$\int 8 \quad \frac{1}{2}$

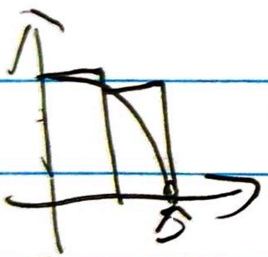
Exam 3 5th Jan 2025 DD

1 a) let $u = 1+x^2$. $du = 2x\,dx$

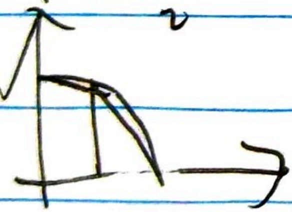
$$\int_0^2 \frac{x\,dx}{(1+x^2)^2} = \int_1^5 \frac{du}{2u^2} = \frac{1}{2}\left(-\frac{1}{u}\right)\Big|_1^5 = \frac{1}{2}\left(-\frac{1}{5}+1\right) = \frac{2}{5}$$

b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^6 x \cos x\,dx$ let $u = \sin x$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^6 x\, d\sin x = \int_{-1}^{1} u^6\,du = \frac{1}{7}u^7\Big|_{-1}^{1} = \frac{2}{7}$$

2. a) 

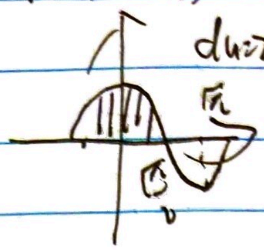
$$\int_0^{\frac{\pi}{2}} \cos x\,dx \approx \frac{\pi}{4}\left(1+\cos\frac{\pi}{4}\right) = \left(1+\frac{\sqrt{2}}{2}\right)\frac{\pi}{4} = \frac{\pi}{4}+\frac{\sqrt{2}\pi}{8}$$

b) 

$$\int_0^{\frac{\pi}{2}} \cos x\,dx \approx \frac{\pi}{4}\left(\frac{1}{2}\cos 0 + \cos\frac{\pi}{4} + \frac{1}{2}\cos\frac{\pi}{2}\right) = \left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\frac{\pi}{4} = \frac{\pi+\sqrt{2}\pi}{8}$$

c) $\int_0^{\frac{\pi}{2}} \cos x\,dx \approx \frac{\frac{\pi}{4}}{3}\left(1+4\cos\frac{\pi}{4}+\cos\frac{\pi}{2}\right) = \frac{\pi}{12}\left(1+2\sqrt{2}\right) = \frac{\pi+2\sqrt{2}\pi}{12}$

3 $y=\cos(x^2)$



$du = 2x\,dx$

$V = \int_0^{\sqrt{\frac{\pi}{2}}} \cos(x^2)\,2\pi x\,dx$

$= \int_0^{\frac{\pi}{2}} \cos(u)\,\pi\,du$

$= \pi \int_0^{\frac{\pi}{2}} \cos(u)\,du = \pi \sin u \Big|_0^{\frac{\pi}{2}} = \pi$

4 a) $\dfrac{\int_0^{\frac{a}{2}} cx\,dx}{\int_0^a cx\,dx} = \dfrac{\frac{c}{2}\cdot\frac{a^2}{4}}{\frac{c}{2}a^2} = \frac{1}{4}$

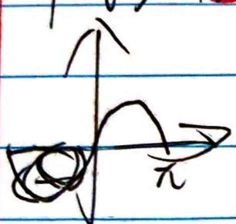
b) $\dfrac{1}{\int_0^a cx\,dx}\int_0^a cx\cdot\frac{100x}{a}\,dx$

$= \frac{2}{a^2c}\,\frac{100c}{a}\int_0^a x^2\,dx = \frac{2}{a^2}\,\frac{100c}{a}\cdot\frac{x^3}{3}\Big|_0^a = \frac{200c}{3}$

5 a) $F'(x)$ ~~$= \sqrt{t}\sin t \ldots$~~ $=$

$F'(x)=\sqrt{x}\sin x \qquad a = k\pi,\ k\in\mathbb{Z}$

b)



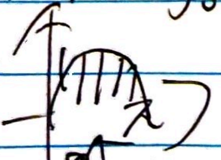
$F''(x) = \frac{1}{2\sqrt{x}}\sin x + \sqrt{x}\cos x$

$F''(\pi) = -\sqrt{\pi} < 0$

∴ Yes there is a local maximum.

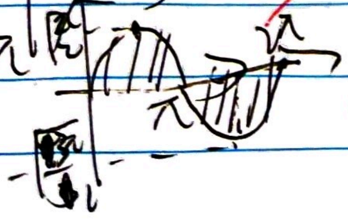
c) i) $x=0$. $F(0)=\int_0^0 \sqrt{t}\sin t\,dt = 0$.

ii) $x=\pi$



positive.

iii) $x=2\pi$



Negative.

d) $G(x) = \int_0^{x^2} \frac{1}{2}\sqrt{t}\,\sin t\,dt$ $u^2 = t$ $dt = 2u\,du$ $du = \frac{dt}{2u}$

$= \frac{1}{2}\int_0^{x^2} \sqrt{t}\sin t\,dt$

$= \frac{1}{2}F(x^2)$.