Application of Millepede algorithm to Time and Position Calibration of NeuLAND

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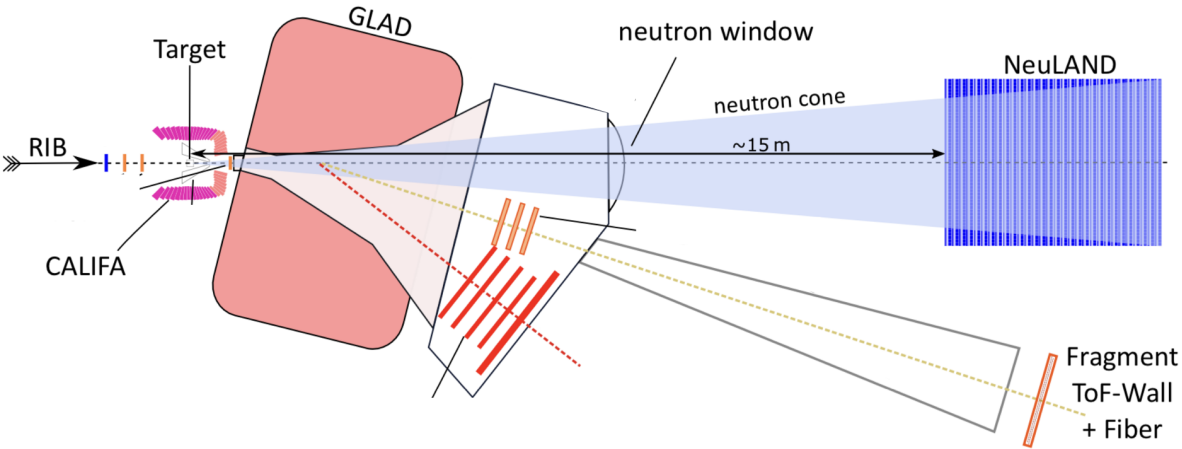
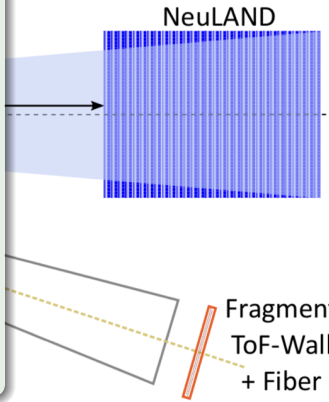
NeuLAND setup in R^3B^1



NeuLAND setup in R^3B^1

Geometry:

- 26 planes
- $250 \times 250\,\mathrm{cm}^2$
- 50 scintillators each plane
- 100 PMTs each plane



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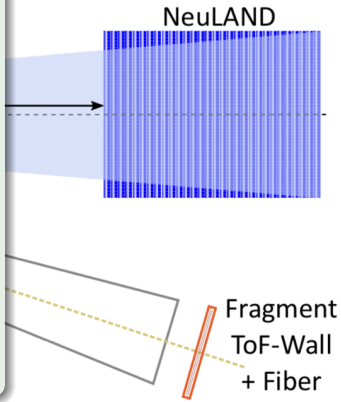
Measurements:

- interaction position
- interaction time
- energy deposition



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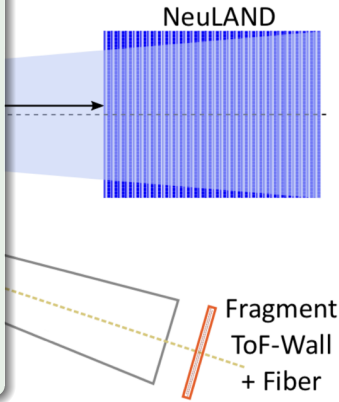
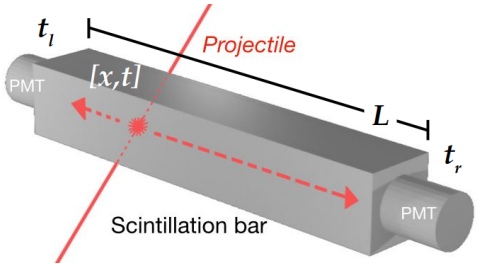


Position and time calibration



Symbols:

x : position of the interaction

t : time of the interaction

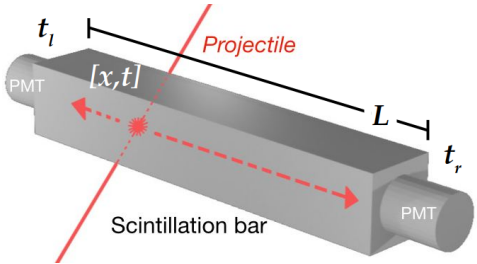
L : length of the scintillator

t_l : time of the left PMT signal

t_r : time of the right PMT signal

C_e : effective speed of light

Position and time calibration



Time relation:

$$t = \frac{t_r + t_l}{2} - \frac{L}{2 \cdot C_e}$$

Position relation:

$$x = \frac{C_e}{2}(t_r - t_l)$$

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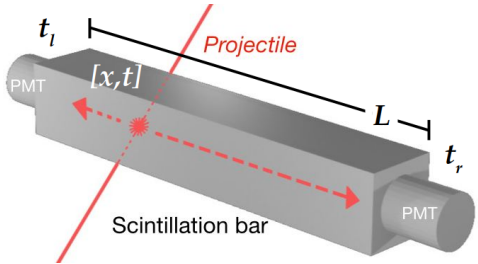

Time relation:

$$t = \frac{t_r + t_l}{2} - \frac{L}{2 \cdot C_e} + t_{\text{sync}}$$

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Symbols:

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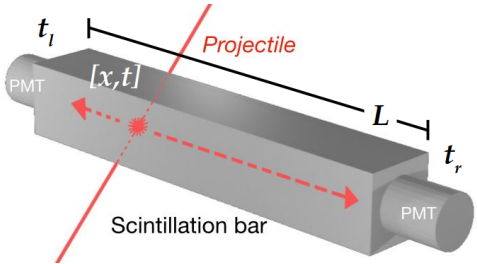
Additional calibration parameters:

- t_{sync} : time synchronization among scintillators

Position and time calibration



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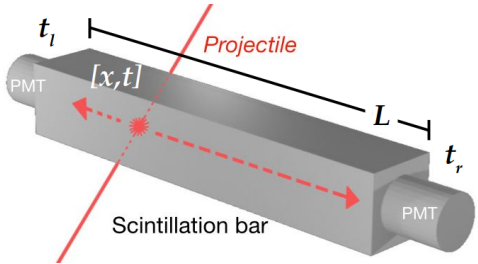

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Additional calibration parameters:

- t_{sync} : time synchronization among scintillators
- t_{offset} : time offset between adjacent PMTs

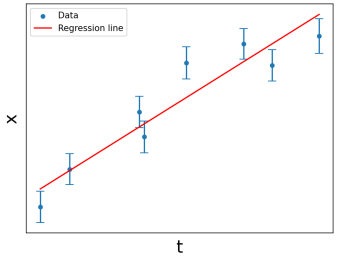
Total number of calibration parameters: 3900

Calibration principle

Calibration relation

$$x = C_1 \cdot t + C_2$$

Data fitting:



$$(t_1,\ x_1)$$
$$(t_2,\ x_2)$$
$$\ldots$$
$$(t_i,\ x_i)$$
$$\ldots$$
$$(t_n,\ x_n)$$

Minimize

$$\text{residual} = \sum_i \frac{(x_i - x(t_i, C_1, C_2))}{2 * \sigma_i^2}$$

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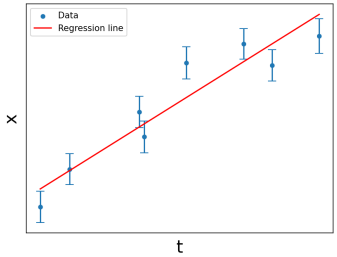

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$(t_2,\ x_2)$
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...
$(t_n,\ x_n)$

Minimize

$$\text{residual} = \sum_i \frac{(x_i - x(t_i, C_1, C_2))}{2 * \sigma_i^2}$$

Calibration with muon tracks

$$t = (t_r + t_l)/2 - L/(2 \cdot C_e) + t_{\text{sync}} \quad (1)$$

$$x = C_e \cdot (t_r - t_l + t_{\text{offset}})/2 \quad (2)$$

$$x_\mu = a_x^i \cdot z_\mu + b_x^i \quad (3)$$

$$y_\mu = a_y^i \cdot z_\mu + b_y^i \quad (4)$$

$$t_\mu = a_t^i \cdot z_\mu + b_t^i \quad (5)$$

Calibration parameters for the ith event:

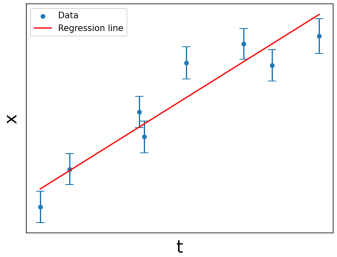
$$C_e, t_{\text{sync}}, t_{\text{offset}}, a_x^i, a_y^i, a_t^i, b_x^i, b_y^i, b_t^i$$

Calibration principle

Calibration relation

$$x = C_1 \cdot t + C_2$$

Data fitting:



$$(t_1,\ x_1)$$
$$(t_2,\ x_2)$$
$$\dots$$
$$(t_i,\ x_i)$$
$$\dots$$
$$(t_n,\ x_n)$$

Minimize

$$\text{residual} = \sum_i \frac{(x_i - x(t_i, C_1, C_2))}{2 * \sigma_i^2}$$

Calibration with muon tracks

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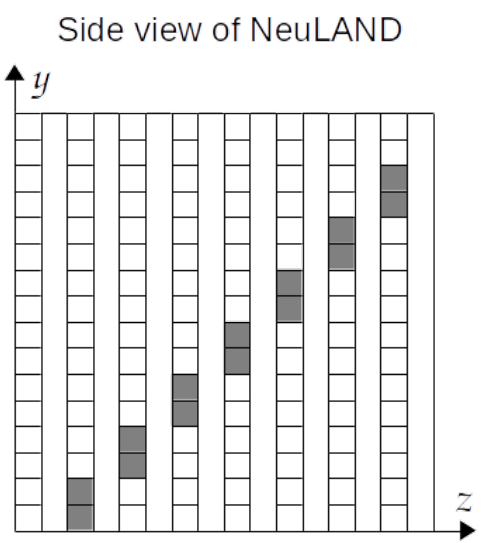
Calibration parameters for the ith event:

$$C_e, t_{\text{sync}}, t_{\text{offset}}, a_x^i, a_y^i, a_t^i, b_x^i, b_y^i, b_t^i$$

With 10'000 events, the total number of calibration parameters: *63'900!*

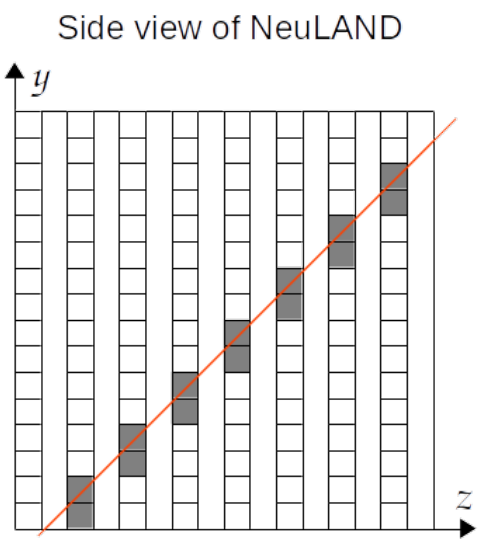
Current calibration method



Procedures

1. Obtain the positions of bars with signals

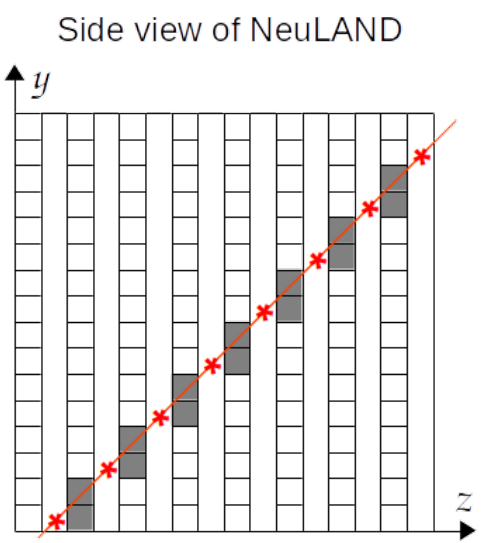
Current calibration method



Procedures

1. Obtain the positions of bars with signals
2. Reconstruct the muon track from the bar positions

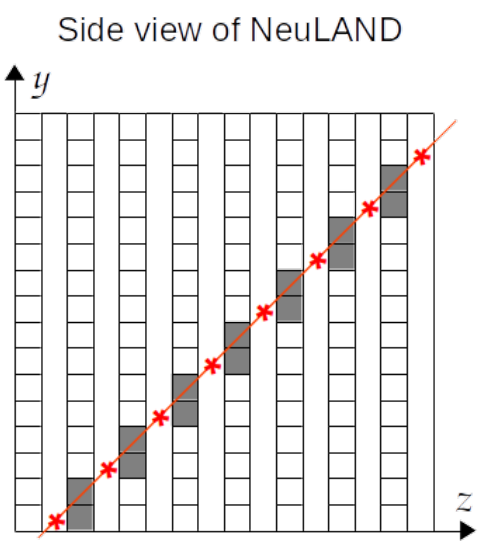
Current calibration method



Procedures

1. Obtain the positions of bars with signals
2. Reconstruct the muon track from the bar positions
3. Calculate positions of interaction point of the muon

Current calibration method



Procedures

1. Obtain the positions of bars with signals
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3. Calculate positions of interaction point of the muon
4. Obtain calibration parameters via data fitting

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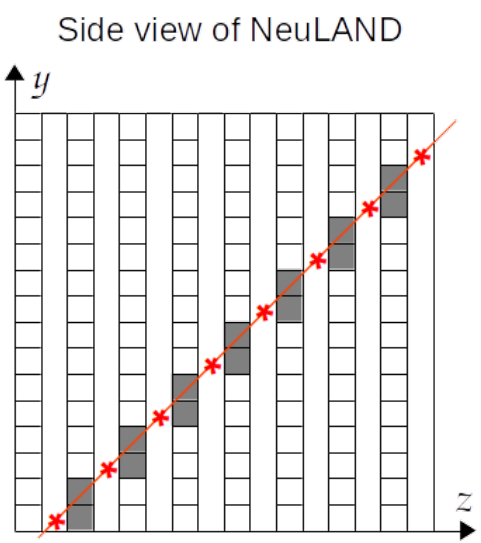

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Data fitting on positions:

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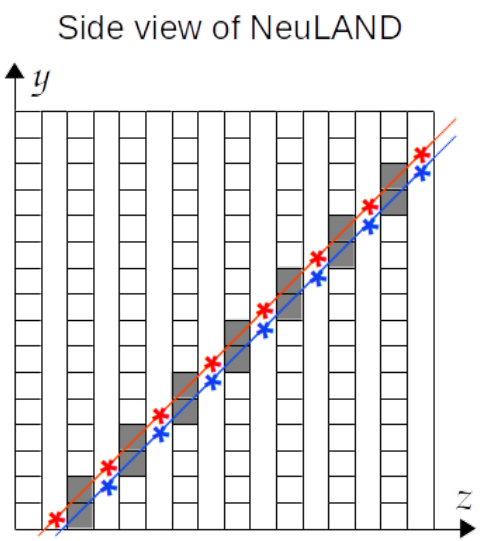

Procedures

1. Obtain the positions of bars with signals
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Data fitting on positions:

Simultaneous fitting of global and local parameters

Residual minimization

$$\partial \sum_{j=0}^{n} \sum_{i} \frac{\mathcal{Z}_i^j(g_1, ..., g_m, p_1^j, ..., p_l^j)}{2(\sigma_i^j)^2} = 0$$

$g_{1...m}$: m *global parameters*

$p_{1...l}^j$: l *local parameters* for the jth μ track

n : the total number of μ tracks

Newton's method:

$$\overset{\longleftarrow m \longrightarrow}{\overset{\longleftarrow \sim n \longrightarrow}{\begin{bmatrix} \sum_j \mathcal{C}_j & \dots & \mathcal{G}_j & \dots \\ \vdots & \ddots & 0 & 0 \\ \mathcal{G}_j^T & 0 & \Gamma_j & 0 \\ \vdots & 0 & 0 & \ddots \end{bmatrix}} \cdot \begin{bmatrix} \Delta\mathbf{g} \\ \vdots \\ \Delta\mathbf{p}^j \\ \vdots \end{bmatrix} = - \begin{bmatrix} \partial_{\mathbf{g}} \mathcal{Z} \\ \vdots \\ \partial_{\mathbf{p}^j} \mathcal{Z} \\ \vdots \end{bmatrix}}$$

Matrix Dimension reduction! (Schur complement method)

$$\tilde{\mathcal{C}} \cdot \Delta\mathbf{g} = \mathcal{D}$$

where

$$\tilde{\mathcal{C}} = \sum_j \mathcal{C}_j + \sum_j \left(-\mathcal{G}_j \Gamma_j^{-1} \mathcal{G}_j^T\right)$$

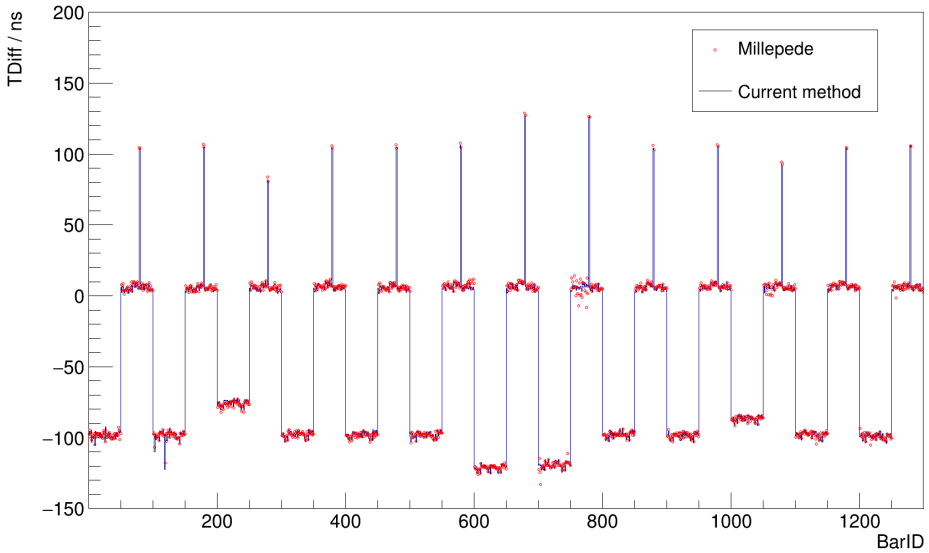
Advantages

- Simultaneous fitting of all parameters
- Computation complexity independent of local parameter size
- Direct fitting without any approximation

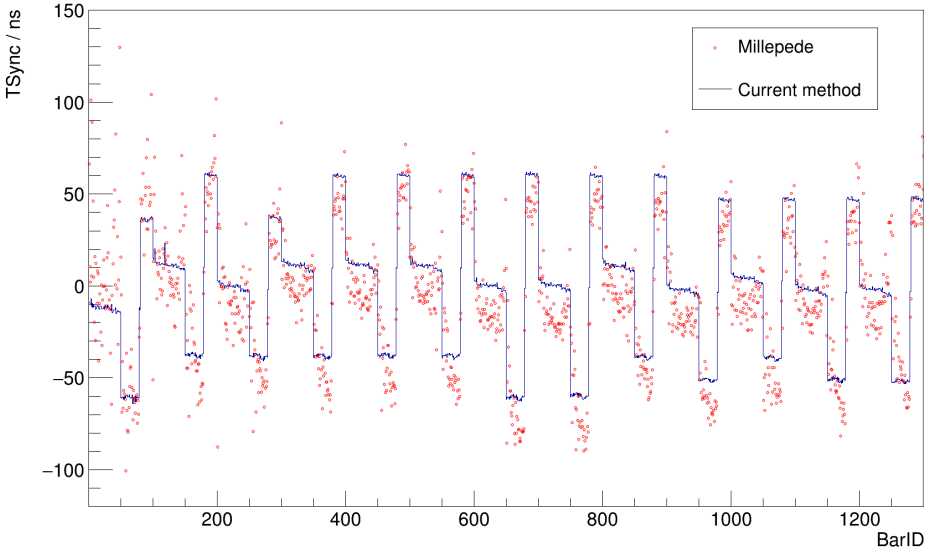
Software implementation: **Millepede-II**[1]

[1] *Millepede-ii*, https://www.desy.de/~kleinwrt/MP2/doc/html/index.html, [Online; accessed 2024-03-04]

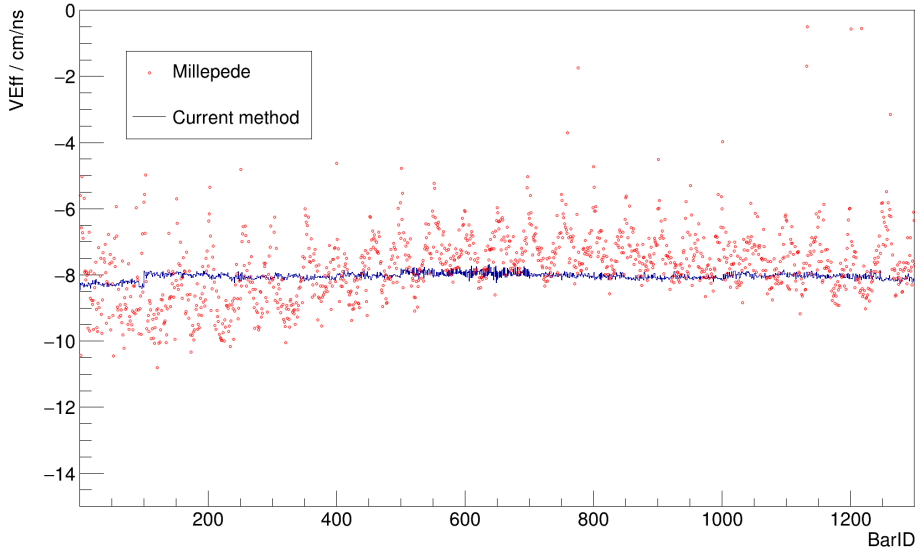
Comparisons on PMT time offsets



Comparisons on effective speed of light



Comparisons on time synchronization



Summary and outlook