Exam 2 26/3/2025 100

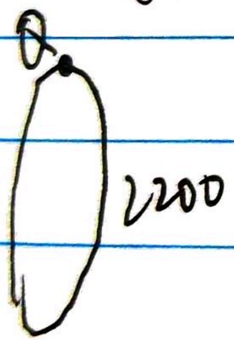
P1 a) $\nabla f = \langle y - 4x^3, x \rangle$ at $P(1,1)$

$\nabla f = \langle -3, 1 \rangle$

b) $\Delta w \approx \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y$

$= -3\Delta x + \Delta y$

P2 a) $\cancel{\frac{100}{500}}$ $\left(\frac{dh}{ds}\right)_{\hat{u}} \approx \frac{100}{500} = 0.2$

b) 

$\left(\frac{\partial h}{\partial y}\right)_Q \approx \frac{100}{300} \approx 0.3$

P3 $\nabla f = \langle 3yx^2, x^3, 2z \rangle$ at $(-1,1,2)$, $\nabla f = \langle 3, -1, 4 \rangle$

$\therefore T: 3(x+1) - (y-1) + 4(z-2) = 0$

$3x+3 - y + 1 + 4z - 8$

$3x - y + 4z = 4$

P4 a) $p(x,y,z) = x^2+y^2+z = 1$

$V(x,y,z) = xyz$

$z = 1-(x^2+y^2)$ plug in to V

$f(x,y) = V(x,y,1-(x^2+y^2)) = xy(1-(x^2+y^2))$

$= xy - x^3y - xy^3$

$f_x = y - 3x^2y - y^3$ $\quad f_y = x - x^3 - 3xy^2$

$\begin{cases} y - 3x^2y - y^3 = 0 \\ x - x^3 - 3xy^2 = 0 \end{cases}$ $\quad x - 3x^3 - x^3 \quad x - 4x^3 = 0 \quad x(1-4x^2) \quad x(1-2x)(\ldots$

b) $x = y = \frac{1}{2}$

c) $f_{xx} = -6yx \quad f_{xy} = 1 - 3x^2 - 3y^2 \quad f_{yy} = -6xy$

$A = -\frac{3}{2} \quad B = -\frac{1}{2} \quad C = -\frac{3}{2}$

$AC - B^2 = \frac{9}{4} - \frac{1}{4} = 2 > 0$, $A < 0$

$\therefore$ local Max

d) $f_{max}(x,y) = f(\frac{1}{2},\frac{1}{2}) = \frac{1}{4} - \frac{1}{16} - \frac{1}{16} = \frac{1}{8}$

P5 a) $f(x,y,z) = xyz$ to maximize

$g(x,y,z) = x^2+y^2+z = 1$

$\nabla f = \langle yz, xz, xy \rangle$

$\nabla g = \langle 2x, 2y, 1 \rangle$

$\nabla f = \lambda \nabla g$ $\quad \begin{cases} yz = 2\lambda x \\ xz = 2\lambda y \\ xy = \lambda \end{cases}$

b) $\cancel{2\lambda} x^2 = 2\lambda y^2$, $(x>0, y>0)$

$\therefore x = y$

$xy = x^2 = \lambda$

$yz = 2\lambda x$

$xz = 2\lambda x$

$z = 2\lambda = 2x^2$

$x^2 + y^2 + z^2 = 4x^2 = 1$

$$\begin{cases} x = \frac{1}{2} \\ y = \frac{1}{2} \\ z = \frac{1}{2} \end{cases}$$

$V = f(x, y, z) = \frac{1}{8}$

P6

$w = f(u, v)$

$u = xy$

$v = \frac{x}{y}$

$\cancel{\frac{\partial w}{\partial x}} = \quad dw = \frac{\partial f}{\partial u} du + \frac{\partial f}{\partial v} dv$

$du = y\,dx + x\,dy$

$dv = \frac{1}{y} dx + \frac{x}{y^2} dy$

$\cancel{\therefore dw = \frac{\partial f}{\partial u}(y\,dx + x\,dy) + \frac{\partial f}{\partial v}\left(\frac{1}{y} dx + \frac{x}{y^2} dy\right)}$

$\cancel{= \left(\frac{\partial f}{\partial u} y + \frac{\partial f}{\partial v} \frac{1}{y}\right) dx +}$

$\frac{\partial w}{\partial x} = \frac{\partial f}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x}$

$= \frac{\partial f}{\partial u}(y + \ldots) + \frac{\partial f}{\partial v} \cdot \left(\frac{1}{y} \ldots\right)$

$= \frac{\partial f}{\partial u} \cdot y + \frac{\partial f}{\partial v} \cdot \frac{1}{y}$

$\frac{\partial w}{\partial y} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial y} = \frac{\partial f}{\partial u} \cdot x + \frac{\partial f}{\partial v}\left(\frac{x}{y^2}\right)$

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$\cancel{2xy + z^2}$

$(2xy+z^2)dx + x^2 dy + 2xz\,dz = 0$

$dw = 3x^2y\,dx + x^3\,dy$

$$dx = \frac{-x^2dy - 2xz\,dz}{2xy+z^2}$$

pue $(x,y,z) = (1,1,2)$.

$$dx = \frac{-dy - 4dz}{2+4} = -\frac{1}{6}dy - \frac{2}{3}dz$$

$dw = 3dx + dy$

$= -\frac{1}{2}dy - 2dz + dy$

$= \frac{1}{2}dy - 2dz$

y constant

$\left(\frac{\partial w}{\partial z}\right)_y = -2$