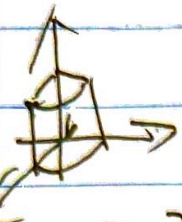
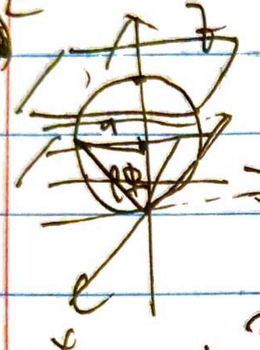
Exam 4 4/4/2025 $\underline{\underline{100}}$

P1 $\int_0^{\frac{\pi}{2}}\int_0^1\int_0^1 r^2\, r\, \cancel{dz\,dr\,d\theta}\, dz\,dr\,d\theta$



P2 

a) $\rho = 2a\cos\phi$

b) $\rho = a\csc\phi \quad a\sec\phi$

c) $\cancel{\int_0^{2\pi}\int_{a\sec\phi}^{2a\cos\phi}}\quad \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_{a\sec\phi}^{2a\cos\phi}\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$

P3 a) $\nabla\times\vec{F} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ 2xy+z^3 & x^2+2yz & y^2+3xz^2-1\end{vmatrix} = \underbrace{(2y-2y)\hat{i} - (3z^2-3z^2)\hat{j} + (2x-2x)}_{=0}$

b) $f(x,y,z) = \int \frac{\partial f}{\partial x}dx = \int 2xy+z^3\, dx = x^2y + z^3x + g(y,z).$

$\frac{\partial f}{\partial y} = x^2 + \frac{\partial g}{\partial y} = x^2+2yz$

$\frac{\partial g}{\partial y} = 2yz$

$g(y,z) = \int\frac{\partial g}{\partial y}dy = y^2z + h(z).$

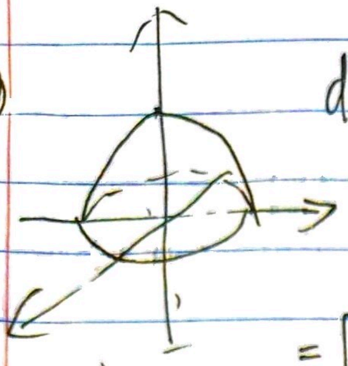
$\frac{\partial f}{\partial z} = 3z^2x + y^2 + \frac{\partial h}{\partial z} = y^2+3xz^2-1$

$\frac{\partial h}{\partial z} = -1$

$h(z) = \int\frac{\partial h}{\partial z}dz = -z + C$

$f(x,y,z) = x^2y + z^3x + y^2z - z + C$

P4



$d\vec{S} = \langle 2x, 2y, 1\rangle\, dx\,dy \quad 1-x^2-y^2 > 0$

$r<1$

a) $\iint_S \vec{F}\cdot d\vec{S} = \iint_S \left[2x^2+2y^2+2(x^2+y^2)\right] dx\,dy$

$= \iint_S (4r^2)\, r\,dr\,d\theta$

$\int_0^{2\pi}\int_0^1 4r^3\,dr\,d\theta$

$= \int_0^{2\pi} d\theta = 2\pi$

b) $\iint_{S}\vec{F}\cdot d\vec{S} = \iiint_D \nabla\cdot\vec{F}\,dV - \iint_{S'}\vec{F}\cdot d\vec{S} = \cancel{\iiint_D (\ldots)\,dV} = 0$

$= 0 - \iint_{S'}\vec{F}\cdot(-\hat{k})\, r\,dr\,d\theta$

$= \cancel{-\iint_{S'} 2r^2\, r\,dr\,d\theta} = \int_0^{2\pi}\int_0^1 2r\,dr\,d\theta$

$= \cancel{\int_0^{2\pi}\int_0^1 r^3\,dr\,d\theta} = 2\pi$

P5 a) $\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xz & 0 & y^2 \end{vmatrix} = 2y\hat{i} + 2x\hat{j}$

b) $\iint_R \nabla\times\vec{F}\cdot d\vec{S} = \cancel{\oint_C \vec{F}\cdot d\vec{r} = \oint_C 2xz\,dx + y\,dz}$

$= \iint_R \langle 2y, 2x, 0\rangle\cdot\langle x, y, z\rangle \sin\phi\, d\phi\, d\theta$

$= \cancel{\iint_R 4xy\sin\phi\,d\phi\,d\theta} = \cancel{\iint_R 4\sin\phi\cos\theta\sin\phi\sin\theta\,d\phi\,d\theta} = 0$

$\cancel{= \int_0^{2\pi}\int_0^{\pi/2} \sin^2\phi\,d\phi\,d\theta = \int_0^{2\pi} 2\cos\theta\sin\theta\,d\theta} = 0$

c) $\oint_C \vec{F}\cdot d\vec{r} = \iint_R \nabla\times\vec{F}\cdot d\vec{S} = 0$