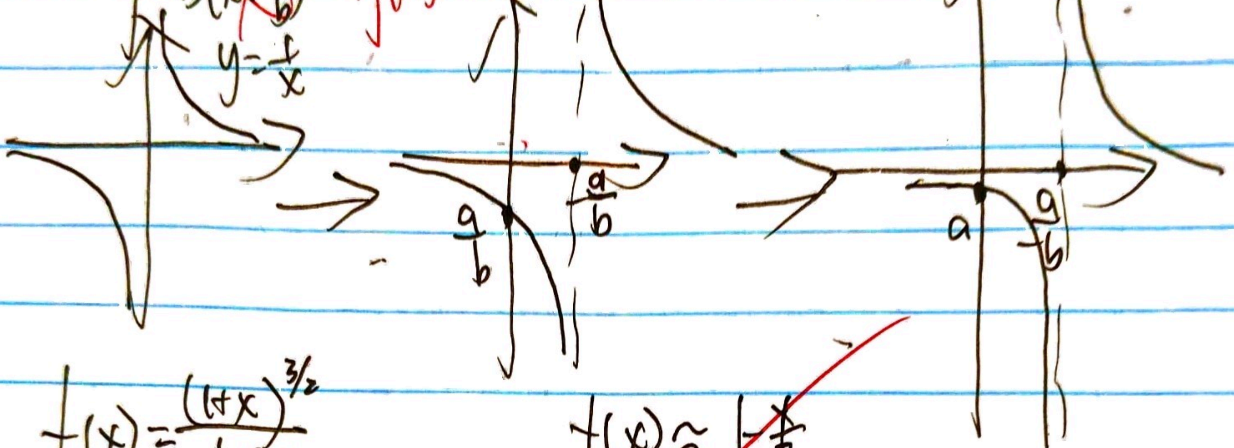
$\ln(u_1 u_2 \cdots u_n) = \ln u_1 + \ln u_2 + \cdots + \ln u_n$.

$D(u_1 u_2 \cdots u_n) \cdot \frac{1}{u_1 \cdots u_n} = \frac{u_1'}{u_1} + \frac{u_2'}{u_2} + \cdots \frac{u_n'}{u_n}$

$D(u_1 u_2 \cdots u_n) = u_1' u_2 \cdots u_n + u_1 u_2' \cdots u_n + \cdots + u_1 u_2 \cdots u_n'$

Lecture 9 8 Aug.

A-2. $y = \frac{1}{b(x+\frac{a}{b})}$ $\quad f(x) \approx \frac{1}{a} - \frac{b}{a^2}x$



3 $\quad f(x) = \frac{(1+x)^{3/2}}{1+2x}$ $\qquad f(x) \approx \cancel{1 - \frac{x}{2}}$

$f(0) = 1$

$f'(x) = \frac{\frac{3}{2}(1+x)(1+2x) - 2(1+x)^{\frac{3}{2}}}{(1+2x)^2} = \frac{(1+x)(\frac{3}{2} + 3x - 2\sqrt{1+x})}{(1+2x)^2}$ $\qquad f'(0) = \frac{3}{2}$

7. $f(x) = \frac{\sec x}{\sqrt{1-x^2}} = \frac{1}{\cos x \sqrt{1-x^2}}$ $\qquad f(x) \approx f(0) + f'(0) \cdot x + f'' x^2$

$f(0) = \frac{1}{1} = 1$

$(\sqrt{1-x^2})' = \frac{1}{2} \cdot \frac{1}{\sqrt{1-x^2}} \cdot (-2x)$ $\quad (\sqrt{1-x^2})'' = -\frac{1}{2} \cdot (-\frac{1}{2}) (\frac{1}{(1-x^2)^{3/2}})$

$(\sqrt{1-x^2})''|_{x=0} = -\frac{1}{4}(-2) = \frac{1}{2}$ $\quad + \frac{1}{\sqrt{1-x}}$

~~$f'(x) = \ln(\ldots)$~~

$f'(x) = -\frac{1}{\cos^2 x (1-x^2)} \left[-\sin x \sqrt{1-x^2} + \cos x \left(\frac{1}{2\sqrt{1-x^2}} \cdot (-2x) \right) \right]$

$(\cos x)' = -\sin x$ $\quad 0$

$(-\sin x)' = -\cos x$ $\quad 1$

$= \frac{\sin x}{\cos^2 x \sqrt{1-x^2}} + \frac{x}{\cos x \cdot \sqrt{1-x^2}^3} = \frac{\sin x + x\cos x}{\cos^2 x \sqrt{1-x^2}}$ $\qquad f'(0) = 0$.

~~$f''(x) = \frac{\cos x \sqrt{1-x} - \sin(2\cos x \cdot (-\sin x)\sqrt{1-x} + \cos^2 x \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{1-x}})}{\cos^4 x (1-x^2)} + \frac{2\cos x \sqrt{1-x^2} - 2x(-\sin x)}{\cos^2 x \cdot 4(1-x^2)}$~~

$\ln f'(x) = \ln(\sin x + x\cos x) - \ln(\cos^2 x \sqrt{1-x^2})$

$\frac{f''(x)}{f'(x)} = \frac{\cos x + \cos x - x\sin x}{\sin x + x\cos x} - \frac{2\cos x \cdot (-\sin x)\sqrt{1-x^2} + \cos^2 x \cdot \frac{1}{2} \frac{1}{\sqrt{1-x^2}}}{\cos^2 x \sqrt{1-x^2}}$

$f''(0) = \frac{\cancel{(\ldots)}}{\cos^2 x \sqrt{1-x^2}}$ $\quad 0$. $\quad \therefore f(x)$ where $x \geq 0$

11. $pv^k = C$

$p(v) = \frac{C}{v^k}$

$p'(v) = -k \cdot \frac{C}{v^{k+1}} = \frac{-kC}{v^{k+1}}$

$p''(v) = \frac{k(k+1)C}{v^{k+2}}$

$p(v_0 + \Delta v) \approx \frac{C}{v_0^k} - \frac{kC}{v_0^{k+1}}(\Delta v) + \frac{k(k+1)C}{2v_0^{k+2}}(\Delta v)^2$

12. a) $\frac{e^x}{1-x}$ (quadratic) $\approx (1+x+\frac{x^2}{2})(1+x+x^2) \approx 1+2x+\frac{5x^2}{2}$ $(x \approx 0)$

d) $\ln \cos x$ (q) $\approx (1-\frac{x^2}{2}-1) - \frac{1}{2}(-\frac{x^2}{2})^2 \approx -\frac{x^2}{2}$

e) $x \ln x$ (q) (part $x = 1+h$) $(x \approx 1) \approx x[(x-1) - \frac{1}{2}(x-1)^2]$ $\approx x(\ln x)$

a) $f(0) = 1$

$f'(x) = \frac{e^x(1-x) - e^x \cdot (-1)}{(1-x)^2} = \frac{e^x(2-x)}{(1-x)^2} = \frac{2e^x - xe^x}{(1-x)^2}$ $f'(0) = 2$.

$f''(x) = \frac{2e^x(1-x)^2 \ldots}{(1-x)^3} - \frac{(e^x + xe^x)(1-x) \ldots}{(1-x)^3}$

$= \frac{2e^x(1-x+2) - e^x[(1-x)+x(1-x)+2x]}{(1-x)^3}$

$= \frac{e^x(6-2x-1-x-x+x^2)}{(1-x)^3} = \frac{e^x(5-4x+x^2)}{(1-x)^3}$ $f''(0) = \frac{5}{1} = 5$

$f(x) \approx 1 + 2x + \frac{5}{2}x^2$, $x \approx 0$.

d) $f(0) = 0$

$f'(x) = \frac{-\sin x}{\cos x} = -\tan x$ $f'(0) = 0$

$f''(x) = \frac{-\cos^2 x - \sin^2 x}{\cos^2 x}$ $= \frac{-1}{\cos^2 x}$ $f''(0) = -1$

$f(x) \approx -\frac{x^2}{2}$

e) $f(1) = 0$

$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$, $f'(1) = 1$

$f''(x) = \frac{1}{x}$ $f''(1) = 1$

$f(x) \approx (x-1) + \frac{(x-1)^2}{2}$

2B-1 $y=0$. ^]

a) $y=x^3-3x+1$ b) $y=x^4-4x+1$

c) $y'(x)=\frac{1}{1+x^2}$, $y(0)=0$ d) $y=x^2/(x-1)$

e) $y=\frac{x}{x+4}$ f) $y=\sqrt{x+1}/(x-3)$

g) $y=3x^4-16x^3+18x^2+1$ h) $y=e^{-x^2}$

i) $y'=e^{-x^2}$ $y(0)=0$

2. a) e) h)

2B-4 f contain $x\in[0,10]$

$f=0$ at 4, 7, 9. $f'(x)>0$ on $x\in(0,5)\cup(8,10)$

$f'(x)<0$ on $5<x<8$

max, min + place attained

2B-6 a) cubic polynomial with local maximum at $x=-1$ and local min at $x=1$.

b) draw the cubic on $-3\le x\le 3$

7 a) Pv: if $f(x)\uparrow$ and has a ' at a, then $f'(a)>0$.

b) Proof: if $f'(a)>0$, a) fails. give a counterexample

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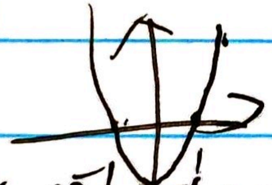
2B-1 a)

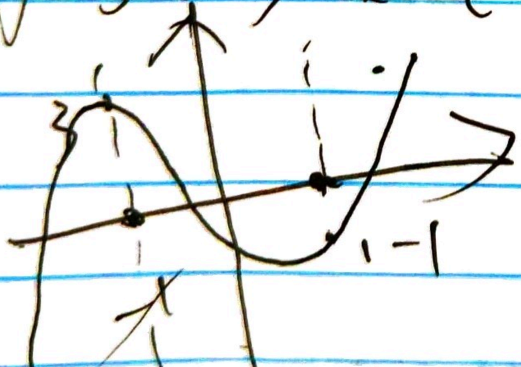
$f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$



$f'(x) > 0, \ x \in (-\infty, -1) \cup (1, +\infty)$

$f'(x) < 0, \ x \in (-1, 1)$



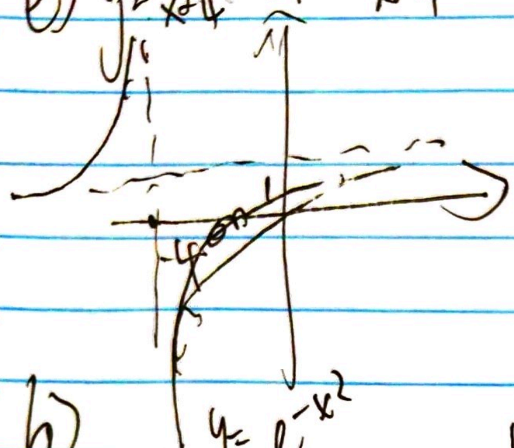
$f(-1) = 3 \quad f(1) = -1$

$\therefore$ there one 3 roots.

b) ~~$f'(x) = 4x^3 - 4 = 4(x^3 - 1)$~~

~~$f'(x) > 0, \ x \in (1, +\infty)$~~ $\quad f'(x) < 0, \ x \in (-\infty, 1)$

e) $y = \frac{x}{x+4} = 1 - \frac{4}{x+4}$



$y' = +\frac{4}{(x+4)^2}\cdot 1 = \frac{4}{(x+4)^2}$

∴ 1 root.

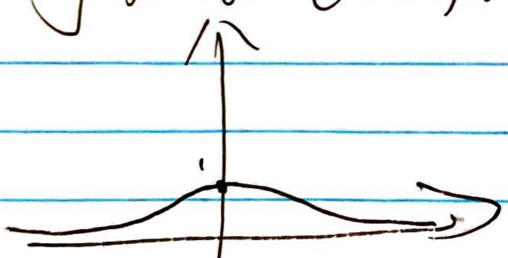
↗ on $(-\infty,-4)$, $(-4,+\infty)$

h) $y = e^{-x^2}$ $\quad y' = e^{-x^2}\cdot(-2x) = -2xe^{-x^2}$

$y'>0$, $x<0$

$y'<0$, $x>0$.

∴ y ↗ on $(-\infty,0)$. ↘ on $(0,+\infty)$.



no root.

UB-2. c) $y' = 3x^2-3$ $\quad y'' = 6x = 0$, $x=6$.

inflection point $x=6$.

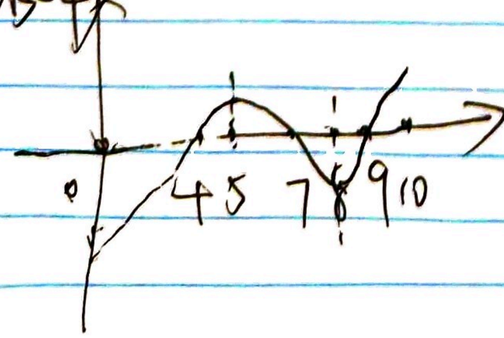
e). $y' = \frac{4}{(x+4)^2}$ $\quad y'' = 4\times(-2)\times\frac{1}{(x+4)^3} = \frac{-8}{(x+4)^3}$

no inflection point.

h) $y'' = -2(e^{-x^2})\cdot(-2x^2e^{-x^2}) = 4x^2e^{-2x^2} = 0$

$x=0$ inflection point $x=0$.

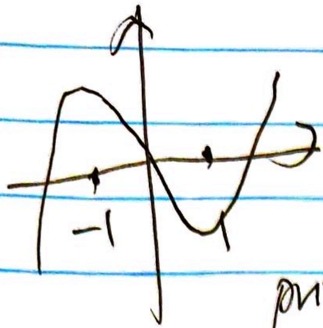
2.5/4 UB-4.



$f_{max} = \max\{f(10), f(5)\}$

$f_{min} = \min\{f(8), f(0)\}$

7.3-6 a)

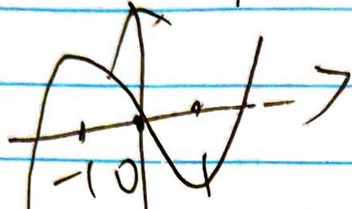


$y' = a(x+1)(x-1) = a(x^2-1)$, $a>0$.

$y = a(\frac{x^3}{3} - x) + c$

put $a=3$, $c=0$, $y = x^3 - 3x$

b)



7 a) ~~$f(a)$~~ $f'(a) = \lim\limits_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$

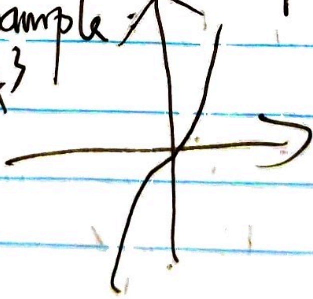
$\because f(x)\uparrow \quad \therefore \frac{f(a+\Delta x) - f(a)}{\Delta x} > 0$.

$\therefore \lim\limits_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x} \geq 0$.

$\therefore f'(a) \geq 0$.

b). Because the fore limie of a positive function is greater or equal to 0.

counter example: $f(x) = x^3$

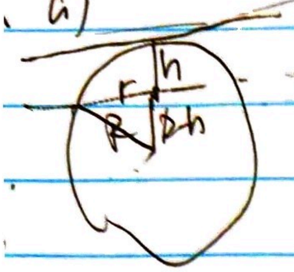


$f'(0) = 0$.

PART II

. None,

a)



$$r^2+(R-h)^2=R^2$$

$$r^2+\cancel{R^2}+h^2-2Rh=\cancel{R^2}$$

$$h^2-2Rh+r^2=0$$

$$h=\frac{2R\pm\sqrt{4R^2-4r^2}}{2}=R\pm\sqrt{R^2-r^2}$$

$\because$ above horizontal plane

$$\therefore h=R-\sqrt{R^2-r^2}$$

$$A=2\pi\left(R-\sqrt{R^2-r^2}\right)R$$

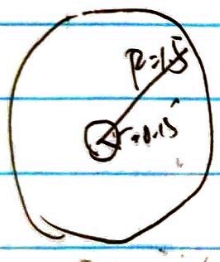
b)

$$A=2\pi\left(1-\sqrt{1-\left(\tfrac{r}{R}\right)^2}\right)R^2$$

Let $\frac{r}{R}=x$,

$$A(x)=2\pi\left(1-\sqrt{1-x^2}\right)R^2 \approx \pi x^2R^2=\pi r^2$$

c) i)



$$4\pi R^2-100\cdot\cancel{2}\pi r^2+100\cdot 2\pi r^2$$

$$=4\pi R^2-\cancel{100\pi r(r-1)}+100\pi r^2$$

ii)

$$\sqrt{1-x^2}\approx 1-\frac{1}{2}\frac{x^2}{1}+\left(\frac{1}{2}\right)\left(-\frac{1}{4}\right)x^4=1-\frac{x^2}{2}-\frac{x^4}{8}$$

$$\cancel{(\sqrt{1-x^2})'=\frac{1}{2}\cdot\frac{-2x}{\sqrt{1-x^2}}=-\frac{x}{\sqrt{1-x^2}}}\Big|_{x=0}=0.$$

$$\cancel{(\sqrt{1-x})''=\frac{\sqrt{1-x^2}-x\frac{x}{\sqrt{1-x^2}}}{1-x}=\frac{1-x^2+x^2}{\sqrt{1-x^2}(1-x)}=\frac{2x^2-1}{(1-x)\sqrt{1-x^2}}}\Big|_{x=0}=-1$$

$$A(x)\approx 2\pi\left(\frac{x^2}{2}-\frac{x^4}{8}\right)R^2 \quad \left(x=\frac{r}{R}\right)$$

$$=4\pi R^2-100\cdot 2\pi\left(\frac{r^2}{2R^2}R^2-\frac{r^4}{8R^2}\right)+100\pi r^2$$

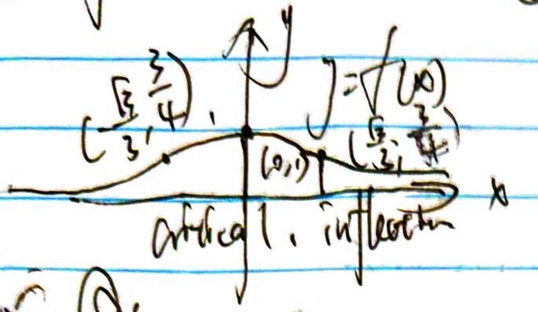
$$=4\pi R^2-100\pi\left(r^2-\frac{r^4}{4R^2}\right)+100\pi r^2$$

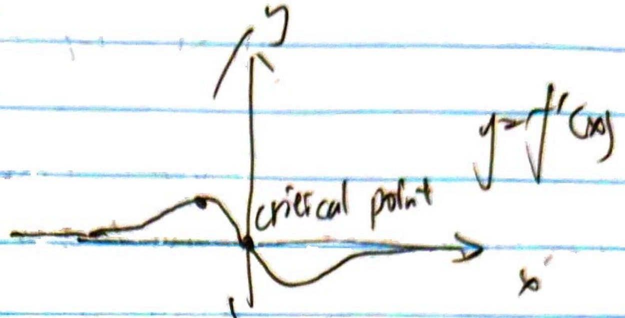
$$=4\pi R^2+100\pi\frac{r^4}{R^2}$$

iii) i): $4\pi R^2+100\pi r^2=4\pi(R^2+25r^2)\approx 35.34291735$

ii): $4\pi\left(R^2+25\frac{r^4}{R^2}\right)\approx 28.37065.$

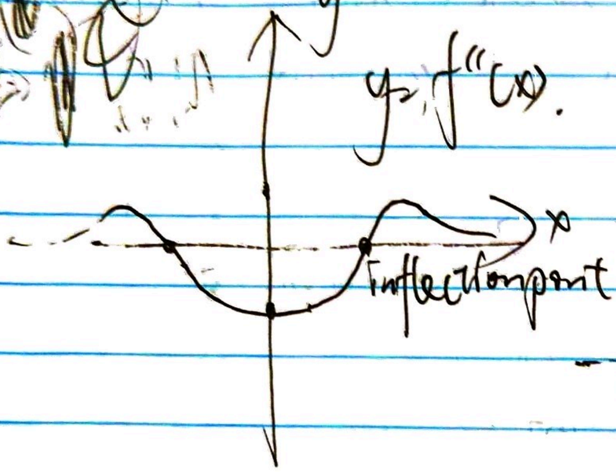
2 (b) $f(x) = \frac{1}{1+x^2}$





$= -\frac{1}{(1+x^2)^2} \cdot 2x$

$= -\frac{2x}{x^4+2x^2+1}$



$= -\left(\frac{2x^4+4x^2+2 - 2x(4x^3+4x)}{(x^4+2x^2+1)^2}\right)$

$= -\left(\frac{2(1+x^2)^2 - 2x\cdot 4x(x^2+1)}{(x^4+2x^2+1)^2}\right)$

$= -\frac{(1+x^2)(\overset{2-6x^2}{\cancel{2+2x^2-8x^2}})}{(1-\sqrt{3}x)(1+\sqrt{3}x)}$