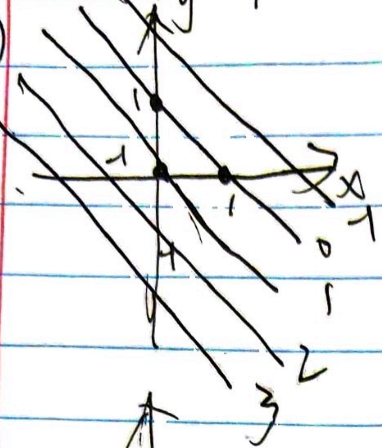
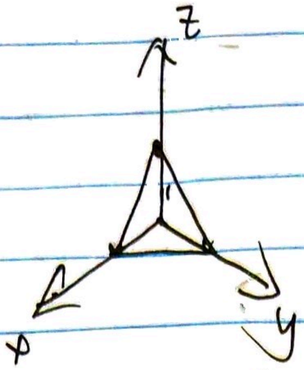
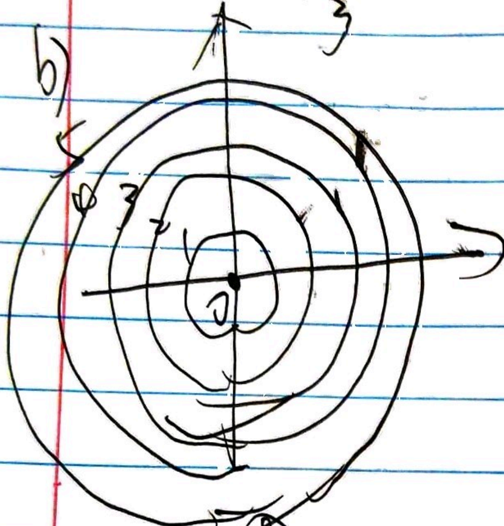
Lec 9 11/3/2025

2A-1 a)

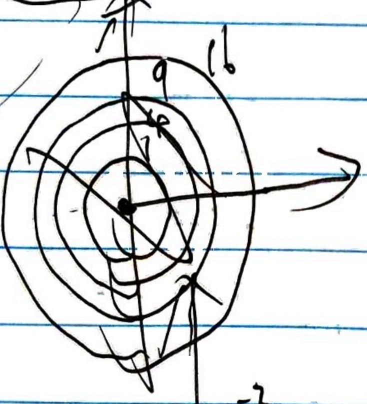


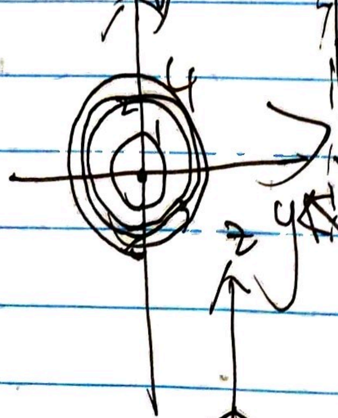


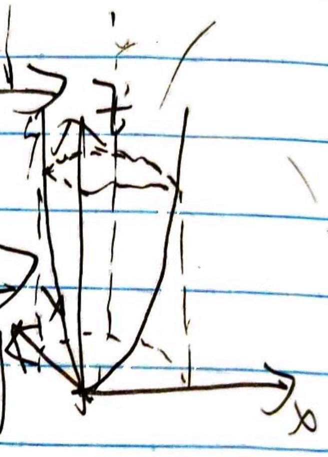
b)



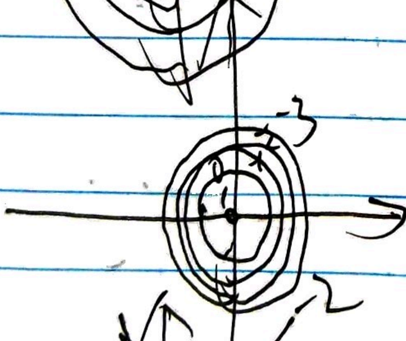
c)

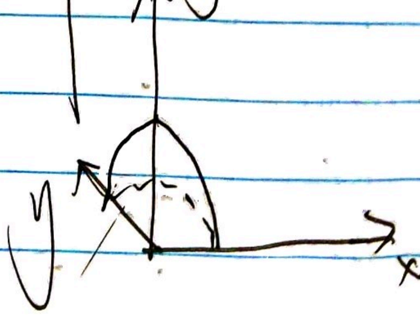




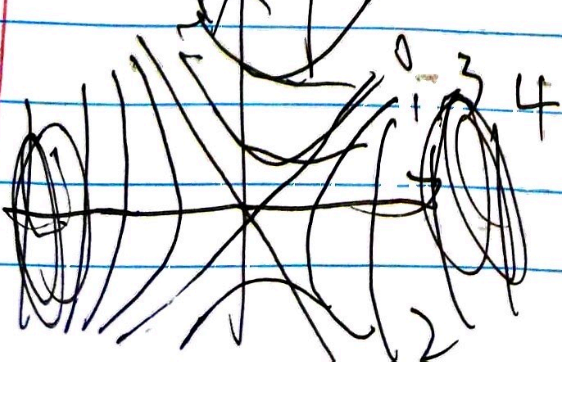


d





e)



2 a) $\frac{\partial w}{\partial x} = 3yx^2 - 3y^2$

$\frac{\partial w}{\partial y} = x^3 - 6xy + 4y$

b) $\frac{\partial z}{\partial x} = \frac{1}{y}$ $\frac{\partial z}{\partial y} = -\frac{x}{y^2}$

c) $\frac{\partial}{\partial x} \sin(3x+2y) = 3\cos(3x+2y)$

$\frac{\partial}{\partial y} \sin(2y+3x) = 2\cos(2y+3x)$

d) $\frac{\partial}{\partial x} e^{x^2y} = 2yx \cdot e^{x^2y}$ $\frac{\partial}{\partial y} e^{x^2y} = x^2 e^{x^2y}$

e) $\frac{\partial z}{\partial x} = \ln(2x+y) + x \cdot \frac{1}{2x+y} \cdot 2 = \frac{2x}{2x+y} + \ln(2x+y)$

$\frac{\partial z}{\partial y} = x \cdot \frac{1}{2x+y} = \frac{x}{2x+y}$

3 a) $\frac{\partial}{\partial x} x^m y^n = m y^n x^{m-1}$ $\frac{\partial}{\partial y}\frac{\partial}{\partial x} x^m y^n = mn x^{m-1} y^{n-1}$

$\frac{\partial}{\partial y} x^m y^n = n y^{n-1} x^m$ $\frac{\partial}{\partial x}\frac{\partial}{\partial y} x^m y^n = mn y^{n-1} x^{m-1}$

b) $\frac{\partial}{\partial x} \frac{y}{x+y} = -\frac{y}{(x+y)^2}$ $\frac{\partial}{\partial y}\frac{\partial}{\partial x} \frac{y}{x+y} = -\frac{(x+y)^2 - y \cdot 2(x+y)}{(x+y)^{\cancel{4}3}} = -\frac{x-y}{(x+y)^3}$

$\frac{\partial}{\partial y} \frac{y}{x+y} = \frac{x+y-y}{(x+y)^2} = \frac{x}{(x+y)^2}$ $\frac{\partial}{\partial x}\frac{\partial}{\partial y} \frac{y}{x+y} = \frac{(x+y)^2 - x \cdot 2(x+y)}{(x+y)^{\cancel{4}3}} = \frac{y-x}{(x+y)^3}$

c) $\frac{\partial}{\partial x} \cos(x^2y) = 2x\sin(x^2y)$ $\frac{\partial}{\partial y}\frac{\partial}{\partial x} \cos(x^2y) = -2x\cos(x^2y)$

$\frac{\partial}{\partial y} \cos(x^2y) = -\sin(x^2y)$ $\frac{\partial}{\partial x}\frac{\partial}{\partial y} \cos(x^2y) = -2x\cos(x^2y)$

5 a) $w_{xx} = \frac{\partial}{\partial x}(ae^{ax}\sin ay) = a^2e^{ax}(\sin ay)$

$w_{yy} = \frac{\partial}{\partial y}(e^{ax} \cdot a\cos ay) = e^{ax}a^2\sin ay$

$w_{xx} + w_{yy} = 0$

1. b) $\frac{\partial w}{\partial x} = \frac{1}{x^2+y^2}\cdot 2x$ $\quad \frac{\partial}{\partial x}\frac{\partial w}{\partial x} = \frac{2(x^2+y^2)-2x(2x)}{(x^2+y^2)^2} = \frac{2y^2-2x^2}{(x^2+y^2)^2}$

$\frac{\partial w}{\partial y} = \frac{1}{x^2+y^2}\cdot 2y$ $= \frac{\partial}{\partial y}\frac{\partial w}{\partial y} = \frac{2(x^2+y^2)-2y\cdot 2y}{(x^2+y^2)^2} = \frac{2x^2-2y^2}{(x^2+y^2)^2}$

$w_{xx}+w_{yy}=0$.

2B-1 a) $\frac{\partial z}{\partial x} = y^2 \quad \frac{\partial z}{\partial y} = 2xy$

$z = 1 + z_x(1,1)x + z_y(1,1)y$

$= 1 + (x-1) + 2(y-1)$ $\therefore$ $=1$ $\quad x+2y-3=z$

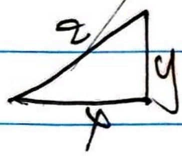
b) $\frac{\partial w}{\partial x} = -\frac{y^2}{x^2} \quad \frac{\partial w}{\partial y} = \frac{2y}{x}$

$w = 4 + z_x(1,2)x + z_y(1,2)y$

$= 4 - 4(x-1) + 4(y-2)$ $\therefore$ $=4$

$\therefore$ $w = -4x+4y$

3



$z = \sqrt{x^2+y^2}$

$\frac{\partial z}{\partial x} = \frac{1}{2}\cdot\frac{1}{\sqrt{x^2+y^2}}\cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$

$\frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$

$z \approx \sqrt{3^2+4^2} + z_x(3,4)\cdot\Delta x + z_y(3,4)\cdot\Delta y$

$= 5 + \frac{3}{5}\Delta x + \frac{4}{5}\Delta y$

$= 5.014$

6 $V = \pi r^2 h \quad \frac{\partial V}{\partial r} = 2\pi hr \quad \frac{\partial V}{\partial h} = \pi r^2$

$\Delta V \approx z_x(2,3)\Delta x + z_y(2,3)\Delta y \qquad \Delta x = \Delta y$

$= 12\pi\Delta x + 4\pi\Delta x$

$= 16\pi\Delta x \le 0.1$

$\Delta x \le \frac{0.1}{16\pi} \approx 2.0\times 10^{-3}$

9 a) $\frac{\partial w}{\partial x} = 2(y+1)x \quad \frac{\partial w}{\partial y} = x^2$

$w_x(1,0) = 2 > w_y(1,0) = 1$

b) $\Delta z = w_x(1,0)\Delta x + w_y(1,0)\Delta y$

$= 2\Delta x + \Delta y$

$\cancel{\frac{\partial z}{\partial x} = 2 + \frac{\Delta y}{\Delta x}}$ $\quad 2\Delta x + \Delta y = 0$

$\cancel{\frac{\Delta y}{\partial x} = -2}$ $\quad \frac{\Delta y}{\Delta x} = -2.$

PART B 14/3/2025

a) $x_0 < 2$ since you cannot see it if the object is behind your eyes.

$\vec{EP} = (x_0 - 2, y_0, z_0)$

$(2 + (x_0-2)t, y_0 t, z_0 t) = 0$

$2 + (x_0 - 2)t = 0$

$t = \frac{2}{x_0 - 2}$

$\therefore Q\left(\frac{y_0}{x_0-2}, \frac{z_0}{x_0-2}\right)$

b) $P_1\left(\frac{y_1}{x_1-2}, \frac{z_1}{x_1-2}\right)$

$\cancel{\vec{QP} = \left(0, \frac{y_1}{x_1-2} - \frac{y_0}{x_0-2}, 0, \frac{z_1}{x_1-2} - \frac{z_0}{x_0-2}\right)}$

$\cancel{\vec{EQ} = (x_0 - 2, y_0, z_0)} \quad \cancel{\vec{EP} = (x_1 - 2, y_1, z_1)}$

$\cancel{\vec{EQ} \times \vec{QP} = \begin{vmatrix} i & j & k \\ x_0-2 & y_0 & z_0 \\ x_1-2 & y_1 & z_1 \end{vmatrix} = (y_0 z_1 - y_1 z_0, x_0 z_1 - 2z_1 - x_1 z_0 + 2z_0, x_0 y_1 - 2y_1 - y_0 x_1 + 2y_0)}$

$\therefore$ It should be like a line segment or a point.

c) $Q_0\left(\frac{3}{-3}, \frac{1}{-3}\right) = \left(1, -\frac{1}{3}\right)$

$Q_1\left(\frac{4}{-4}, \frac{6}{-4}\right) = \left(-1, -\frac{3}{2}\right)$

$Q_0 Q_1$ is a line segment

d) $\overrightarrow{P_0P_1} = (-1,7,5)$

$P_B(-1-t, -3+7t, 1+5t)$

$Q_B\left(\frac{-3+7t}{2+1+t}, \frac{1+5t}{2+1+t}\right) = \left(\frac{-3+7t}{3+t}, \frac{1+5t}{3+t}\right)$

$V = \lim_{t\to\infty} Q_B = (7,5)$

e) $\cancel{\overrightarrow{EP_B} = (-3\ldots, -3+7t, 1+5t)}$

$\overrightarrow{EP}_B = (-3-t, -3+7t, 1+5t)$

$Q_B = [2+(-3-t)t_1, (-3+7t)t_1, (1+5t)t_1)$

$2+(-3-t)t_1 = 1$

$t_1 = \frac{1}{t+3}$

$(1+5t)\cdot\frac{1}{t+3} \le 1$

$\frac{5t+1}{t+3} - 1 \le 0$

$\frac{4t-2}{t+3} \le 0$

$-1-t<2 \quad t>-3$

$-3 \quad \frac{1}{2}$

$t \in [0, \frac{1}{2}]$ was hidden

P2 a) $w = x^y = e^{y\ln x}$

$\frac{\partial w}{\partial x} = \frac{y}{x}\cdot e^{y\ln x} \qquad \frac{\partial w}{\partial y} = \ln x\, e^{\ln x\, y}$

$\Delta w \approx w_x(x_0,y_0)\Delta x + w_y(x_0,y_0)\Delta y$

$x_0 = y_0 = 2$. $\Delta x = -0.02$, $\Delta y = 0.01$

$1.98^{2.01} \approx 2^2 - 2^2\cdot 0.02 + \ln 2 \cdot 2^2 \cdot 0.01$

$\approx 4 - 0.08 + 0.69\times 0.04 = \cancel{3.95}\ 3.947$

b) $w_x(2,2) = 2^2 = 4 \qquad w_y(2,2) = 4\ln 2 < w_x$

$\therefore$ more sensitive to Δx.

P3 a) $f_x < 0$, $f_y < 0$.

~~f_y~~ $f_x < 0$, $f_y \approx 0$.

b) $\frac{\partial}{\partial x} f = 3x^2 - y^2 - 8x + 3 + 2yx$

$\frac{\partial}{\partial y} f = -2xy + x^2$

$f_x(1, 1.5) = 3 - 2.25 - 8 + 3 + 3 = -1.15$

$f_y(1, 1.5) = -3 + 1 = -2$.

$f_x(1.2, 0.6) = 3 \times 1.44 - 0.36 - 9.6 + 3 + 1.44 = -1.2$

$f_y(1.2, 0.6) = -1.44 + 1.44 = 0$.

c)

PS4 14/3/2015 Lec 10

14-1 a) $d^2 = x^2 + y^2 + z^2$

$= x^2 + y^2 + \frac{1}{xy}$

$\frac{\partial d^2}{\partial x} = 2x + \frac{1}{y} \cdot \left(-\frac{1}{x^2}\right)$ $\quad \frac{\partial d^2}{\partial y} = 2y + \frac{1}{x} \cdot \left(-\frac{1}{y^2}\right)$

$\begin{cases} 2x - \frac{1}{x^2 y} = 0 \\ 2y - \frac{1}{x y^2} = 0 \end{cases}$ $\quad x = y = 2^{-\frac{1}{4}}$

$2x = \frac{1}{x^3}$ $\quad 2x^4 = 1$

$d^2 = 2\sqrt{\frac{1}{2}} + \sqrt{2} = 2\sqrt{2}$, $\quad d = \sqrt{2\sqrt{2}}$ $\quad z = \pm\frac{1}{\sqrt{xy}} = 2^{\frac{1}{4}}$

$\therefore (x, y, z) = (2^{-\frac{1}{4}}, 2^{-\frac{1}{4}}, 2^{\frac{1}{4}})$

b) $d^2 = x^2 + y^2 + z^2 = y^2 + z^2 + yz + 1$

$\frac{\partial d^2}{\partial y} = 2y + z$ $\quad \frac{\partial d^2}{\partial z} = 2z + y$

$\begin{cases} 2y + z = 0 \\ 2z + y = 0 \end{cases}$ $\quad y = z = 0$. $\quad x = yz + 1 = 1$

$\therefore (1, 0, 0)$.

2 

$A = 3xy + 4xz + 2yz$ $\quad V = xyz = 1$

$= 3xy + \frac{4}{y} + \frac{2}{x}$