



7. A body of rest mass  $m_0$  moving at speed  $v$  collides with and sticks to an identical body at rest. What is the mass and momentum of the final clump?
8. A body of rest mass  $m_0$  moving at speed  $v$  approaches an identical body at rest. Find  $V$ , the speed of a frame in which the total momentum is zero. Do this first by the law of composition of velocities starting with how you would do this non-relativistically. Next, repeat using the transformation law of for the components of the Energy-momentum vector. (You may need to come back to the second part after I do energy-momentum vector in class.)
9. Show that a photon cannot break up into an electron and a positron. For our purposes the electron and positron are identical particles with four-momenta (with  $c = 1$ )  $P_1 = m_0(\gamma_1; \gamma_1 \mathbf{v}_1)$  and  $P_2 = m_0(\gamma_2; \gamma_2 \mathbf{v}_2)$  where  $\gamma = 1/\sqrt{1 - v^2}$ . The photon four-momentum is  $K = (\omega; \mathbf{k})$  with  $\omega = |\mathbf{k}|$ . I suggest you use energy-momentum conservation and the dot products of four vectors to show that this process is kinematically forbidden.
10. **In this problem let  $c = 1$ .** A particle of rest mass  $m_0$  decays into a photon and loses rest mass  $\pm$  in the bargain. Show that the photon energy is  $\omega = \pm(1 \pm \frac{\pm}{2m_0})$ . Note that the photon has zero rest mass, that is, the square of its four momentum vanishes. The problem can be solved most efficiently if you use four vectors and dot products between them, but I do not insist you do. Denote the four vector of the photon by  $K = (\omega; \mathbf{k})$  and that of the particle before and after by  $P = (E; \mathbf{p})$  and  $P' = (E'; \mathbf{p}')$ .

## TWO ADDITIONAL PROBLEMS

11. Consider a particle of rest mass  $m_0$  moving at velocity  $v$  in your frame  $S$ . Write down expression for the components of its energy momentum vector  $P = (p_0; \mathbf{p})$  in terms of  $m_0; v$ . Now see this particle from a frame  $S'$  moving at velocity  $u$ . What will be its velocity  $w$  and what will be the components of  $P' = (p'_0; \mathbf{p}')$ , first in terms of  $w$  and then in terms of  $w$  written in terms of  $u$  and  $v$ ? Show that the primed coordinates are related to unprimed ones by the same Lorentz Transformation that relates  $x_0; x_1$  to  $(x'_0; x'_1)$ .
12. **OPTIONAL, SOLUTIONS WILL BE POSTED** Consider two rockets  $A$  and  $B$  of rest length  $L_0 = 1$  m travelling towards each other with a tiny shift in the  $y$ -direction so they do not collide, as in part (i) of Figure 1. Each sees the other approach it with speed  $u$ . According to  $A$ , when the tail of  $B$  passed the tip of  $A$ , a missile was fired from the tail of  $A$  towards  $B$  as in part (ii) of the figure. It will clearly miss due to length contraction of  $B$  as seen by  $A$ . But  $B$  will see the event as in part (ii) of the Figure and expect a hit. Who is right? (Ignore the time it takes the missile to hit its target).

It will be most educational to assign space-time coordinates to five events in each frame: tip of  $A$  passes tip of  $B$  (set it to  $(0,0)$  for both), tail of  $B$  passes tip of  $A$ , missile is fired, tip of  $B$  passes tail of  $A$  and finally tails pass. It will be useful to know that if an event occurs at either end of either rockets its spatial coordinates are no-brainers *in that rocket frame* since the tips are always at  $x = 0$ ;  $x' = 0$  and the

tails are at  $x = -1$  and  $x' = +1$ . It should also be easy to find time elapsed between tip of your rocket passing my tip and my tail since I am of unit length and you are moving at speed  $u$ . Ditto for tail. As a check of your coordinates you may want to see that the space-time interval between any event and  $(0;0)$  comes out same for both.

FIG. 1. The point of view of  $A$  is in (i). We think  $B$  will see it as in (ii). But something is wrong since there was no hit as per  $A$ . The three funny lines coming transversely out of the tip of  $A$  are supposed to be the missile aimed at  $B$ .