Application of the Millepede algorithm to the Time and Position Calibration of NeuLAND

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Gießen 2024

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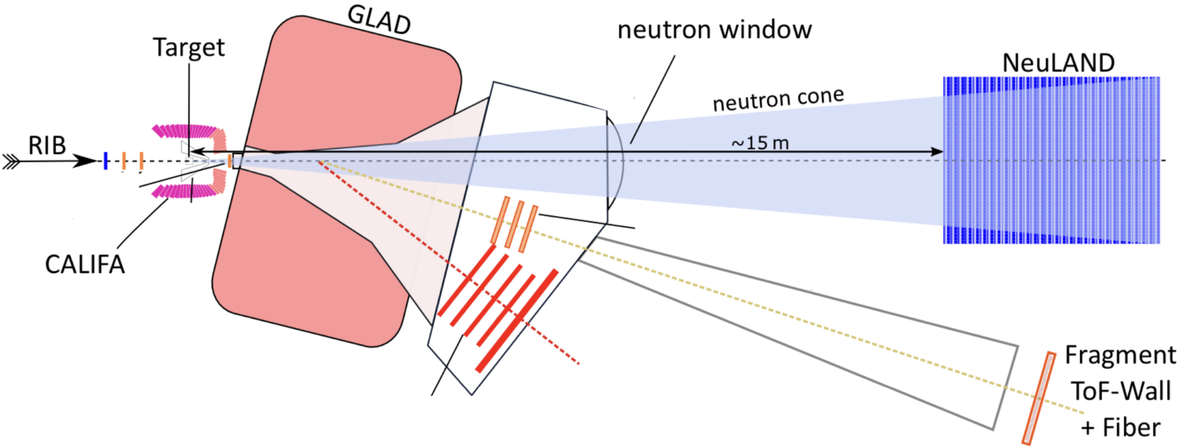
NeuLAND setup in R^3B

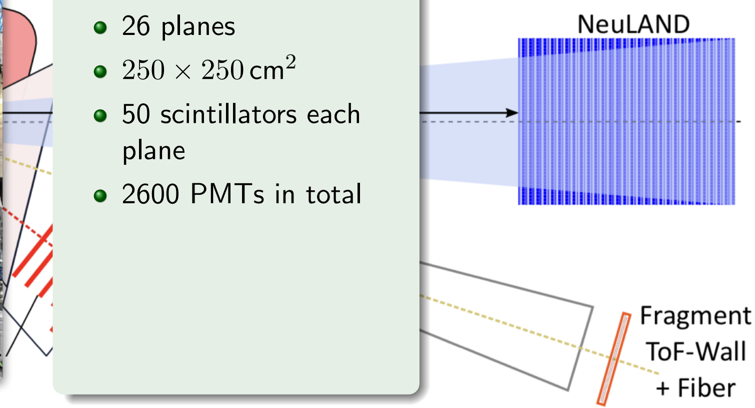


K. Boretzky *et al.*, Nucl. Instrum. Methods. Phys. Res. B **1014**, 165701 (2021)

NeuLAND setup in R^{3}B

Geometry:

- 26 planes
- $250 \times 250\,\mathrm{cm}^2$
- 50 scintillators each plane
- 2600 PMTs in total



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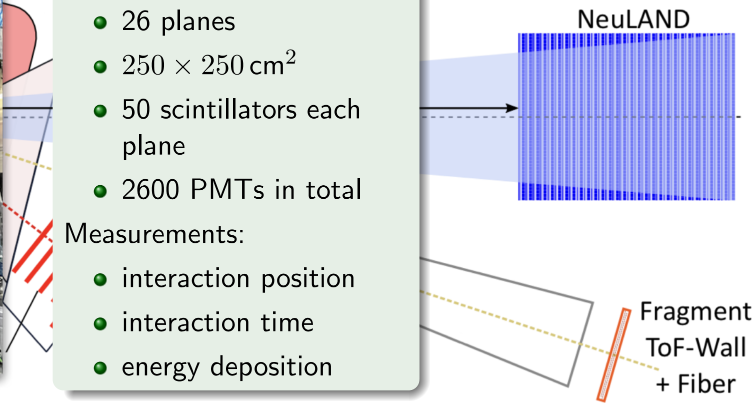
- interaction position
- interaction time
- energy deposition



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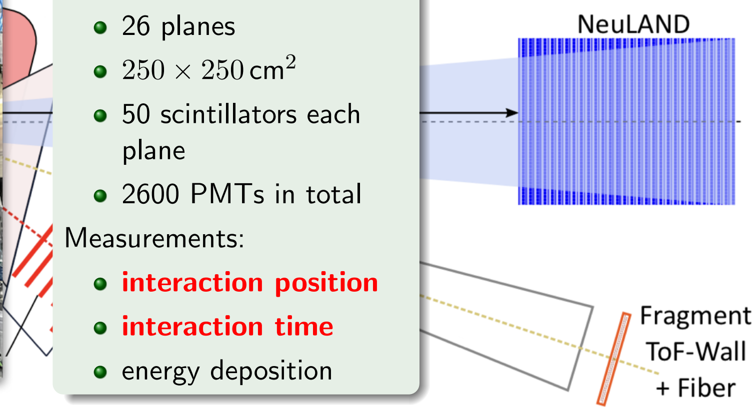

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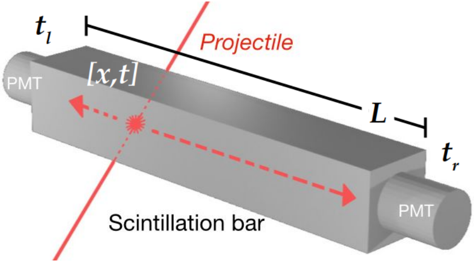
Position and time calculation



Symbols:

x : position of the interaction

t : time of the interaction

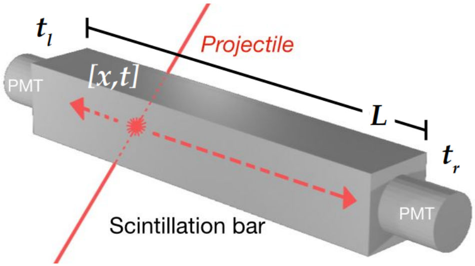
L : length of the scintillator

t_l : time of the left PMT signal

t_r : time of the right PMT signal

C_e : effective speed of light

Position and time calculation



Time relation:

$$t = \frac{t_r + t_l}{2} - \frac{L}{2 \cdot C_e}$$

Position relation:

$$x = \frac{C_e}{2}\left(t_r - t_l\right)$$

Symbols:

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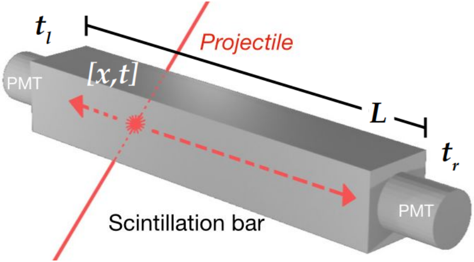

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C_e : effective speed of light

Time relation:

$$t = \frac{t_r + t_l}{2} - \frac{L}{2 \cdot C_e} + t_{\text{sync}}$$

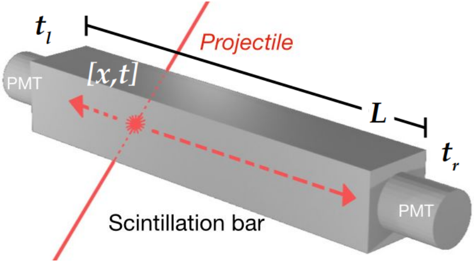
Position relation:

$$x = \frac{C_e}{2}(t_r - t_l)$$

Additional calibration parameters:

- t_{sync} : time synchronization among scintillators

Position and time calculation



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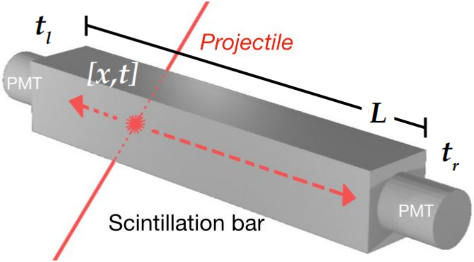
Additional calibration parameters:

- t_{sync} : time synchronization among scintillators
- t_{offset} : time offset between adjacent PMTs

Position and time calculation



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Additional calibration parameters:

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- t_{offset} : time offset between adjacent PMTs

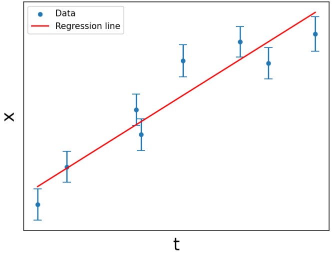
Total number of calibration parameters: *3900*

Calibration principle

Calibration relation

$$x = C_1 \cdot t + C_2$$

Data fitting:



$$(t_1,\ x_1)$$
$$(t_2,\ x_2)$$
$$\dots$$
$$(t_i,\ x_i)$$
$$\dots$$
$$(t_n,\ x_n)$$

Minimize

$$\text{residual} = \sum_i \frac{(x_i - x(t_i, C_1, C_2))^2}{2 * \sigma_i^2}$$

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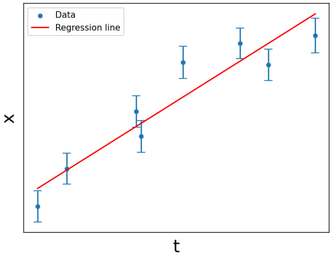

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Calibration with muon tracks

$$t = (t_r + t_l)/2 - L/(2 \cdot C_e) + t_{\text{sync}} \quad (1)$$

$$x = C_e \cdot (t_r - t_l + t_{\text{offset}})/2 \quad (2)$$

$$x_\mu = a_x^i \cdot z_\mu + b_x^i \quad (3)$$

$$y_\mu = a_y^i \cdot z_\mu + b_y^i \quad (4)$$

$$t_\mu = a_t^i \cdot z_\mu + b_t^i \quad (5)$$

Calibration parameters for the ith track:

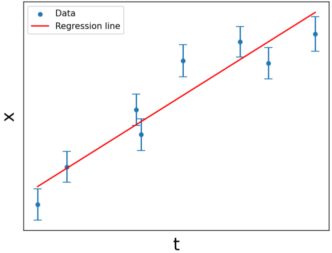
global parameters : $C_e, t_{\text{sync}}, t_{\text{offset}}$

local parameters : $a_x^i, a_y^i, a_t^i, b_x^i, b_y^i, b_t^i$

Calibration relation

$$x = C_1 \cdot t + C_2$$

Data fitting:



$$(t_1,\ x_1)$$
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$$(t_i,\ x_i)$$
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$$(t_n,\ x_n)$$

Minimize

$$\text{residual} = \sum_i \frac{(x_i - x(t_i, C_1, C_2))^2}{2 * \sigma_i^2}$$

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$$t = (t_r + t_l)/2 - L/(2 \cdot C_e) + t_{\text{sync}} \quad (1)$$
$$x = C_e \cdot (t_r - t_l + t_{\text{offset}})/2 \quad (2)$$
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$$t_\mu = a_t^i \cdot z_\mu + b_t^i \quad (5)$$

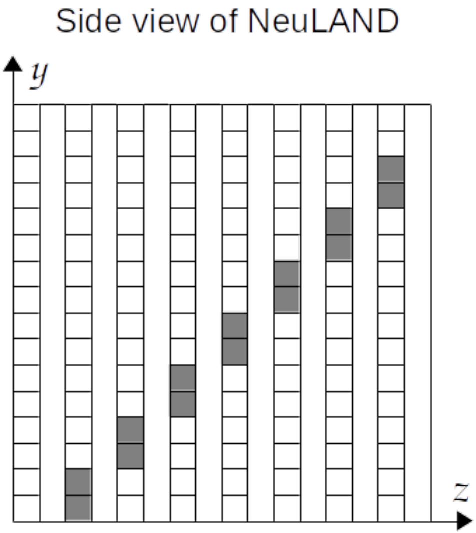
Calibration parameters for the i*th track:*

global parameters : $C_e, t_{\text{sync}}, t_{\text{offset}}$

local parameters : $a_x^i, a_y^i, a_t^i, b_x^i, b_y^i, b_t^i$

With 10'000 tracks, the total number of calibration parameters is ***63'900!***

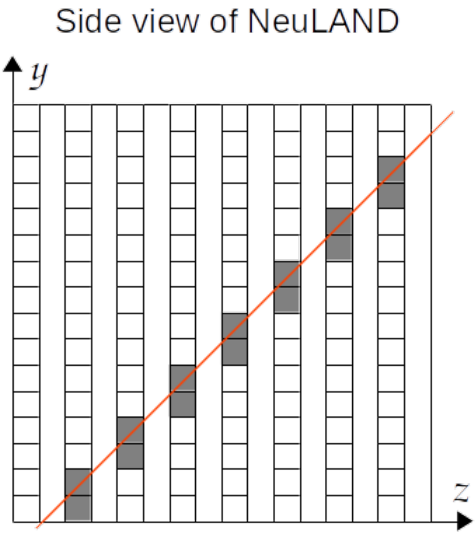
Current calibration method



Procedures

1. Obtain the positions of bars with signals

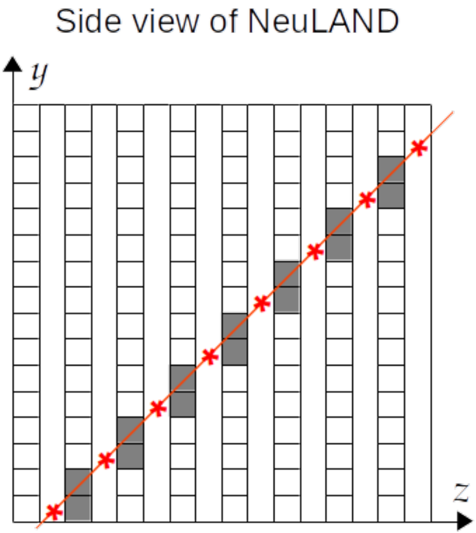
Current calibration method



Procedures

1. Obtain the positions of bars with signals
2. Reconstruct the muon track from the bar positions

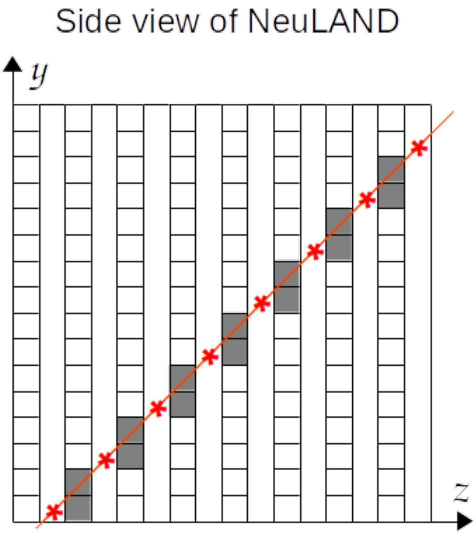
Current calibration method



Procedures

1. Obtain the positions of bars with signals
2. Reconstruct the muon track from the bar positions
3. Calculate the positions of the interaction points of the muon

Current calibration method



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1. Obtain the positions of bars with signals
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4. Calculate the calibration parameters via data fitting

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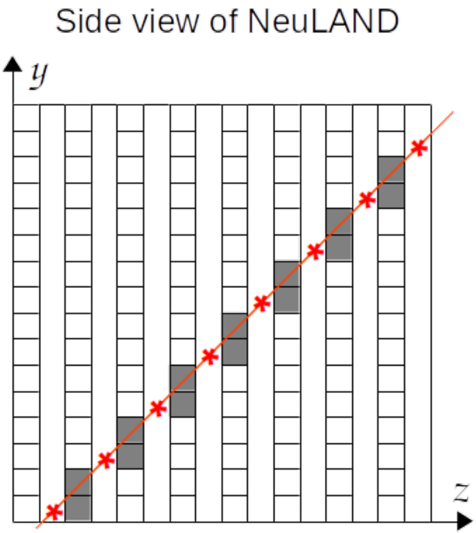

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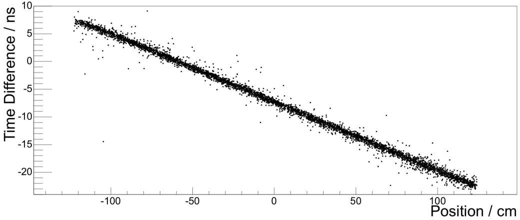
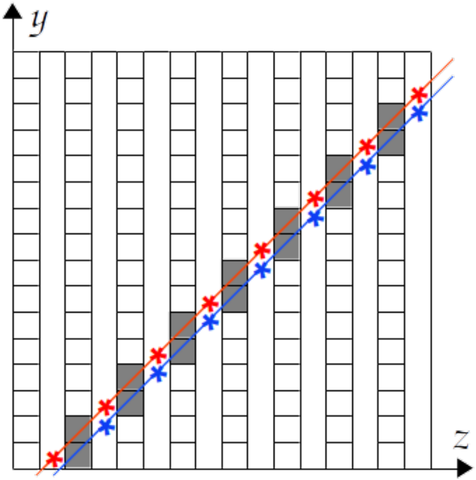
Data fitting in the position calibration:



Side view of NeuLAND



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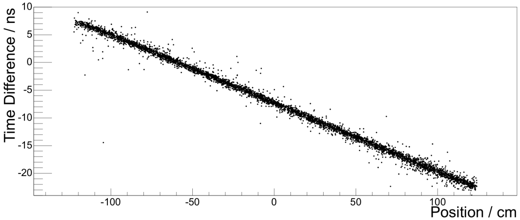

Simultaneous fitting of global and local parameters

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Residual minimization

$$\partial \sum_{j=0}^{n} \sum_{i} \frac{(\mathcal{Z}_i^j(g_1, ..., g_m, p_1^j, ..., p_l^j))^2}{2(\sigma_i^j)^2} = 0$$

$g_{1...m}$: m *global parameters*

$p^j_{1...l}$: l *local parameters* for the jth μ track

n : the total number of μ tracks

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Newton's method:

$$\begin{array}{cc} \overbrace{\phantom{\sum_j \mathcal{C}_j}}^{m} & \overbrace{\phantom{\dots \quad \mathcal{G}_j \quad \dots}}^{\sim n \cdot l} \end{array}$$

$$\left[\begin{array}{c||c:c:c} \sum_j \mathcal{C}_j & \dots & \mathcal{G}_j & \dots \\ \hline\hline \vdots & \ddots & 0 & 0 \\ \hdashline \mathcal{G}_j^T & 0 & \Gamma_j & 0 \\ \hdashline \vdots & 0 & 0 & \ddots \end{array}\right] \cdot \left[\begin{array}{c} \Delta\mathbf{g} \\ \hline\hline \vdots \\ \hdashline \Delta\mathbf{p}^j \\ \hdashline \vdots \end{array}\right] = - \left[\begin{array}{c} \partial_{\mathbf{g}}\mathcal{Z} \\ \hline\hline \vdots \\ \hdashline \partial_{\mathbf{p}^j}\mathcal{Z} \\ \hdashline \vdots \end{array}\right]$$

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Matrix Dimension reduction! (Schur complement method)

$$\tilde{\mathcal{C}} \cdot \Delta\mathbf{g} = \mathcal{D}$$

where

$$\tilde{\mathcal{C}} = \sum_j \mathcal{C}_j + \sum_j (-\mathcal{G}_j \Gamma_j^{-1} \mathcal{G}_j^T)$$

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Advantages

- Simultaneous fitting of all parameters
- Computation complexity independent of local parameter size
- No muon track reconstruction

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Algorithm implementation: **Millepede-II**[1]

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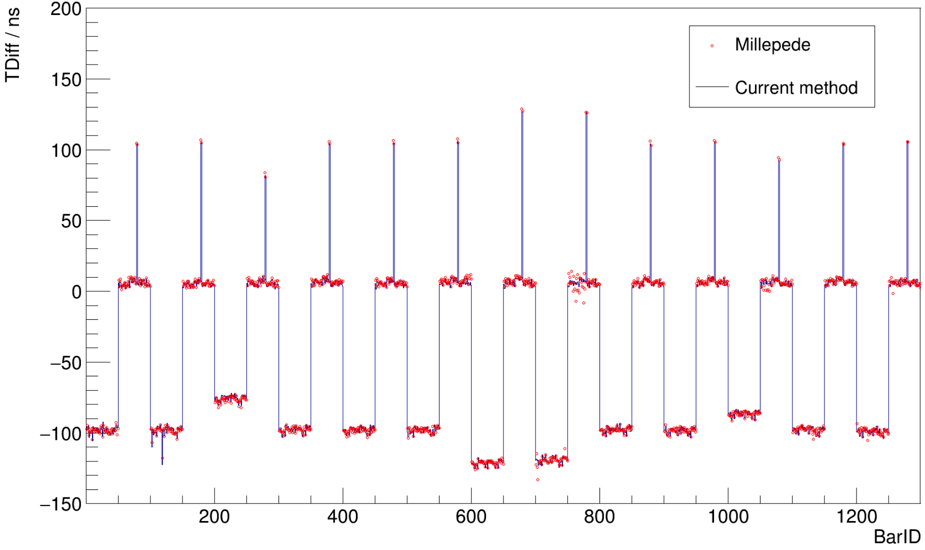
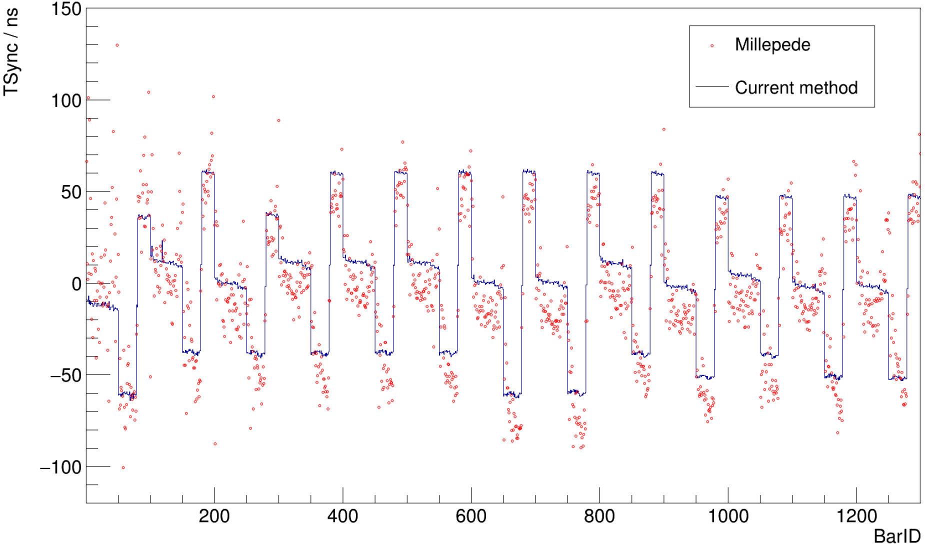
[1] *Millepede-ii*, https://www.desy.de/~kleinwrt/MP2/doc/html/index.html, [Online; accessed 2024-03-04]

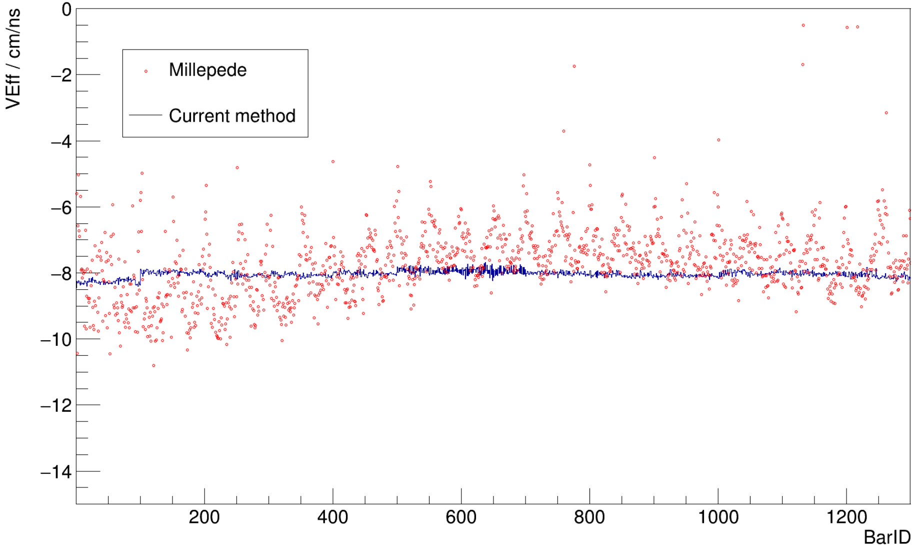
Comparisons of the PMT time offsets



Comparisons on time synchronization



Comparisons of the effective speed of light



Summary and outlook

Summary

- **Large number** of fitting parameters in time and position calibration
- **Simultaneous fitting** of local and global parameters using the Millepede algorithm
- **Consistent results** compared to the current method

Outlook

- Apply Millepede algorithm to energy calibration
- Improve precision of calibration parameters
- Possible applications on other detectors in the R^3B experiment