{2}{k}} = 1 \quad x^{\frac{2}{k}} + y^{\frac{2}{k}} = a^{\frac{2}{k}}$

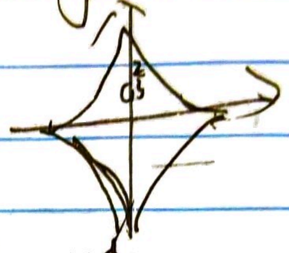
i) $k=1$ ii) $k=2$ iii) $k=3$

$x+y=a$ $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$





$x, y \in [0,1]$



b) i) $k=1$ $x^2+y^2=a^2$ $2xdx + 2ydy = 0$

$ds = \sqrt{(dx)^2+(dy)^2}$ $y=\sqrt{a^2-x^2}$ $dy = \frac{-xdx}{\sqrt{a^2-x^2}}$

$= \sqrt{(dx)^2 + \frac{x^2(dx)^2}{a^2-x^2}}$

$= \sqrt{\frac{a^2(dx)^2}{a^2-x^2}} = \frac{adx}{\sqrt{a^2-x^2}}$

$S = 2\int_{-a}^{a} \frac{a\,dx}{\sqrt{a^2-x^2}} = 2\left(a \arcsin\frac{x}{a}\right)\Big|_{-a}^{a}$

$= 2a\pi$.

ii) $k=2$. $S = \int_0^a \sqrt{1+1}\, dx = \sqrt{2}x\Big|_0^a = \sqrt{2}a$

c) $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \sqrt{(ak\cos t\cdot(-\sin t))^2 + (ak\sin t\cdot\cos t)^2}\, dt$

$= \sqrt{2a^2k^2\cos^2 t\sin^2 t}\, dt = \sqrt{2}\, ak\cos t\sin t\, dt = \frac{\sqrt{2}ak}{2}\sin 2t\, dt$

$S = \int_0^{\frac{\pi}{2}} \frac{\sqrt{2}ak}{2}\sin 2t\, dt = \frac{\sqrt{2}ak}{2}\int_0^{2\pi}\sin 2t\, dt = \frac{\sqrt{2}ak}{2}\left(-\frac{1}{2}\cos 2t\right)\Big|_0^{\frac{\pi}{2}}$

$= \frac{\sqrt{2}ak}{2}\cdot\frac{1}{2} = \frac{\sqrt{2}ak}{4}$

c) $\frac{2}{n}x^{\frac{2}{n}-1} + \frac{2}{n}y^{\frac{2}{n}-1}\frac{dy}{dx} = 0$

$$dy = -\frac{x^{\frac{2}{n}-1}}{y^{\frac{2}{n}-1}}\,dx \qquad y^{\frac{2}{n}-1} =$$

$$(dy)^2 = \left(-\frac{x}{y}\right)^{2\left(\frac{2}{n}-1\right)}(dx)^2$$

$$S = \int_0^a \sqrt{1+\left(\frac{x}{y}\right)^{\frac{4}{n}-2}}\,dx \qquad y = \left(a^{\frac{2}{n}} - x^{\frac{2}{n}}\right)^{\frac{n}{2}}$$

$$dy = \frac{n}{2}\left(a^{\frac{2}{n}} - x^{\frac{2}{n}}\right)^{\frac{n}{2}-1}\cdot\left(-\frac{2}{n}x^{\frac{2}{n}-1}\right)dx$$

$$S = \int_0^a \sqrt{1+\frac{n^2 4}{4 n^2}\left(x^{\frac{2}{n}-1}\right)\left(a^{\frac{2}{n}} - x^{\frac{2}{n}}\right)^{n-2}}\,dx$$

$$= \int_0^a \sqrt{1+x^{\frac{2}{n}-1}\left(a^{\frac{2}{n}} - x^{\frac{2}{n}}\right)^{n-2}}\,dx.$$

2a i) $\frac{d}{dx}\sinh x = \frac{1}{2}(e^x - e^{-x}) = \sinh x$

$\frac{d}{dx}\cosh x = \frac{1}{2}(e^x + e^{-x}) = \sinh x$

ii) $\cosh^2 x - \sinh^2 x$

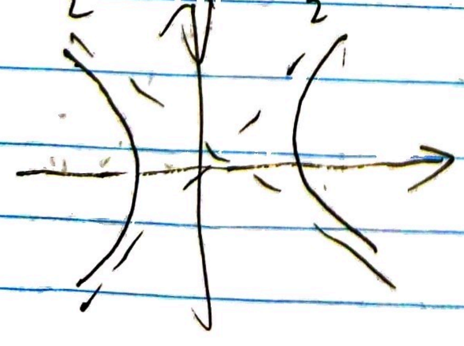
$$= \frac{e^{2x}+e^{-2x}+2}{4} - \frac{e^{2x}+e^{-2x}-2}{4}$$

$$= \frac{4}{4} = 1$$

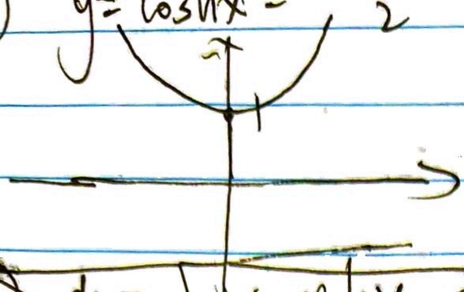
iii) $\cosh^2 x = \frac{e^{2x}+e^{-2x}+2}{4} = \frac{\frac{e^{2x}+e^{-2x}}{2}+1}{2} = \frac{\cosh 2x + 1}{2}$

b) $x = \frac{e^x+e^{-x}}{2}$ $\quad y = \frac{e^x - e^{-x}}{2}$ $\quad x^2 - y^2 = 1$

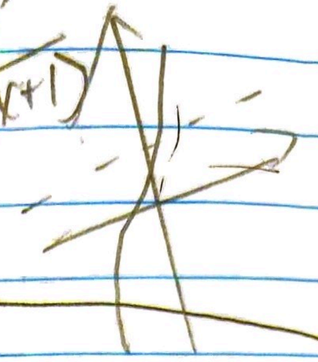
hyperbola.



c) i) $y = \cosh x = \frac{e^{x}+e^{-x}}{2}$



$\frac{2x^3}{(x+2)(x+1)}$

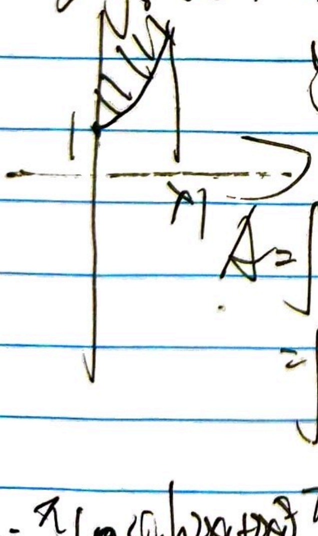


3=

ii) $ds = \sqrt{1+\sinh^2 x}\,dx = \sqrt{\cosh^2 x}\,dx = \cosh x\,dx$

$s = \int_0^{x_1} \cosh x\,dx = \sinh x \Big|_0^{x_1} = \sinh x_1$

d)



$y = \cosh x = \frac{e^{x}+e^{-x}}{2}$

$ds = \cosh x\,dx$

$A = \int_0^{x_1} 2\pi y \cdot ds$

$= \int_0^{x_1} 2\pi \cdot \cosh^2 x\,dx$

$= 2\pi \int_0^{x_1} \cosh^2 x\,dx$

$= \cancel{2\pi\left(x\sinh x\Big|_0^{x_1} - \int_0^{x_1}\sinh x\,dx\right)}$

$= \cancel{2\pi(x_1\sinh x_1 - \sinh x_1)}$

$= \cancel{2\pi\sinh x_1(x_1-1)}$

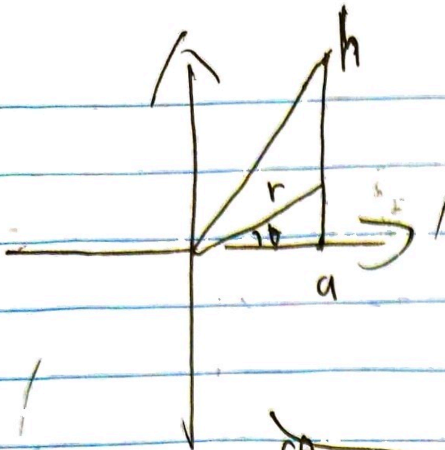
$= \frac{2\pi}{2}\int_0^{x_1} \frac{e^{2x}+e^{-2x}+2}{2}\,dx$

$= \frac{\pi}{2}\left(\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + 2x\right)\Big|_0^{x_1}$

$= \frac{\pi}{2}\left(\frac{e^{2x_1}-e^{-2x_1}}{2} + 2x_1\right)$

$A = \frac{\pi}{2}(\sinh 2x_1 + 2x_1)$

3.a)



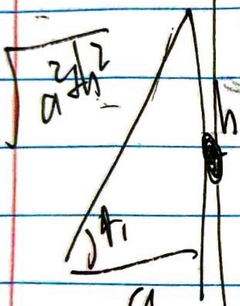
$\cancel{\begin{cases} x = 0 = r\cos\theta \\ y = 0 = r\sin\theta \end{cases} \Rightarrow \begin{cases} r = 0 \\ \theta \in [0, 2\pi] \end{cases}}$

$r = \cancel{a\tan\theta}\ \ a\sec\theta$

$\theta \in [0, \tan^{-1}\frac{h}{a}]$

let θ $\tan^{-1}\frac{h}{a} = \theta_1$

$$A = \int_0^{\theta_1} \frac{1}{2} a^2 \cos^2\theta \, d\theta$$
$$= \frac{1}{2} a^2 \int_0^{\theta_1} \cos^2\theta \, d\theta$$
$$= \frac{1}{2} a^2 \int_0^{\theta_1} \frac{1}{2}(2\cos^2\theta - 1) + \frac{1}{2} \, d\theta$$
$$= \frac{1}{2} a^2 \int_0^{\theta_1} \frac{1}{2}\cos 2\theta + \frac{1}{2} \, d\theta$$
$$= \frac{1}{4} a^2 \int_0^{\theta_1} \cos 2\theta + 1 \, d\theta$$
$$= \frac{1}{4} a^2 \left(\frac{1}{2}\sin 2\theta + \theta\right)\Big|_0^{\theta_1}$$
$$= \frac{1}{4} a^2 \left(\frac{1}{2}\sin 2\theta_1 + \theta_1\right)$$
$$= \frac{1}{4} a^2 \left(\frac{ah}{a^2+h^2} + \tan^{-1}\frac{h}{a}\right)$$



$\cancel{A = \int_0^{\theta_1} \frac{1}{2} r^2 \, d\theta = \frac{1}{2}\int_0^{\theta_1} a^2+h^2 \, d\theta}$

$$A = \int_0^{\theta_1} \frac{1}{2} a^2 \sec^2\theta \, d\theta$$
$$= \frac{a^2}{2} \int_0^{\theta_1} \sec^2\theta \, d\theta$$
$$= \frac{a^2}{2} \tan\theta \Big|_0^{\theta_1} = \frac{a^2}{2}\tan\theta_1 = \frac{a^2}{2}\cdot\frac{h}{a} = \frac{ah}{2}$$

$$r = \frac{1}{1+\sin\theta}$$

$$r + r\sin\theta = 1$$

$$y = r\sin\theta = 1 - r.$$

$$x = r\cos\theta$$

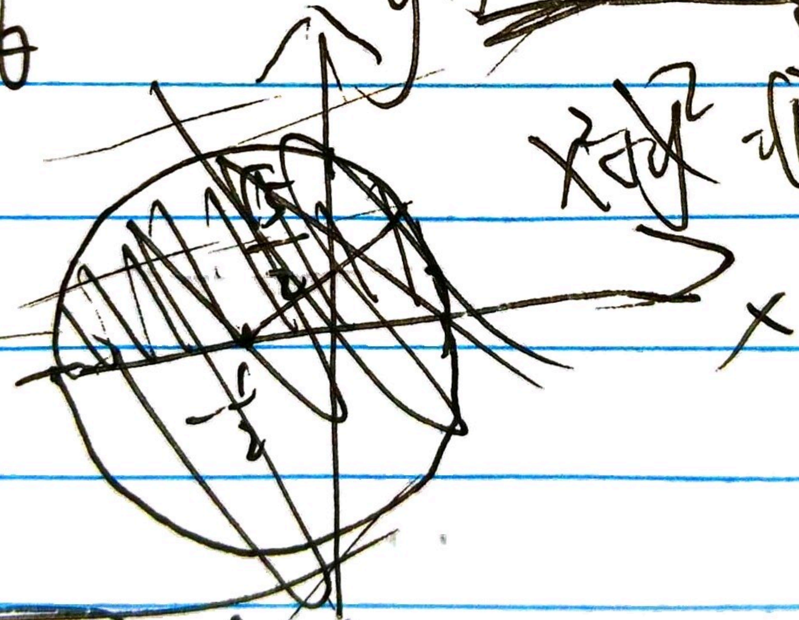
$$x^2+y^2 = r^2 = (-y+1)^2$$

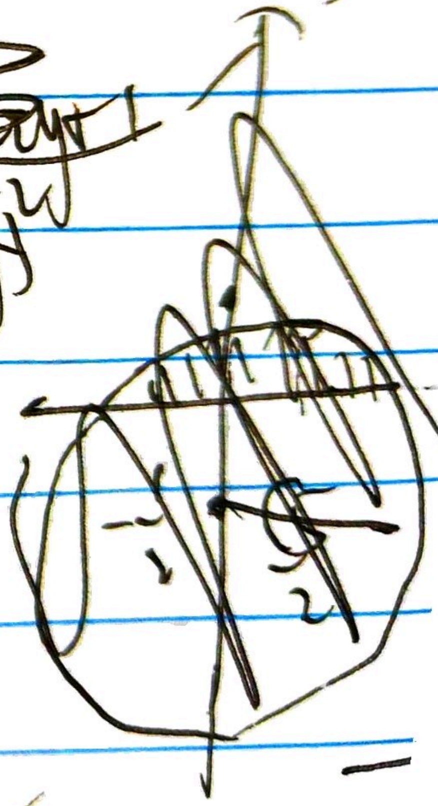
$$x^2+y^2+y+\frac{1}{4} = \frac{5}{4}$$

$$x^2+\left(y+\frac{1}{2}\right)^2 = \frac{5}{4}$$

$$x^2+y^2 = (1-y)^2$$

x^2+2y-1





$$A = \int \left[\sqrt{\frac{5}{4} - x^2} - \frac{1}{2}\right] dx$$

c) $\cancel{A = \int} \quad -\frac{x^2+1}{2} = 0$

$x = \pm 1$

$A = \int_{-1}^{1} y\,dx = \int_{-1}^{1} -\frac{x^2-1}{2}\,dx = \left(-\frac{1}{6}x^3 + \frac{x}{2}\right)\Big|_{-1}^{1}$

$= \left(-\frac{1}{6} + \frac{1}{2} - \frac{1}{6} + \frac{1}{2}\right) = 1 - \frac{1}{3} = \frac{2}{3}$

d) $A = \int_0^{\pi} \frac{1}{2}\left(\frac{1}{1+\sin\theta}\right)^2 d\theta$ let $u = \theta - \frac{\pi}{2}$

$= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{1+\cos u}\right)^2 d\theta$

$= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{1+2\cos^2\frac{u}{2}-1}\right)^2 d\theta$

$= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{4}\sec^4\frac{u}{2}\,du =$

$= \frac{1}{8}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec^4\frac{u}{2}\,du$

$= \frac{1}{8}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(1+\tan^2\frac{u}{2}\right)\left(\sec^2\frac{u}{2}\right) du$

$= \frac{1}{4}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(1+\tan^2\frac{u}{2}\right) d\tan\frac{u}{2}$

$= \frac{1}{4}\left(\tan\frac{u}{2} + \frac{1}{3}\tan^3\frac{u}{2}\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$= \frac{1}{4}\left(1+\frac{1}{3} - (-1) - \left(-\frac{1}{3}\right)\right)$

$= \frac{1}{4}\left(2+\frac{2}{3}\right) = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$