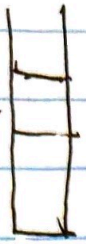
PSX 1/5/2005

1.



$P = \rho g h_1 + \rho_0 g h_2 = \rho_w g(h_w + h_w) = 455\,Pa$

~~$1.82 \rho_w g(h_2 + h_2) = 5 \times 9.8 \times 1.82 = 89.2\,Pa$~~

2. $\Delta P = \rho g h = 1000 \times 9.8 \times 1.2 = 1.18 \times 10^4\,Pa$

3.

$P_0 + \frac{1}{2}\rho v^2 + \rho g(h-y) = P_0 + \rho g(h-y)$

$v = \sqrt{2g(h-y)}$

$\begin{cases} y = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2y}{g}} \\ x = vt = \sqrt{\frac{4g(h-y)y}{g}} = 2\sqrt{(h-y)y} \end{cases}$

i) $\frac{dx}{dy} = \frac{1}{\sqrt{(h-y)y}}(h-2y) = 0$

$y = \frac{h}{2}$

ii) $x = 2\sqrt{(h-y)y} = y$

$4(hy - y^2) = y^2$

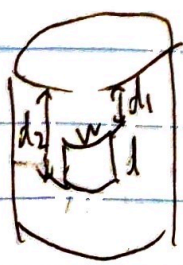
$5y^2 - 4hy = 0$

$y(5y - 4h) = 0$

$y = \frac{4}{5}h$

iii) $x_{max} = x\left(\frac{h}{2}\right) = h$

4.



i) $v = \sqrt{2gd}$

flow rate $= \int_0^w \int_{d_1}^{d_2} \sqrt{2gd}\, dd = w\left[\sqrt{2g} \cdot d^{\frac{3}{2}} \cdot \frac{2}{3}\right]_{d_1}^{d_2}$

$= \frac{2w\sqrt{2g}}{3}\left(d_2^{\frac{3}{2}} - d_1^{\frac{3}{2}}\right)\ m^3/s$

ii) $\frac{2w}{3}\sqrt{2g}\left[(d_1+\delta)^{\frac{3}{2}} - d_1^{\frac{3}{2}}\right]$

$= \frac{2w}{3}\sqrt{2g}\left[d_1^{\frac{3}{2}}\left(1+\frac{\delta}{d_1}\right)^{\frac{3}{2}} - d_1^{\frac{3}{2}}\right]$

~~$= w\delta\sqrt{2g}$~~

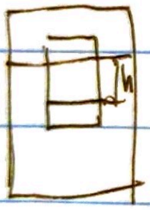
$\simeq \frac{2w}{3}\sqrt{2gd_1}\, d_1\left(\frac{3}{2}\frac{\delta}{d_1}\right)$

$= w\delta\sqrt{2gd_1}$

5. $V = \frac{m}{\rho}$

$T = F_b - mg = \rho_w V g - mg = \frac{\rho_w}{\rho} mg - mg$

$\frac{T}{mg} = \frac{\rho_w}{\rho} - 1 = \frac{1}{0.22} - 1 = 3.5$

6. 

$ma = -\rho V g + mg$

$= -\rho A g h + mg$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\rho A g}{m}}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{\rho A g}}$

7. $\begin{cases} P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \\ v_1 A_1 = v_2 A_2 \end{cases}$

$v_2 = \frac{v_1 A_1}{A_2}$

$\rho v_1^2 = 2(P_2 - P_1) + \rho\left(\frac{v_1 A_1}{A_2}\right)^2$

$\rho v_1^2 = 2(P_2 - P_1) + \rho v_1^2\left(\frac{A_1}{A_2}\right)^2$

$\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)\rho v_1^2 = 2(P_1 - P_2)$

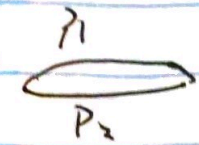
$v_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}}$

~~$v_1 = \sqrt{\frac{1}{\rho}\left(2(P_2 - P_1)\right) + \left(\frac{v_1 A_1}{A_2}\right)^2}$~~

~~$= \sqrt{\frac{2}{1000}\left(-2\times10^4\right)} +$~~

$= \sqrt{\frac{4\times10^4}{10^3\left(\left(\frac{[illegible]}{[illegible]}\right)^2 - 1\right)}}$ ~~$= \sqrt{\frac{[illegible]}{[illegible]}}$~~

$v_1 A_1 = 25\times10^{-4}\, v_1 = [illegible] = 8.1\times10^{-3}$

8. 

$P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(h_1 - h_2)$

$F = (P_2 - P_1)A = \rho A\left(\frac{1}{2}(v_1^2 - v_2^2) + g(h_1 - h_2)\right)$

$= 1.16 \times 50\left(\frac{1}{2}(10^2 - 70^2)\right) = 9.28 \times 10^4\ N$

9. 

$p(h) = \rho g h x$

$dF = p(h)\,dA = \rho g h\,dA$

$F = \int_0^L \int_0^h \rho g x\,dx\,dy$

$= L\int_0^h \rho g x\,dx = \frac{L\rho g}{2}x^2\Big|_0^h = \frac{L\rho g h^2}{2}$

$= \frac{10 \times 10^3 \times 9.8 \times 2^2}{2} = 1.96 \times 10^5\ N$

PS XI 1/5/2025

$C_w = 4.2 \times 10^3\ J/(kg \cdot K)$

$C_{ice} = 2060\ J/(kg \cdot K)$

$C_{vapour} = 1850\ J/(kg \cdot K)$

$L_{fusion} = 334\ kJ/kg$

$L_{vaporisation} = 2260\ kJ/kg$

1. $Q_1 = mC_{ice}\Delta T = 10 \times 2.06 \times 10^3\ J = 2.06 \times 10^4\ J$

$Q_2 = mL_f = 3.34 \times 10^5\ J$

$Q_3 = mC_w\Delta T = 4.2 \times 10^5\ J$

$Q_4 = mL_v = 2260 \times 10^3\ J = 2.26 \times 10^6\ J$

$Q = \sum Q = 3.03 \times 10^6\ J$

+2Cm₁

2. $C_2m_1(T-T_1)+m_1L_f=C_2m_2(T_2-T)$

$m_1(C_1(T-T_1)+L_f)=C_2m_2(T_2-T)$

$0.4(2.06\times10^3(T+2)+334\times10^3)=4200\times8(21-T)$

$T(C_2m_1+C_2m_2)=C_2m_1T_2+C_2m_2T_1-m_1L_f-2C_2m_1$

$T=\dfrac{C_2m_1T_2+C_2m_2T_1-m_1L_f-2C_2m_1}{C_2(m_1+m_2)}$

$=\dfrac{-2060\times(0.4\times2)+4200\times21-0.4\times334000}{1.4\times42000}$

$Q_{ice}=C_{ice}m_{ice}\times2=2.1\times0.4\times2=1.7kJ$

$Q_{water}=C_wm_w\times21=4.2\times21=88kJ$

$Q_f=Q_w-Q_i=86.5kJ=m_fL_f$

$m_f=\dfrac{86.5}{L_f}=\dfrac{865}{334}=0.26kg$

$m_{ice}'=m_{ice}-m_f=0.14kg$.

∴ product is 0.14kg ice and 1.26kg water.

3. $C_{Cu}=0.385kJ/(kg\cdot K)$

$C_{Cu}m_{Cu}(T-T_1)+C_wm_w(T-T_1)+C_xm_x(T-T_1)=C_wm_w(T_2-T)$

$(T-T_1)C_xm_x=C_wm_w(T_2-T_1)-(T-T_1)+C_{Cu}m_{Cu}(T_1-T)$

$C_x=\left(\dfrac{C_wm_w(T_2-T_1)}{T-T_1}+C_{Cu}m_{Cu}-C_wm_w\right)/m_x$

$C_x=\left(\dfrac{4.2\times100\times31}{29}+0.39\times30-4.2\times45\right)/75=2.5kJ$

4. $PV=nRT$

$n=\dfrac{PV}{RT}=\dfrac{100\times30}{300\times8.31}=2000mol$

5. $P_1V_1 = nRT_1$

$n = \frac{P_1V_1}{RT_1}$

$P_2V_2 = nRT_2$

$V_2 = \frac{nRT_2}{P_2} = \frac{P_1V_1T_2}{P_2T_1} = \frac{(P_0+\rho gh)VT_2}{P_0T_1}$

$= \frac{(100\times10^3\times9.8\times37)\pi(0.02)^2\times300}{100\times300\times280}$

$= 1.4\times10^{-4}\,m^3$

6. $V = \frac{nRT}{P}$

$= \frac{8.31\times273}{1.01\times10^5} = 0.0225\,m^3 = 22.5\,L$

7. $T = \frac{PV}{nR} = \frac{2.02\times10^5\times10}{8.31} = 2.43\times10^5\,K$

$U = \frac{3}{2}kT = \frac{3}{2}\times1.38\times10^{-23}\times2.43\times10^5 = 5.03\times10^{-18}\,J$

$= \frac{1}{2}mv^2 \quad v = \sqrt{\frac{2U}{m}} = 1.47\times10^5\,m/s$

8. $\frac{dQ}{dt} = -KA\frac{\Delta T}{L}$

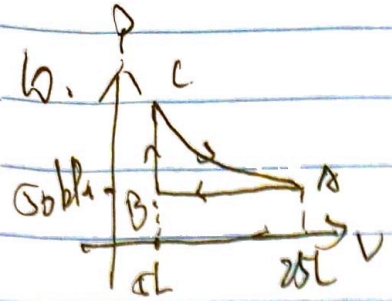
$= -47\times(\pi\cdot(0.02)^2)\frac{100}{0.5}$

$= -101\,W$

9. $\frac{dQ}{dt} = -KA\frac{\Delta T}{L}$

$= -1\times100\times\frac{17}{0.2} = 8500\,W$

10.

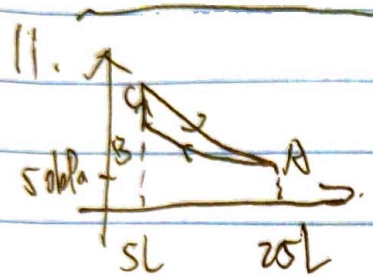


$P_A V_A = P_C V_C$

$P_C = \frac{P_A V_A}{V_C} = \frac{5\times10^4 \times 25\times10^{-3}}{5\times10^{-3}} = 2.5\times10^5 \text{ Pa}.$

$\therefore C : (2.5\times10^5 \text{Pa}, 5\text{L})$

11.



C→A adiabatic

A→B isothermal

$P_A V_A = P_B V_B$

$P_B = \frac{P_A V_A}{V_B}$

$= \frac{5\times10^4 \times 25\times10^{-3}}{5\times10^{-3}}$

$= 250 \text{kPa}$

$T_{AB} = \frac{P_A V_A}{nR} = \frac{5\times25\times10^{-3}}{8.31} = 150 \text{ K}$

$P_A V_A^\gamma = P_B V_B^\gamma$

$P_B = \frac{P_A V_A^\gamma}{V_B^\gamma} = \frac{5\times10^4 \times (25\times10^{-3})^{\frac{7}{5}}}{(5\times10^{-3})^{\frac{7}{5}}}$

$= 5\frac{7}{9}\times10^4 \text{ Pa} = 427 \text{ kPa}$

$T_C = \frac{P_B V_B}{nR} = 257 \text{K}$

$W_{C\to A}^T = P_A V_A \ln\left(\frac{V_A}{V_C}\right)$

$= 5\times10^4 \times 25\times10^{-3} \times \ln\left(\frac{25}{5}\right)$

$= 2012 \text{ J}$

$W_{A\to B}^P = P\Delta V = 5\times10^4 \times (25-25)\times10^{-3} = -10^3 \text{ J}$

$W_{B\to C}^V = 0$

$W = 1012 \text{ J}$

$U = Q - W$

$Q = W = 1012 \text{ J}$ absorbed.

$W_{C\to A}^Q = \frac{P_B V_C - P_A V_A}{\gamma - 1}$

$= \frac{2}{5}\left(427\times10^3 \times 5\times10^{-3} - 5\times10^4 \times 25\times10^{-3}\right)$

$= 2655 \text{ J}$

$W_{A\to B}^T = P_A V_A \ln\left(\frac{V_B}{V_A}\right)$

$= 50\times25 \times \ln\left(\frac{5}{25}\right) = -2012$

$W_{B\to C}^V = 0$

$Q = W = 643 \text{ J}$ absorbed.