OP202 HW

Yanyu ZHOU

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1 Q1

Example 1. $-\log(x)$ *is self-confordant on its own domain but not strongly convex.*

Proof. Let $f(x) = -\log(x), x > 0$, then $f'(x) = -\frac{1}{x}$, $f''(x) = x^{-2}$, $f'''(x) = -2x^{-3}$. Since $2x^{-3} \leq 2x^{-3}$, $|-2x^{-3}| \leq f''(x)^{\frac{3}{2}}$.

Also, if there exists a $m > 0$ such that f is m-strongly convex, we have $f''(x) \geq m\text{In}$. It's impossible since $x > 0$. □

2 Q2

Theorem 2. *All positive semi-definite matrices form a convex set.*

Proof. Let A, B be two positive semi-definite matrices, $\lambda \in [0, 1]$. Then we have

$$x^T[\lambda A + (1-\lambda)B]x = x^T(\lambda Ax + (1-\lambda)Bx) = \lambda x^T Ax + (1-\lambda)x^T Bx \geq 0.$$

□