$x_m - x_0 - \frac{2Mg}{k}$

PS IV 10/4/2025

1\. i) $(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B}) = 0$

$\Rightarrow A^2 - B^2 = 0$

$A^2 = B^2$

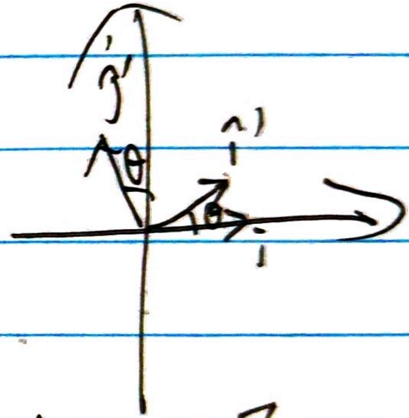
$|A| = |B|$

ii) $A_x' = \vec{A}\cdot\hat{i}' = \vec{A}\cdot(\hat{i}\cos\theta + \hat{j}\sin\theta)$

$= A_x\cos\theta + A_y\sin\theta$

$A_y' = \vec{A}\cdot\hat{j}' = \vec{A}\cdot(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

$= -A_x\sin\theta + A_y\cos\theta$



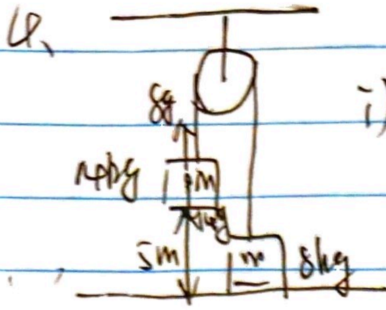
2\. $W = \int_0^A F(x)\,dx = \int_0^A A x^{\frac{3}{2}}\,dx = A\cdot\frac{2}{5}x^{\frac{5}{2}}\Big|_0^A = \frac{2}{5}A^{\frac{7}{2}}$

3\. Bridge 1: $0.11t\cdot 225\times 7.5\times 10 + 1150$

Bridge 2: $475\cdot 0.2t\times 9.5\times 10^{5}t$ = + 850.

$T_1 = T_2$, t = 2880 days.

4.



i) $G - T = (M+m)a$

$a = \frac{G-T}{M+m} = \frac{(M-m)g}{M+m}$

$V = \sqrt{2a \cdot x} = \sqrt{\frac{2\times(M-m)g}{M+m}} = \sqrt{\frac{10\times 6g}{22}}$

$Mgh = mgh + \frac{1}{2}(M+m)V^2$

$V = \sqrt{\frac{2(M-m)gh}{M+m}}$

$= \frac{\sqrt{\ }}{11} \approx 5.17$ m/s

ii) $\frac{1}{2}mv^2 = mg\Delta h$

$\Delta h = \frac{V^2}{2g} = \frac{15}{11} \approx 1.36$ m  $h' = h_0 + \Delta h = 6.36$ m

iii) $E_0 = Mgh$  $E_1 = mgh'$.

$\frac{E_1}{E_0} = \frac{mgh'}{Mgh} = \frac{8\times 6.36}{14\times 5} \approx 1.727$

5. i) $mgh = mg\cdot 2R + \frac{1}{2}mV_{min}^2 = 2mgR + \frac{1}{2}mgR$

$h = \frac{5}{2}R$

ii) $\frac{1}{2}mv^2 = mgh$

$v = \sqrt{2gh} = \sqrt{5gR}$

iii) $F = N - mg = m\frac{v^2}{R}$

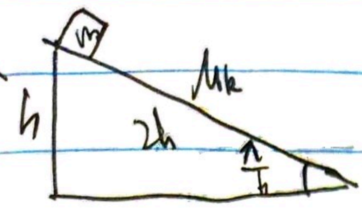
$N = m\frac{5gR}{R} + mg = 6mg$

iv) $mgh = \mu_k mg x$

$x = \frac{h}{\mu_k} = \frac{5R}{2\mu_k}$

$\frac{\sqrt{105}}{25} \approx 2.87$ m

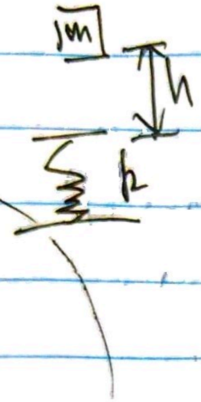
v). $mgh = \frac{1}{2}kx^2$   $x = \sqrt{\frac{2mg\cdot\frac{5}{2}R}{k}} = \sqrt{\frac{5mgR}{k}} = 7$ m

6. 

$mgh = \cancel{\mu_k mg\, 2h}\ \mu_k mg\cos\frac{\pi}{6}\cdot 2h + \frac{1}{2}mv^2$

$v^2 = 2gh - 2\mu_k g\cdot\sqrt{3}h$

$v = \sqrt{2gh(1-\sqrt{3}\mu_k)}$

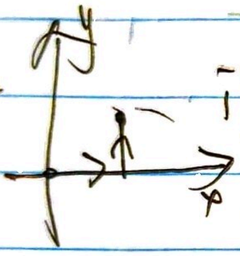
7. 

$mg(h+A) = \frac{1}{2}kA^2$

$\frac{1}{2}kA^2 - mgA - mgh = 0$

$A = \frac{mg \pm \sqrt{m^2g^2 + 2k(mgh)}}{k}$

$= \frac{mg}{k}\left(1+\sqrt{1+\frac{2hk}{mg}}\right)$

8. 

i) $\int_C \vec{F}\cdot d\vec{r} = \int_C x^2y^3dx + x^3y^2dy$

$= \int_0^1 0 + \int_0^1 y^2dy = \frac{1}{3}$

ii) $y=x.\ W=\int_C \vec{F}\cdot d\vec{r} = \int_0^1 x^5dx + x^5dx$

$= \frac{1}{3}x^6\Big|_0^1 = \frac{1}{3}$

iii) $y=x^2\quad W=\int_C \vec{F}\cdot d\vec{r} = \int_0^1 x^8dx + x^7\cdot 2xdx$

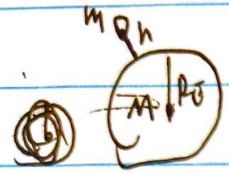
$= \int_0^1 3x^8dx = \frac{1}{3}x^9\Big|_0^1 = \frac{1}{3}$

iv) $\nabla\times\vec{F} = \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = 3x^2y^2 - 3x^2y^2 = 0.$

$U(x,y) = \int\frac{\partial U}{\partial x}dx = \int x^2y^3dx = -\frac{1}{3}x^3y^3 + g(y).$

$\frac{\partial U}{\partial y} = -x^3y^2 + g'(y) = -x^3y^2 \Rightarrow g'(y)=0.\ g(y) = \int g'(y)dy = C.$

$U(x,y) = -\frac{1}{3}x^3y^3 + C.\qquad W = U(0,0) - U(1,1) = 0 + \frac{1}{3} = \frac{1}{3}$

9. 

$$F = G\frac{mM}{(h+R_E)^2}$$

$$g_h = \frac{F}{m} = \frac{GM}{(h+R_E)^2} = \frac{GM}{R_E^2\left(1+\frac{h}{R_E}\right)^2} \approx \frac{GM}{R_E^2+2hR_E}$$

$$g_0 = \frac{GM}{R_E^2}$$

$$\frac{g_h}{g_0} = \frac{R_E^2}{R_E^2+2hR_E}$$

$$g_h = \frac{g_0}{1+\frac{2h}{R_E}} \approx \left(1-\frac{2h}{R_E}\right)g_0$$

10. $$G\frac{mM}{r^2} = m\omega^2 r = \frac{4\pi^2 m r}{T^2}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} \approx 5.82\times10^{15}\,s$$

$\approx$ 184 Million years

$$-\frac{GMm}{R}+\frac{1}{2}mv_0^2=-\frac{GMm}{R_E}+\frac{1}{2}mv^2$$

$$GM\left(\frac{1}{R_E}-\frac{1}{R}\right) \cancel{=\frac{1}{2}(v^2-v_0^2)} \quad +\frac{1}{2}v_0^2=\frac{1}{2}v^2$$

$$\cancel{v^2-v_0^2}=\cancel{\frac{v^2}{2}} \quad v=1.04\times10^4\,m/s$$

2. $$\frac{GM}{R^2}=\frac{4\pi^2R}{T^2}$$

$$R^3=\frac{GMT^2}{4\pi^2} \quad R=\sqrt[3]{\frac{GMT^2}{4\pi^2}}=4.23\times10^7\,m$$

3. $$\lim_{R\to\infty}-\frac{GMm}{R}+\frac{1}{2}mv_0^2=\frac{1}{2}mv^2 \geq 0$$

$$\cancel{-\frac{GMm}{R_E}+\frac{1}{2}mv_0^2=0}$$

$$-\frac{GMm}{R}+\frac{1}{2}mv^2=0$$

$$v^2=\frac{2GM}{R} \quad v=\sqrt{\frac{2GM}{R}}$$

4. $$v_h dt\cdot r_h=v_L dt\cdot r_L$$

$$v_L=\frac{v_h\cdot r_h}{r_L}=\frac{7.23\times(840+6.4\times10^3)}{230+6.4\times10^3}$$

$$=7.95\,km/s$$