

1: To calculate b from the $\text{Lap}(x|b)$ that satisfies ϵ -differential privacy with an l_1 -sensitivity of 1 we can use this formula: $b = \frac{\Delta f}{\epsilon}$

where Δf is the l_1 sensitivity of the function f that we want to compute on the data, ϵ is the privacy budget

In this case, we have $\Delta f = 1$. so $b = \frac{1}{\epsilon}$

2: DP-SGD is a variant of the standard SGD algorithm that adds noise to gradients to achieve differential privacy.

Here are the steps

① Initialize the model parameters

② For each epoch, shuffle the training data

And for each batch:

select
random
data
point

① Compute the gradient of the loss function with respect to the model parameters on the current batch

② Add Laplace noise to gradient to achieve differential privacy. The noise depends on ϵ and the sensitivity of the gradient

③ randomly select unbiased gradient estimate

④ Update the model parameter using the noisy gradient and a learning rate

⑤ return the final model parameters



3: Here is an algorithm based on the requirements:

- ① Compute gradient matrix $G \in \mathbb{R}^{n \times p}$ using the training data
- ② Perform SVD on G to obtain $G = U \Sigma V^T$
- ③ Choose the top k rows and let them be $B = V_k$
- ④: $\hat{G} = GB$, using this formula to compress G
- ⑤: Doing per-example clipping and use \hat{G} in DP-SGD
to add Gaussian noise
- ⑥: Inject \hat{G} back to RP using B^T
- ⑦: update the model parameters

