Exam 1 A 11/3/2015 90

P1 a) $\vec{OQ} = \hat{i} + \hat{j} + \hat{k}$

$\vec{OR} = \hat{i} + \frac{1}{2}\hat{j} + \frac{\hat{k}}{2}$

b) $\cos\theta = \frac{\vec{OQ}\cdot\vec{OR}}{|\vec{OQ}||\vec{OR}|} = \frac{\frac{1}{2}+1+\frac{1}{2}}{\sqrt{3}\sqrt{\frac{3}{2}}} = \frac{2\sqrt{2}}{3}$

$\theta = \arccos\frac{2\sqrt{2}}{3} \approx 0.340$

P2 $\vec{v} = \frac{d\vec{R}}{dt} = -3\sin t\,\hat{i} + 3\cos t\,\hat{j} + \hat{k}$

$|v| = \sqrt{9\sin^2 t + 9\cos^2 t + 1} = \sqrt{10}$

P3 a) $a = -\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = -1$ 2

Inverse!

$b = \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2$ -1 2 -3

b) $X = A^{-1}B = \frac{1}{2}\begin{bmatrix} 1 & & \\ -1 & -2 & 5 \\ 2 & 2 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1+2+2 \\ -1+4+5 \\ 2-4+6 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 5 \\ 8 \\ 4 \end{bmatrix}$ $\begin{bmatrix} -3 \\ 4 \\ 4 \end{bmatrix}$

c) $M = \begin{bmatrix} 1 & 3 & c \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ $|A| = \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} + 3\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} + c\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = \begin{bmatrix} 2.5 \\ 4 \\ 2 \end{bmatrix}$

$= 1 + 3(+1) + c(2)$

$= 4 + 2c = 0$

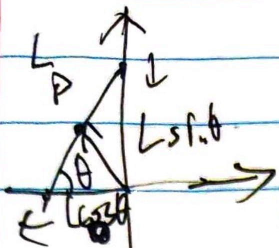
$m = \begin{bmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \end{bmatrix}$ $\det(m) = \det(\vec{u}, \vec{v}, \vec{w}) = 0$ -2

$\therefore \vec{u}, \vec{v}, \vec{w}$ coplane.

~~$\vec{x}\cdot\vec{u} = 0$~~

$X = n\ \vec{v}\times\vec{w}$ ~~$= \hat{i} + \hat{j} + 2\hat{k}$~~ $n\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $n \in R$

P4



$\vec{OP} = \langle -\frac{L\cos\theta}{2}, \frac{L\sin\theta}{2} \rangle$

$P(x,y) \begin{cases} x(\theta) = -\frac{L\cos\theta}{2} \\ y(\theta) = \frac{L\sin\theta}{2} \end{cases}$

P5 a) $\vec{P_0P_1} = \langle -1,-1,1 \rangle$

$\vec{P_0P_2} = \langle 0,-2,1 \rangle$

$A = \frac{1}{2}|\vec{P_0P_1} \times \vec{P_0P_2}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 1 \\ 0 & -2 & 1 \end{vmatrix} = |\hat{i} + \hat{j} + 2\hat{k}| \cdot \frac{1}{2}$

$= \frac{\sqrt{2+4}}{2} = \frac{\sqrt{6}}{2}$

b) $\vec{n} = \vec{P_0P_1} \times \vec{P_0P_2}$

$= \langle 1,1,2 \rangle$

$x-2 + (y-1) + 2z = 0$

$x + y + 2z = 3$

c) $x = t-1$

$y = t$

$z = t$

$t-1+t+2t = 3$

$4t = 4$

$t = 1$

$\therefore (0,1,1)$

P6 a) $\frac{d}{dt}(\vec{R}\cdot\vec{R}) = \frac{d\vec{R}}{dt}\cdot\vec{R} + \vec{R}\cdot\frac{d\vec{R}}{dt} = 2\vec{R}\cdot\frac{d\vec{R}}{dt}$

~~$= 2\langle x', y', z' \rangle \cdot \langle x, y, z \rangle$~~

b) $\frac{d}{dt}(|R|^2) = 0 \Rightarrow 2\vec{R}\cdot\frac{d\vec{R}}{dt} = 0$

$\therefore \vec{R} \perp \vec{V}$

c) $\vec{R}\cdot\vec{A} = -V^2$

$\frac{d}{dt}(\vec{R}\cdot\vec{V}) = V^2 + \vec{R}\cdot\vec{A} = 0$