

It follows

$$P^2 = P_0^2 - P_1^2 = P_0'^2 - P_1'^2$$

is an invariant. What is this invariant value? You can show by explicit calculation using Eq. (5) that

$$P^2 = m^2 c^2$$

Or you can be clever and say that since it is invariant, I will calculate it in the frame moving with the particle. There only $P_0 = mc$ is nonzero and the result follows. We can also rewrite

$$P^2 = (E/c)^2 - p^2 = m^2 c^2$$

as

$$E^2 = (cp)^2 + m^2 c^4$$

Photons also have energy, which we denote by ϵ instead of E , and momentum, which we denote by k instead of p . We assemble these onto a four-vector

$$K = (K_0; K_1) = (\epsilon/c; k)$$

whose components obey

$$\epsilon = kc$$

This is the same as

$$K \cdot K = K^2 = 0$$

In other words $P^2 = m^2 c^2$ becomes $K^2 = 0$ when applied to massless particles like photons. However massless does not mean momentum-less or energy-less.

In any collision, you set the sum of initial four momenta equal to the sum of the final four-momenta. This is really four equations (or two if we set motion along y and z to zero) and sometimes rather than juggle these equations you should think in terms of four vectors and their dot products, as illustrated in class and the notes.

Remember P^2 same in all frames whether it refers to the momentum of one particle or the sum over many. When P is the momentum of a single particle $P^2 = c^2 m^2$ regardless how it is moving. When P is the sum of many momenta, such as total of all incoming momenta, P^2 can be anything, but the same anything for all observers. (Recall the anti-proton creation experiment where the square of the total momentum was evaluated in the CM frame).