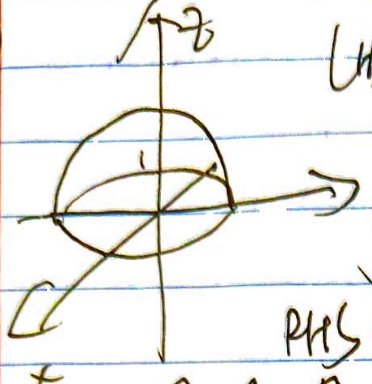
PS 12 5/4/2005

Lec 34 ~~4/7~~ 5/4/2005

6F-1(a)



$LHS = \oint_C \vec{F}\cdot d\vec{r} = \oint_C x\,dx + y\,dy$

$= \oint_C \cancel{\sin\theta\cos\theta\,d\theta} - \sin\theta\cos\theta\,d\theta + \sin\theta\cos\theta\,d\theta$

$RHS = \iint_S \overset{=0}{\nabla\times\vec{F}}\cdot d\vec{S} = 0 = LHS$

$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix} = 0$

b) $LHS = \oint_C \vec{F}\cdot d\vec{r} = \oint_C y\,dx = \int_0^{2\pi} -\sin^2\theta\,d\theta = -\pi$

$RHS = \iint_S \nabla\times\vec{F}\cdot d\vec{S} = \iint_S \langle -1, 1, -1\rangle\langle x, y, z\rangle\, dS$

$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & z & x \end{vmatrix} = -\hat{i} - \hat{j} - \hat{k}$

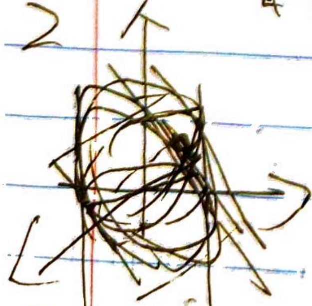
$= \int_0^{2\pi}\int_0^{\frac{\pi}{2}} (-\sin\phi\cos\theta - \sin\phi\sin\theta - \cos\phi)\sin\phi\,d\phi\,d\theta$

$= -\int_0^{2\pi}\int_0^{\frac{\pi}{2}} (\sin\theta + \cos\theta)\sin^2\phi\,d\phi\,d\theta - \cos\phi\,d\cos\phi\,d\theta$

$= \int_0^{2\pi} \frac{\pi}{4}(\sin\theta + \cos\theta)\,d\theta - \frac{1}{2}\,d\theta$

$= \frac{\pi}{4}\left[-\cos\theta + \sin\theta\right]_0^{2\pi} - \pi$

2



$\cancel{x = \sqrt{1-y^2}}$ $r^2 = 1$ $r = 1$ let $x = \cos\theta$, $y = \sin\theta$

$\cancel{\sqrt{1-y^2} + y + z = 0}$ $r\cos\theta + r\sin\theta + z = 0$ $z = -\cos\theta - \sin\theta$

$\oint_C \vec{F}\cdot d\vec{r} = \oint_C y\,dx + z\,dy + x\,dz$

$= \oint_C -\sin^2\theta\,d\theta - (\cos\theta + \sin\theta)\cos\theta\,d\theta + \cos\theta(\sin\theta - \cos\theta)\,d\theta$

$= \int_0^{2\pi} -d\theta - \cos^2\theta\,d\theta = -2\pi - \pi = -3\pi$

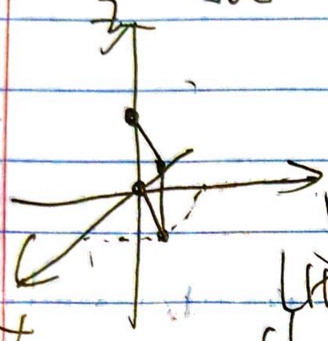
$$RHS = \iint_S \nabla\times\vec{F}\cdot d\vec{S}$$

$$= \iint_S \langle -1,-1,-1\rangle \cdot \frac{\langle 1,1,1\rangle}{1}\, dx\,dy \qquad \nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ y & z & x \end{vmatrix}$$

$$= \iint_S -3\, dx\,dy = -3\pi \qquad = -\hat{i} - \hat{j} - \hat{k}$$

$$\cancel{\int_0^{2\pi}\int}$$

3



$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ yz & xz & xy \end{vmatrix} = (x-x)\hat{i} - (y-y)\hat{j} + (z-z)\hat{k} = 0$$

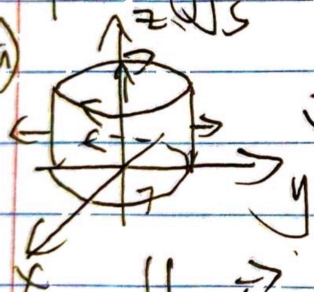
$$LHS = \oint_C \vec{F}\cdot d\vec{r}$$

$$= \int_0^1 0 + \int_0^1 dz + \int_1^0 x\,dx + x\,dx + \int_1^0 0$$

$$= z\Big|_0^1 + x^2\Big|_1^0 = 0$$

$$RHS = \iint_S \nabla\times\vec{F}\cdot d\vec{S} = 0$$

5(a)



$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ -y & x & x^2 \end{vmatrix} = -2x\hat{j} + 2\hat{k}$$

$$\iint_S \nabla\times\vec{F}\cdot d\vec{S} = \iint_{S_1} \langle 0,-2x,2\rangle \cdot \frac{\langle x,y,0\rangle}{a}\, a\, d\theta$$

$$+ \iint_{S_2} \langle 0,-2x,2\rangle \cdot \hat{k}\, dx\,dy$$

$$\iint_{S_1} = \int_0^{2\pi}\int_0^h -2xy\, dz\, d\theta = \int_0^{2\pi} -2a^2 \sin\theta\cos\theta\, h\, d\theta$$

$$= -2a^2h \int_0^{2\pi} \sin\theta\cos\theta\, d\theta = -a^2h\sin^2\theta\Big|_0^{2\pi} = 0$$

$$\iint_{S_2} = 2\pi a^2$$

$$\iint_S \nabla\times\vec{F}\cdot d\vec{S} = 2\pi a^2$$

b) $\iint_S \nabla\times\vec{F}\cdot d\vec{S}$

$= \oint_C -y\,dx + x\,dy + x^2 dz = \oint_C a\sin\theta \cdot a\sin\theta + a\cos\theta \cdot a\cos\theta \, d\theta$

$= \int_0^{2\pi} a^2 d\theta = 2\pi a^2$

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6G-1 a) yes

b) no

c) yes

d) i) not ii) yes iii) ~~yes~~ no IV) yes v) ~~yes~~ no

6H-1 a)

$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ F_x & F_y & F_z \end{vmatrix} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{i} - \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right)\hat{j} + \left(\frac{\partial F_y}{\partial x} -\right.$

$\nabla\cdot\nabla\times\vec{F} = \frac{\partial F_z}{\partial x\partial y} - \frac{\partial F_y}{\partial x\partial z} - \cancel{\frac{\partial F_z}{\partial x\partial y}} + \cancel{\frac{\partial F_x}{\partial y\partial z}} + \cancel{\frac{\partial F_y}{\partial x\partial z}} - \cancel{\frac{\partial F_x}{\partial y\partial z}}$

$= 0$

2a) $\oiint_S \nabla\times\vec{F}\cdot d\vec{S} = \iiint_D \nabla\cdot\nabla\times\vec{F}\, dV = 0$

b) $\oiint_S \nabla\times\vec{F}\cdot d\vec{S}$ ~~$= \oint_C \vec{F}\cdot d\vec{r}$~~

$= \iint_{S_1} \nabla\times\vec{F}\cdot d\vec{S} + \iint_{S_2} \nabla\times\vec{F}\cdot d\vec{S}$

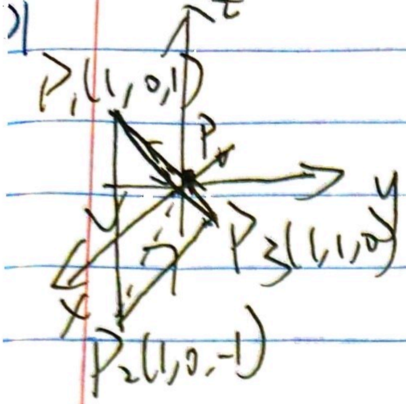
$= \oint_C F\cdot d\vec{r} - \oint_C F\cdot d\vec{r} = 0$

PART B 5/4/2025

~~1 a)~~

PARTB 5/4/2015

1)



a) as shown

b) $\int_{P_2P_1P_3} \vec{F}\cdot d\vec{r} = \int_{P_2P_1P_3} yz\,dy - y^2 dz$

$= \int_0^1 0 + \int_0^1 (1-z)z(-dz) - (1-z)^2 dz + \int_1^0 0$

$= \int_0^1 \left(z^2 - z - z^2 + 2z - 1\right) dz$

$= \int_0^1 (z-1)\,dz = \frac{z^2}{2} - z\Big|_0^1 = -\frac{1}{2}$

c) $\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ 0 & yz & y^2 \end{vmatrix} = (2y - y)\hat{i} = -3y\hat{i}$

$\oint_{C_1} \vec{F}\cdot d\vec{r} = \iint_{S_1} \nabla\times\vec{F}\cdot d\vec{S} = -3\cdot 0 = 0.$

$\oint_{C_2} \vec{F}\cdot d\vec{r} = \iint_{S_2} \nabla\times\vec{F}\cdot d\vec{S} = -3\left(\frac{1}{3}\right) = -1$

$\oint_{C_3} \vec{F}\cdot d\vec{r} = \iint_{S_3} \nabla\times\vec{F}\cdot d\vec{S} = -3\cdot\left(-\frac{1}{6}\right) = \frac{1}{2}$

$\oint_{C_4} \vec{F}\cdot d\vec{r} = \iint_{S_4} \nabla\times\vec{F}\cdot d\vec{S} = -3\left(-\frac{1}{6}\right) = \frac{1}{2}.$

d) i) each pair of corresponding edges are cancelled out,

ii) $\oint_{C_1} + \oint_{C_2} + \oint_{C_3} + \oint_{C_4} = \iint_{S_1} + \iint_{S_2} + \iint_{S_3} + \iint_{S_4}$

$= \oiint_S \nabla\cdot\nabla\times\vec{F}\,dV = 0$