

**Total: \_\_\_\_/40**

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 Section: AB2

**Question 1. \_\_\_\_/15**
**Theoretical/Experimental Results \_\_\_\_/5**

$$M_p = (MaxValue - SteadyState) / SteadyState * 100$$

$\zeta$	$M_p$ Theory %	$M_p$ Expmt %	$t_r$ Theory (s)	$t_r$ Expmt (s)	$t_s$ Theory (s)	$t_s$ Expmt (s)
2.0	0.00	0.00	8.20	8.02	11.60	11.14
1.5	0.00	0.00	5.85	5.80	8.30	8.32
1.0	0.00	0.10	3.35	3.32	5.00	4.76
0.8	1.52	1.77	2.50	2.54	3.68	3.38
0.7	4.60	5.62	2.16	2.28	3.02	4.86
0.5	16.30	17.24	1.62	2.04	6.28	5.32
0.3	37.23	40.37	1.30	1.88	10.14	10.62
0.2	52.66	70.56	1.21	1.86	15.08	17.46

Table 1: Theoretical/Experimental Results

Attach one sample plot from your StepResponseMetrics file that shows how you obtained the experimental results for one of the values of  $\zeta$ .

**Comparison of Theoretical vs. Experimental Results \_\_\_\_/5**

Hint: Does it look like the theoretical equations on page 11 of the lab manual match the experimental values?

Put Discussion Here

Yes, they match.

**Discussion of variation with  $\zeta$  of  $M_p$ ,  $t_s$ ,  $t_r$  \_\_\_\_/5**

Put Discussion Here

When  $\zeta$  decreases,  $M_p$  increases,  $t_r$  decreases, and  $t_s$  first decreases and then increases.

**Question 2. \_\_\_\_/15**
**Effect of  $\zeta$  on Pole Locations (Derive Equation and Explain) \_\_\_\_/5**

Put Discussion of  $\zeta$ 's Effect Here. Include the equation of the two pole locations in terms of  $\zeta$  (you may assume  $\omega_n = 1$ ). Include either a sketch/graph of the pole locations as  $\zeta$  increases, or a description of what this graph would look like.

$$s_{1,2} = -\zeta \pm j\sqrt{1 - \zeta^2}$$

As  $\zeta$  increases, the poles go more left on the left half of the unit circle, and then go left on the x axis.

### Effect of Pole Locations on $M_p$ , $t_s$ , $t_r$ for an Underdamped System \_\_\_/5

Hint: An underdamped system has  $\zeta$  \_\_\_

As  $\zeta$  increases, the poles do \_\_\_ which makes  $M_p$ ,  $t_s$ ,  $t_r$  do \_\_\_\_  
(Double Hint: moving the poles causes two different effects on  $t_s$ )

An underdamped system has  $\zeta$  less than 1.

As  $\zeta$  increases, the poles go left on the left half of unit circle, which makes  $M_p$  smaller,  $t_s$  smaller and  $t_r$  greater

### Effect of Pole Locations on $M_p$ , $t_s$ , $t_r$ for an Overdamped/Critically Damped System \_\_\_/5

Hint: An over-damped system has  $\zeta$  \_\_\_

A critically damped system has  $\zeta$  \_\_\_

As  $\zeta$  increases, the poles do \_\_\_ which makes  $M_p$ ,  $t_s$ ,  $t_r$  do \_\_\_\_

An overdamped system has  $\zeta$  greater than 1.

As  $\zeta$  increases, the poles go left on the x axis, which makes  $M_p$  be zero,  $t_s$  greater and  $t_r$  greater

### Question 3. \_\_\_/10

Investigate the effects of approximating an overdamped 2<sup>nd</sup> order system with a 1<sup>st</sup> order system. The approximation will be done by using a transfer function with only the pole that is closer to the origin,  $p_{min}$ .

$$H_1(s) = \frac{p_1 p_2}{(s + p_1)(s + p_2)} \Rightarrow H_2(s) = \frac{p_{min}}{s + p_{min}}$$

The response speed would be underestimated. The settling time would be overestimated. The rise time would be underestimated. And the overshoot would be overestimated.

### Similarities/Differences on Overdamped 2<sup>nd</sup>-Order system to a 1<sup>st</sup>-Order System with the less negative of the 2<sup>nd</sup>-Order's poles \_\_\_/6

Plot the step responses for the 2<sup>nd</sup> order systems and their 1<sup>st</sup> order approximations for  $\zeta = 1.5$ ,  $\zeta = 5$ , and  $\zeta = 40$ . Assume  $\omega_n = 1$ . How are the step responses of the 1<sup>st</sup> order approximations similar to and different from the step responses of the original 2<sup>nd</sup> order systems?

The 1<sup>st</sup> approximations' step responses have a similar shape as the original ones'. But they rise faster.

**Effect of magnitude of  $\zeta$  on the accuracy of the approximations \_\_\_\_/4**

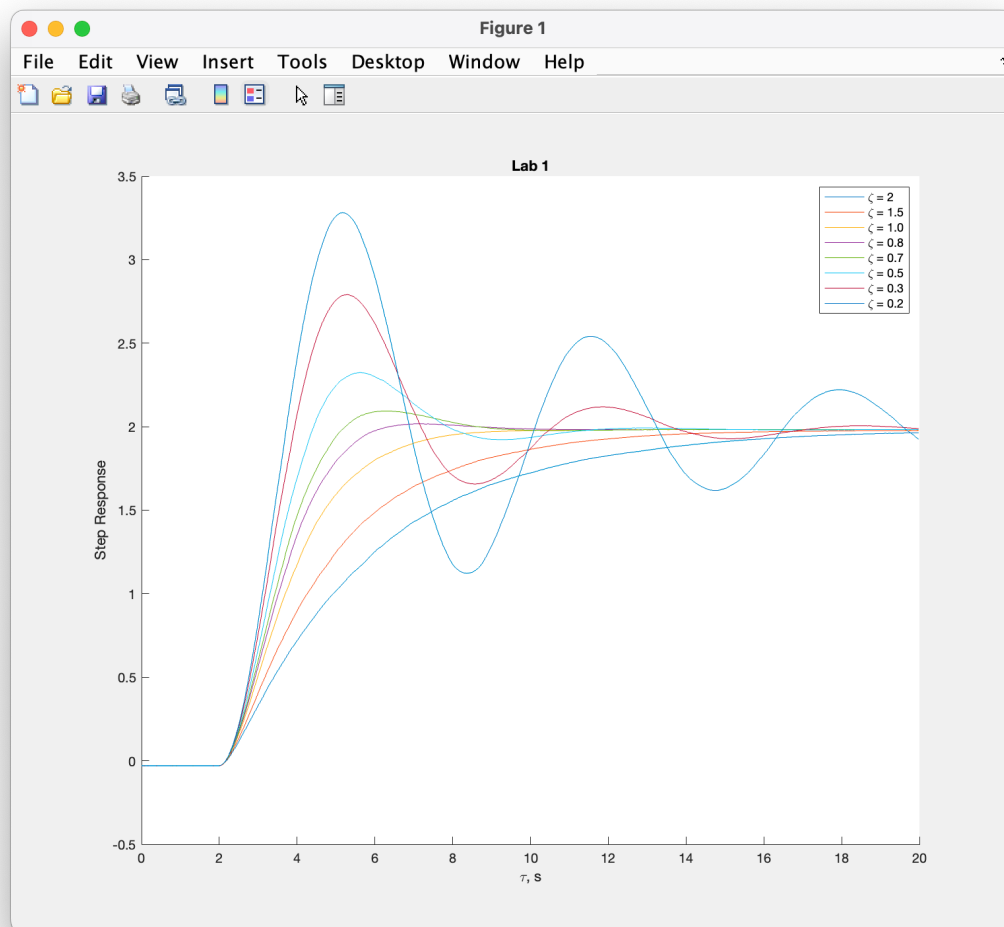
*How does  $\zeta$  affect the accuracy of the 1<sup>st</sup> order approximations?*

The greater  $\zeta$  is, the higher the accuracy is.

**Attachments (3)**

- Plots obtained during lab
- Sample response with relevant points for calculating  $M_p$ ,  $t_s$  and  $t_r$  marked
- Step Responses comparing 2<sup>nd</sup> order systems and 1<sup>st</sup> order approximations

Plots obtained during lab:



Sample response with relevant points for calculating  $M_p$ ,  $t_s$  and  $t_r$  marked:

```

19 fprintf("Experiment\n");
20 for i = 1:length(ys)
21     zeta = zetas(i);
22     yn = ys(i);
23     t = yn(:,1);
24     y = yn(:,2);
25     yss = y(end);
26     ymax = max(y);
27     mp = (ymax - yss) / yss * 100;
28     t10_index = find(y > 0.1 * ymax, 1, "first");
29     t90_index = find(y > 0.9 * ymax, 1, "first");
30     tr = t(t90_index) - t(t10_index);
31     x = length(y);
32     while y(x) < 1.05 * yss && y(x) > 0.95 * yss
33         x = x - 1;
34     end
35     ts = t(x) - 2; % start at 2 seconds
36     fprintf("zeta=%1f: M_p=%1f%%, t_r=%1f, t_s=%1f\n", zeta, mp, tr, ts);
37 end
38
39 fprintf("\nTheory\n");
40 omega = 1;
41 for i = 1:length(zetas)
42     zeta = zetas(i);
43     if zeta < 1
44         mp = 100 * exp(-pi * zeta / sqrt(1 - zeta^2));
45     else
46         mp = 0;
47     end
48     if zeta < 1.2
49         tr = (1.2 - 0.45 * zeta + 2.6 * zeta^2) / omega;
50     else
51         tr = (4.7 * zeta - 1.2) / omega;
52     end
53     if zeta < 0.69
54         ts = -0.5 / (zeta * omega) * log((1 - zeta^2) / 400);
55     else
56         ts = (6.6 * zeta - 1.6) / omega;
57     end
58     fprintf("zeta=%1f: M_p=%1f%%, t_r=%1f, t_s=%1f\n", zeta, mp, tr, ts);
59 end
60 end
61

```

```

Experiment
zeta=2.0: M_p=0.00%, t_r=8.02s, t_s=11.14s
zeta=1.5: M_p=0.00%, t_r=5.80s, t_s=8.32s
zeta=1.0: M_p=0.10%, t_r=3.32s, t_s=4.76s
zeta=0.8: M_p=1.77%, t_r=2.54s, t_s=3.38s
zeta=0.7: M_p=5.62%, t_r=2.28s, t_s=3.86s
zeta=0.5: M_p=17.24%, t_r=2.04s, t_s=5.32s
zeta=0.3: M_p=40.37%, t_r=1.88s, t_s=10.62s
zeta=0.2: M_p=70.56%, t_r=1.86s, t_s=17.46s

Theory
zeta=2.0: M_p=0.00%, t_r=8.20s, t_s=11.60s
zeta=1.5: M_p=0.00%, t_r=5.85s, t_s=8.30s
zeta=1.0: M_p=0.00%, t_r=3.35s, t_s=4.80s
zeta=0.8: M_p=1.52%, t_r=2.50s, t_s=3.68s
zeta=0.7: M_p=4.60%, t_r=2.16s, t_s=3.82s
zeta=0.5: M_p=16.30%, t_r=1.62s, t_s=5.28s
zeta=0.3: M_p=37.23%, t_r=1.30s, t_s=10.14s
zeta=0.2: M_p=52.66%, t_r=1.21s, t_s=15.08s
>>

```

Step Responses comparing 2<sup>nd</sup> order systems and 1<sup>st</sup> order