Exam 4 13/2/2015

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P1 $\frac{1}{x(x+1)^2} = \frac{A}{x} + \frac{B_1}{x+1} + \frac{B_2}{(x+1)^2}$

$1 = A(x+1)^2 + B_1 x + B_2$ let $x=-1$

$1 = B_2$

$\frac{1}{(x+1)^2} = A + \frac{B_1 x}{x+1} + \frac{B_2 x}{(x+1)^2}$ let $x=0$

$A = 1$

$1 = A(x+1) + B_1(x) + B_2$ let $x=1$

$\frac{1}{2} = 2 + B_1 + B_2$

$2B_1 + B_2 = -3$

$B_1 = -1$

$B_2 = -1$

$\therefore \int \frac{dx}{x(x+1)^2} = \int \left(\frac{1}{x} + \frac{-1}{x+1} + \frac{-1}{(x+1)^2}\right) dx$

$= \ln|x| - \ln|x+1| + \frac{1}{(x+1)} + C$

P2 $\int (\ln x) x^2 \, dx$

$= \frac{x^3}{3}\ln x - \int \frac{1}{x} \cdot \frac{x^3}{3} \, dx$

$= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$

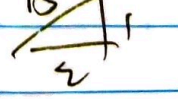
P3 $\int_0^1 \frac{dx}{(4+x^2)^{\frac{3}{2}}} = \frac{1}{8}\int_0^1 \frac{dx}{\left(1+\left(\frac{x}{2}\right)^2\right)^{\frac{3}{2}}}$

let $\frac{x}{2} = \tan\theta$ $dx = 2\sec^2\theta \, d\theta$

$= \frac{1}{8}\int_0^{\tan^{-1}\frac{1}{2}} \frac{2\sec^2\theta \, d\theta}{(1+\tan^2\theta)^{\frac{3}{2}}} = \frac{1}{4}\int_0^{\tan^{-1}\frac{1}{2}} \frac{d\theta}{\sec\theta} = \frac{1}{4}\int_0^{\tan^{-1}\frac{1}{2}} \cos\theta \, d\theta$

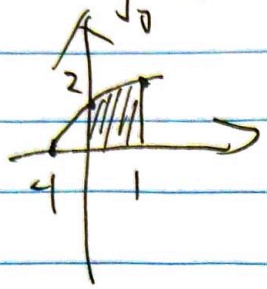
$= \frac{1}{4}\sin\theta \Big|_0^{\tan^{-1}\frac{1}{2}} = \frac{1}{4}\sin\left(\tan^{-1}\frac{1}{2}\right)$

$= \frac{1}{4}\cdot\frac{1}{\sqrt{5}}$



P4 a) $ds = \sqrt{1+\left(2\frac{1}{\sqrt{x+1}}\cdot\frac{1}{2}\right)^2}\,dx$

$S = \int_0^1 ds = \int_0^1 \sqrt{1+\frac{1}{x+1}}\,dx$

b) 

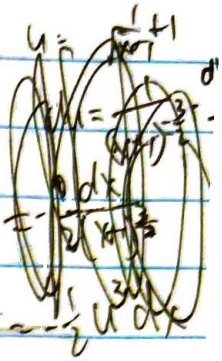
$A = \int_0^1 2\pi\sqrt{2x+2}\sqrt{1+\frac{1}{x+1}}\,dx$

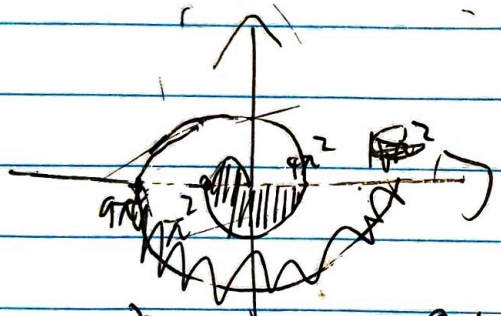
$= 4\pi\int_0^1 \sqrt{x+2}\,dx$

$= 4\pi\left(\frac{2}{3}(x+2)^{\frac{3}{2}}\right)\Big|_0^1$

$= 4\pi\left(\frac{2}{3}(3)^{\frac{3}{2}} - \frac{2}{3}(2)^{\frac{3}{2}}\right)$

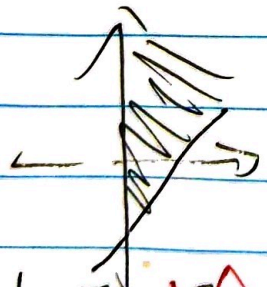
$= \frac{8\pi}{3}\left(3\sqrt{3}-2\sqrt{2}\right)$



P5 a) 

b) $A = \int_0^{2\pi}\frac{1}{2}\theta^4\,d\theta = \frac{1}{2}\left(\frac{1}{5}\theta^5\right)\Big|_0^{2\pi}$

$= \frac{1}{10}(32\pi^5) = \frac{16\pi^5}{5}$

P6 a) 

$r\sin\theta = r\cos\theta - 1$

$r(\sin\theta - \cos\theta) = -1$

$r = \frac{-1}{\sin\theta - \cos\theta}$

b) $\theta \in [0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$