PS VI

1. i) $I = 3\int_0^L \frac{m}{L} r^2\,dr = 3\left(\frac{mr^3}{3L}\Big|_0^L\right) = mL^2$

ii) $\alpha = \frac{\tau}{I} = \frac{\tau}{mL^2}$

$\omega = \alpha t$ $\quad t_0 = \frac{\omega}{\alpha} = \frac{m\omega L^2}{\tau}$

iii) $\cancel{\omega = \alpha t = \frac{\tau}{mL^2} t}$

$\cancel{\theta = \int_0^{t_0} \frac{\tau}{mL^2} t\,dt = \frac{\tau}{mL^2}\frac{t_0^2}{2}}$

$\cancel{\theta = \frac{1}{2}\alpha t^2 = \frac{\tau t_0^2}{2mL^2}}$

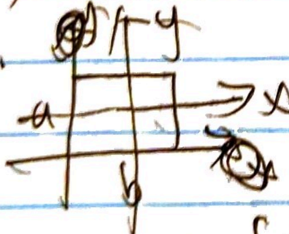
$\cancel{\text{rev} = \frac{\theta}{2\pi} = \frac{\tau t^2}{4\pi mL^2}}$

$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha\left(\frac{\omega}{\alpha}\right)^2 = \frac{1}{2}\frac{\omega^2}{\alpha}$

$= \frac{1}{2}\frac{m\omega^2 L^2}{\tau}$

$\text{rev} = \frac{\theta}{2\pi} = \frac{m\omega^2 L^2}{4\pi\tau}$

iv) rev = 2000

2. 

$= \int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{M}{ab}\, y^2\,dy\,dx$

$= \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{M}{ab}\cdot\frac{1}{3}\left(\frac{2a^3}{8}\right)$

$I_x = \int_P y^2\,dm = \cancel{\frac{ab}{3}\delta m\, a^2} = \frac{1}{3}Ma^2$

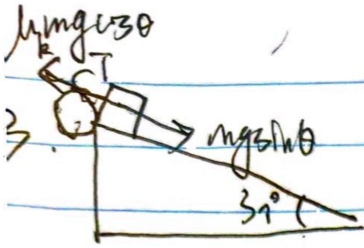
$= \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{1}{12}\frac{M}{b}\cdot a^2\,dx$

$= \frac{1}{12}Ma^2$

$I_y = \int_P x^2\,dm = \frac{1}{12}Mb^2.$

$I_{cm} = \int (x^2+y^2)\,dm = \int x^2\,dm + \int y^2\,dm = I_x + I_y$

$= \frac{1}{12}M(a^2+b^2).$

3. 

$$= \Delta r \cdot r\Delta\theta \cdot \frac{M_p}{\pi r^2} \qquad dm = \frac{M_p}{\pi r^2} r\,dr\,d\theta$$

$$\Delta m_p = \Delta x \Delta y \cdot \frac{M_p}{\pi r^2} \qquad dm = \frac{M_p}{\pi r^2} dx\,dy$$

$$I_p = \int_0^{r_0} \cancel{2\pi r^2\,dm} = \frac{1}{2} M_p r_0^2$$

$$= \int_0^{r_0} \cancel{2\pi \cdot \frac{M_p}{\pi r}} r^2\,dr$$

$$\cancel{= 2M_p \cdot \frac{1}{2} r_0^2 = M_p r_0^2}$$

$$\cancel{\tau = T r_0} \qquad \tau = \alpha \cdot I = \frac{a}{r_0} \cdot \frac{1}{2} \ldots = \frac{M_p \cdot a \cdot r_0}{2}$$

$$T = \frac{\tau}{r_0} = \frac{M_p a}{2}$$

$$\mu_k mg\cos + T = mg\sin\theta + ma$$

$$\mu_k = \tan\theta - \frac{T}{mg\cos\theta} + \frac{ma}{mg\cos\theta}$$

$$= \frac{\sqrt{3}}{3} - \frac{M a}{2mg\cos\theta} + \frac{a}{g\cos\theta}$$

$$= 0.31$$

$$v = \sqrt{2ad} = 2.8\ m/s$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \mu_k mg\cos\theta \cdot d$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2 + \mu_k mg\cos\theta \cdot d.$$

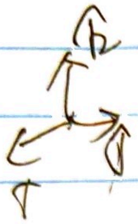
$$v^2 = \frac{mgh - \mu_k mg\cos\theta \cdot d}{\left(\frac{1}{2}m + \frac{1}{4}M\right)} \qquad v = 2.8\ m/s$$

4. $(\vec{A}\times\vec{B}) \perp \vec{A}$ $\therefore \vec{A}\cdot(\vec{A}\times\vec{B}) = 0$.

$\vec{A} = \langle A_x, A_y, A_z\rangle$ $\vec{B} = \langle B_x, B_y, B_z\rangle$

$\vec{A}\times\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\times(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= (A_xB_y)(\hat{i}\times\hat{j}) + (A_xB_z)(\hat{i}\times\hat{k}) + (A_yB_x)(\hat{j}\times\hat{i}) + (A_yB_z)(\hat{j}\times\hat{k}) + (A_zB_x)(\hat{k}\times\hat{i}) + (A_zB_y)(\hat{k}\times\hat{j})$



$= (A_xB_y - A_yB_x)(\hat{i}\times\hat{j}) + (A_xB_z - A_zB_x)(\hat{i}\times\hat{k}) + (A_yB_z - A_zB_y)(\hat{j}\times\hat{k})$

$= (A_xB_y - A_yB_x)\hat{k} - (A_xB_z - A_zB_x)\hat{j} + (A_yB_z - A_zB_y)\hat{i}$

5. $L_0 = L_1 + L_2$

$\frac{1}{2}MR^2\omega_0 = \frac{1}{2}MR^2\omega + \frac{1}{2}mr^2\omega$

$\omega = \frac{MR^2\omega_0}{MR^2 + mr^2}$

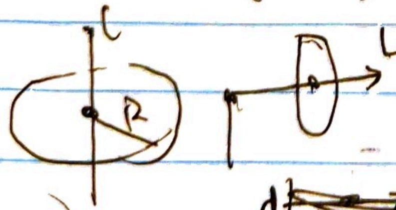
$E_1 = \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2 = \frac{1}{2}(I_1+I_2)\omega^2$.

$E_0 = \frac{1}{2}I_1\omega_0^2$ $\quad \Delta E = \frac{1}{2}(I_1+I_2)\omega^2 - \frac{1}{2}I_1\omega_0^2$

$\frac{E_1}{E_0} = \frac{(I_1+I_2)\omega^2}{I_1\omega_0^2} = \frac{(I_1+I_2)\cdot\frac{(I_1\omega_0)^2}{(I_1+I_2)^2}}{I_1\omega_0^2}$

$= \frac{I_1}{(I_1+I_2)} - 1 = \frac{MR^2}{MR^2+mr^2}$

6.



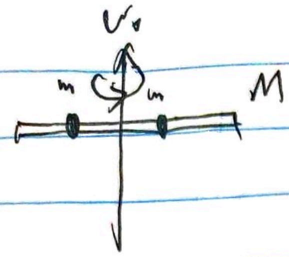
$L = I\omega$ $\quad d\theta =$

$\tau = lmg$

$\frac{dL}{dt} = \tau$ $\quad \tau dt = L d\theta$

$\frac{d\omega}{dt}$ $\quad \omega_p = \frac{d\theta}{dt} = \frac{\tau}{L} = \frac{lmg}{\frac{1}{2}MR^2\omega} = \frac{2lg}{R^2\omega}$

$l = \frac{\frac{1}{2}\omega_p R^2\omega}{g}$

7. 

i) $I_R = 2 \cdot \int_0^{\frac{L}{2}} \frac{M}{L} r^2 dr = \cancel{\frac{2}{3}ML^2}\ \frac{1}{12}ML^2$

$I_B = 2 \cdot mr^2$

$L_0 = I_R \omega_0 = \cancel{\frac{2}{3}ML^2\omega_0}\ \frac{1}{12}ML^2\omega_0$

$(I_R + I_B)\omega(r) = L_0$

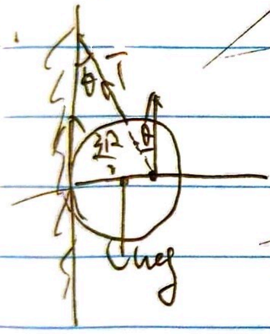
$\omega(r) = \dfrac{\frac{1}{12}ML^2\omega_0}{\frac{1}{12}ML^2 + 2mr^2} = \dfrac{ML^2\omega_0}{ML^2 + 24mr^2}$

ii) $\omega\left(\frac{L}{2}\right) = \dfrac{ML^2\omega_0}{ML^2 + 6mL^2} = \dfrac{M\omega_0}{M + 6m} \left(\dfrac{1}{1 + \frac{6m}{M}}\right)\omega_0$

$\vec{L} = \vec{r} \times \vec{p}$ $\because$ beads fly tangentially

$\therefore \vec{r} \times \vec{p}$ doesn't change.

iii) $\vec{f} \parallel \vec{r}$ $\therefore \tau_f = \cancel{\vec{f} \times \vec{r}} = 0$. $\vec{r} \times \vec{f} = 0$.



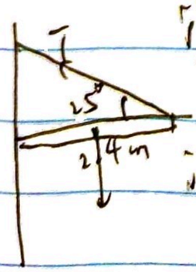
$\cancel{mgR} = \begin{cases} \mu_B T\sin\theta + T\cos\theta = mg \\ \frac{3R}{2} T\cos\theta = mgR \end{cases}$

$\cancel{T\cos\theta = \frac{2}{3}mg}$ $mg = \frac{3}{2}T\cos\theta$

$\mu_B T\sin\theta = \frac{1}{2}T\cos\theta$

$\mu_B \cancel{\tan} = \frac{1}{2}\cot\theta$.

$\cos\theta = 30°$ $\mu_B = \frac{1}{2}\cot\frac{\pi}{6} = \frac{\sqrt{3}}{2} \approx 0.87$.

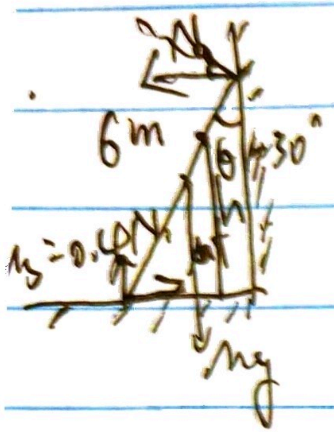
9. 

i) $mg \cdot \frac{L}{2} = T\sin\theta \cdot L$

$T = \frac{1}{2\sin\theta}mg = \cancel{92.76}\ 92.8\ N$

ii) $mg \cdot \frac{L}{2} + MgL = T\sin\theta \cdot L$

$T = \frac{1}{\sin\theta}\left(\frac{1}{2}mg + Mg\right) = \frac{g}{\sin\theta}\left(\frac{1}{2}m + M\right) = 672N$



$N = f = \mu_s N_1 = \mu_s(Mg + mg)$

$LN\cos\theta = Mg\sin\theta \cdot \frac{L}{2} + mg\sin\theta \cdot \frac{h}{\cos\theta}$

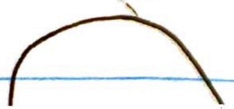
$LN\cos\theta = \frac{1}{2}MgL\sin\theta + mgh\tan\theta$

$$h = \frac{LN\cos\theta - \frac{1}{2}MgL\sin\theta}{mg\tan\theta}$$

$$= \frac{L\mu_s(M+m)g\cos\theta - \frac{1}{2}MgL\sin\theta}{mg\tan\theta}$$

$= 3.81\text{m}$

Midterm Exam ~~12/4/2015~~ Q5



i) ~~momentum and~~ angular momentum are conserved