Microeconomic Theory HW4 Solutions

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Exercise 1

(a) Andy's income is given by:

$$m = p_x w_x + p_y w_y = 4 * 20 + 5 * 8 = 120$$

(b) Since Andy's utility function is Cobb-Douglas, we use the tangency condition to find Andy's optimal consumption of goods x and y:

$$MRS = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{y}{2x} = \frac{4}{5} = \frac{p_x}{p_y} \implies y = \frac{8x}{5}$$

By plugging y into the budget constraint $120 = 4x + 5y$ we get Andy's optimal consumption of goods x and y:

$$(x^*, y^*) = (10, 16)$$

(c) Andy is a net seller of good x because:

$$x^* - w_x = 10 - 20 < 0$$

(d) After the price change, Andy's income is given by:

$$m' = p'_x w_x + p_y w_y = 10 * 20 + 5 * 8 = 240$$

(e)

$$MRS = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{y}{2x} = \frac{10}{5} = \frac{p_x}{p_y} \implies y = 4x$$

By plugging y into the budget constraint $240 = 10x + 5y$ we get Andy's optimal consumption of goods x and y after the price change:

$$(x^*, y^*) = (8, 32)$$

(f) In the following graph, the budget lines and the optimal consumption found in (b) and (e) are represented:

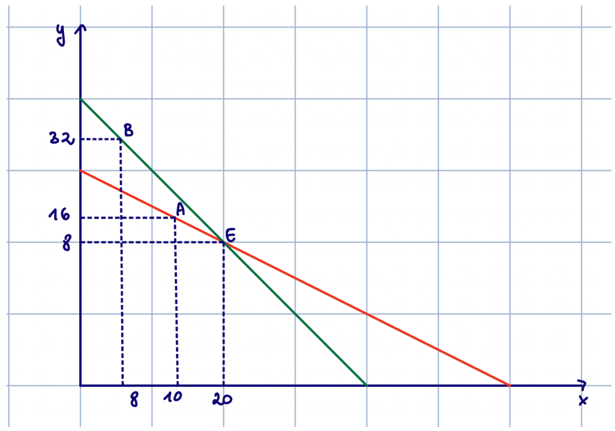

Figure 1:

(g) Since Andy is a net seller of good x, therefore he is better off when the price of good x increases.

(h)

$$p'_x w_x + p_y w_y = 10 * 10 + 5 * 16 = 180$$

(i)

$$MRS = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{y}{2x} = \frac{10}{5} = \frac{p_x}{p_y} \implies y = 4x$$

By plugging y into the budget constraint $180 = 10x+5y$ we get the optimal solution:

$$(x^*, y^*) = (6, 24)$$

The budget line with the same income level before the price change is $10x + 5y = 120$. The optimal solution is

$$(x^*, y^*) = (4, 16)$$

The substitution effect is given by 6-10=-4. The income effect is given by 4-6=-2. The endowment effect is given by 8-4=4.

Exercise 2

(a) Andy's income is given by:

$$m = p_x w_x + p_y w_y = 12 * 20 + 4 * 10 = 280$$

(b) Since good x and y are perfect complements for Andy, we have:

$$3y = x \implies y = \frac{x}{3}$$

By plugging y into the budget constraint $280 = 12x+4y$ we get the optimal solution:

$$(x^*, y^*) = (21, 7)$$

(c) Andy is a net buyer of x because:

$$x^* - w_x = 21 - 20 > 0$$

(d) After the price change, Andy's income is given by:

$$m' = p'_x w_x + p_y w_y = 2 * 20 + 4 * 10 = 80$$

(e)

$$y = \frac{x}{3}$$

By plugging y into the budget constraint $80 = 2x + 4y$ we get the optimal solution:

$$(x^*, y^*) = (24, 8)$$

(f) In the following graph, the budget lines and the optimal consumption found in (b) and (e) are represented:

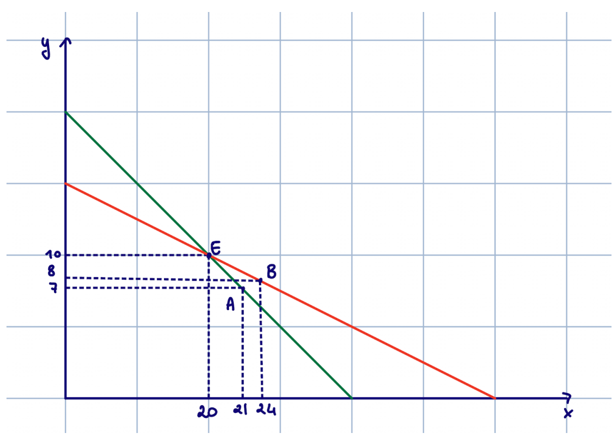

Figure 2:

(g) Since Andy is a net buyer of good x, therefore he is better off when the price of good x decreases.

(h)

$$p'_x w_x + p_y w_y = 21 * 2 + 7 * 4 = 70$$

(i) By plugging $y = \frac{x}{3}$ into the budget constraint $70 = 2x + 4y$ we get the optimal solution:

$$(x^*, y^*) = (21, 7)$$

Since the two goods are perfect complements, the substitution effect is zero. When the budget constraint is given by $280 = 2x + 4y$, $x^* = 84$. The income effect is 84-21=63. The endowment effect is given by 24-84=-60.

Exercise 3

(a) Andy's income is given by:

$$m = p_x w_x + p_y w_y = 4 * 20 + 4 * 10 = 120$$

(b) Since good x and y are perfect complements for Andy, we have:

$$2y = 4x \implies y = \frac{x}{2}$$

By plugging y into the budget constraint $120 = 4x + 4y$ we get the optimal solution:

$$(x^*, y^*) = (20, 10)$$

(c) Andy is neither a net buyer nor a net seller of x because:

$$x^* - w_x = 0$$

(d)

$$m' = p'_x w_x + p_y w_y = 2 * 20 + 4 * 10 = 80$$

(e)

$$y = \frac{x}{2}$$

By plugging y into the budget constraint $80 = 2x + 4y$ we get the optimal solution:

$$(x^*, y^*) = (20, 10)$$

(f) In the following graph, the budget lines and the optimal consumption found in (b) and (e) are represented:

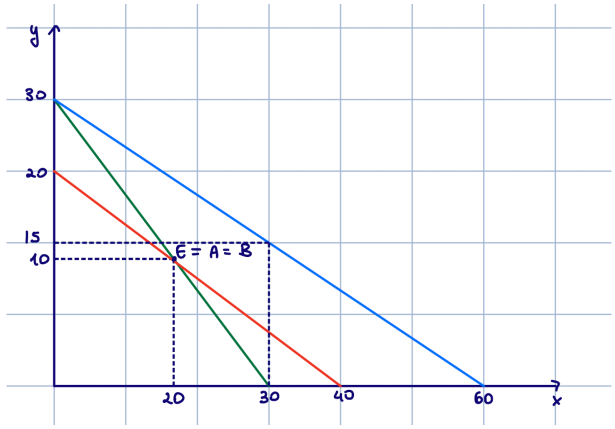

Figure 3:

(g) Since Andy is neither a net buyer of good x nor a net seller of good x, he is neither better or worse off when the price of good x increases.

(h)

$$p'_x w_x + p_y w_y = 2 * 20 + 4 * 10 = 80$$

(i) By plugging $y = \frac{x}{2}$ into the budget constraint $80 = 2x + 4y$ we get the optimal solution:

$$(x^*, y^*) = (20, 10)$$

Since there was no change in demand, the substitution effect, income effect and endowment income effect are all equal to zero. Alternatively, when the budget constraint is given by $120 = 2x + 4y$, $x^* = 30$ The income effect is given by $30 - 20 = 10$ and the endowment income effect is given by $20 - 30 = -10$. This implies that the two effects offset each other.

Exercise 4

(a)

$$x_{tot} = x_A + x_B = 10 + 2 = 12$$

$$y_{tot} = y_A + y_B = 2 + 8 = 10$$

(b) In the following graph, the Edgeworth box is represented:

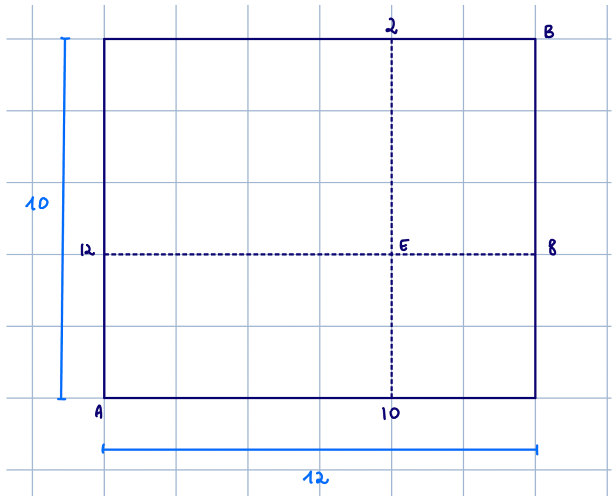

Figure 4: Edgeworth Box

(c) A bundle is said to be Pareto efficient if there is no way to make someone better off without making someone else worse off i.e. we can't make everyone better off. The set of all Pareto efficient bundles constitutes the contract curve.

(d) In the case of Cobb-Douglas utility functions, to find the contract curve, we set $MRS_A = MRS_B$:

$$MRS_A = \frac{y_A}{2x_A}$$

$$MRS_B = \frac{y_B}{2x_B}$$

$$\frac{y_A}{2x_A} = \frac{y_B}{2x_B}$$

Since $x_A + x_B = 12$ and $x_A + x_B = 10$, we can write:

$$\frac{y_A}{x_A} = \frac{y_B}{x_B} = \frac{10 - y_A}{12 - x_A}$$

$$x_A(10 - y_A) = y_A(12 - x_A) \implies y_A = \frac{5x_A}{6}$$

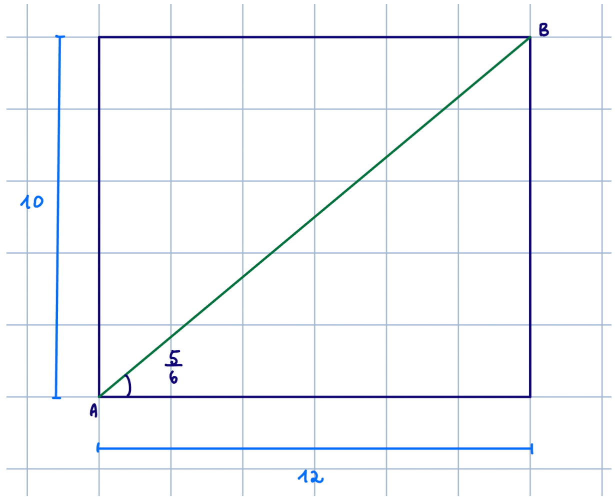

Figure 5: Edgeworth Box

(e) In the case of perfect complements, we find the contract curve as follows:

$$x_A = 6y_A \implies y_A = \frac{5x_A}{6}$$

$$x_B = 6y_B \implies y_B = \frac{5x_B}{6}$$

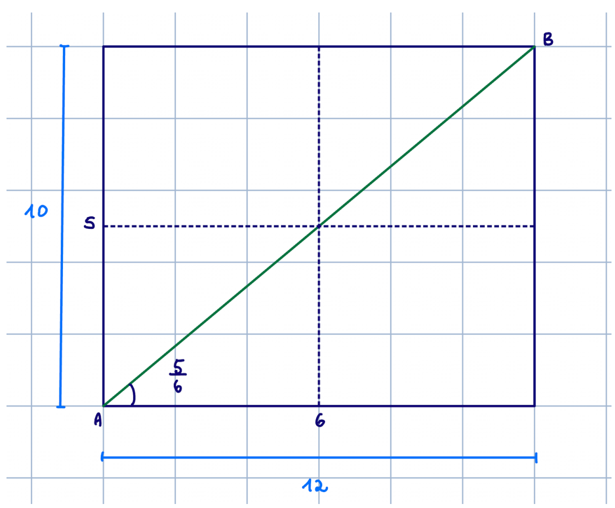

Figure 6: Edgeworth Box

(f) This is another case of perfect complements:

$$x_A = y_A \qquad and \qquad x_B = y_B$$

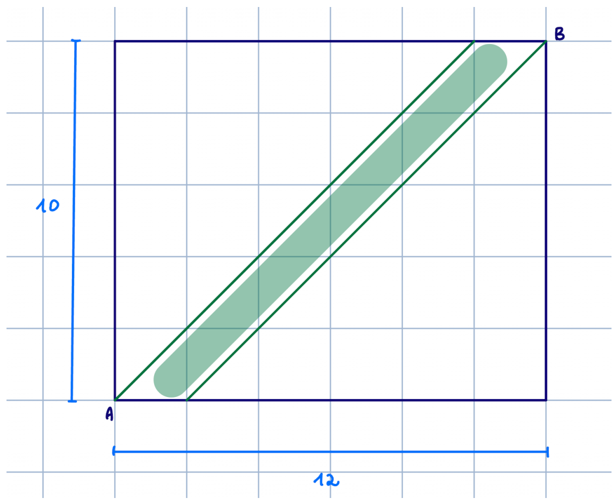

Figure 7: Edgeworth Box

(g) In the case of perfect substitutes, the contract curve is the following:

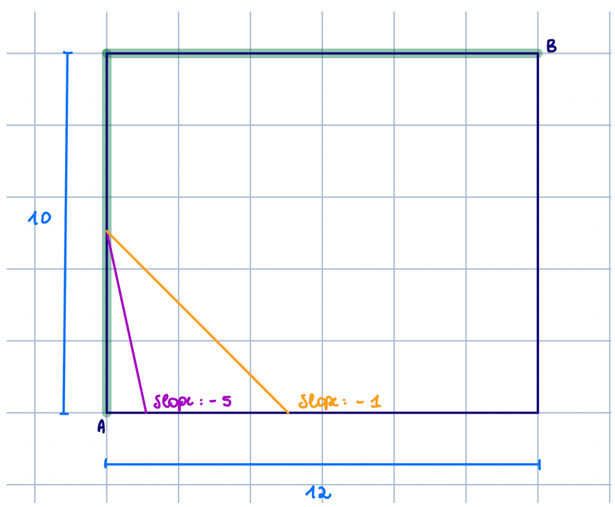

Figure 8: Edgeworth Box

(h) Using the tangency condition and the budget constraint, we obtain:

$$\frac{y_A}{2x_A} = \frac{p_x}{p_y} = \frac{5}{12}$$

The equilibrium price is:

$$\frac{p_x}{p_y} = \frac{5}{12}$$

The optimal consumption bundles are:

$$(x_A^*, y_A^*) = (\frac{74}{15}, \frac{37}{9})$$

$$(x_B^*, y_B^*) = (\frac{106}{15}, \frac{53}{9})$$

(i) In this case, several prices support an equilibrium. That is, as long as there is a price line that passes through the endowment and the point where the two utility functions are "tangent", trade can happen and there is an equilibrium. Therefore, we have multiple equilibrium prices and multiple optimal consumption bundles.

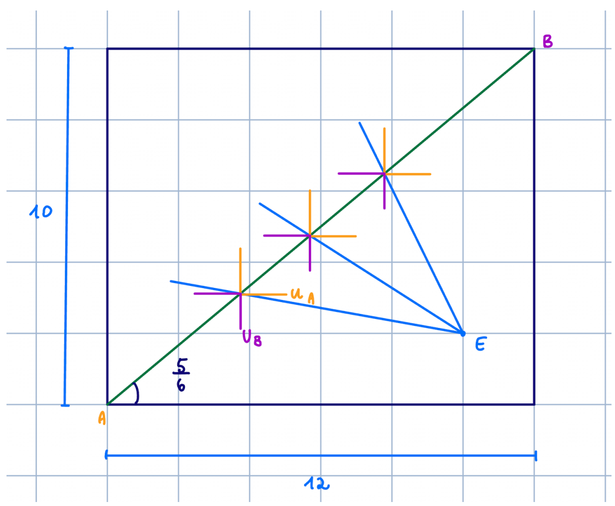

Figure 9: Edgeworth Box