Midterm Exam 12/9/2015

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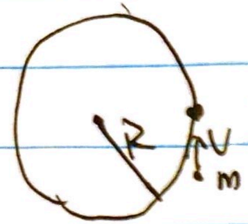
I.

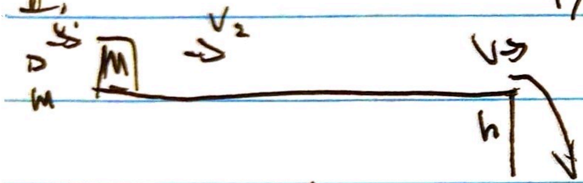


i) ~~momentum and~~ angular momentum are conserved.

ii) $I_w \cdot 0 + mR^2\omega_0 = I \cdot \omega_f$

$$\omega_f = \frac{mR^2 \cdot \frac{v}{R}}{I} = \frac{mRv}{mR^2 + \frac{1}{2}MR^2} = \frac{v}{R + \frac{1}{2}MR/m}$$

II.



i) $mv_1 = mv_2 + Mv$

$$V = \frac{mv_1 - mv_2}{M}$$

$$t_1 = \frac{L}{V} = \frac{ML}{mv_1 - mv_2}$$

ii) $\begin{cases} \frac{1}{2}gt^2 = h \\ x = vt \end{cases}$ $\quad t = \sqrt{\frac{2h}{g}}$

$$x = vt = \frac{mv_1 - mv_2}{M}\sqrt{\frac{2h}{g}}$$

III. ~~$\frac{\partial F_x}{\partial x} = 6y^3 \quad \frac{\partial F_y}{\partial y} = 6x^2y$~~

i) $\nabla \times \vec{F} = \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = 6xy^2 - 6xy^2 = 0$. $\therefore \vec{F}$ is conserved.

ii) $U(x,y) = -\int F_x\,dx = -\int 2xy^3\,dx = -x^2y^3 + g(y)$

$F_y = -\frac{\partial U}{\partial y} = 3x^2y^2 - g'(y) = 3x^2y^2 \qquad g'(y) = 0. \quad g(y) = C.$

$\therefore U(x,y) = -x^2y^3$

iii) $W = -\Delta U = U(0,0) - U(1,1)$

$= 0 - 1 = -1$

IV. i) $E = K + U$

$= \frac{1}{2}mv^2 - \frac{GmM}{R}$

$E < 0$

$\lim_{R \to +\infty} E = \frac{1}{2}mv^2 < 0$

∴ cannot escape

ii) $k(\Delta x)_1 = mg$

block. $k(\Delta x)_2 = Mg$

$k(\Delta x)_2 = f = \mu_s$

Δx: the longest distormation made before the block moves.

$k\Delta x = f = \mu_s Mg = k(\Delta x)_2\ \mu_s$

ii) $I_1 = \int_0^{\pi}\int_0^{2\pi}\int_0^{R} (r\sin\theta)^2 \cdot \frac{M}{\frac{4}{3}\pi R^3} r^2 \sin\theta \, dr \, d\phi \, d\theta$

$= \frac{2}{5}MR^2$ $\quad I_1' = I_1 + MR^2 = \frac{7}{5}MR^2$

$I_2 = \frac{2}{5}(M+m)R^2 - \frac{2}{5}mr^2$

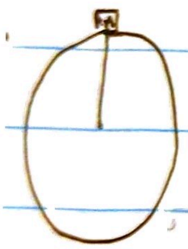
$= \frac{2}{5}MR^2 + \frac{2}{5}m(R^2 - r^2)$ $\quad I_2' = I_2 + MR^2 = \frac{7}{5}MR^2 + \frac{2}{5}m(R^2 - r^2)$

$\therefore I_2' > I_1'$

$\tau = Mg\sin\theta \cdot R$

$\alpha_2 = \frac{\tau}{I_2'}$ $\quad \alpha_1 = \frac{\tau}{I_1'}$

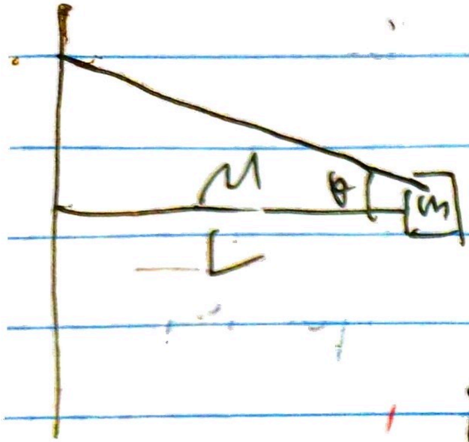
$\therefore \alpha_2 < \alpha_1$ $\quad \therefore$ the ~~sho~~ slower one is the hollow one



$mv_{top}^2/R = mg + T_1$ $\quad mv_b^2/R = T_2 - mg$

$\frac{1}{2}mv_{top}^2 + 2mgR = \frac{1}{2}mv_{Bottom}^2$ $\quad V_t^2 - V_B^2 = -4gR$

$T_1 - T_2 = \frac{m}{R}(v_t^2 - v_b^2) - 2mg = -6mg$



i) $Mg \cdot \frac{L}{2} \cancel{= T\sin\theta} + mgL = T\sin\theta\, L$

$T = \csc\theta\, \cancel{gL}\,(M/2 + m)$ $\csc\theta\, g\,(M/2+m)$

ii) $CM = \frac{1}{M+m} \cdot L \cdot (M/2 + m)$

$(M+m) g\, CM = \frac{1}{2} I \omega^2 + \frac{1}{2} m (\omega L)^2$

$gL(M/2+m) = \frac{1}{2}\left(\frac{1}{3} ML^2 + mL^2\right)\omega^2$

$\omega^2 = \frac{2g}{L} \frac{M/2 + m}{M/3 + m}$

$\omega = \sqrt{\frac{2g}{L} \frac{M/2+m}{M/3+m}}$