Exam 3

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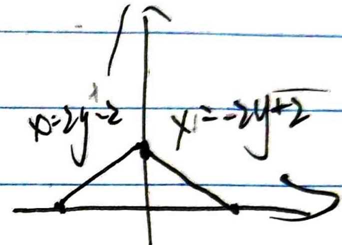
1 a) Mass = Area = 2

$\bar{y} = \frac{1}{Mass} \iint_R y \delta \, dA$



$= \frac{1}{2} \iint_R y \, dA = \cancel{\frac{1}{2}\int_0^1 \int_{-2}^{2} \ldots \frac{1}{2}\int_{-2}^{2}\int_0^1 (\ldots) \, dy \, dx}$

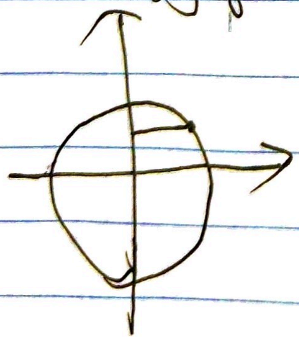
$= \frac{1}{2} \int_0^1 \int_{2y-2}^{-2y+2} y \, dx \, dy$

b) $\bar{x} = \frac{1}{mass} \iint_R x \delta \, dA$

$= \frac{1}{2} \int_0^1 \int_{2y-2}^{-2y+2} x \, dx \, dy = \frac{1}{2} \int_0^1 \frac{x^2}{2} \Big|_{2y-2}^{-2y+2} dy$

$= \frac{1}{2} \int_0^1 \cancel{2}\left((y-1)^2 - (y-1)^2 \right) dy = \cancel{\ldots \int_0^1 2y \, dy} \quad 0$

2.



$\delta = |x|$

$I = \iint_R r^2 \delta \, dA$

$= 2 \int_0^{\pi} \int_0^1 r^2 \cdot r\cos\theta \cdot r \, dr \, d\theta$

$= 4 \int_0^{\frac{\pi}{2}} \int_0^1 r^4 \cos\theta \, dr \, d\theta = 4 \int_0^{\frac{\pi}{2}} \cos\theta \left[\frac{1}{5} r^5\right]_0^1 d\theta$

$= \cancel{\frac{1}{5} \sin\theta \Big|_0^{2\pi}} \quad \frac{4}{5} \sin \Big|_0^{\frac{\pi}{2}} = \frac{4}{5}$

3 a) $\frac{\partial}{\partial y}\vec{F}_x = ax^2 + 3y^2$

$\frac{\partial}{\partial x}\vec{F}_y = 6x^2 + by^2 = ax^2 + 3y^2$

$\therefore a=6,\ b=3$

b) $\vec{F} = (6x^2y + y^3 + 1)\hat{i} + (2x^3 + 3xy^2 + 2)\hat{j}$

$f(x,y) = \int \frac{\partial f}{\partial x}dx = \int (6x^2y + y^3 + 1)dx$

$= 2x^3y + y^3x + x + g(y)$

$\frac{\partial f}{\partial y} = 2x^3 + 3y^2x + g'(y) = 2x^3 + 3xy^2 + 2$

$g'(y) = 2$

$g(y) = \int (2)dy = 2y$

$\therefore f(x,y) = 2x^3y + y^3x + x + 2y + C$

c) $\int_C \vec{F}\cdot d\vec{r}$

$= \cancel{\int_C (6x^2y + y^3 + 1)dx + (2x^3 + 3xy^2 + 2)dy}$

$= f(x(\pi), y(\pi)) - f(x(0), y(0))$

$= 2e^{\pi} - \cancel{1}$ $\quad -e^{\pi} - 1$

$x(0) = 1$ $\quad -5$

$y(0) = 0$

$x(\pi) = 0$ $\quad -e^{\pi}$

$y(\pi) = \cancel{e^{\pi}}$ $\quad 0$

4. $\int_C \vec{F}\cdot d\vec{r} = \int_C yx^3 dx + y^2 dy$

$y = x^2$

$dy = 2x\,dx$

$= \int_0^1 x^5 dx + x^4 \cdot 2x\,dx$

$= \int_0^1 3x^5 dx = \frac{1}{2}x^6 \Big|_0^1 = \frac{1}{2}$

5.a) let $u=\frac{x^2}{y}$, $v=xy$

$J=\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{2x}{y} & -\frac{x^2}{y^2} \\ y & x \end{vmatrix} = \frac{2x^2}{y}+\frac{x^2}{y} = \frac{3x^2}{y}$

$du\,dv = \frac{3x^2}{y}dx\,dy$

$dx\,dy = \frac{y}{3x^2}du\,dv = \frac{du\,dv}{3u}$

b) $u=1$, $u=5$, $v=2$, $v=4$

$\iint_R du\,dv = \int_1^5\int_2^4 dv\,du$

6.a) $\oint_C M\,dx = \iint_R -\frac{\partial M}{\partial y}\,dA$

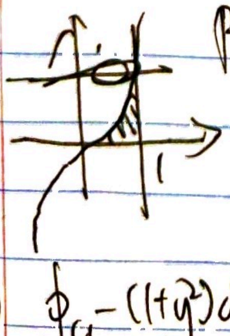
b) ~~$\oint_C M\,dx = \iint_R \frac{\partial M}{\partial y}\,dA$~~

$Mass = \iint_R \delta\,dA = \iint_R (x+y)^2 dA = \iint_R -\frac{\partial M}{\partial y}$

$-\frac{\partial M}{\partial y} = (x+y)^2$

$M = \int \frac{\partial M}{\partial y}dy = \int -(x+y)^2 dy$

$= -x^2y - xy^2 - \frac{1}{3}y^3 + g(x)$

7. a) 

$Flux = \oint_C -(1+y^2)dx$

$= \iint_R 2y\,dA = \int_0^1\int_0^{x^3} 2y\,dy\,dx = \int_0^1 x^6 dx = \frac{1}{7}$

b) $\oint_{C_1} -(1+y^2)dx = \int_0^1 -dx = -1$ $\quad \oint_{C_2} -(1+y^2)dx = 0$

c) $\oint_{C_3} = \oint_C - \oint_{C_1} - \oint_{C_2} = \frac{8}{7}$