IO2 problem set 1

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1 Question 1

The buyer's utility maximization problem is:

$$max_{q(\theta),T(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} [V(q(\theta)) - T(\theta)] f(\theta) d\theta$$

subject to

(1) Individual Rationality constraint (IR):

$$T(\theta) - C(q(\theta), \theta) \geq 0 \; \forall \theta$$

(2) Incentive Compatibility constraint (IC):

$$T(\theta) - C(q(\theta), \theta) \geq T(\theta^{'}) - C(q(\theta^{'}), \theta) \; \forall \theta, \theta^{'}$$

Denote $\pi(\theta, \hat{\theta}) = T(\hat{\theta}) - C(q(\hat{\theta}), \theta)$ To satisfy the local IC, the FOC with respect to $\hat{\theta}$ is zero.

$$\frac{d\pi(\theta, \hat{\theta})}{d\theta} = -C_\theta(q, \theta) < 0$$

$$\pi(\theta, \hat{\theta}) \equiv \pi(\theta) = \pi(\overline{\theta}) + \int_\theta^{\overline{\theta}} C_\theta(q, \theta)$$

Since the IR constraint binds for the highest type if IC holds, $\pi(\overline{\theta}) = 0$.

$$T(\theta) = C(q, \theta) + \int_\theta^{\overline{\theta}} C_\theta(q, \theta)$$

Now the relaxed problem of the buyer is:

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} [V(q(\theta)) - C(q, \theta) - \int_\theta^{\overline{\theta}} C_\theta(q, \theta)] f(\theta) d\theta$$

Using integration by parts

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_\theta^{\overline{\theta}} C_\theta(q(x), x) f(\theta) dx d\theta = F(\theta) \int_\theta^{\overline{\theta}} C_\theta(q(x), x) dx \Big|_{\underline{\theta}}^{\overline{\theta}} + \int_{\underline{\theta}}^{\overline{\theta}} C_\theta(q(\theta), \theta) F(\theta) d\theta$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} F(\theta) C_\theta(q(\theta), \theta) d\theta$$

The relaxed problem is simplified to be:

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} [V(q(\theta)) - C(q(\theta), \theta) - \frac{F(\theta)}{f(\theta)} C_\theta(q(\theta), \theta)] f(\theta) d\theta$$

Denote

$$\psi(q, \theta) = V(q(\theta)) - C(q(\theta), \theta) - \frac{F(\theta)}{f(\theta)} C_\theta(q(\theta), \theta)$$

To solve this unconstrained problem, the following conditions should hold:

(1) FOC

$$V_q(q) - C_q(q,\theta) - \frac{F(\theta)}{f(\theta)} C_{\theta q}(q,\theta) = 0$$

(2) SOC

$$V_{qq} - C_{qq} - \frac{F(\theta)}{f(\theta)} C_{\theta qq}(q,\theta) \leq 0$$

which is satisfied if $C_{\theta qq} \geq 0$

(3) Global IC

A sufficient condition for global IC is $\frac{\partial^2 \pi(\theta,\hat{\theta})}{\partial\hat{\theta}\partial\theta} \geq 0$. Because $\frac{\partial \pi(\theta,\theta)}{\partial\hat{\theta}} = 0$, if $\theta > \hat{\theta}$, $\frac{\partial \pi(\theta,\hat{\theta})}{\partial\hat{\theta}} \geq 0$ so sellers who reported their type below their true type want to increase their reported type $\hat{\theta}$. And if $\theta < \hat{\theta}$, $\frac{\partial \pi(\theta,\hat{\theta})}{\partial\hat{\theta}} \leq 0$ so sellers who reported their type above their true type want to decrease their reported type $\hat{\theta}$. Since

$$\frac{\partial^2 \pi(\theta,\hat{\theta})}{\partial\hat{\theta}\partial\theta} = -C_{q\theta}(q(\hat{\theta}),\theta)\frac{dq(\hat{\theta})}{d\hat{\theta}}$$

plus $C_{q\theta} > 0$, the sufficient condition is $\frac{dq(\hat{\theta})}{d\hat{\theta}} \leq 0$. By monotone comparative statics, $\frac{\partial^2 \psi(q,\theta)}{\partial q \partial\theta} \leq 0$.

$$-C_{q\theta}(q,\theta) - \frac{d}{d\theta}[\frac{F(\theta)}{f(\theta)}]C_{q\theta}(q,\theta) - \frac{F(\theta)}{f(\theta)}C_{q\theta\theta} \leq 0$$

To satisfy the above inequality, we need to assume $C_{q\theta\theta} \geq 0$.

From the FOC,

$$P'(q) = V_q(q) = C_q(q,\theta) + \frac{F(\theta)}{f(\theta)}C_{q\theta}(q,\theta)$$

Since $C_{q\theta}(q,\theta)$ is positive, the monopsony purchasing price is higher than the first best case where marginal price equals marginal cost. Because $V_{qq}(q) \leq 0$, the monopsony quantity $q(\theta)$ is lower than the first best quantity q^{FB}. Since $F(\underline{\theta}) = 0$, the lowest type is not distorted so $q(\underline{\theta}) = q^{FB}$.

2 Question 2

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3 Question 3