18.02. Rec 1 ~~Lec 1~~ 25/2/2025

1A-1 a) $|\hat{i}+\hat{j}+\hat{k}| = \sqrt{1+1+1} = \sqrt{3}$

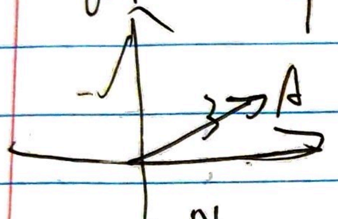
$$\frac{\hat{i}+\hat{j}+\hat{k}}{|\hat{i}+\hat{j}+\hat{k}|} = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$$

b) $|2\hat{i}-\hat{j}+2\hat{k}| = \sqrt{4+1+4} = 3$

$$\text{dir} = \frac{2\hat{i}-\hat{j}+2\hat{k}}{3}$$

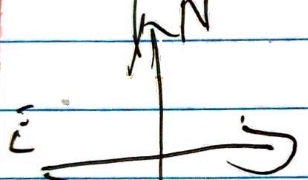
c) $|3\hat{i}-6\hat{j}-2\hat{k}| = \sqrt{9+36+4} = 7$

$$\text{dir} = \frac{3\hat{i}-6\hat{j}-2\hat{k}}{7}$$

5. 

$A = 3\cos 30^\circ\,\hat{i} + 3\sin 30^\circ\,\hat{j}$

$= \frac{3\sqrt{3}}{2}\hat{i} + \frac{3}{2}\hat{j}$

6. 

$v = 200\vec{j} - 50\vec{i}$

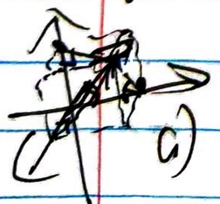
7. $\vec{A} = a\hat{i} + b\hat{j}$

a) $\vec{A}' = -a\hat{j} + b\hat{i}$

b) $\vec{A}'' = a\hat{j} - b\hat{i}$

c) $\hat{j}' = \frac{3}{5}\hat{j} - \frac{4}{5}\hat{i}$

8. $\text{dir}\,\vec{A} = \frac{a\hat{i}+b\hat{j}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$



a) $= \frac{a}{\sqrt{a^2+b^2+c^2}}\hat{i} + \frac{b}{\sqrt{a^2+b^2+c^2}}\hat{j} + \frac{c}{\sqrt{a^2+b^2+c^2}}\hat{k}$

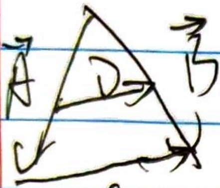
$= \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$

b) $\cos\alpha = \frac{a}{\sqrt{a^2+b^2+c^2}}$ $\cos\beta = \frac{b}{\sqrt{a^2+b^2+c^2}}$ $\cos\gamma = \frac{c}{\sqrt{a^2+b^2+c^2}}$

$|-\hat{i}+2\hat{j}+2\hat{k}| = \sqrt{1+4+4} = 3$

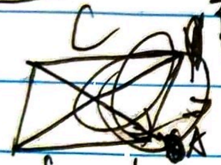
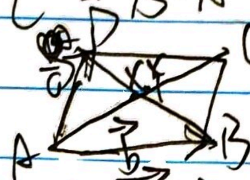
$\therefore \cos\alpha = -\frac{1}{3}$ $\cos\beta = \frac{2}{3}$ $\cos\gamma = \frac{2}{3}$

c) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{a^2+b^2+c^2}{a^2+b^2+c^2} = 1$

9 

$\vec{D} = \frac{\vec{B}-\vec{A}}{2}$ $\vec{D} = \frac{1}{2}\vec{C}$

$\vec{C} = \vec{B}-\vec{A}$

11  

$\vec{BD} = \vec{a}-\vec{b}$

$\vec{AC} = \vec{a}+\vec{b}$

Let the midpoint of $\vec{BD}$ is X, midpoint of $\vec{AC}$ is Y.

$\vec{AY} = \frac{1}{2}\vec{AC} = \frac{\vec{a}+\vec{b}}{2}$

$\vec{AX} = \vec{AD}+\vec{DX} = \vec{AD}+\frac{1}{2}\vec{DB} = \vec{a}+\frac{\vec{b}-\vec{a}}{2} = \frac{\vec{a}}{2}+\frac{\vec{b}}{2} = \vec{AY}$

$\therefore X = Y$

$\therefore$ AC and BD bisect each other.

Lec 1 25/2/2025

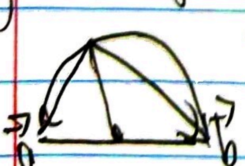
13-1a) $\langle c, 2, -1\rangle \cdot \langle 1, -1, 2\rangle$

$= c-2-2 = 0$ $c=4$.

b) $c-2-2 > 0$

$c > 4$.

5 b) $\cancel{3\hat{i}+2\hat{j}}$ $\frac{3\hat{i}+2\hat{j}-6\hat{k}}{\sqrt{9+4+36}} = \frac{3\hat{i}+2\hat{j}-6\hat{k}}{7}$

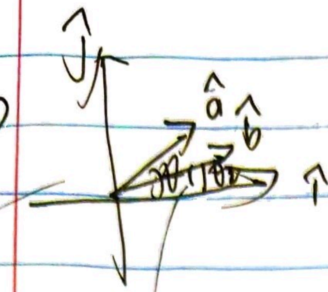
12 

$\left|\frac{\vec{a}+\vec{b}}{2}\right| = \left|\frac{\vec{b}-\vec{a}}{2}\right|$ $a^2+b^2+2\vec{a}\cdot\vec{b} = a^2+b^2-2\vec{a}\cdot\vec{b}$

$4\vec{a}\cdot\vec{b} = 0$

$\vec{a}\cdot\vec{b} = 0$

$\angle\langle\vec{a},\vec{b}\rangle = 90°$

13 

~~$\cos\theta_1 = |a||i|\ \hat{a}\cdot\hat{i}$~~ $\sin\theta_1 = \hat{a}\cdot\hat{j}$

~~$\cos\theta_2 = \hat{b}\,\hat{i}$~~ $\sin\theta_2 = \hat{b}\cdot\hat{j}$

$$\cos(\theta_1-\theta_2) = \frac{\hat{a}\cdot\hat{b}}{|a||b|} = \hat{a}\cdot\hat{b}$$

$$= \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$$

$\vec{a} = \cos\theta_1\hat{i} + \sin\theta_1\hat{j}$

$\vec{b} = \cos\theta_2\hat{i} + \cos\theta_2\hat{j}$

Lec 2

1-1 a) $\begin{vmatrix} 1 & 4 \\ 2 & -1 \end{vmatrix} = -1-8 = -9$

b) $\begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = -6-4 = -10$

2 a) $\begin{vmatrix} -1 & 0 & 4 \\ 1 & 2 & 2 \\ 3 & -2 & -1 \end{vmatrix} = -1\begin{vmatrix} 2 & 2 \\ -2 & -1 \end{vmatrix} + 4\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix}$

$= -1(-2+4) + 4(-2-6) = -2-32 = -34$

b) $\begin{vmatrix} -1 & 0 & 4 \\ 1 & 2 & 2 \\ 3 & -2 & -1 \end{vmatrix} = -1\begin{vmatrix} 2 & 2 \\ -2 & -1 \end{vmatrix} - 1\begin{vmatrix} 0 & 4 \\ -2 & -1 \end{vmatrix} + 3\begin{vmatrix} 0 & 4 \\ 2 & 2 \end{vmatrix}$

$= -1(-2+4) - (+8) + 3(-8)$

$= -2-8-24 = -34$

3 a) $A = \frac{1}{2}\left|\begin{pmatrix} 1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right| = \frac{1}{2}(-1+2) = \frac{1}{2}$

b) $A = \frac{1}{2}\left(\begin{pmatrix} 1-1 \\ 1-2 \end{pmatrix} \times \begin{pmatrix} 2-1 \\ 3-2 \end{pmatrix}\right) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ -3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = \frac{1}{2}(+3) = \frac{3}{2}$

5 a) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc$

$ad+\lambda ab - (bc+\lambda ab) = ad-bc$

$\begin{vmatrix} a & b \\ c & d+\lambda b \end{vmatrix} =$ ~~$ad+\lambda ac - (bc+\lambda ac) = ad-bc = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$~~

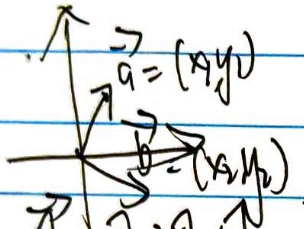
11.2

6 $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$

$\begin{vmatrix} a & b & c \\ d+\lambda g & e+\lambda h & f+\lambda i \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e+\lambda h & f \\ h+\lambda i & i \end{vmatrix} \cancel{- b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e+\lambda h \\ g & h+\lambda i \end{vmatrix}}$

$= a\begin{vmatrix} e+\lambda h & f+\lambda i \\ h & i \end{vmatrix} - b\begin{vmatrix} d+\lambda g & f+\lambda i \\ g & i \end{vmatrix} + c\begin{vmatrix} d+\lambda g & e+\lambda h \\ g & h \end{vmatrix}$

$= a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$

7 

$f(x_1, x_2, y_1, y_2) = \vec{a} \times \vec{b} = |a||b| \sin\theta$

$f(x_1, x_2, y_1, y_2) = |\alpha| = 1$

10-1 a) $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ $\quad \vec{B} = 2\hat{i} - \hat{j} - \hat{k}$

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 2 & -1 & -1 \end{vmatrix} = \hat{i}\begin{vmatrix} -2 & 1 \\ -1 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix}$

$= 3\hat{i} + 3\hat{j} + 3\hat{k}$

b) $\vec{A} = 2\hat{i} - 3\hat{k}$ $\quad \vec{B} = \hat{i} + \hat{j} - \hat{k}$

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -3 \\ 1 & 1 & -1 \end{vmatrix} = \hat{i}\begin{vmatrix} 0 & -3 \\ 1 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & -3 \\ 1 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$

$= 3\hat{i} - \hat{j} + 2\hat{k}$

2 $\vec{PQ} = \langle 1, 1, 1 \rangle$

$\vec{PR} = \langle -3, 1, -2 \rangle$

$A = \frac{1}{2} \vec{PQ} \times \vec{PR} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -3 & 1 & -2 \end{vmatrix} = \frac{1}{2}\left(\hat{i}\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 1 \\ -3 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 1 \\ -3 & 1 \end{vmatrix}\right)$

$= \frac{1}{2}\left[\hat{i}(-2-1) - \hat{j}(-2-(-3)) + \hat{k}(1-(-3))\right] = \frac{1}{2}(2\hat{i} - \hat{j} + 4\hat{k})$

$= \hat{i} - \frac{1}{2}\hat{j} + 2\hat{k}$

1D-3 $\vec{A} = 2\hat{i} - \hat{j}$ $\quad \vec{B} = \hat{i} + 2\hat{j} + \hat{k}$

$\hat{i}' = \frac{2\hat{i}-\hat{j}}{\sqrt{4+1}} = \frac{2}{\sqrt5}\hat{i} - \frac{1}{\sqrt5}\hat{j}$

$\hat{j}' = \frac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{1+4+1}} = \frac{1}{\sqrt6}\hat{i} + \frac{2}{\sqrt6}\hat{j} + \frac{1}{\sqrt6}\hat{k}$

$\hat{k}' = \hat{i}' \times \hat{j}' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{2}{\sqrt5} & -\frac{1}{\sqrt5} & 0 \\ \frac{1}{\sqrt6} & \frac{2}{\sqrt6} & \frac{1}{\sqrt6} \end{vmatrix}$

$= \hat{i}\begin{vmatrix} -\frac{1}{\sqrt5} & 0 \\ \frac{2}{\sqrt6} & \frac{1}{\sqrt6} \end{vmatrix} - \hat{j}\begin{vmatrix} \frac{2}{\sqrt5} & 0 \\ \frac{1}{\sqrt6} & \frac{1}{\sqrt6} \end{vmatrix} + \hat{k}\begin{vmatrix} \frac{2}{\sqrt5} & -\frac{1}{\sqrt5} \\ \frac{1}{\sqrt6} & \frac{2}{\sqrt6} \end{vmatrix}$

$= \hat{i}\left(-\frac{1}{\sqrt{30}}\right) - \hat{j}\left(\frac{2}{\sqrt{30}}\right) + \hat{k}\left(\frac{4}{\sqrt{30}} - \left(-\frac{1}{\sqrt{30}}\right)\right)$

$= -\frac{1}{\sqrt{30}}\hat{i} - \frac{2}{\sqrt{30}}\hat{j} + \frac{5}{\sqrt{30}}\hat{k}$

4 Let $\hat{i} = \langle 1, 0 \rangle$ $\quad \hat{j} = \langle 0, 1 \rangle$

$(\hat{i}\times\hat{i})\times\hat{j} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{vmatrix} \times \hat{j} = \vec{0} \times \hat{j} = \vec{0}$

$\hat{i}\times(\hat{i}\times\hat{j}) = \langle 1, 0, 0 \rangle \times \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$

$= \hat{i} \times (\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \hat{j} \neq (\hat{i}\times\hat{i})\times\hat{j}$

5a) If $|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|$ $\quad |\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$ $\quad \vec{A}\perp\vec{B}$ ∴ $\sin\theta_2$ $\theta_2 = \frac{\pi}{2}$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \sqrt{\hat{i}(a_2b_3 - a_3b_2) + \hat{j}(a_1b_3 - a_3b_1) + \hat{k}(a_1b_2 - a_2b_1)}$

$= \sqrt{a_2^2b_3^2 + a_3^2b_2^2 - 2a_2a_3b_2b_3 + a_1^2b_3^2 + a_3^2b_1^2 - 2a_1a_3b_1b_3 + a_1^2b_2^2 + a_2^2b_1^2 - 2a_1a_2b_1b_2}$

$|\vec{A}||\vec{B}| = \sqrt{(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)}$ //

$a_1^2b_1^2 + a_2^2b_2^2 + a_3^2b_3^2 = -2(a_2a_3b_2b_3 + a_1a_3b_1b_3 + a_1a_2b_1b_2)$

b) $|A \times B| = A \cdot B$

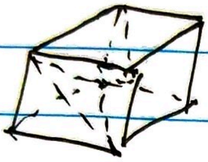
$|A \times B| = |A||B| \sin\theta = |A||B| \cos\theta$

$\sin\theta = \cos\theta$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

7 $\vec{PQ} = \langle -2, 1, 1 \rangle$ $\quad \vec{PR} = \langle -1, 0, 1 \rangle$ $\quad \vec{PS} = \langle 2, 1, -2 \rangle$



$V = \frac{1}{6} \det(\vec{PQ}, \vec{PR}, \vec{PS})$

$= \frac{1}{6} \begin{vmatrix} -2 & 1 & 1 \\ -1 & 0 & 1 \\ 2 & 1 & -2 \end{vmatrix}$

$= \frac{1}{6}\left(-2(0-1) - 1(2-2) + 1(-1)\right)$

$= \frac{1}{6}(2-1) = \frac{1}{6}$

Lec 3 25/2/2025

(F-5a) $A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

$A^3 = A^2 A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$\cancel{A^n = \begin{bmatrix} n-2 & n-1 \\ n-1 & n \end{bmatrix}}$ $\quad \cancel{A^{k+1} = \begin{bmatrix} k-2 & k-1 \\ k-1 & k \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}}$

$\cancel{= \begin{bmatrix} k-1 & 2k-3 \end{bmatrix}}$

b) $A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ $\quad A^{k+1} = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$

8a) $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & 1 \\ 1 & 4 & -1 \end{bmatrix}$

9a) $A = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & x_{ij} & \vdots \\ x_{n1} & \cdots & x_{nn} \end{bmatrix}$ $\quad A^T = \begin{bmatrix} x_{11} & \cdots & x_{n1} \\ \vdots & x_{ji} & \vdots \\ x_{1n} & \cdots & x_{nn} \end{bmatrix}$

$= \begin{bmatrix} \vec{a}_1 \\ \vec{a}_i \\ \vec{a}_n \end{bmatrix}$ $\quad = \begin{bmatrix} \vec{a}_1 & \cdots & \vec{a}_i & \cdots & \vec{a}_n \end{bmatrix}$,

$A \cdot A^T = \begin{bmatrix} a_1^2 & \cdots & \vec{a}_i\vec{a}_1 & \cdots & \vec{a}_n a_1 \\ \vec{a}_1\vec{a}_i & \cdots & a_i^2 & & \vdots \\ \vdots & & & \ddots & a_n^2 \\ \vec{a}_1\vec{a}_n & & \cdots & & a_n^2 \end{bmatrix} = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & & & \vdots \\ 0 & & 1 & \ddots & 0 \\ 0 & \cdots & & & 1 \end{bmatrix} = I_n$

$\therefore a_i^2 = 1, \quad \vec{a}_i \cdot \vec{a}_j = 0,$

$\therefore$ each row of A is a row vector

and two different rows are othogonal vectors.

14-3 $M = \begin{pmatrix} +3 & -1 & +1 \\ +1 & +3 & +2 \\ -2 & -1 & +2 \end{pmatrix}$ $\quad A^{-1} = \dfrac{\operatorname{adj}(A)}{\det(A)} = \begin{pmatrix} 1 & \frac{1}{3} & -\frac{2}{3} \\ -\frac{1}{3} & 1 & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix}$

$\operatorname{adj}(A) = M^T = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 3 & -1 \\ 1 & 2 & 2 \end{pmatrix}$ $\quad \det(A) = (2+1) - (1) + 1$

$= 3$

$x = A^{-1}b = \begin{pmatrix} 0 \\ \frac{5}{3} \\ \frac{8}{3} \end{pmatrix}$

4 $$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A^{-1}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \frac{1}{3} & -\frac{2}{3} \\ -\frac{1}{3} & 1 & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$= \begin{pmatrix} y_1 + \frac{1}{3}y_2 - \frac{2}{3}y_3 \\ -\frac{1}{3}y_1 + y_2 - \frac{1}{3}y_3 \\ \frac{1}{3}y_1 + \frac{2}{3}y_2 + \frac{2}{3}y_3 \end{pmatrix}$$

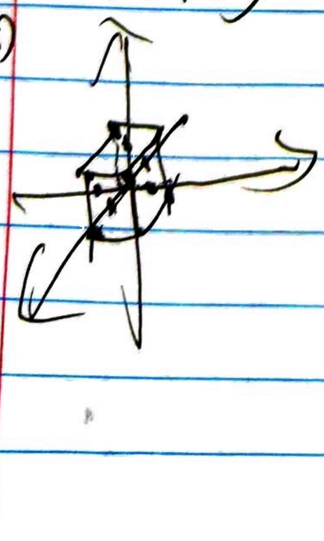
5 $\cancel{B(AB)^{-1}A = \dots = I}A$ $\quad (AB)^{-1}A = B^{-1}I$

$\cancel{(AB)^{-1}A = B^{-1}}I \iff (AB)^{-1}AB = I$

$\cancel{(AB)^{-1}AB = I}$

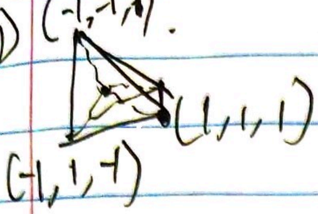
PART B 26/2/2015

P1(a)



$\sqrt{4+4} = \sqrt{2}$.

every edge is the diagonal of a square of the edge length of 2.

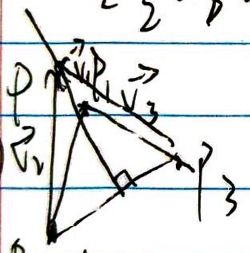
b) 

$$\theta = \arccos\left(\frac{\langle -1,-1,1\rangle \cdot \langle 1,1,1\rangle}{|\langle -1,-1,1\rangle|\,|\langle 1,1,1\rangle|}\right)$$
$$= \arccos\left(\frac{-1-1+1}{3}\right)$$
$$= \arccos\left(\frac{-1}{3}\right) \approx 109.5^\circ$$

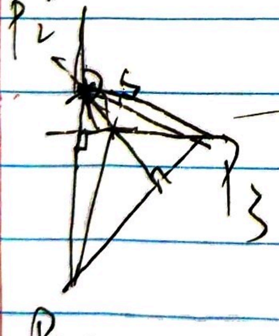
c) $$\theta = \arccos\left(\frac{\langle 2,0,2\rangle \cdot \langle 0,-2,2\rangle}{|\langle 2,0,2\rangle|\,|\langle 0,-2,2\rangle|}\right)$$
$$= \arccos\left(\frac{4}{8}\right) = \arccos\left(\frac{1}{2}\right) = 60^\circ$$

d) $$A = \frac{1}{2}\left|\langle 2,0,2\rangle \times \langle 0,-2,2\rangle\right|$$
$$= \frac{1}{2}\,|\langle 2,0,2\rangle|\,|\langle 0,-2,2\rangle| \sin\theta$$
$$= \frac{1}{2}\cdot 8 \cdot \sin 60^\circ = 2\sqrt{3}$$

P2

a) 

$$(\vec{v_2} - \vec{v_3}) \cdot \vec{v_1} = 0$$

b) 

$$\Rightarrow \begin{cases} \vec{v_2} \cdot (\vec{v_1} - \vec{v_3}) = 0 \\ (\vec{v_2} - \vec{v_3}) \cdot \vec{v_1} = 0 \end{cases}$$

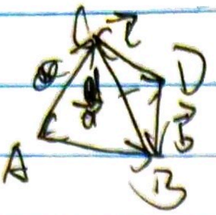
$$\Rightarrow \begin{cases} \vec{v_2} \cdot \vec{v_1} = \vec{v_2} \cdot \vec{v_3} \\ \vec{v_2} \cdot \vec{v_1} = \vec{v_1} \cdot \vec{v_3} \end{cases}$$

c) $\cancel{\vec{v_3}(\vec{v_1} - \vec{v_2})}$

$$\therefore \vec{v_1} \cdot \vec{v_2} = \vec{v_2} \cdot \vec{v_3} = \vec{v_1} \cdot \vec{v_3}$$
$$\vec{v_1} \cdot \vec{v_2} = \vec{v_2} \cdot \vec{v_3}$$
$$\vec{v_3}(\vec{v_1} - \vec{v_2}) = 0$$

$\therefore$ P is also ...

P3



$\vec{v}_1 = \frac{1}{2}\vec{c}\times\vec{b}$

$\vec{v}_2 = \frac{1}{2}\vec{a}\times\vec{c}$

$\vec{v}_3 = \frac{1}{2}\vec{b}\times\vec{a}$

d)

$\vec{v}_4 = \frac{1}{2}(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})$

$= \frac{1}{2}(\vec{b}\times\vec{c} - \vec{a}\times\vec{c} + \vec{a}\times\vec{a} - \vec{a}\times\vec{b})$

$\vec{v}_1+\vec{v}_2+\vec{v}_3+\vec{v}_4 = 0$

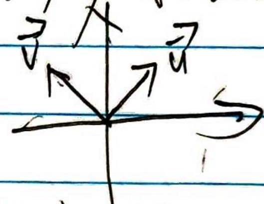
P4 $AA^T = I$

a) $\vec{u} = A_\theta \hat{i} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$

$\vec{v} = A_\theta \hat{j} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$

for $\theta = \frac{\pi}{4}$

$\vec{u} = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix} \quad \vec{v} = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}$



b) $A_{\theta_1}A_{\theta_2} = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix}\begin{pmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{pmatrix}$

$= \begin{pmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & -\sin\theta_2\cos\theta_1 - \sin\theta_1\cos\theta_2 \\ \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2 & -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 \end{pmatrix}$

$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix} = A_{\theta_1+\theta_2}$

c) $A_\theta^{-1} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = A^T$

$= \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix} = A_{-\theta}$

d) $-\frac{1}{\sqrt{2}} = \cos\left(\frac{3\pi}{4}\right)$ ~~$\cos\left(\frac{3\pi}{4}\right)$~~ $\sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$$\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \quad \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix} \quad \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \quad = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

e)

15 a) total amount of $\begin{pmatrix} \text{flour} \\ \text{sugar} \\ \text{egg} \\ \text{butter} \end{pmatrix}$ required respectively.

b) ~~ABA~~ NMX

c) $NMx = y$

$x = (NM)^{-1} y$

$A = (NM)^{-1}$

$$NM = \begin{pmatrix} 0.10 & 0 & 0.13 & 0 \\ 1.76 & 1.00 & 0.01 & 0 \\ 0.01 & 0 & 0.10 & 0.82 \end{pmatrix} \begin{pmatrix} 22 & 40 & 50 \\ 18 & 10 & 3 \\ 5 & 14 & 5 \\ 10 & 10 & 22 \end{pmatrix}$$

$$= \begin{bmatrix} 2.85 & 5.82 & 5.65 \\ 34.77 & 80.54 & 91.05 \\ 8.92 & 10 & 19.04 \end{bmatrix}$$

$$A = (NM)^{-1} = \begin{bmatrix} -0.469 & 0.070 & -0.013 \\ 0.384 & \cancel{5.077\times10^{-3}}\ 0.005 & -0.103 \\ 0.018 & -0.030 & 0.113 \end{bmatrix}$$

d) the amount of reaction is not exactly proportional.