PS IX 1/5/2015

1. $y(x,t) = 0.4\cos\left(\frac{17\pi}{3}x - 34\pi t\right)$

$\omega = 2\pi f = 34\pi = kv$

$k = \frac{34\pi}{v} = \frac{17\pi}{3}$

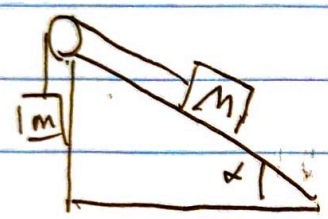
$\bar{u} = \frac{1}{2}\mu A^2\omega^2 = \frac{1}{2} \times 0.02 \times 0.4^2 \times (34\pi)^2$

$= 106\ W/m$

2. $\frac{x}{v_{air}} - \frac{x}{v_w} = 5$

$x = \frac{5}{\frac{1}{v_{air}} - \frac{1}{v_w}} = \frac{5}{\frac{1}{330} - \frac{1}{1450}} = 213\ m$

3.



$mg = Mg\sin\alpha$

$m = M\sin\alpha$

$T = mg$

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\mu}}$

$\delta v = \sqrt{\frac{mg\delta l}{\mu}}$

$6400 = \frac{9.8m}{0.03}$

$m = 19.6\ kg$

$M = \frac{m}{\sin\alpha} = \frac{19.6}{\frac{1}{2}} = 27.7\ kg$

4. $\Delta\beta = 1\ dB$

$\beta = 1\ dB$

$10\log_{10}\frac{I_2}{I_0} - 10\log_{10}\frac{I_1}{I_0} = 1$

$\log_{10}\frac{I_2}{I_1} = \frac{1}{10}$

$\frac{I_2}{I_1} = 10^{0.1} = 1.3$

5. $\beta_1 - \beta_2 = 4$

$$\frac{I_1}{I_2} = 10^{0.4}$$

$$I_1 = \frac{P}{4\pi r^2} \quad I_2 = \frac{P}{4\pi(r+\Delta r)^2}$$

$$I_1 = 10^{0.4} I_2$$

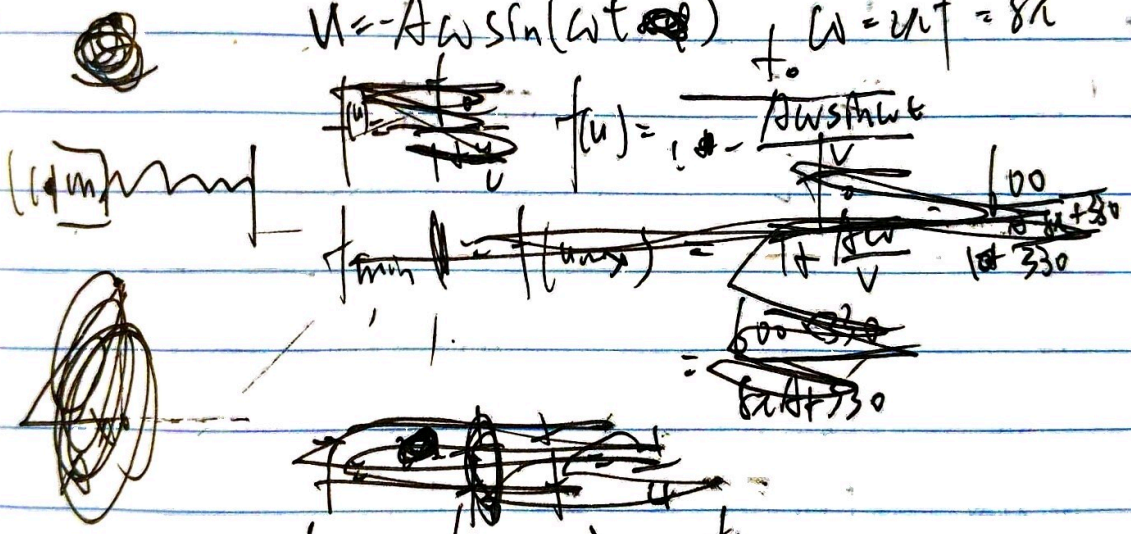
$$\frac{1}{r^2} = 10^{0.4} \frac{1}{(r+\Delta r)^2}$$

$$r + \Delta r = 10^{0.2} r$$

$$(10^{0.2} - 1) r = 2$$

$$r = \frac{2}{10^{0.2} - 1} = 5.42\ m$$

6. $u = -A\omega \sin(\omega t + \varphi)$ $\omega = 2\pi f = 8\pi$



$$f_{max} = f(u_{max}) = \frac{f_0}{1 + \frac{A\omega}{v}}$$

$$f_{min} = f(u_{min}) = \frac{f_0}{1 - \frac{A\omega}{v}}$$

$$f_{max} - f_{min} = v f_0 \left(\frac{1}{v - A\omega} - \frac{1}{v + A\omega} \right)$$

$$2 = v f_0 \frac{2A\omega}{v^2 - A^2\omega^2}$$

$$v^2 - A^2\omega^2 = v f_0 A\omega$$

$$\omega^2 A^2 + v f_0 \omega A - v^2 = 0. \qquad A = -0.022\ m$$

7. i) $\mu = \frac{m}{L}$

$T = Mg$

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{MLg}{m}}$

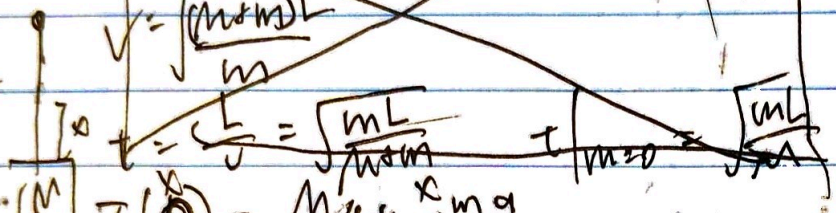
$t = \frac{L}{v} = \sqrt{\frac{mL}{Mg}}$

ii) $T = Mg + mg$

$v = \sqrt{\frac{(M+m)L}{m}}$

$t = \frac{L}{v} = \sqrt{\frac{mL}{(M+m)g}}$

$t|_{m=0} \to \sqrt{\frac{mL}{Mg}}$



$T(x) = Mg + \frac{x}{L}mg$

$v(x) = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg + \frac{x}{L}mg}{m/L}} = \sqrt{(M/m)Lg + xg}$

$\frac{dx}{dt} = \sqrt{(M/m)Lg + xg} = \sqrt{\frac{MLg + mxg}{m}} = \sqrt{ML + mx} \cdot \sqrt{\frac{g}{m}}$

$\int \frac{dx}{\sqrt{ML + mx}} = \int \sqrt{\frac{g}{m}}\, dt$

$\frac{1}{m}\sqrt{ML + mx} = \sqrt{\frac{g}{m}}\, t + C$

~~$t(x) = \sqrt{\frac{m}{mg}} \cdot \frac{2}{m}\sqrt{ML + mx} - C$~~

~~$t(L) = \sqrt{\frac{m}{mg}} \cdot \frac{2}{m}\sqrt{ML + mL} - C$~~

~~$t(0) = \sqrt{\frac{m}{mg}} \cdot \frac{2}{m}\sqrt{ML} - C$~~

~~$t = \sqrt{\frac{m}{mg}} \cdot \frac{2}{m}\left(\sqrt{ML + mL} - \sqrt{ML}\right)$~~

$x(0)=0$.

$\left(\frac{2}{m}\sqrt{ML}\right)^2=C^2 \qquad L=\pm\frac{2}{m}\sqrt{ML}$

$\frac{2}{m}\sqrt{ML+mx}=\sqrt{\frac{g}{m}}t\pm\frac{2}{m}\sqrt{ML}$

$\sqrt{ML+mx}=\frac{1}{2}\sqrt{mg}t\pm\sqrt{ML}$

$ML+mx=\frac{1}{4}mgt^2\pm\sqrt{mMgL}\,t+ML$

$mx=\frac{1}{4}mgt^2\pm\sqrt{mMgL}\,t$

$x=\frac{1}{4}gt^2\pm\sqrt{\frac{MgL}{m}}\,t$

~~$L=t\left(\frac{1}{4}gt\pm\sqrt{\frac{MgL}{m}}\right)$~~

$L=\frac{1}{4}gt^2\pm\sqrt{\frac{MgL}{m}}\,t=\frac{1}{4}\frac{mg}{ML}t^2\pm\sqrt{\frac{Mg}{M}}\cdot t$

~~$t=\frac{\pm\sqrt{\frac{MgL}{m}}\pm\sqrt{\frac{MgL}{m}+gL}}{\frac{g}{2}}$~~

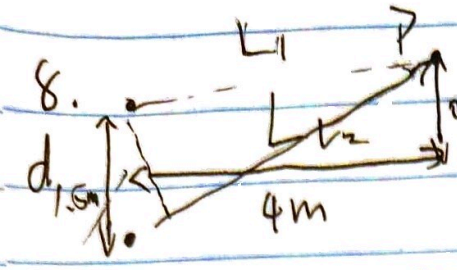
$t=\frac{\pm\sqrt{\frac{Mg}{M}}\pm\sqrt{\frac{Mg}{M}+\frac{mg}{M}}}{\frac{1}{2}\frac{mg}{ML}}=2\left(\pm\sqrt{\frac{ML}{mg}}\pm\sqrt{\frac{ML}{mg}+\frac{L}{g}}\right)$

if $m\to 0$, $L=\pm\sqrt{\frac{Mg}{M}}\cdot t$, $t>0$, $L=\sqrt{\frac{Mg}{M}}\cdot t$

$t=\frac{\frac{m}{L}\cdot L^2}{\sqrt{Mg}}=\sqrt{\frac{ML}{mg}}$

$\therefore t=2\left(\sqrt{\frac{ML}{mg}}\pm\sqrt{\frac{ML}{mg}+\frac{L}{g}}\right)>0 \quad \therefore t=2\left(\sqrt{\frac{ML}{mg}}+\sqrt{\frac{ML}{mg}+\frac{L}{g}}\right)$

8.



$L_1^2 = L^2 + \left(x-\frac{d}{2}\right)^2$

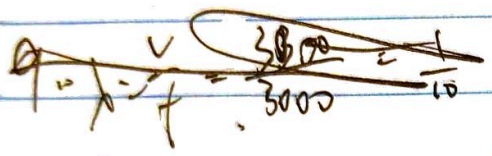
$L_2^2 = L^2 + \left(x+\frac{d}{2}\right)^2$

$\Delta L = (n+\frac{1}{2})\lambda \quad \frac{1}{2}\lambda$

$\sqrt{L^2+\left(x+\frac{d}{2}\right)^2} - \sqrt{L^2+\left(x-\frac{d}{2}\right)^2} = (\quad)\frac{1}{2}\lambda$

$x = 0.58m \quad 0.579m$

$v = \lambda f = 3.19m/s \quad 3.18m/s$



$\frac{1}{2}\lambda = d\sin\theta$

$v = 2df\sin\theta$

$= 3\times 550 \times \left(\frac{0.8}{\sqrt{x^2+L^2}}\right)$

$= 32.6m/s$

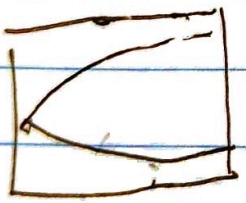
$\Delta v = 8m/s$

9.

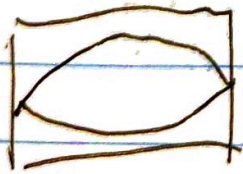
$\lambda = 4L = \frac{v}{f}$

$L = \frac{v}{4f} = \frac{330}{4\times 3000} = 0.0275m = 2.75cm$

10.



$f = \frac{v}{\lambda} = \frac{v}{4L} = \frac{340}{8} = 43.1Hz$



$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{340}{4} = 85.3Hz$

11.

$\lambda = \frac{v}{f} = \frac{320}{1600} = 0.2m$

$x = \frac{\lambda}{2} = 0.1m$