PS4 14/3/2015 Lec 10

2f-1a) $d^2 = x^2+y^2+z^2$

$d^2 = x^2+y^2+\frac{1}{xy}$

$\frac{\partial d^2}{\partial x} = 2x + \frac{1}{y}\cdot\left(-\frac{1}{x^2}\right)$ $\frac{\partial d^2}{\partial y} = 2y + \frac{1}{x}\cdot\left(-\frac{1}{y^2}\right)$

$\begin{cases} 2x - \frac{1}{x^2y} = 0 \\ 2y - \frac{1}{xy^2} = 0 \end{cases}$ $x=y=2^{-\frac{1}{4}}$

$2x = \frac{1}{x^3}$ $2x^4=1$

$d^2 = 2\sqrt{\frac{1}{2}} + \sqrt{2} = 2\sqrt{2}$ $d = \sqrt{2\sqrt{2}}$ $z = \sqrt{\frac{1}{xy}} = 2^{\frac{1}{4}}$

$\therefore (x,y,z) = (2^{-\frac{1}{4}}, 2^{-\frac{1}{4}}, 2^{\frac{1}{4}})$

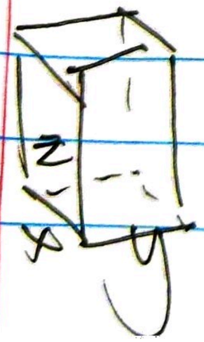
b) $d^2 = x^2+y^2+z^2 = y^2+z^2+yz+1$

$\frac{\partial d^2}{\partial y} = 2y+z$ $\frac{\partial d^2}{\partial z} = 2z+y$

$\begin{cases} 2y+z=0 \\ 2z+y=0 \end{cases}$ $y=z=0$. $x = yz+1 = 1$

$\therefore (1,0,0)$

2



$A = 3xy+4xz+2yz$ $V = xyz = 1$

$= 3xy + \frac{4}{y} + \frac{2}{x}$

$3xy + \frac{4}{y} + \frac{2}{x}$

$\frac{\partial}{\partial x}A = 3y - \frac{2}{x^2} \qquad \frac{\partial}{\partial y}A = 3x - \frac{4}{y^2}$

$$\begin{cases} 3y - \frac{2}{x^2} = 0 \\ 3x - \frac{4}{y^2} = 0 \end{cases} \qquad 3y = \frac{2}{x^2} \qquad 3x^2y = 2 \qquad y = \frac{2}{3x^2} \quad y^2 = \frac{4}{9x^4}$$

$3x - 9x^4 = 0$

$3x^3 - 1 = 0$

$x^3 = \frac{1}{3} \quad x = \sqrt[3]{\frac{1}{3}} \qquad y = \frac{2}{3^{\frac{1}{3}}} \qquad 2 \times 3^{-\frac{1}{3}} = 2\sqrt[3]{\frac{1}{3}}$

$\frac{x}{y} = \frac{1}{2}$

2(1+a) $w = \sum_{i}^{n} (y_i - (ax_i + b))^2$

$\frac{\partial w}{\partial a} = \sum_{i}^{n} 2(y_i - (ax_i + b))(-x_i)$

$= \sum_{i}^{n} -2x_iy_i + 2ax_i^2 + 2bx_i = 0$

$a\sum_{i}^{n} x_i^2 + b\sum_{i}^{n} x_i = \sum_{i}^{n} x_iy_i$

$\frac{\partial w}{\partial b} = \sum_{i}^{n} 2(y_i - (ax_i + b)) = 0$

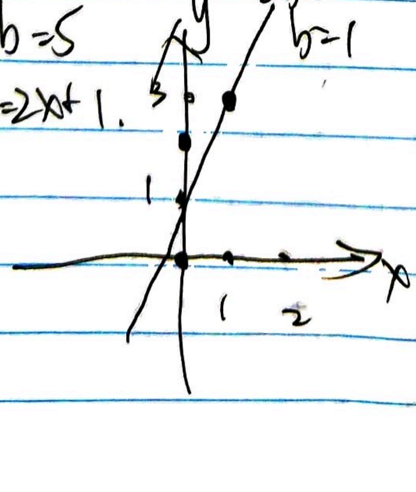
$\sum_{i}^{n} y_i = a\sum_{i}^{n} x_i + nb$

$$\begin{cases} (\sum x_i^2)a + (\sum_{i}^{n} x_i)b = \sum x_iy_i \\ (\sum x_i)a + nb = \sum y_i \end{cases}$$

$\sum x_i = 1 \quad \sum x_i^2 = 1 \quad \sum y_i = 5 \quad \sum x_iy_i = 3$

$$\begin{cases} a + b = 3 \\ a + 3b = 5 \end{cases} \qquad 2b = 2 \quad b = 1 \quad a = 2.$$

$\therefore y = 2x + 1.$



b) $\sum x_i = 3 \quad \sum x_i^2 = 5 \quad \sum y_i = 3 \quad \sum x_i y_i = 4 \quad n = 3.$

$$\begin{cases} 5a + 3b = 4 \\ 3a + 3b = 3 \end{cases} \qquad 2a = 1 \quad a = \tfrac{1}{2}, \quad b = \tfrac{1}{2}$$

4 ~~$\sum d^2$~~ z_i

$$w = \sum d^2 = \sum (z_i - (a + bx_i + cy_i))^2$$

$$\frac{\partial w}{\partial a} = \sum 2(z_i - (a + bx_i + cy_i))(-1) = 0$$

$$\sum z_i - na - \sum bx_i - \sum cy_i = 0$$

$$\frac{\partial w}{\partial b} = \sum 2(z_i - (a + bx_i + cy_i))(-x_i) = 0$$

$$\sum x_i z_i - \sum a x_i - \sum b x_i^2 - \sum c x_i y_i = 0$$

$$(\sum x_i) a + (\sum x_i^2) b + (\sum x_i y_i) c = \sum x_i z_i$$

$$\frac{\partial w}{\partial c} = \sum 2(z_i - (a + bx_i + cy_i))(-y_i) = 0$$

$$\sum y_i z_i = \sum a y_i + \sum b x_i y_i + \sum c y_i^2$$

$$(\sum y_i) a + (\sum x_i y_i) b + (\sum y_i^2) c = \sum y_i z_i$$

$$\begin{cases} na + (\sum x_i) b + (\sum y_i) c = \sum z_i \\ (\sum x_i) a + (\sum x_i^2) b + (\sum x_i y_i) c = \sum x_i z_i \\ (\sum y_i) a + (\sum x_i y_i) b + (\sum y_i^2) c = \sum y_i z_i \end{cases}$$

Lec 11 14/3/2025

2H-1 a) $f_x = 2x - y - 3$

$f_y = -x - 4y - 3$

$$\begin{cases} 2x - y = 3 \\ -x - 4y = 3 \end{cases} \Rightarrow \begin{cases} y = -1 \\ x = 1 \end{cases}$$

$f_{xx} = 2 \qquad f_{xy} = -1 \qquad f_{yy} = -4$

~~$f_{xx} f_{yy} - (f_{xy})^2 = -8 - 1 = -9 < 0$~~

$AC - B^2 < 0 \qquad \therefore$ saddle point.

(b) $f_x = 8x^3 - y$ $\quad \begin{cases} 8x^3 - y = 0 \\ y - \frac{x}{2} = 0 \end{cases}$ $\quad 16x^3 - x = 0$

$f_y = 2y - x$ $\quad 16x(x^2-1) = 0 \quad x = 0, \pm 1$

$\therefore (x,y) = (0,0), (1, \frac{1}{2}), (-1, -\frac{1}{2}) \quad y = 0, \pm\frac{1}{2}$

$f_{xx} = 24x^2 \quad f_{xy} = -1 \quad f_{yy} = 2$

1) $x = 0 \quad A = 0$.

$AC - B^2 = 0 - 1 = -1 < 0 \quad \therefore (0,0)$ is a saddle point.

2) $x = 1$. $A = 24$

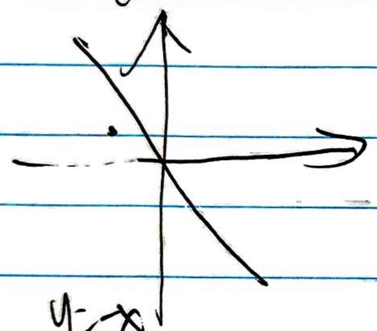
$AC - B^2 = 48 - 1 = 47 \quad A > 0$,

$(1, \frac{1}{2})$ is a local minimum

3) $x = -1 \quad A = 24$

$\therefore (-1, -\frac{1}{2})$ is a local minimum

3



$y = -x$

$f_x = 2x + 2$

$f_y = 2y + 4$

$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \Rightarrow \begin{cases} x = -1 \\ y = -\frac{1}{2} \end{cases}$

$f(x,y) = x^2 + x^2 + 2x - 4x - 1 = 2x^2 - 2x - 1 = g(x)$

$g_{max}(x)$ DNE

$g_{min}(x) = \frac{1}{2} - 1 - 1 = -\frac{3}{2}$

$\lim_{x \to \infty} f(x,y) = \infty \quad \lim_{y \to \infty} f(x,y) = \infty$.

$\therefore$ No maximum

minimum: $-\frac{3}{2}$

4 $f(x,y) = xy - x - y + 2$

$\begin{cases} f_x = y - 1 = 0 \\ f_y = x - 1 = 0 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}$

$f_{xx}=0$, $f_{xy}=1$, $f_{yy}=0$
$=A$ $\quad =B$ $\quad =C$

$AC-B^2=-1<0$. $\therefore (1,1)$ is a saddle point.

a) $x\geq 0, y\geq 0$

No ~~max~~ local max or min in R

~~$\therefore f(0,y)$ $f(x,y)_{max}=\max\{f(0,y), f(x,0)\}$~~

~~$f(x,y)_{min}=\min\{f(0,y), f(x,0)\}$~~

~~$\therefore f_{min}(x,y)=\min\{-y+2, -x+2\}$~~

$f_{min}(x,y)=\min\{-y+2, -x+2, \lim_{x\to\infty} f(x,y), \lim_{y\to\infty} f(x,y), \lim_{x\to\infty, y\to\infty}$

$\therefore$ No minimum

$f_{max}(x,y)=\max\{-y+2, -x+2, \ldots, \lim_{x\to\infty, y\to\infty} f(x,y)$

$\therefore$ No maximum.

b) $f_{min}(x,y)=\min\{f(0,y), f(2,y), f(x,2), f(y,0)\}$

$f(0,y)=-y+2$, $y\in[0,2]$, $\therefore f(0,y)\in[0,2]$

$f(2,y)=2y-2-y+2=y$ $\quad \therefore f(2,y)\in[0,2]$

$f(x,0)=-x+2$ $\quad \in[0,2]$

$f(x,2)=x$ $\quad \in[0,2]$

$\therefore f_{min}(x,y)=0$, $f_{max}(x,y)=2$.

c) saddle.

6

$\begin{cases} x+y+z=4 \\ ~~f(x,y,z)~~ A=xy+\frac{z^2}{4} \end{cases}$

$x=4-y-z$

~~$f(x,y,z)$~~ ~~$f(x)$~~ $A=f(y,z)=4y-y^2-yz+\frac{z^2}{4}$ $\quad y\geq 0, z\geq 0$

$f_y=4-2y-z$ $\quad \begin{cases} f_y=0 \\ f_z=0 \end{cases} \Rightarrow \begin{cases} 2y+z=4 \\ 2y-z=0 \end{cases} \Rightarrow \begin{cases} y=1 \\ z=2 \end{cases}$

$f_z=-y+\frac{z}{2}$

$f(1,2) = 4 - 1^2 - 2 + 1 = 2$

$\lim_{y \to 0} f(y,z) = \frac{z^2}{4} \in (0, +\infty)$

$\lim_{z \to 0} f(y,z) = 4y - y^2 = y(4-y) \in (0, 4)$

$\therefore$ no min. $f(x,y)_{max} = 4$.

b) $f_y = 4 - 2y - z \quad f_z = -y + \frac{z}{2} \quad (1,2)$

$f_{yy} = -2 \quad f_{yz} = -1 \quad f_{zz} = \frac{1}{2}$

$AC - B^2 = -1 - 1 = -2 < 0$.

$\therefore$ saddle point.

Lec12 20/3/2025.

2(-1 a) $w_x = \frac{1}{x} \quad w_y = \frac{1}{y} \quad w_z = \frac{1}{z}$

$dw = \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}$

b) $\frac{\partial w}{\partial x} = y^2 z \cdot 3x^2 = 3y^2 z x^2$

$\frac{\partial w}{\partial y} = x^3 z \cdot 2y = 2x^3 z y$

$\frac{\partial w}{\partial z} = x^3 y^2$

$dw = 3y^2 z x^2 dx + 2x^3 z y\, dy + x^3 y^2 dz$

c) $\frac{\partial z}{\partial x} = \frac{(x+y) - (x-y)}{(x+y)^2} = \frac{2y}{(x+y)^2}$

$\frac{\partial z}{\partial y} = -\frac{2x}{(x+y)^2}$

$dz = \frac{2y}{(x+y)^2} dx - \frac{2x}{(x+y)^2} dy$

d $\cancel{\frac{\partial w}{\partial u}}$

$$\frac{\partial w}{\partial u} = \frac{1}{\sqrt{1-\frac{u^2}{t^2}}}\cdot\frac{1}{t} = \frac{1}{\sqrt{t^2-u^2}}$$

$$\frac{\partial w}{\partial t} = \frac{1}{\sqrt{1-\frac{u^2}{t^2}}}\cdot\left(-\frac{u}{t^2}\right) = \left(-\frac{u}{t}\right)\left(\frac{1}{\sqrt{t^2-u^2}}\right)$$

$$dw = \frac{1}{\sqrt{t^2-u^2}}\,du - \frac{u}{t}\cdot\frac{1}{\sqrt{t^2-u^2}}\,dt$$

2 $f(x,y,z) = xyz \qquad (x,y,z) = (5,10,20)$

$$\Delta f = \frac{\partial f}{\partial x}\cdot\Delta x + \frac{\partial f}{\partial y}\cdot\Delta y + \frac{\partial f}{\partial z}\Delta z$$

$$= yz\Delta x + xz\Delta y + xy\Delta z$$

$$= 200\,\Delta x + 100\,\Delta y + 50\,\Delta z$$

$$= \pm 20 \pm 10 \pm 5 = \pm 35\ cm^3$$

3 $A = \frac{1}{2}ab\sin\theta$

a) $\frac{\partial A}{\partial a} = \frac{1}{2}b\sin\theta, \quad \frac{\partial A}{\partial b} = \frac{1}{2}a\sin\theta, \quad \frac{\partial A}{\partial \theta} = \frac{1}{2}ab\cos\theta$

$dA = \frac{1}{2}b\sin\theta\,da + \frac{1}{2}a\sin\theta\,db + \frac{1}{2}ab\cos\theta\,d\theta$

b) $\frac{\partial A}{\partial a} = \frac{1}{2} \quad \frac{\partial A}{\partial b} = \frac{1}{4} \quad \frac{\partial A}{\partial \theta} = \frac{\sqrt{3}}{4}\times 1\times 2 = \frac{\sqrt{3}}{2}$

$\therefore$ θ is most sensitive.

b is least sensitive.

c) $\Delta A = \frac{1}{2}\times .02 + \frac{1}{4}\times .02 + \frac{\sqrt{3}}{2}\times .02$

$\approx .01 + .005 + .017$

$\approx .032 \approx .03$

5 a) $d\left(\frac{1}{w}\right) = d\left(\frac{1}{t} + \frac{1}{u} + \frac{1}{v}\right)$

$\frac{dw}{w^2} = \frac{dt}{t^2} + \frac{du}{u^2} + \frac{dv}{v^2}$

$d(u^2 + 2v^2 + 3w^2) = 0$

b) $2u\,du + 4v\,dv + 3w\,dw = 0$

$dw = \frac{1}{3w}(2u\,du + 4v\,dv)$

2E-1 a) $dw = yz\,dx + xz\,dy + xy\,dz$

$\frac{dw}{dt} = yz + xz(2t) + xy(3t^2)$

$= t^5 + 2t^5 + 3t^5 = 6t^5$

b) $\frac{dw}{dt} = 2x\frac{dx}{dt} - 2y\frac{dy}{dt}$

$= 2\cos t \cdot (-\sin t) - 2\sin t \cdot \cos t$

$= -4\sin t\cos t = -2\sin 2t$

c) $\frac{dw}{dt} = \frac{2u}{u^2+v^2}\frac{du}{dt} + \frac{2v}{u^2+v^2}\frac{dv}{dt}$

$= \frac{4\cos t}{4}\cdot(-2\sin t) + \frac{4\sin t}{4}\cdot(2\cos t)$

$= 0$

2 b) $\frac{dw}{dt} = y\frac{dx}{dt} + x\frac{dy}{dt}$

$= \cancel{-y\sin t} - \sin^2 t + \cos^2 t = \cos 2t = 0 \Rightarrow 2t = \frac{\pi}{2}$

$t =$

c) $\nabla f = \langle 1, -1, 2 \rangle$

$\frac{df}{dt} = \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} = 1 + 2t + 3t^2 = 6$

5 $dw = w_x\,dx + w_y\,dy = w_x(\cos\theta\,dr - r\sin\theta\,d\theta) + w_y($

$dw = w_r\,dr + w_\theta\,d\theta$

$= (w_x\cos\theta + w_y\sin\theta)\,dr + (r\cos\theta\,w_y - r\sin\theta\,w_x)\,d\theta$

$w_r = w_x\cos\theta + w_y\sin\theta$

$w_\theta = r\cos\theta\,w_y - r\sin\theta\,w_x$

1.

$(w_r)^2 = (w_x)^2 + (w_y)^2 + 2\sin\theta\cos\theta\, w_x w_y$

$(w_\theta)^2 = r^2\left((w_x)^2 + (w_y)^2 - 2\cos\theta\sin\theta\, w_x w_y\right)$

$(w_r)^2 + \frac{1}{r^2}(w_\theta)^2 = (w_x)^2 + (w_y)^2.$

2(a) $dw = f'(u)\cdot du$

$= f'(u)\cdot\left(-\frac{y}{x^2}dx + \frac{1}{x}dy\right)$

$\frac{\partial w}{\partial x} = f'(u)\cdot\left(-\frac{y}{x^2}\right) \quad \frac{\partial w}{\partial y} = f'(u)\cdot\left(\frac{1}{x}\right)$

$\therefore x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} = 0.$

b) $dw = df = f'(u)\cdot du$

$\frac{\partial w}{\partial x} = f'(u)\cdot\frac{\partial u}{\partial x} = 2x\, f'(u)$

$\frac{\partial w}{\partial y} = f'(u)\cdot\frac{\partial u}{\partial y} = -2y\, f'(u)$

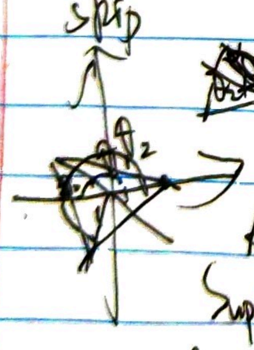
$\therefore y\frac{\partial w}{\partial x} + x\frac{\partial w}{\partial y} = 0.$

P1

PART II 24/3/2025

P1

P2

a) 

$A = \frac{1}{2}\sin\theta_1 + \frac{1}{2}\sin|\theta_2-\theta_1| + \frac{1}{2}\sin\left[(|\theta_2-\theta_1|+\theta_1)\right]$

Suppose $\theta_2 \geq \theta_1$ $\quad 0<\theta_1\leq\theta_2<\pi$

$A = \frac{1}{2}\sin\theta_1 + \frac{1}{2}\sin(\theta_2-\theta_1) - \frac{1}{2}\sin(\theta_2)$

b) $A_{\theta_1} = \frac{1}{2}\cos\theta_1 + \frac{1}{2}(-\cos(\theta_2-\theta_1))$ or $A_{\theta_2} = \frac{1}{2}\cos(\theta_2-\theta_1) - \frac{1}{2}\cos(\theta_2)$

$\begin{cases} A_{\theta_1}=0 \\ A_{\theta_2}=0 \end{cases} \Rightarrow \begin{cases} \cos\theta_1 = \cos(\theta_2-\theta_1) \\ \cos\theta_2 = \cos(\theta_2-\theta_1) \end{cases}$

$\cos\theta_1 - \cos\theta_2 = 0$

$\theta_2 = \theta_1 \quad \cos\theta_1 = -\cos 2\theta_1$

$\theta_1 + \theta_2, \; \theta_2 = 2\pi - \theta_1$

P3

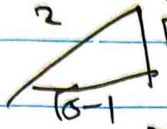
$\cos\theta_1 = \cancel{\cos 2\theta_2} - \cos 2\theta_1 = -(2\cos^2\theta_1 - 1) = -2\cos^2\theta_1 + 1$

$\cos\theta_2 = -\cos 2\theta_2 = 2\cos^2\theta_2 - 1$

2t 1

t -1 $\quad 2t^2 - t - 1 = (2t+1)(t-1)$ $\quad \cancel{\frac{1\pm\sqrt{5}}{2}}$

$2t^2 + t - 1 = (2t-1)(t+1)$ $\quad \cancel{\frac{-1\pm\sqrt{5}}{2}}$



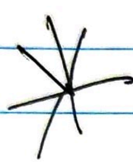
2t 1

t 1 $\quad \therefore t \in [-1, 1]$

$\cancel{\cos\theta_1 = \frac{-1-\sqrt{5}}{2}}$ $\quad \theta_2 = 2\pi - \theta_1$

$4 - (\sqrt{5}+1-\sqrt{6})$

$2\sqrt{3}-2$ $\quad \cancel{\cos\theta_2 = \frac{\sqrt{5}-1}{2}}$



$(\cos\theta_1, \cos\theta_2) = (-\frac{1}{2}, -\frac{1}{2}), \cancel{(1,1)}$

$\therefore \theta_1 = \frac{2\pi}{3}, \quad \theta_2 = \frac{4\pi}{3}$

c) $A(\frac{2\pi}{3}, \frac{4\pi}{3}) = \frac{1}{2}\sin(\frac{2\pi}{3}) + \frac{1}{2}\sin(\frac{2\pi}{3}) - \frac{1}{2}\sin\frac{4\pi}{3}$

$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$

$A(0,0) = 0$

$A(\theta_1, \theta_2) = \frac{1}{2}\sin\theta_1 + \frac{1}{2}\sin(\theta_2 - \theta_1) - \frac{1}{2}\sin\theta_2$

$A(2\pi, 2\pi) = 0$

$A(0, 2\pi) = 0$.

$\therefore A_{max} = \frac{3\sqrt{3}}{4}$ equilateral triangle.

d) $\frac{\partial A}{\partial\theta_1} = \frac{1}{2}\cos\theta_1 - \frac{1}{2}\cos(\theta_2 - \theta_1)$

$\frac{\partial A}{\partial\theta_2} = \frac{1}{2}\cos(\theta_2 - \theta_1) - \frac{1}{2}\cos\theta_2$

$\frac{\partial^2 A}{\partial\theta_1^2} = -\frac{1}{2}\sin\theta_1 - \frac{1}{2}\sin(\theta_2 - \theta_1)$

$\frac{\partial^2 A}{\partial\theta_1\partial\theta_2} = \frac{1}{2}\sin(\theta_2 - \theta_1)$

$\frac{\partial^2 A}{\partial\theta_2^2} = -\frac{1}{2}\sin(\theta_2 - \theta_1) + \frac{1}{2}\sin\theta_2$.

$A = \frac{\partial^2}{\partial\theta_1^2}A(\frac{2\pi}{3}, \frac{4\pi}{3}) = -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$ $\quad B = \frac{\sqrt{3}}{4}$ $\quad C = -\frac{\sqrt{3}}{4}$

$AC - B^2 < 0$ $\quad A < 0$ local Max

P3 a) ~~$w_r = w_x \cdot \frac{\partial x}{\partial r}$~~

$$dw = \frac{\partial w}{\partial x} \cdot dx + \frac{\partial w}{\partial y} \cdot dy$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial r}$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial \theta}$$

$$\begin{bmatrix} w_r \\ w_\theta \end{bmatrix} = \begin{bmatrix} x_r & y_r \\ x_\theta & y_\theta \end{bmatrix} \begin{bmatrix} w_x \\ w_y \end{bmatrix}$$

$$A = \begin{bmatrix} x_r & y_r \\ x_\theta & y_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{bmatrix}$$

b) $dw = w_r\, dr + w_\theta\, d\theta$;

$$r_x = \frac{2x}{2\sqrt{x^2+y^2}} \qquad \theta_x = \frac{1}{1+\frac{y^2}{x^2}} \cdot \left(-\frac{y}{x^2}\right)$$

$$w_x = w_r \cdot r_x + w_\theta \cdot \theta_x \qquad = -\frac{y}{x^2+y^2}$$

$$w_y = w_r \cdot r_y + w_\theta \cdot \theta_y \qquad r_y = \frac{y}{\sqrt{x^2+y^2}} \qquad \theta_y = \frac{1}{1+\frac{y^2}{x^2}} \cdot \left(\frac{1}{x}\right)$$

$$\begin{bmatrix} w_x \\ w_y \end{bmatrix} = \begin{bmatrix} r_x & \theta_x \\ r_y & \theta_y \end{bmatrix} \begin{bmatrix} w_r \\ w_\theta \end{bmatrix} \qquad = \frac{1}{x+\frac{y^2}{x}}$$

$$\therefore B = \begin{bmatrix} r_x & \theta_x \\ r_y & \theta_y \end{bmatrix} = \begin{bmatrix} \frac{x}{\sqrt{x^2+y^2}} & -\frac{y}{x^2+y^2} \\ \frac{y}{\sqrt{x^2+y^2}} & \frac{x}{x^2+y^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{x}{r} & -\frac{y}{r^2} \\ \frac{y}{r} & \frac{x}{r^2} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\frac{\sin\theta}{r} \\ \sin\theta & \frac{\cos\theta}{r} \end{bmatrix}$$

c) $$AB = \begin{bmatrix} \frac{x}{r}\cdot\left(\cos\theta + \frac{y}{x}\sin\theta\right) & \frac{x}{r^2}\left(-\frac{y}{x}\cos\theta + \sin\theta\right) \\ x\left(-\sin\theta + \frac{y}{x}\cos\theta\right) & \frac{x}{r}\left(\frac{y}{x}\sin\theta + \cos\theta\right) \end{bmatrix} = \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

~~$= x\begin{bmatrix} \frac{1}{x}\cos^2\theta & 0 \\ 0 & \frac{x}{\cos^2\theta} \end{bmatrix} = \frac{x}{x}\begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix}$~~ $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.$

d) $\begin{bmatrix} w_x \\ w_y \end{bmatrix} = \begin{bmatrix} r_x & \theta_x \\ r_y & \theta_y \end{bmatrix} \begin{bmatrix} 2 \\ 10 \end{bmatrix}$ $\quad r = 5,\ \theta = \frac{\pi}{2}$

$$= \begin{bmatrix} \cos\theta & -\frac{\sin\theta}{r} \\ \sin\theta & \frac{\cos\theta}{r} \end{bmatrix} \begin{bmatrix} 2 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\frac{1}{5} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 10 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$