pf: $\frac{d}{dx} a^x = \lim_{\Delta x \to 0} \frac{a^{x+\Delta x} - a^x}{\Delta x} = \lim_{\Delta x \to 0} a^x \frac{a^{\Delta x} - 1}{\Delta x}$

Let $M(a) = \lim_{\Delta x \to 0} \frac{a^{\Delta x} - 1}{\Delta x}$

$\therefore \frac{d}{dx} a^x = M(a) a^x$.

$\square$

Lecture 5. Study

1F-3 $y = x^{\frac{1}{n}}$

$y^n = x$

$(y^n)' = 1$

$ny^{n-1} \cdot y' = 1$

$y' = \frac{1}{ny^{n-1}} = \frac{1}{n x^{\frac{n-1}{n}}}$

5. $\frac{dy}{dx}(\sin x + \sin y) = 0$

$\cos x + \cos y \cdot y' = 0$

$y' = \frac{-\cos x}{\cos y} = 0$

$x = (2k+1)\pi$, $k \in \mathbb{Z}$

IB Physics Content G

Units

8c) $\frac{d}{db}(c^2) = \frac{d}{db}(a^2+b^2-2ab\cos\theta)$.

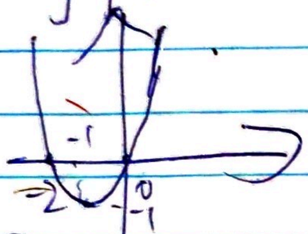
$0 = 2a\cdot a' + 2b - 2\cos\theta(ab+a)$

$0 = 2a\cdot a' + 2b - (2ab\cos\theta + 2a\cos\theta$

$(2a - 2b\cos\theta)a' = -2b + 2a\cos\theta$

$a' = \frac{-2b+2a\cos\theta}{2a-2b\cos\theta} = \frac{a\cos\theta - b}{a - b\cos\theta}$

4-5 b) $y = f(x) = x^2+2x$



Domain: $x \in \mathbb{R}$

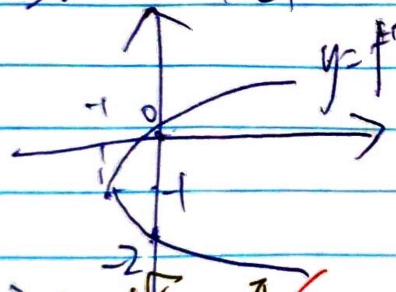
$x^2+2x-y=0$

$x = \frac{-2\pm\sqrt{4+4y}}{2}$

$4+4y \ge 0$

$y \ge -1$

$y = f^{-1}(x) = \frac{-2\pm\sqrt{4+4x}}{2}$



Domain: $x \in [-1, +\infty)$.

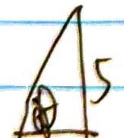
restrict domain to $x \le -1$

$y = \sqrt{x+1} - 1$.

5A-1 a) $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$

b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

c)



$\sqrt{1+25} = \sqrt{26}$.

$\sin\theta = \frac{5}{\sqrt{26}}$ $\cos\theta = \frac{1}{\sqrt{26}}$ $\sec\theta = \sqrt{26}$.

3 f) $\frac{d}{dx}\left(\sin^{-1}\left(\frac{a}{x}\right)\right)$

$y = \sin^{-1}\left(\frac{a}{x}\right)$

$\sin(y) = \frac{a}{x}$

$\cos y \cdot y' = -\frac{a}{x^2}$

$y = -\frac{a}{\cos y \cdot x^2}$

$= \frac{a}{\cos\left(\sin^{-1}\left(\frac{a}{x}\right)\right)x^2}$

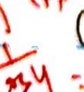
$= \frac{-a}{x\sqrt{x^2-a^2}}$.

$\frac{d}{dx}(\sin^{-1}(x))$

$g(\sin y) = 1$

$y'\cos y = 1$

$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$



b) $y = \sin^{-1}\sqrt{1-x}$

$\sin y = \sqrt{1-x}$

$\cos y \cdot y' = \frac{1}{2} \cdot \frac{1}{\sqrt{1-x}}$

$y' = \frac{1}{2\sqrt{1-x} \cdot \cos(\sin^{-1}\sqrt{1-x})} = \frac{a}{x\sqrt{x^2-a^2}}$

Lecture 6 0 Aug.

1H-1 a) $y = y_0 e^{kt} = \frac{1}{2} y_0$

$e^{kt} = \frac{1}{2}$

$\ln\frac{1}{2} = kt$

$\cancel{\lambda = \frac{\ln\frac{1}{2}}{k}}$

b) $y_1 = y_0 e^{kt_1}$ $\quad kt_1 = \ln\frac{y_1}{y_0}$ $\quad t_1 = \frac{\ln\frac{y_1}{y_0}}{k}$

$y_2 = y_0 e^{k(t_1+\lambda)} = y_0 e^{k\left(\frac{\ln 2\frac{y_1}{y_0}}{k}\right)}$ $\cancel{= y_0 \cdot \frac{y_1}{2y_0}} = \frac{y_1}{2}$

2 $(pH)_{\text{original}} = -\log_{10}(H^+)$

$C'(H^+) = \frac{C(H^+)}{2}$

~~original~~

$(pH)_{\text{diluted}} = -\log_{10}\left(\frac{C(H^+)}{2}\right) = \cancel{(pH)_{\text{original}}} + \log_{10} 2$

3 c) $\ln(y^2-1) = \ln x + 2x$

$xe^{2x} = y^2 - 1$

$y^2 = xe^{2x}+1$

$y = \pm\sqrt{xe^{2x}+1}$ $\quad y > 0.$

5(b) pre $u = e^x \in (0, +\infty)$.

$y = u + \frac{1}{u}$ $\quad u + \frac{1}{u} \geq 2\sqrt{1} = 2$

$u^2 - yu + 1 = 0$ $\quad \Delta = y^2 - 4 \geq 0$

$u = \frac{-1 \pm \sqrt{1+4y}}{2} = e^x > 0$ $\quad x = \ln\frac{y \pm \sqrt{y^2-4}}{2}$

$\therefore u = \frac{-1+\sqrt{1+4y}}{2}$

$\cancel{x = \ln\frac{-1+\sqrt{1+4y}}{2}}$

II-1 c) $y' = e^{-x^2} \cdot (-2x) = -2x \cdot e^{-x^2}$

d) $y' = \ln x + 1 - 1 = \ln x$

e) $y' = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$

f) $y' = 2\ln x \cdot \frac{1}{x} = \frac{2\ln x}{x}$

h) $y' = \frac{(-e^x)(1+e^x) - (1-e^x)e^x}{1+2e^x+e^{2x}} = \frac{-e^x - e^{2x} - e^x + e^{2x}}{e^{2x}+2e^x+1} = \frac{-2e^x}{e^{2x}+2e^x+1}$

-4 a) $= \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^n\right)^3 = e^3$

Lecture 7 5 Aug

5A-5 a) $y = \sinh x = \frac{e^x - e^{-x}}{2}$

$y' = \frac{e^x + e^{-x}}{2} = \cosh x$

$y'' = \frac{e^x - e^{-x}}{2} = \sinh x.$

$f(x) = \sinh(x)$. $f(0) = 0$. $f(+\infty) = +\infty$. $f(-\infty) = -\infty$.

$f(x) = -f(-x)$. $\therefore$ symmetry about origin.

$f'(x) = 0$. x DNE. since $e^x + e^{-x} > 0$.

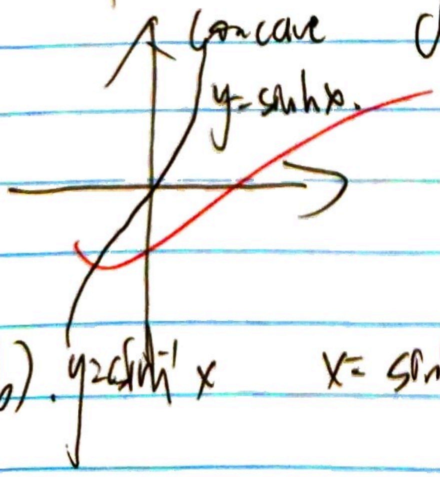
$f''(x) = 0$. $x = 0$. is the inflection point.

$f'(x) > 0 \Rightarrow x \in \mathbb{R}$. $f'(x) < 0 \Rightarrow x$ DNE.

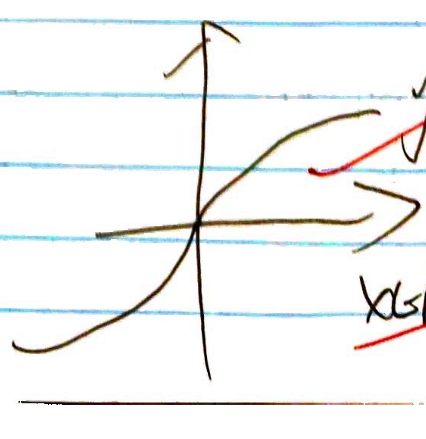
$f''(x) > 0 \Rightarrow x > 0$. $f''(x) < 0 \Rightarrow x < 0$.

$\therefore f(x)$ concave up. as $x > 0$.

concave down as $x < 0$.



b). $y = \sinh^{-1} x$ $x = \sinh y$



$x \in \mathbb{R}$.

i). ~~$\frac{d}{dx}\sinh^{-1}x$~~ $y = \sinh^{-1}x$ $x \in \mathbb{R}$.

$x = \sinh y$ $x = \frac{e^{y} - e^{-y}}{2}$

~~$1 = \cosh y \cdot y'$~~ ~~$2x = e^{y} - e^{-y}$~~

$y' = \frac{1}{\cosh y} =$ ~~$2 = e^{y} y' + e^{-y}(y')$~~

$2 = y'(e^{y} + e^{-y})$

$y' = \frac{1}{\cosh y}$ $\cosh^2 x - \sinh^2 x = 1$ $\cosh x = \sqrt{1 + \sinh^2 x}$

$= \frac{1}{\sqrt{1+x^2}}$

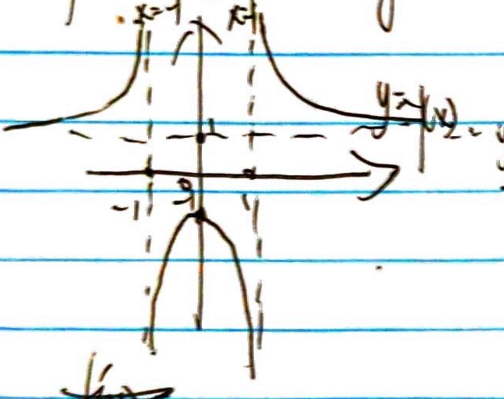
PART II 5 Aug.

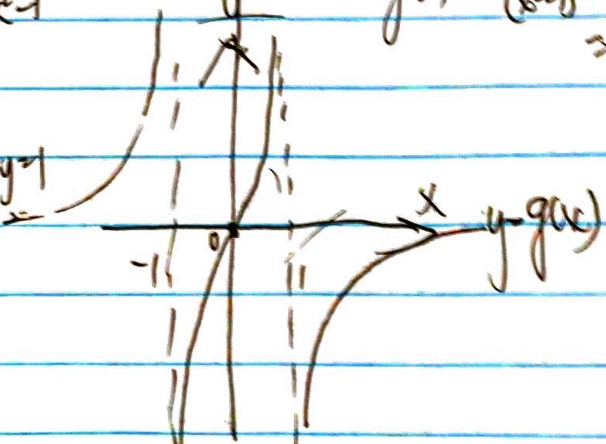
0. None

1. $f(x) = \frac{x^2+1}{x^2-1}$ $g(x) = \frac{-2x}{x^2-1}$

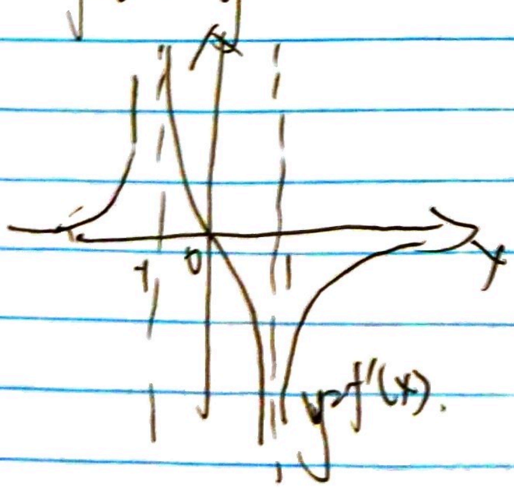
$g'(x) = \frac{-2(x^2-1) - (-2x)2x}{(x^2-1)^2} = \frac{-2x^2+2+4x^2}{(x^2-1)^2} = \frac{2x^2+2}{(x^2-1)^2}$

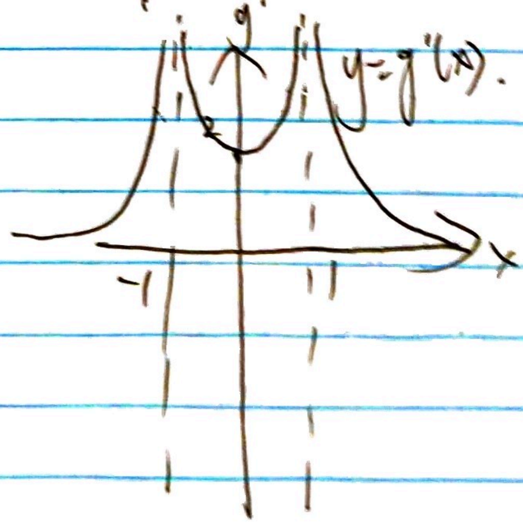
$g'(0) = \frac{2}{1} = 2$.





~~$f'(x) =$~~





2a) $\frac{d}{dx}(\tan^3(x^4))$

$= \frac{d}{dx}\left(\frac{\sin^3(x^4)}{\cos^3(x^4)}\right)$

$= \frac{d}{dx}\left(\frac{\sin(x^4)}{\cos(x^4)}\right)^3$

$= \left(\frac{\cos x^4 \cdot 4x^3 \cdot \cos x^4 + \sin x^4 \cdot \sin x^4 \cdot 4x^3}{\cos^2(x^4)}\right) \cdot 3\left(\frac{\sin(x^4)}{\cos(x^4)}\right)^2$

$= \frac{4x^3(\cos^2 x^4 + \sin^2 x^4)}{\cos^2(x^4)} \cdot 3\tan^2(x^4)$

$= \frac{12x^3 \cdot \tan^2(x^4)}{\cos^2(x^4)}$

b) 1° $\frac{d}{dy}(\sin^2 y \cos^2 y) = 2\sin y \cdot \cos y \cdot \cos^2 y + \sin^2 y \cdot 2\cos y \cdot (-\sin y)$

$= 2\sin y \cdot \cos^3 y - 2\sin^3 y \cos y$

$= 2\sin y \cos y(\cos^2 y - \sin^2 y)$

$= \sin 2y \cdot \cos 2y$

$= \frac{\sin 4y}{2}$

2° $\frac{d}{dy}(\sin^2 y \cos^2 y) = \frac{d}{dy}\left(\frac{\sin^2 2y}{4}\right)$

$= \frac{2\sin 2y \cdot \cos 2y}{2} = \frac{\sin 4y}{2}$

3. a) $y' = u'v + uv'$

$y'' = u''v + u'v' + u'v' + uv'' = u''v + 2u'v' + uv''$

b) $y''' = u'''v + u''v' + 2u''v' + 2u'v'' + u'v'' + uv'''$

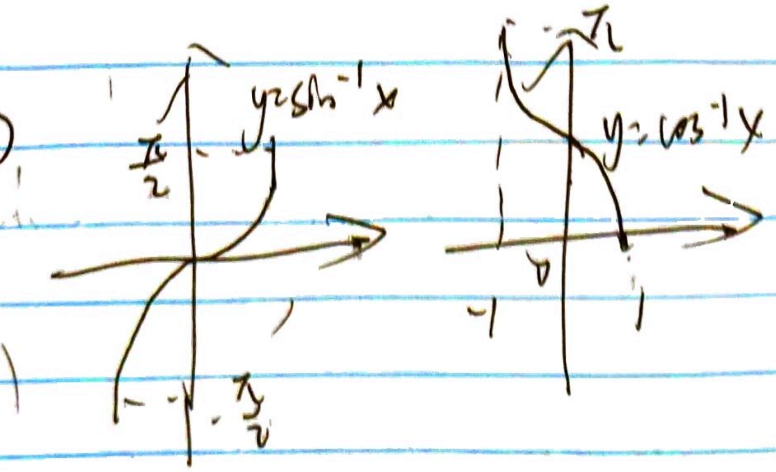
$= u'''v + 3u''v' + 3u'v'' + uv'''$

4. a) $x = \cos y$ $\quad\quad -\sin y \in [-1, 0]$

$1 = -\sin y \cdot y'$

$y' = \frac{1}{-\sin y} = \frac{1}{-\sqrt{1-\cos^2 y}} = \frac{1}{-\sqrt{1-x^2}}$

b)



(c) $\sin(x+\frac{\pi}{2}) = \cos x$ where $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

5. 8.2-8

a) $M_1 = M_2 - 1$

$\frac{2}{3}\lg\frac{E_1}{E_0} = \frac{2}{3}\lg\frac{E_2}{E_0} - 1$

$\lg\left(\frac{E_1}{E_0}\right)^{\frac{2}{3}} = \lg\frac{E_2^{\frac{2}{3}}}{E_0^{\frac{2}{3}}\cdot 10}$

$\frac{E_1^{\frac{2}{3}}}{\cancel{E_0^{\frac{2}{3}}}} = \frac{E_2^{\frac{2}{3}}}{\cancel{E_0^{\frac{2}{3}}}\cdot 10}$

$E_1 = E_2 \cdot 10^{-\frac{3}{2}}$

$E_1 \cdot 10^{\frac{3}{2}} = E_2$.

c) $6 = \frac{2}{3}\lg\frac{E}{E_0}$

$10^9 = \frac{E}{E_0}$

$E = 7\times 10^6$ kWh

$\frac{E}{3\times 10^5} = \frac{7}{3}\times 10$ ~~days~~

≈ 23 days.

10. Assume tha $\log_3 2$ can be expressed as $\frac{p}{q}$ where p and q are positive integers.

$3^{\frac{p}{q}} = 2$

$3^p = 2^q$

Since p and q are positive integers, 3^p is impossible to equal 2^q which is contrary.

$\therefore \log_3 2$ is irrational! □.

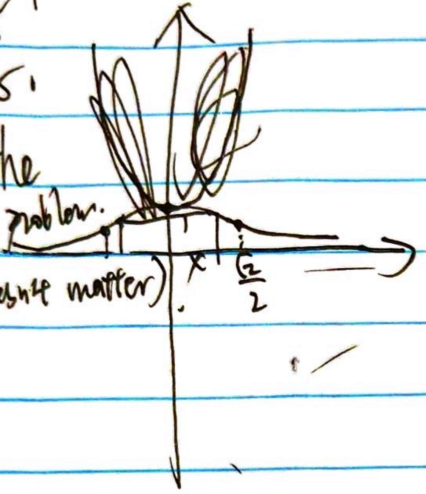
11. $\log\frac{1}{2}<0$

$\therefore$ when it is multiplied both sides, the sign must to be changed from "<" to ">".

~~8.4-18~~

8.3-18.

I do the wrong problem. but doesn't matter)



$f(x)=e^{-x^2}$

$f(0)=1 \quad f(+\infty)=0 \quad f(-\infty)=0.$

$f(x)=f(-x)$

$f'(x)=e^{-x^2}\cdot(-2x)$

$\cancel{f'(0)=0}$

$f'(x)>0$ when $x\in(-\infty,0)$

$f'(x)<0$ when $x\in(0,+\infty)$

$f''(x)=e^{-x^2}\cdot(-2x)\cdot(-2x)-2e^{-x^2}$

$=e^{-x^2}\cdot 4x^2-2e^{-x^2} =2e^{-x^2}(2x^2-1)$

$f''(\pm\frac{\sqrt{2}}{2})$

$A(x)=2x\cdot f(x)$

$=2x\cdot e^{-x^2} \quad x\in(0,+\infty)$

$A'(x)=2e^{-x^2}+2x\cdot e^{-x^2}\cdot(-2x)$

$=2e^{-x^2}(1-2x^2)=0$

$x=\pm\frac{\sqrt{2}}{2} \quad \because x>0, \therefore x=\frac{\sqrt{2}}{2}$

$A(0^+)=0. \quad A(\frac{\sqrt{2}}{2})=\sqrt{2}\cdot e^{-\frac{1}{2}}$

$\therefore$ the largest rectangle is with height $2\sqrt{2}e^{-\frac{1}{2}}$ and width $\sqrt{2}$.

8.4-18

$y'\cdot\frac{1}{y}=\frac{1}{3}[\frac{1}{x+1}+\frac{1}{x-2}+\frac{2}{2x+7}]$

$y'=\boxed{\frac{\sqrt[3]{(x+1)(x-2)(2x+7)}}{3}(\frac{1}{x+1}+\frac{1}{x-2}+\frac{2}{2x+7})}$

19(a) $\ln y=x+\ln(x^2-1)-\frac{1}{2}\ln(6x-2).$

$\ln y=x+\ln(x+1)+\ln(x-1)-\frac{1}{2}\ln(6x-2).$

$y'\cdot\frac{1}{y}=1+\frac{1}{x+1}+\frac{1}{x-1}-\frac{1}{2}\cdot\frac{6}{6x-2}$

$y'=(\frac{1}{x+1}+\frac{1}{x-1}-\frac{3}{6x-2}+1)(e^x(x^2-1))/\sqrt{6x-2}$

$$\ln(u_1 u_2 \cdots u_n) = \ln u_1 + \ln u_2 + \cdots + \ln u_n.$$

$$D(u_1 u_2 \cdots u_n) \cdot \frac{1}{u_1 \cdots u_n} = \frac{u_1'}{u_1} + \frac{u_2'}{u_2} + \cdots \frac{u_n'}{u_n}$$

$$D(u_1 u_2 \cdots u_n) = u_1' u_2 \cdots u_n + u_1 u_2' \cdots u_n + \cdots + u_1 u$$

Lecture 9 8 Aug.

2A-2. $y = 5(x + \frac{a}{b})$ $f(x) \approx \frac{1}{a} - \frac{b}{a^2} x$