Total ____/45

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Section: AB2

Part 1. ___/15

Plots ___/6

(Attach time response plots to end or include them here)

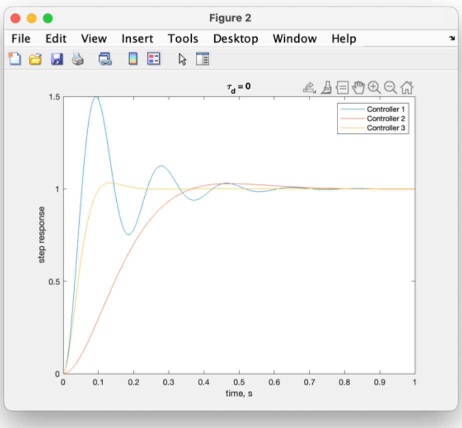

Fig. 1

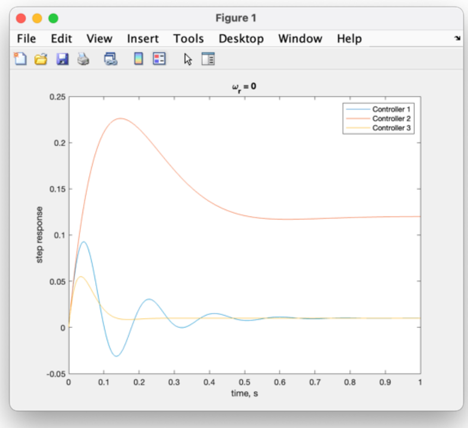

Fig. 2

Time Response to a Unit step for ω_r ___/6

	Controller 1		Controller 2		Controller 3	
	Prelab	Lab	Prelab	Lab	Prelab	Lab
M_p (%)	49.8%	49.8%	2.8%	2.84%	2.85%	2.83%
t_r (s)	0.052	0.035	0.23	0.228	0.07	0.067
t_s (s)	0.4	0.390	0.33	0.312	0.097	0.089
K	19.4		1.07		19.4	
K_r	1.03		1.56		1.03	
K_d	0		0		0.03	

Table 1

Compare/contrast ___/3

- *M_p, t_r, and t_s from Prelab with those from Lab*
- *Which controllers met the specifications?*

- *Those from Prelab are very close to the ones from Lab.*
- *The third.*

Part 2. ___/12 Deriving e_{ss} components

For the system in Figure 3.1, derive the relationship between steady-state error ($e_{ss} = \omega_r - \omega$) and natural frequency, ω_n. Consider the error as a function of both ω_r and τ_d, and model these as step inputs. Since the system is linear, superposition allows the two components to be calculated separately and then summed. Notice that e_{ss} is not the same thing as "e" in the block diagram ($e = K_r\,\omega_r - \omega$).

$$\omega_n^2 = 60K + 36 \quad (1)$$

$$\omega_r(s) - \omega(s) = \omega_r - \frac{60KK_r}{(s+3)(s+12)+60K}\omega_r + \frac{4(s+3)}{(s+3)(s+12)+60K}\tau_r \quad (2)$$

Hint: Use the Final Value Theorem, (page 93, FPE).

Make sure you answer the following questions:

e_{ss} due to a step in ω_r ($\tau_d = 0$) is: $\omega_r - \frac{60KK_r}{(s+3)(s+12)+60K}\omega_r$

To minimize this error component, ω_n should be $\sqrt{60KK_r}$

e_{ss} due to a step in τ_d ($\omega_r = 0$) is: $\frac{4(s+3)}{(s+3)(s+12)+60K}\tau_r$

To minimize this error component, ω_n should be *as large as possible*

Part 3. ___/18

For controller 3, derive the relationship between ζ, ω_n, and the gains K and K_d. ___/12

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + (15 + 60KK_d)s + 60K + 36 \quad (3)$$

$$\omega_n = \sqrt{60K + 36} \quad (4)$$

$$\xi = \frac{15 + 60KK_d}{2\sqrt{60K+36}} \quad (5)$$

Discussion: Increasing K does what to ω_n^2, what to ζ
(and at what rate: linearly, exponentially, as K^2, etc.)
Increasing K_d does what to ω_n^2, what to ζ
(and at what rate)

Increasing K increase ω_n^2 and ζ at the rate of $\sqrt{K}$
Increasing K_d doesn't affect ω_n^2 but increase ζ at the rate of $\frac{30K}{\sqrt{60K+36}}$

Using these equations, show how the pole locations change as K_d>0 increases in value. ___/6

$$\text{Poles} = -\zeta\,\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \quad (6)$$

Discuss: As K_d grows, the poles move…(remember there are two components of this, depending on the damping)
When ζ < 1, as K_d grows, the poles move closer to the real axis. When ζ > 1, as K_d grows, the poles move further from the origin on the real axis.