HW0

Problem 1

A

Yes, Q is an inductive invariant of A.

The definition of inductive invariant is:

If $Q_0 \subseteq I$ and $Post(I) \subseteq I$ then I is an invariant, i.e., $Reach_A \subseteq I$.

Such invariants are called inductive invariants.

$Q_0 \subseteq Q$ and $Post(Q) \subseteq Q$ are true, so Q is an inductive invariant of A.

B

For I to be an inductive invariant of A, $Q_0 \subseteq I$ and $Post(I) \subseteq I$ must be true.

To help prove safety, I should not intersect with the unsafe set U.

Problem 1

Yes, $0 \leq v_1(t) \leq v_0$ is an invariant of A.

The vehicle starts with a speed of v_0 and can only decelerate or maintain its speed.

Thus, the speed of the vehicle will always be between 0 and v_0.

Problem 2

Yes, $timer(t) \leq v_0/a_b$ is an invariant of A.

The $timer(t)$ represents how long the vehicle has been braking.

So, we have $timer(t) = (v_0 - v_1(t))/a_b$.

According to the invariant $0 \leq v_1(t) \leq v_0$ and induction,

we have $timer(t) \leq (v_0 - 0)/a_b = v_0/a_b$.

Problem 3

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SimpleCar(Dsense, v0, x10, x20, ab), x20 > x10
Initially: x1(0) = x10, x2(0) = x20, v1(0) = v0, v2(0) = 0
s(0) = 0, timer(0) = 0, timer2(0) = 0, aware(0) = 0
d(t) = x2(t) − x1(t)
if d(t) ≤ Dsense
    s(t + 1) = 1
    aware(t + 1) = aware(t) + 1
    if aware(t + 1) >= Treact
        if v1(t) ≥ ab
            v1(t + 1) = v1(t) − ab
            timer(t + 1) = timer(t) + 1
            timer2(t + 1) = timer2(t)
        else
            v1(t + 1) = 0
            timer(t + 1) = timer(t)
            timer2(t + 1) = timer2(t)
    else
        v1(t + 1) = v1(t) + as
        timer(t + 1) = timer(t)
        timer2(t + 1) = timer2(t) + 1
else
    s(t + 1) = 0
    v1(t + 1) = v1(t) + as
    timer(t + 1) = timer(t)
    timer2(t + 1) = timer2(t) + 1
x1(t + 1) = x1(t) + v1(t + 1)
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Problem 4