

Application of the Millepede algorithm to the Time and Position Calibration of NeuLAND

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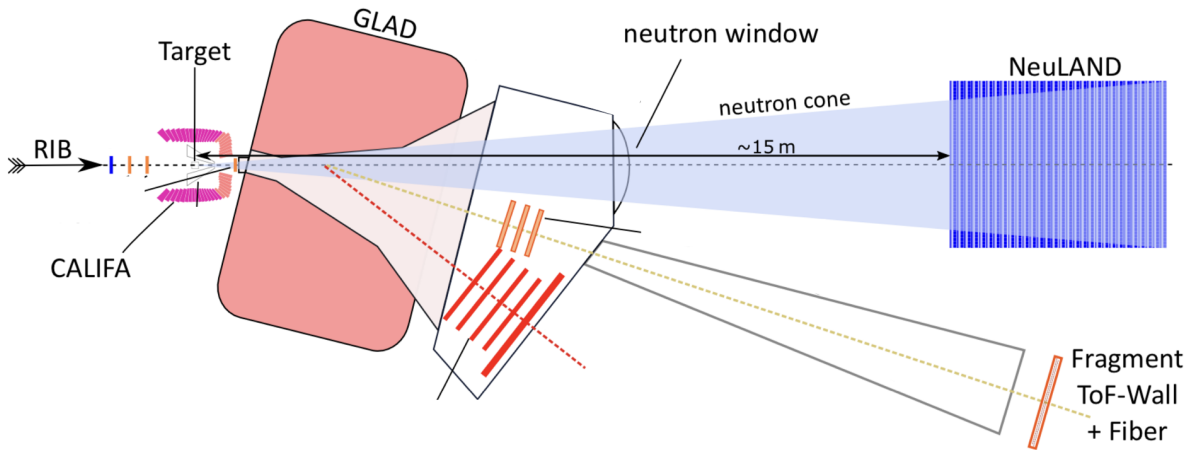
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NeuLAND setup in R³B

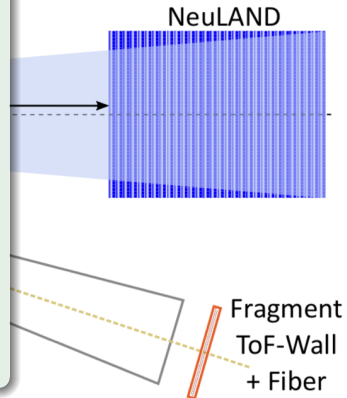


NeuLAND setup in R³B



Geometry:

- 26 planes
- $250 \times 250 \text{ cm}^2$
- 50 scintillators each plane
- 2600 PMTs in total



NeuLAND setup in R³B

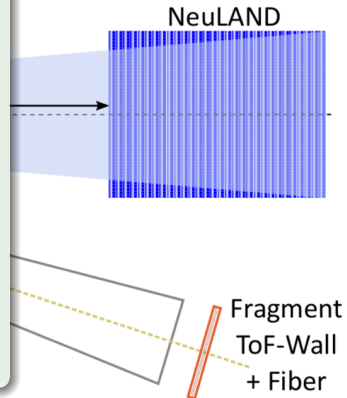


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Measurements:

- interaction position
- interaction time
- energy deposition



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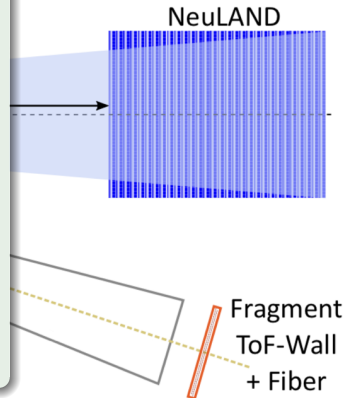


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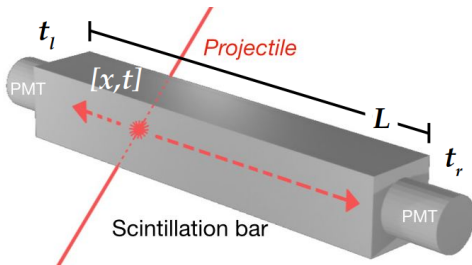
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Measurements:

- **interaction position**
- **interaction time**
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Position and time calculation



Symbols:

x : position of the interaction

t : time of the interaction

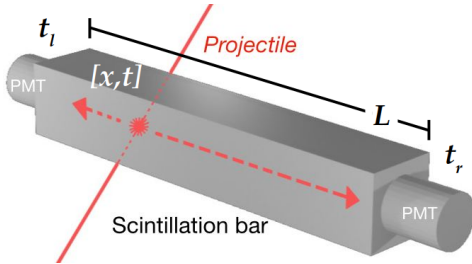
L : length of the scintillator

t_l : time of the left PMT signal

t_r : time of the right PMT signal

C_e : effective speed of light

Position and time calculation



Time relation:

$$t = \frac{t_r + t_l}{2} - \frac{L}{2 \cdot C_e}$$

Position relation:

$$x = \frac{C_e}{2} (t_r - t_l)$$

Symbols:

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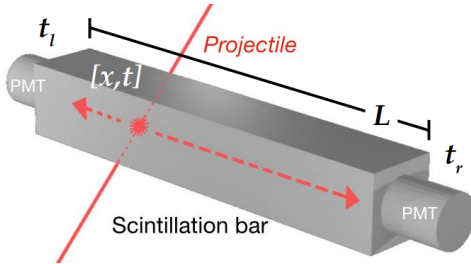
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Position and time calculation



Time relation:

$$t = \frac{t_r + t_l}{2} - \frac{L}{2 \cdot C_e} + t_{\text{sync}}$$

Position relation:

$$x = \frac{C_e}{2} (t_r - t_l)$$

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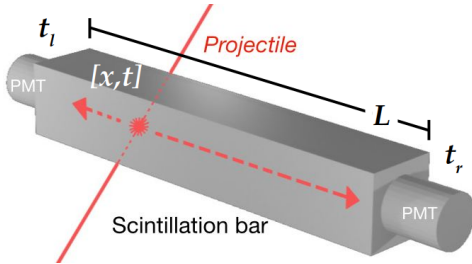
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Additional calibration parameters:

- t_{sync} : time synchronization among scintillators

Position and time calculation



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Position relation:

$$x = \frac{C_e}{2} (t_r - t_l + t_{\text{offset}})$$

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L : length of the scintillator

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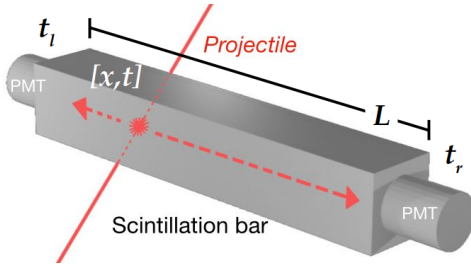
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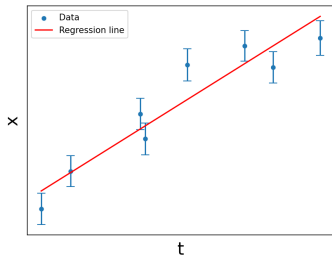
- t_{sync} : time synchronization among scintillators
- t_{offset} : time offset between adjacent PMTs

Total number of calibration parameters: **3900**

Calibration relation

$$x = C_1 \cdot t + C_2$$

Data fitting:



(t_1, x_1)

(t_2, x_2)

...

(t_i, x_i)

...

(t_n, x_n)

Minimize

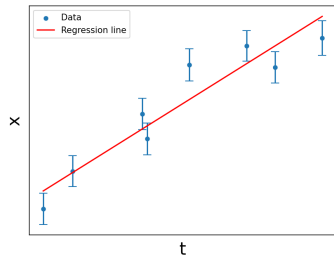
$$\text{residual} = \sum_i \frac{(x_i - x(t_i, C_1, C_2))^2}{2 * \sigma_i^2}$$

Calibration principle

Calibration relation

$$x = C_1 \cdot t + C_2$$

Data fitting:



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Calibration with muon tracks

$$t = (t_r + t_l)/2 - L/(2 \cdot C_e) + t_{\text{sync}} \quad (1)$$

$$x = C_e \cdot (t_r - t_l + t_{\text{offset}}) / 2 \quad (2)$$

$$x_\mu = a_x^i \cdot z_\mu + b_x^i \quad (3)$$

$$y_\mu = a_y^i \cdot z_\mu + b_y^i \quad (4)$$

$$t_\mu = a_t^i \cdot z_\mu + b_t^i \quad (5)$$

Calibration parameters for the i th track:

global parameters : $C_e, t_{\text{sync}}, t_{\text{offset}}$

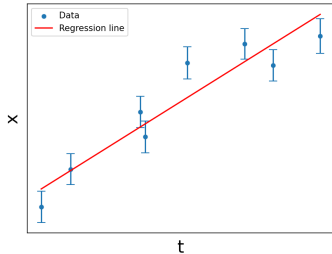
local parameters : $a_x^i, a_y^i, a_t^i, b_x^i, b_y^i, b_t^i$

Calibration principle

Calibration relation

$$x = C_1 \cdot t + C_2$$

Data fitting:



$$(t_1, x_1)$$

$$(t_2, x_2)$$

...

$$(t_i, x_i)$$

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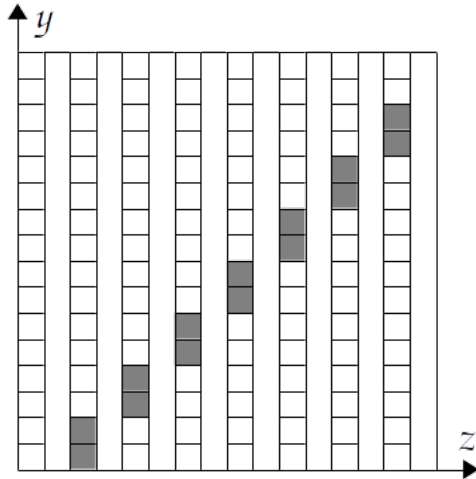
Calibration parameters for the i th track:

global parameters : $C_e, t_{\text{sync}}, t_{\text{offset}}$

local parameters : $a_x^i, a_y^i, a_t^i, b_x^i, b_y^i, b_t^i$

With 10'000 tracks, the total number of calibration parameters is **63'900!**

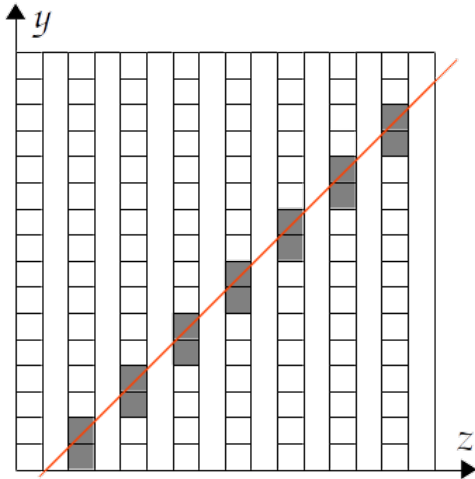
Side view of NeuLAND



Procedures

- 1 Obtain the positions of bars with signals

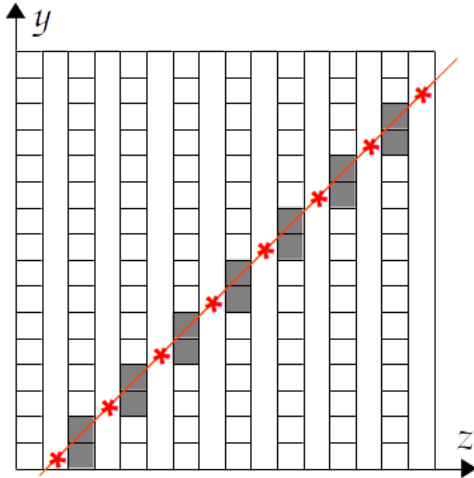
Side view of NeuLAND



Procedures

- 1 Obtain the positions of bars with signals
- 2 Reconstruct the muon track from the bar positions

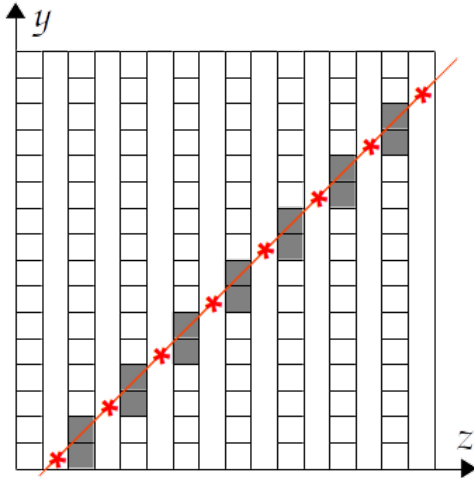
Side view of NeuLAND



Procedures

- 1 Obtain the positions of bars with signals
- 2 Reconstruct the muon track from the bar positions
- 3 Calculate the positions of the interaction points of the muon

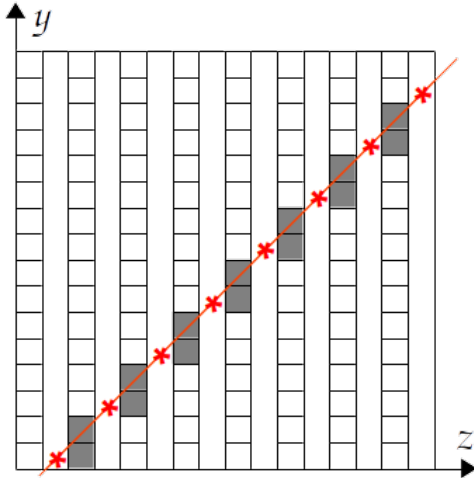
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Procedures

- 1 Obtain the positions of bars with signals
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- 4 Calculate the calibration parameters via data fitting

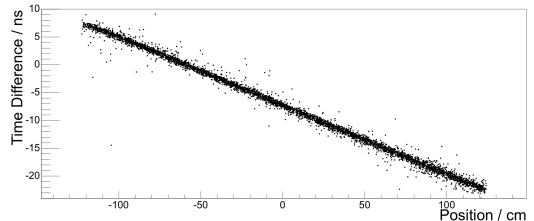
Side view of NeuLAND



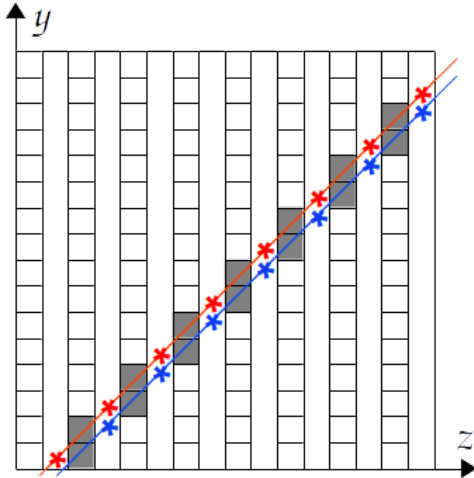
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Data fitting in the position calibration:



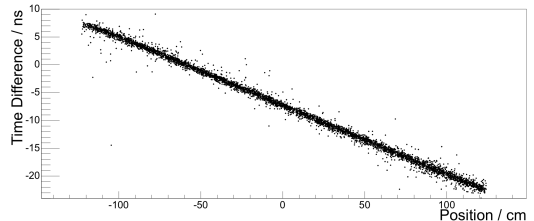
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Data fitting in the position calibration:



Simultaneous fitting of global and local parameters

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Residual minimization

$$\partial \sum_{j=0}^n \sum_i \frac{(\mathcal{Z}_i^j(g_1, \dots, g_m, p_1^j, \dots, p_l^j))^2}{2(\sigma_i^j)^2} = 0$$

$g_{1\dots m} : m$ global parameters

$p_{1\dots l}^j : l$ local parameters for the j th μ track

n : the total number of μ tracks

Simultaneous fitting of global and local parameters

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Newton's method:

← m → ← $\sim n \cdot l$ →

$$\begin{bmatrix} \sum_j \mathcal{C}_j & \dots & \mathcal{G}_j & \dots \\ \vdots & \ddots & 0 & 0 \\ \hline \mathcal{G}_j^T & 0 & \Gamma_j & 0 \\ \hline \vdots & 0 & 0 & \ddots \end{bmatrix} \cdot \begin{bmatrix} \Delta \mathbf{g} \\ \vdots \\ \Delta \mathbf{p}^j \\ \vdots \end{bmatrix} = - \begin{bmatrix} \partial_{\mathbf{g}} \mathcal{Z} \\ \vdots \\ \partial_{\mathbf{p}^j} \mathcal{Z} \\ \vdots \end{bmatrix}$$

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Matrix Dimension reduction! (Schur complement method)

$$\tilde{\mathcal{C}} \cdot \Delta \mathbf{g} = \mathcal{D}$$

where

$$\tilde{\mathcal{C}} = \sum_j C_j + \sum_j (-G_j \Gamma_j^{-1} G_j^T)$$

Simultaneous fitting of global and local parameters

Residual minimization

$$\partial \sum_{j=0}^n \sum_i \frac{(Z_i^j(g_1, \dots, g_m, p_1^j, \dots, p_l^j))^2}{2(\sigma_i^j)^2} = 0$$

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Advantages

- Simultaneous fitting of all parameters
- Computation complexity independent of local parameter size
- No muon track reconstruction

Simultaneous fitting of global and local parameters

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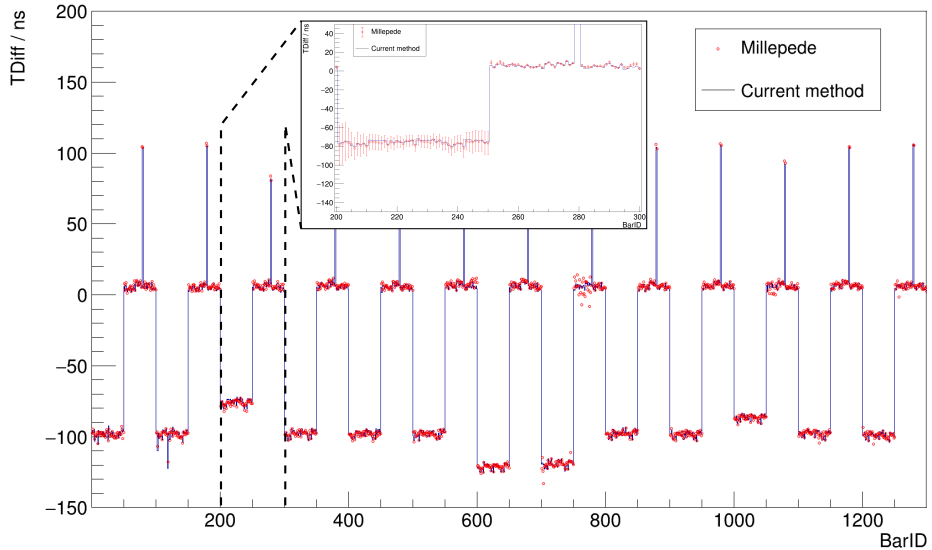
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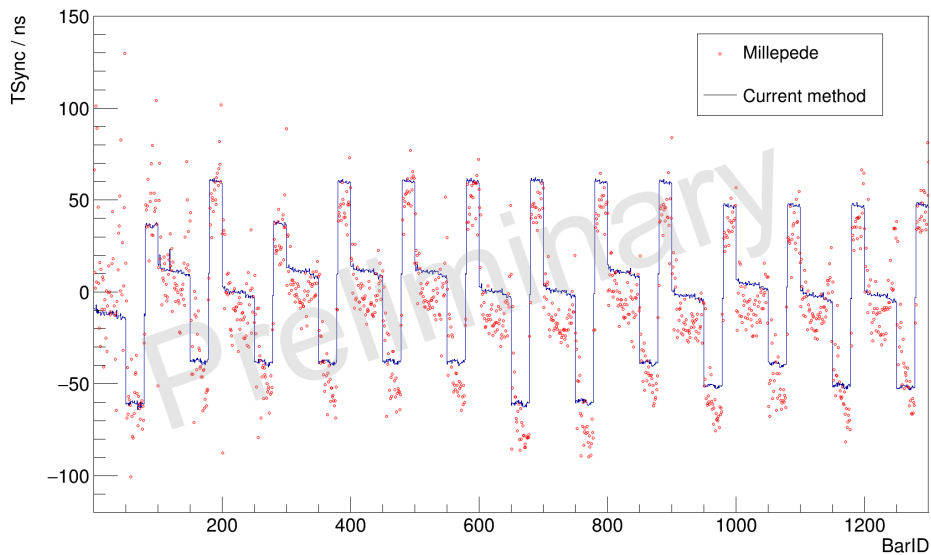
Algorithm implementation: **Millepede-II**

Millepede-II, <https://www.desy.de/~kleinwrt/MP2/doc/html/index.html>, [Online; accessed 2024-03-04]

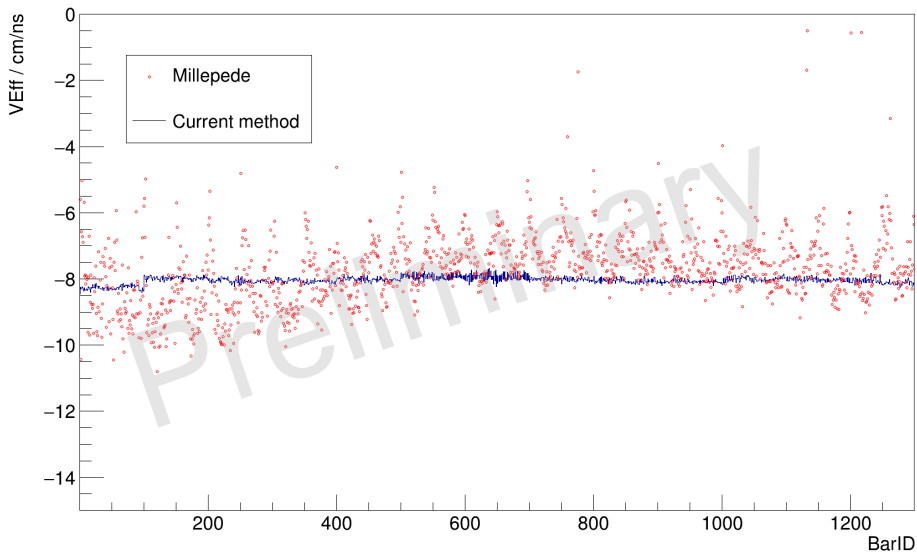
Comparisons of the PMT time offsets



Comparisons on time synchronization



Comparisons of the effective speed of light



Summary and outlook

Summary

- **Large number** of fitting parameters in time and position calibration
- **Simultaneous fitting** of local and global parameters using the Millepede algorithm
- **Consistent results** compared to the current method

Outlook

- Apply Millepede algorithm to energy calibration
- Improve precision of calibration parameters
- Possible applications on other detectors in the R³B experiment

