# ECON 8854: Nonlinear Pricing I

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# Welcome

- Introductions
- Regular Feedback
- Research Proposal Assignments

# Acknowledgement


- Slides draw from Alessandro Bonatti and Glenn Ellison's lecture notes
- Slides draw from Tirole (1988, Sec 3.5)

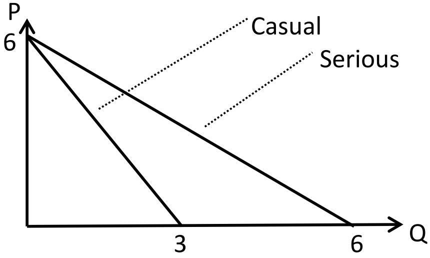

- Good references
  - Fudenberg and Tirole's (1991) *Game Theory*, Chapter 7
  - Wilson's (1993) *Nonlinear Pricing*.
- Seminal references
  - Mussa and Rosen (1978)
  - Maskin and Riley (1984)

# Undergraduate Treatment of 2-types Model

- Warning: I expect homework with graduate level rigour
  - See lecture notes on Canvas for grad treatment of 2-type model.
- Tennis Club Example
  - $P =$ price for one hour of court time
  - $Q =$ hours of court time purchased
  - 1000 serious players and 1000 casual players—cannot distinguish
  - Individual demand curves
    - Serious: $Q_s = 6 - P_s \Leftrightarrow P_s = 6 - Q_s$
    - Casual: $Q_c = 3 - \frac{1}{2}P_c \Leftrightarrow P_c = 6 - 2Q_c$
  - Costs: $FC = 5000/week$, $MC = 0/hour$

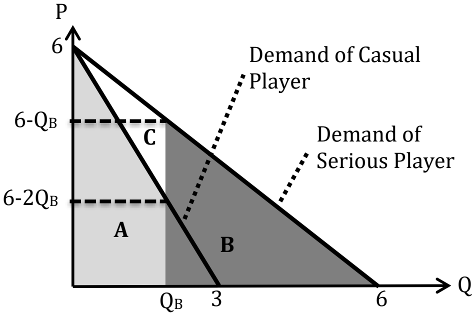
# Tennis Club Example

- Package Pricing Menu
  - Bronze Membership: Pay $P_B$ for 3 hours/wk.
  - Gold Membership: Pay $P_G$ for 6 hours/wk.
- Players choose which (if any) membership to buy
- Q: Optimal $P_B$ and $P_G$?



# Tennis Club Example

- Package Pricing Menu
  - Bronze Membership: Pay $P_B$ for $Q_B$ hours/wk.
  - Gold Membership: Pay $P_G$ for $Q_G$ hours/wk.
- Idea: Have casual buy bronze and serious buy gold.
- Step 1: $Q_G = Q_S^{FB} = 6$
- Step 2: $P_B = A$ and $P_G = A + B = (A + B + C) - C$
- Step 3: Max profits wrt $Q_B \to Q_B^* = 2$.



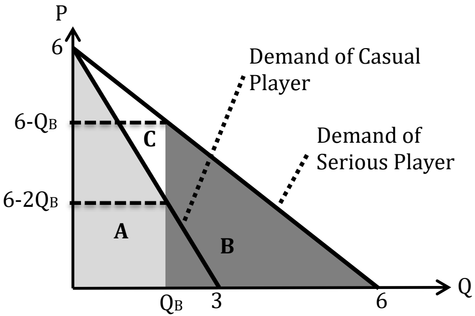

Area A = $Q_B(½)(6 + (6\text{-}2Q_B)$
= $Q_B\,(6 - Q_B)$

Area B = $(½)(6\text{-}Q_B)^2$

Area C = 18-A-B.

# Tennis Club Example

- Package Pricing Menu
  - Bronze Membership: Pay $P_B = 8$ for $Q_B = 2$ hours/wk.
  - Gold Membership: Pay $P_G = 16$ for $Q_G = 6$ hours/wk.
- Volume/Quantity Discounting
  - Bronze: $P_B/Q_B = 8/2 = \$4.00/hr$.
  - Gold: $P_G/Q_G = 16/6 = \$2.67/hr$.

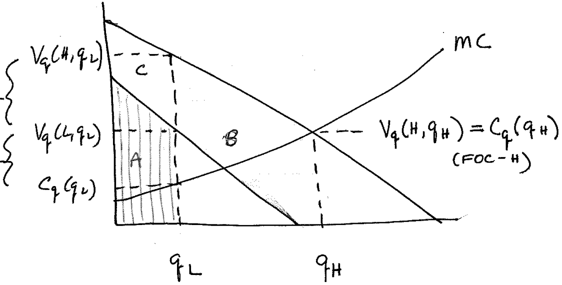



Area A = $Q_B(\frac{1}{2})(6 + (6-2Q_B)$
= $Q_B (6 - Q_B)$

Area B = $(\frac{1}{2})(6-Q_B)^2$

Area C = 18-A-B.

# 2-type model




IR-L: $P_L = V(L,q_L) = A$

IC-H: $P_H - P_L = V(H,q_H) - V(H,q_L) = B$

FOC-H: $V_q(H,q_H) = C_q(q_H) \rightarrow q_H = q_H^{FB}$

FOC-L: $f(\theta_L)\cdot\underbrace{\left(V_q(L,q_L) - C_q(q_L)\right)}_{\text{marginal DWL } \theta_L} = f(\theta_H)\underbrace{\left(V_q(H,q_L) - V_q(L,q_L)\right)}_{\text{marginal information rent } \theta_H}$

# Continuous Types Model

- Measure 1 consumers with density of types $f(\theta)$ on $[\underline{\theta}, \overline{\theta}]$
- Monopolist offers a tariff $P(q)$
- Consumer of type $\theta$ can purchase quantity (quality) $q$:

$$u(q,\theta) = V(q,\theta) - P(q)$$

  or take the outside option: $\underline{u}(\theta) = 0$
- The tariff $P(q)$ induces type $\theta$ to buy

$$q(\theta) = \arg\max_q u(q,\theta)$$

  and pay $P(q(\theta))$ if $u(q(\theta),\theta) \geq \underline{u}(\theta)$
- Firm costs $= C(q)$ and expected profits:

$$E[P(q(\theta)) - C(q(\theta))]$$

# Functional Form Assumptions

1. We will assume $V_\theta > 0$ (a normalization)
2. $V_{q\theta} > 0$ (single crossing)
3. $\frac{d}{d\theta}\left[f(\theta)/(1-F(\theta))\right] \geq 0$ (non-decreasing hazard rate)
4. $V_{qq} \leq 0$ (decreasing marginal value of quantity)
5. $C_{qq} \geq 0$ (convex costs)
6. $V_{qq\theta} \geq 0$ and $V_{q\theta\theta} \leq 0$ (technical/uninterpretable).

# Mechanism Design Approach

- Reality: Firm chooses tariff $P(q)$
- But we imagine: Firm designs a *direct revelation mechanism*
  - Firm chooses pair of functions $\{q(\theta), T(\theta)\}$
  - Consumer reports type $\theta$ to firm
  - Consumer allocated $q(\theta)$ and pays $T(\theta)$
- Letting $\theta(q)$ be the inverse of $q(\theta)$, $P(q) = T(\theta(q))$
- *Revelation Principle*: We need only consider truthful mechanisms. Any mechanism in which agents misreport their types as $\tau(\theta)$ and are assigned $\{q(\tau), T(\tau)\}$ could be replaced by one in which agents are truthful and are assigned $\{\hat{q}(\theta), \hat{T}(\theta)\} = \{q(\tau(\theta)), T(\tau(\theta))\}$.

# Monopolist's Problem

- If type $\theta$ reports $\hat{\theta}$, utility is

$$u(\theta,\hat{\theta}) = V(q(\hat{\theta}),\theta) - T(\hat{\theta})$$

- Then the firm's profit maximization problem is:

$$\max_{q(\theta),T(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[T\left(\theta\right) - C\left(q\left(\theta\right)\right)\right] f\left(\theta\right) d\theta$$

such that:

IR: $u\left(\theta,\theta\right) \geq \underline{u}\left(\theta\right) \quad \forall\theta$

IC: $u\left(\theta,\theta\right) \geq u\left(\theta,\hat{\theta}\right) \quad \forall\theta,\hat{\theta}$

- IR: *individual rationality* or *participation* constraint
- IC: *incentive compatibility* constraint

# Simplifying the Problem: Step 1 of 3 (Local IC)

- Idea: Separate IC constraint into local-IC and global-IC, substitute local-IC into the problem for marginal prices.
- Local IC: Consumer prefers not to misreport her type by $\varepsilon$
  $\rightarrow$ reporting truthfully must satisfy local FOC:

$$\frac{\partial}{\partial \hat{\theta}} u(\theta, \theta) = 0,$$

$\rightarrow$ which implies

$$\frac{d}{d\theta} u(\theta, \theta) = \frac{\partial}{\partial \theta} u(\theta, \theta) = V_\theta(q(\theta), \theta).$$

(An application of the envelope theorem.)

# Simplifying the Problem: Step 1 of 3 (Local IC)

- Let $u(\theta) = u(\theta, \theta)$. As $u(\theta) = u(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \frac{d}{d\theta} u(x)\, dx$,

$$u(\theta) = u(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} V_{\theta}(q(x), x)\, dx. \tag{1}$$

- This is an important equation! It illustrates the trade-off between efficiency and rent extraction. I'd like to sell the efficient quantity $q^*(\theta)$ to maximize surplus if I didn't have to give any of the surplus to consumers. But this equation shows that the higher $q(\theta)$ the higher is $u(\theta')$ for all $\theta' > \theta$. I have no reason to distort quantity of the highest type, but as we get lower in the type space there is more and more of a motivation to extract rents.

# Simplifying the Problem: Step 1 of 3 (Local IC)

- As, $u(\theta) = V(q(\theta), \theta) - T(\theta)$, we can solve for $T(\theta)$:

$$
\begin{aligned}
T(\theta) &= V(q(\theta), \theta) - u(\theta) \\
&= -u(\underline{\theta}) + V(q(\theta), \theta) - \int_{\underline{\theta}}^{\theta} V_{\theta}(q(x), x)\, dx.
\end{aligned}
$$

- $\rightarrow$ given allocation rule $q(\theta)$, the payment rule $T(\theta)$ is pinned down up to a constant $u(\underline{\theta})$.
- $\rightarrow$ marginal prices are determined by the allocation $q(\theta)$.

# Simplifying the Problem: Step 2 of 3 (IR)

- Assume a zero outside option ($\underline{u}(\theta) = 0$)
- If IR is satisfied at the bottom ($u(\underline{\theta}) \geq 0$)
  then local IC $\rightarrow$ IR is satisfied $\forall$ higher types ($u(\theta) \geq 0 \ \forall \theta$).
- Follows from equation (1) as $\int_{\underline{\theta}}^{\theta} V_\theta (q(x), x)\, dx \geq 0$.
  (Recall we assumed $V_\theta \geq 0$).
- Thus IR reduces to $u(\underline{\theta}) = 0$.
  (If the constraint were not binding the firm could raise all prices by $\varepsilon$ without violating IR or IC.)
- $\rightarrow$ $T(\theta)$ is determined by $q(\theta)$ + local IC + IR:

$$T(\theta) = V(q(\theta), \theta) - \int_{\underline{\theta}}^{\theta} V_\theta (q(x), x)\, dx.$$

- Given Local IC, a sufficient condition for Global IC is

$$\frac{\partial^2}{\partial\theta\partial\hat{\theta}} u\left(\theta, \hat{\theta}\right) \geq 0.$$

- Why? As $\frac{\partial}{\partial\hat{\theta}} u(\theta, \theta) = 0$ (Local IC), the condition implies:
  1. $\frac{\partial}{\partial\hat{\theta}} u(\theta, \hat{\theta}) \geq 0$ for $\theta > \hat{\theta}$ (meaning it is optimal to increase the reported type $\hat{\theta}$ if it is below the true type)
  2. $\frac{\partial}{\partial\hat{\theta}} u(\theta, \hat{\theta}) \leq 0$ for $\theta < \hat{\theta}$ (meaning it is optimal to decrease the reported type $\hat{\theta}$ if it is above the true type)

# Simplifying the Problem: Step 3 of 3 (Relaxing Global IC)

- Local IC, $\frac{\partial}{\partial\theta}u(\theta,\hat{\theta}) = V_\theta(q(\hat{\theta}),\theta)$, implies

$$\frac{\partial^2}{\partial\theta\partial\hat{\theta}}u(\theta,\theta) = V_{q\theta}(q(\hat{\theta}),\theta)\frac{d}{d\hat{\theta}}q(\hat{\theta})$$

- Given single crossing ($V_{q\theta} > 0$), our sufficient second-order condition is satisfied as long as $q(\theta)$ is non-decreasing (*monotonicity*).
- Approach: Impose Local IC + IR and solve relaxed problem that ignores Global IC. Then check to see if the solution $q^*(\theta)$ is non-decreasing (and hence solves the original problem).

# Simplified and Relaxed Problem:

- Given 3 simplifying steps, re-write the (relaxed) problem as an *unconstrained* maximization over the allocation $q(\theta)$:

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[ V(q(\theta), \theta) - \int_{\underline{\theta}}^{\theta} V_{\theta}(q(x), x)\, dx - C(q(\theta)) \right] f(\theta)\, d\theta.$$

- Payments $T(\theta)$ have totally dropped out. They are determined entirely by $q(\theta)$, local IC, and IR. Thus profits depend only on the chosen allocation rule $q(\theta)$.
  [This got Myerson the Nobel...].

# Integration By Parts:

- Integrate by parts to eliminate the nested integral
- Recall that $\int_a^b u dv = uv|_a^b - \int_a^b v du$. Thus

$$\begin{aligned}
&\int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} V_\theta\left(q\left(x\right), x\right) dx f\left(\theta\right) d\theta \\
&\quad = \int_{\underline{\theta}}^{\theta} V_\theta\left(q\left(x\right), x\right) dx F\left(\theta\right) \Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} V_\theta\left(q\left(\theta\right), \theta\right) F\left(\theta\right) d\theta \\
&\quad = \int_{\underline{\theta}}^{\bar{\theta}} V_\theta\left(q\left(\theta\right), \theta\right) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} V_\theta\left(q\left(\theta\right), \theta\right) F\left(\theta\right) d\theta \\
&\quad = \int_{\underline{\theta}}^{\bar{\theta}} \left(1 - F\left(\theta\right)\right) V_\theta\left(q\left(\theta\right), \theta\right) d\theta.
\end{aligned}$$

# Simplified$^2$ and Relaxed Problem:

- Firm's problem:

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[ V\left(q\left(\theta\right),\theta\right) - C\left(q\left(\theta\right)\right) - \frac{1-F\left(\theta\right)}{f\left(\theta\right)} V_{\theta}\left(q\left(\theta\right),\theta\right) \right] f\left(\theta\right) d\theta.$$

- $V\left(q,\theta\right) - C\left(q\right)$ is surplus. The adjusted term inside the integrand is called virtual surplus:

$$\psi\left(q,\theta\right) = V\left(q,\theta\right) - C\left(q\right) - \frac{1-F\left(\theta\right)}{f\left(\theta\right)} V_{\theta}\left(q,\theta\right).$$

- Rather than maximizing expected surplus, the firm maximizes expected virtual surplus $E\left[\psi\left(q\left(\theta\right),\theta\right)\right]$.
- Beautiful thing: This expression can be maximized point-wise.

# Remaining Solutions Steps:

1. Solve FOC $\frac{d}{dq}\psi(q,\theta)=0$ to find the optimal $q(\theta)$
2. Check SOC $\frac{d^2}{dq^2}\psi(q,\theta)\leq 0$.
3. Check monotonicity $\frac{d^2}{dqd\theta}\psi(q,\theta)\geq 0$ to verify Global IC
   - Standard monotone comparative statics (MCS) result:
     If $\frac{d^2}{dqd\theta}\psi(q,\theta)\geq 0$
     then $q(\theta)=\arg\max_q \psi(q,\theta)$ is non-decreasing in $\theta$.
   - Hence $\frac{d^2}{dqd\theta}\psi(q,\theta)\geq 0$ implies $q(\theta)$ is non-decreasing which we showed is sufficient for Local IC to imply Global IC.

.

# Remaining Solutions Steps:

1. Solve FOC:

$$\frac{d}{dq}\psi(q,\theta) = V_q(q,\theta) - C_q(q) - \frac{1-F(\theta)}{f(\theta)} V_{q\theta}(q,\theta) = 0$$

2. Check SOC

$$\frac{d^2}{dq^2}\psi(q,\theta) = V_{qq}(q,\theta) - C_{qq}(q) - \frac{1-F(\theta)}{f(\theta)} V_{qq\theta}(q,\theta) \leq 0$$

3. Check monotonicity

$$\frac{d^2}{dqd\theta}\psi(q,\theta) = V_{q\theta}(q,\theta)\left(1 - \frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right)\right) - \frac{1-F(\theta)}{f(\theta)} V_{q\theta\theta}(q,\theta) \geq 0$$

# Solutions Steps 2–3:

2. Check SOC

$$\frac{d^2}{dq^2}\psi(q,\theta) = V_{qq}(q,\theta) - C_{qq}(q) - \frac{1-F(\theta)}{f(\theta)} V_{qq\theta}(q,\theta) \leq 0$$

3. Check monotonicity

$$\frac{d^2}{dqd\theta}\psi(q,\theta) = V_{q\theta}(q,\theta)\left(1 - \frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right)\right) - \frac{1-F(\theta)}{f(\theta)} V_{q\theta\theta}(q,\theta) \geq 0$$

Recall functional form assumptions: $\frac{d}{d\theta}[f(\theta)/(1-F(\theta))] \geq 0$, $C_{qq} \geq 0$, $V_\theta > 0$, $V_{q\theta} > 0$, $V_{qq} \leq 0$, $V_{qq\theta} \geq 0$, and $V_{q\theta\theta} \leq 0$.

# Solution FOC

$$V_q(q,\theta) = C_q(q) + \frac{1 - F(\theta)}{f(\theta)} V_{q\theta}(q,\theta). \tag{2}$$

- LHS = marginal price = $P'(q)$.
  - follows from the consumer's maximization problem $q = \arg\max_q (V(q,\theta) - P(q))$ which sets $P'(q) = V_q(q,\theta)$.
- $\rightarrow$ Equation (2) says that marginal price is equal to marginal cost plus an upwards distortion $\frac{1-F(\theta)}{f(\theta)} V_{q\theta}(q,\theta)$,
- $MP > MC$ $\rightarrow$ distorts quantity downwards below first best.

# Solution FOC: Intuition

$$V_q(q,\theta) = C_q(q) + \frac{1-F(\theta)}{f(\theta)} V_{q\theta}(q,\theta).$$

1. $MR = MC$: On the margin of selling an extra unit to type $\theta$:
   - $MR = f(\theta) V_q(q,\theta) - (1-F(\theta)) V_{q\theta}(q,\theta)$
   - $MC = f(\theta) C_q(q)$
   - When we sell an extra unit to type $\theta$, we earn marginal price $V_q(q,\theta)$ and pay marginal cost $C_q(q,\theta)$, with probability $f(\theta)$.
   - However, $MR < MP$ because we also have to give an additional information rent $V_{q\theta}(q,\theta)$ to all higher types, which arise with probability $(1-F(\theta))$.
2. Surplus vs. Rents: FOC balances trade-off between creating surplus and extracting rents. It balances
   - additional surplus $(V_q - C_q)$ for type $\theta$ with probability $f(\theta)$
   - additional information rents $V_{q\theta}$ for higher types with probability $(1-F(\theta))$.

# Solution FOC: Implications of equation (2)

$$V_q(q,\theta) = C_q(q) + \frac{1-F(\theta)}{f(\theta)} V_{q\theta}(q,\theta).$$

1. No distortion at the top: At $\theta = \bar{\theta}$,

$$\frac{1-F(\bar{\theta})}{f(\bar{\theta})} = 0 \rightarrow q^*(\bar{\theta}) = q^{FB}(\bar{\theta})$$

2. Downward distortions all lower types: For all $\theta < \bar{\theta}$,

$$\frac{1-F(\theta)}{f(\theta)} > 0 \rightarrow q^*(\theta) < q^{FB}(\theta)$$

# Solution FOC: Implications of equation (2)

$$V_q(q,\theta) = C_q(q) + \frac{1-F(\theta)}{f(\theta)} V_{q\theta}(q,\theta).$$

3. Quantity discounts, Ex 1:
   - Assume $V = q\theta - q^2/2 \rightarrow V_q = \theta - q$ and $V_{q\theta} = 1$, and strictly increasing hazard: $\frac{d}{d\theta}\frac{1-F(\theta)}{f(\theta)} < 0$.
   - Then $\rightarrow$ distortion decreasing in $\theta$.
   - Why? Distortions imposed to minimize info. rents to higher types. Fewer higher types above $\theta$ as $\theta$ increases.
   - Absolute markup is decreasing in $\theta$ and $q$:

$$p - c = V_q(q,\theta) - C_q(q) = \frac{1-F(\theta)}{f(\theta)}$$

   - Assume constant MC: $C(q) = cq$
   - Then decreasing markup implies $\rightarrow$ marginal price is decreasing in $\theta$ (and hence $q$) $\rightarrow P''(q) < 0$.
   - Concavity of $P(q)$ implies that average price per unit $P(q)/q$ is decreasing. A quantity discount in a stronger sense.

# Solution FOC: Implications of equation (2)

$$V_q(q,\theta) = C_q(q) + \frac{1-F(\theta)}{f(\theta)} V_{q\theta}(q,\theta)$$

4. Quantity discounts, Ex 2:
   - Assume: Constant $MC = c$ and $V = \theta q^{\gamma}$, for $\gamma \in (0,1)$, so that $V_q = \theta\gamma/q^{1-\gamma}$ and $V_{q\theta} = \gamma/q^{1-\gamma}$. Then

$$p = c + \gamma \frac{1-F(\theta)}{f(\theta)\, q^{1-\gamma}(\theta)}$$

   - By monotonicity $q$, the same quantity discount results apply: $p-c$, $p$, and $P(q)/q$ all decreasing.
   - Ex 2 inspired by McManus (2007).

# Example 1

- $C = cq$ and $V = q\theta - \frac{1}{2}q^2 \to V_q = \theta - q$ and $V_{q\theta} = 1$:

$$\theta - q = c + \frac{1 - F(\theta)}{f(\theta)} \to q(\theta) = \theta - c - \frac{1 - F(\theta)}{f(\theta)}$$

- **(1a)** Suppose $\theta \sim U[0,1]$:

$$q(\theta) = 2\theta - 1 - c$$
$$\theta(q) = \frac{1 + c + q}{2}$$

  - Minimum type served $= \theta^* = (1 + c)/2$.
  - Marginal price $p = V_q(q, \theta(q)) = \theta - q$, or

$$P'(q) = \frac{1 + c + q}{2} - q = \frac{1 + c - q}{2}$$

  - Total price

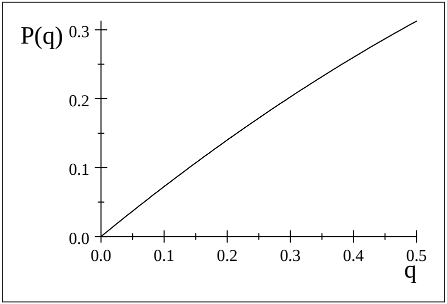
$$P(q) = \frac{1 + c}{2}q - \frac{1}{4}q^2$$

For $c = 1/2$, this is $P(q) = \frac{1}{4}\left(3q - q^2\right)$

# Example 1a continued

- **(1a)** $C = cq$, $V = q\theta - \frac{1}{2}q^2$, $\theta \sim U[0,1]$.
- Special case $c = 1/2$:

$$q(\theta) = 2\theta - 1.5$$
$$P(q) = 0.75q - 0.25q^2$$
$$\theta^* = 0.75$$



# Example 1

- $C = cq$ and $V = q\theta - \frac{1}{2}q^2 \rightarrow V_q = \theta - q$ and $V_{q\theta} = 1$:

$$\theta - q = c + \frac{1 - F(\theta)}{f(\theta)} \rightarrow q(\theta) = \theta - c - \frac{1 - F(\theta)}{f(\theta)}$$

- **(1b)** Suppose $\theta$ is exponential. Then

$$\frac{1 - F(\theta)}{f(\theta)} = \frac{\exp(-\lambda\theta)}{\lambda \exp(-\lambda\theta)} = \frac{1}{\lambda}$$
$$q(\theta) = \theta - c - 1/\lambda$$
$$\theta(q) = q + c + 1/\lambda$$

  and minimum type served is $\theta^* = c + 1/\lambda$

- Implies linear pricing:

$$P'(q) = \theta - q = c + 1/\lambda$$

$$P(q) = (c + 1/\lambda)\, q.$$

# Example 2

- Assume $C = cq$ and $V = q\theta \rightarrow$ no price discrimination
- Virtual surplus $\psi$ is linear (not concave) in $q$
  $\rightarrow$ FOC is "bang-bang":

$$\frac{d}{dq}\psi(q,\theta) = V_q(q,\theta) - C_q(q) - \frac{1-F(\theta)}{f(\theta)}V_{q\theta}(q,\theta)$$
$$= \theta - c - \frac{1-F(\theta)}{f(\theta)}$$

  is independent of $q$.
- It is positive for $\theta > \theta^* = c + \frac{1-F(\theta)}{f(\theta)}$ and negative for $\theta < \theta^*$.
- Thus types $\theta \geq \theta^*$ should all be served the maximum quantity (quality) at the same price with no price discrimination.
- To have meaningful price discrimination we need to have a strictly concave value function $V$ if costs are linear.

# Example 2—Foreshadowing Connection to Leslie (2004)

- Looking ahead to Leslie (2004)
- Model assumes $MC = 0$ and $U_{ij} = q_{ij}\left[B\left(y_i\right) - p_j\right]^{\eta}$.
- When you read Leslie (2004) think about how this functional form relates to assumptions used today, and to Example 2. Think about the role and importance of the parameter $\eta$.
- Think about alternatives such as $U_{ij} = q_{ij}B\left(y_i\right) - \eta q_{ij}^2 - p_j$ or perhaps $U_{ij} = (q_{ij} - \eta q_{ij}^2)B\left(y_i\right) - p_j$ (with $\eta \leq 1$ and $\bar{q} = 1$).

# Applications

- Nonlinear pricing
- Regulated firm (Ramsey pricing)
- Optimal taxation

# When is price discrimination profitable?

Anderson, Eric T. and James D. Dana. 2009. "When Is Price Discrimination Profitable?" Management Science, 55(6), 980-89.

- surplus is log supermodular $\rightarrow$ price discrimination is profitable
- surplus is log submodular $\rightarrow$ price discrimination is unprofitable
- Q for empirical work: Does model assume that surplus is log supermodular or log submodular or nest both as special cases depending on an estimated parameter?
- Continuously differentiable $S(q,\theta)$ is log supermodular iff $\frac{d^2}{dqd\theta}\ln S > 0$ (equivalently $S_{q\theta}S - S_qS_\theta > 0$) and log submodular iff $\frac{d^2}{dqd\theta}\ln S < 0$ ($\Leftrightarrow S_{q\theta}S - S_qS_\theta < 0$).
- The linear example above, $V = q\theta$ and $C = cq$, falls into the unprofitable submodular case: $S = q(\theta - c)$, $S_q = (\theta - c)$, $S_\theta = q$, $S_{q\theta} = 0$ so $S_{q\theta}S - S_qS_\theta = -q(\theta - c) = -S < 0$.
- This result does not directly apply to Leslie (2004) because Anderson and Dana (2009) assume quasi-linear utility but Leslie (2004) does not.

# Nonlinear Pricing versus the Vertical Model

Vertical Model

- $U_{ij} = \delta_j - \alpha_i p_j$
- $\delta_j = \sum_k \beta_k x_{jk}$
- Cost $c_j$

Nonlinear Pricing (multiplicative model)

- $U_{ij} = \theta_i v(q_j) - P(q_j)$
- Cost $C(q_j)$
- Either normalize $C(q_j)$ or $v(q_j)$ to be linear
- For close comparison to vertical model
  Normalize $q_j$ st $v(q_j) = q_j$ & divide by $\theta_i$

$$\hat{U}_{i,j} = q_j - \alpha_i P(q_j),\ \hat{C}(q_j)$$

(Makes most sense for abstract quality
(Crawford and Shum, 2007))

# Nonlinear Pricing versus the Vertical Model

Vertical Model

- $U_{ij} = \delta_j - \alpha_i p_j$, $\delta_j = \sum_k \beta_k x_{jk}$
- Cost $c_j$
- $\delta_j$ (or $x_j k$) exogenous

Nonlinear Pricing (mult. model)

- $\hat{U}_{ij} = q_j - \alpha_i P(q_j)$
- Cost $C(q_j)$
- $q_j$ endogenous
  - Discrete case: extra FOC & extra welfare question (Crawford and Shum, 2007)
  - Connection to endogenous products literature
- $q_j$ may be a continuous choice (Attanasio and Pastorino, 2020)
- ...

# Structural Analysis of Nonlinear Pricing

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# Luo, Perrigne, and Vuong (2018)

| Property | LPV 2018 Assumption |
| --- | --- |
| Fn. form $C(q)$ | $C(q) = \kappa + cq$ |
| Fn. form $V(\theta, q)$ | $V(\theta, q) = \theta v(q)$ |
| Types | continuous $\theta \in \{\underline{\theta}, \bar{\theta}\}$ (Normalisation: min. type served $\theta^* = 1$). |
| Data | price, share, & $q$ =quantity |
| Tariff Menu | Each customer faces a unique tariff that is unobserved. Only the chosen price-quality pair is observed. Implicitly allows for some form of horizontal heterogeneity. |

# Commentary on assumptions

- Approach likely better for quantity rather than quality applications.
  - Quality is less likely observable
  - Linear cost less likely reasonable for quality.
- By observing $q$, LPV relax fn. form restriction on $V(\theta, q)$ relative to Crawford and Shum (2007).
  - Valuable as curvature of $v(q)$ matters for quality/quantity distortion and welfare.
- $V(\theta, q) = \theta v(q)$ still a significant restriction.
  - E.g., rules out $V(\theta, q) = v \min\{\theta, q\}$. (Const. value $v$ per unit up to satiation point $\theta$. Ex. with no quality distortion or DWL.)
  - Requires that increasing $\theta$ vertically stretches inverse demand curves. Horizontal stretching or other shifts are not allowed.

# Identification

- Consider case $T(q)$ known.
- For $T_i(q)$ unobserved case, see paper.

# Identification Part I: Costs

- MC: Follows from no distortion at the top:

$$c = P_q(\bar{q})$$

- FC: Inferred from optimality of firm's choice of $\theta^*$. *FC* are such that the virtual surplus from serving type $\theta^*$ is zero ("optimal exclusion condition" equation (7) in the paper). Paper shows this implies:

$$FC = c\left(\frac{P(\underline{q})}{P_q(\underline{q})} - \underline{q}\right)$$

# Identification Part II: Types

Paper shows that firm FOC and local IC ($P_q(q) = V_q(\theta(q), q)$) imply equation (9) in the paper, which identifies inverse allocation $\theta(q)$

- Perhaps not surprising that this can be backed out from the firm's FOC since the FOC determines the allocation.
- Eq (9) Expresses the $\alpha$ quantile of $\theta$, denoted $\theta(\alpha)$, in terms of MC ($c = P_q(\bar{q})$) and observables, including the marginal price $P_q$, and the quantiles $q(\alpha)$ of the quantity distribution $G_q$:

$$log \theta(\alpha) = \int_0^\alpha \frac{1}{1-x}\left(1 - \frac{c}{P_q(q(x))} dx\right)$$

- $\theta(q)$ given by $q(\alpha)$ and $\theta(\alpha)$

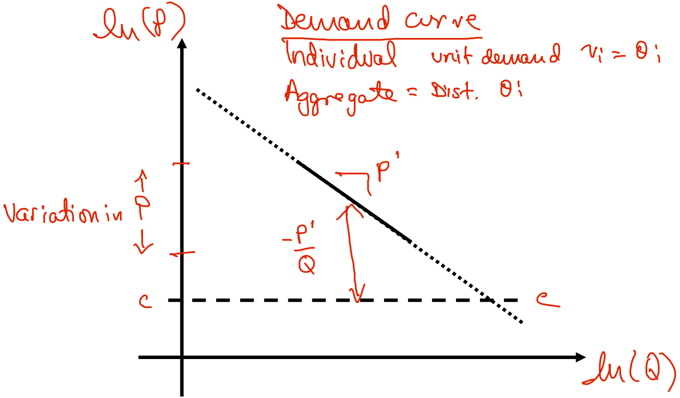
# Identification Part III: Values

- Local IC $\rightarrow P_q(q) = V_q(\theta(q), q)$
- + Fn. form ass. $V(\theta, q) = \theta v(q) \rightarrow$

$$v_q(q) = V_q(\theta(q), q)/\theta(q) = P_q(q)/\theta(q)$$

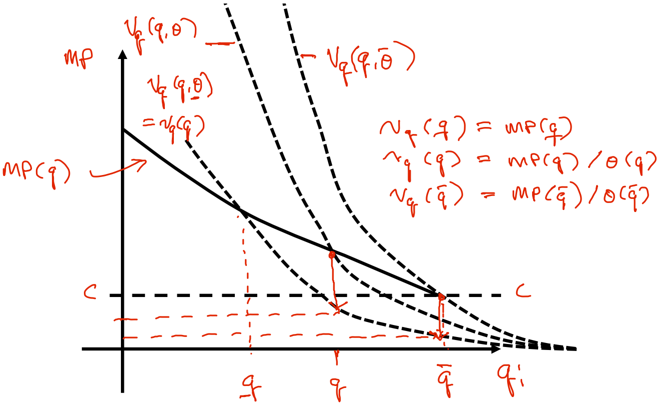
(Integrate to get $v(q)$ given assume $v(0) = 0$.)

# Identification: Classic Demand Est.



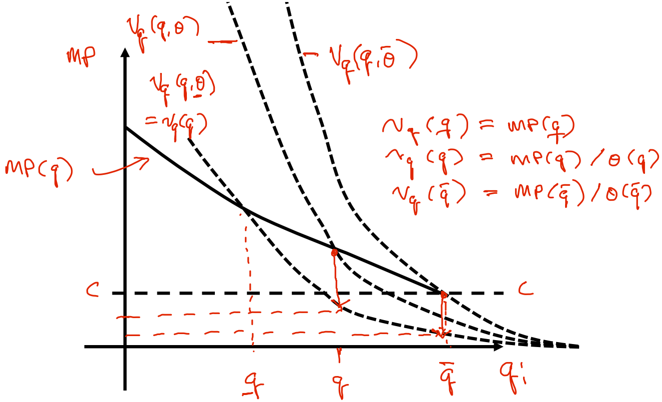

- Fn. form ass. $\rightarrow$ demand shape & extrap beyond range observation
- $p$ variation $\rightarrow$ slope demand
- Firm FOC + demand est. $\rightarrow$ markup $\rightarrow$ MC

# Identification: Luo et al. (2018)




- Ind. demand curve $= V_q(q,\theta)$; Agg. demand integrates over $\theta$
- w/o price var, observe $= V_q(q,\theta)$ at only 1 point for each $\theta$
- Fn. form solution: $V_q(q,\theta) = \theta v_q(q)$
- $v_q(q) = MP(q)/\theta(q)$. But what is $\theta(q)$?

# Identification: Luo et al. (2018)




- Firm FOC: $c = P_q(\bar{q})$ and
  $MP - c = \frac{1-F(\theta)}{f(\theta)} V_{q\theta}(q,\theta) = \frac{1-F(\theta)}{f(\theta)} v_q(q) = \frac{1-F(\theta)}{f(\theta)} \frac{MP}{\theta(q)}$
- Change of var: $\frac{\theta'(q)}{\theta(q)} = \frac{MP-c}{MP} \frac{g(q)}{1-G(q)}$. Paper shows how to solve D.E.

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# Participation Constraints in Adverse Selection Models[1]

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# Virtual Surplus adjusted for heterogeneous outside options:

- Standard Model: $\underline{u}(\theta) = \underline{u}$. Virtual surplus:

$$\psi(q,\theta) = V(q,\theta) - C(q) - \frac{1 - F(\theta)}{f(\theta)} V_\theta(q,\theta).$$

- Jullien (2000): $\underline{u}(\theta)$. Virtual surplus:

$$\psi(q,\theta) = V(q,\theta) - C(q) - \frac{\gamma(\theta) - F(\theta)}{f(\theta)} V_\theta(q,\theta).$$

  - $\gamma(\theta) \in [0,1]$, non-decreasing, shadow value of reducing $\underline{u}(\theta)$ on $[\underline{\theta}, \theta]$
  - For relaxed solution to hold (no bunching), paper imposes sufficient conditions. Requires $\gamma(\theta)$ continuous on $(\underline{\theta}, \bar{\theta})$.

# Jullien (2000): Heterogenous Outside Options

- Standard Problem
  - Relaxed problem $\rightarrow$ monopolist choose $q(\theta)$ to max virtual surplus pointwise.

$$\psi(q,\theta) = V(q,\theta) - C(q) - \frac{1 - F(\theta)}{f(\theta)} V_\theta(q,\theta).$$

  - Intuition: surplus adjusted by an information rent term.
    - Information rent term weighted by no. higher types $(1 - F(\theta))$
    - higher types = those who get higher info. rent if type $\theta$ gets larger $q$. (as dictated by local IC)
  - Follows from assumption all types have **same** outside option $\underline{u}$.
    - $\rightarrow$ participation constraint only binds for bottom type,
    - $\rightarrow$ local IC determines rents for higher types.

# Jullien (2000): Heterogenous Outside Options

- Competition generates outside options increasing in type
  - High types value competitors product more
  - E.g. competitive fringe supplying an imperfect substitute.
- Jullien (2000) allows type dependent $\underline{u}(\theta)$.
  - Assumes no bunching (by imposing "potential separation" (PS), "homogeneity" (H), and full participation (FP)) such that relaxed solutions solves full problem.
- Now virtual surplus is

$$\psi(q,\theta) = V(q,\theta) - C(q) - \frac{\gamma(\theta) - F(\theta)}{f(\theta)} V_\theta(q,\theta).$$

  - $(1 - F(\theta))$ replaced by $(\gamma(\theta) - F(\theta))$
  - $\gamma(\theta)$ is like a CDF (nondecreasing from 0 to 1)
  - shadow value of relaxing IR for $\theta \in [\underline{\theta}, \theta]$.

# Jullien (2000): Heterogenous Outside Options

Virtual surplus:

$$\psi(q,\theta) = V(q,\theta) - C(q) - \frac{\gamma(\theta) - F(\theta)}{f(\theta)} V_\theta(q,\theta).$$

- $\gamma(\theta) =$ shadow value of relaxing IR for $\theta \in [\underline{\theta}, \theta]$.
- $d\gamma(\theta)/d\theta =$ shadow value of IR constraint for type $\theta$.
- $d\gamma(\theta)/d\theta = 0$ and $\gamma(\theta) =$ constant at $\theta$ where IR not binding.
- $\gamma(\theta)$ is discontinuous when participation binds at discrete types.
  - Ruled out except at $\underline{\theta}$ and $\bar{\theta}$ to preclude bunching.
- Possible $\gamma(\theta) < F(\theta) \rightarrow$ quantity distorted upwards above FB.

# Jullien (2000): Heterogenous Outside Options

Virtual surplus:

$$\psi(q,\theta) = V(q,\theta) - C(q) - \frac{\gamma(\theta) - F(\theta)}{f(\theta)} V_\theta(q,\theta).$$

- Example 1: $\gamma$ is $1/2$ on $[0, 0.5]$ and 1 on $[0.5, 1]$,
  - Participation constraint binds at 0 and $1/2$ but not elsewhere.
  - For types below $1/2$, info. rent has lower weight $(1/2 - F(\theta))$.
  - Intuition: Increasing $q$ for $\theta < \frac{1}{2}$ only raises info. rent to $\theta \in [\theta, 1/2]$, as at $1/2$ utility is pinned down by binding IR.
  - Imperfect example because violates no-bunching conditions
- Example 2: $\underline{u}(\theta) = V(\theta, q^{FB}(\theta)) - C(q^{FB}(\theta))$
  - $\gamma(\theta) = F(\theta)$, $q^*(\theta) = q^{FB}(\theta) \to$ comp. outcome.

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# NONLINEAR PRICING IN VILLAGE ECONOMIES

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# Setting: Quantity Discounting in Mexican Villages

TABLE I

PRICE SCHEDULES AND IMPACT OF CASH TRANSFERS ON PRICES (98% TRIMMING)[a]

| | Rice Unit Values | | | Kidney Beans Unit Values | | | Sugar Unit Values | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| Intercept | 1.866 | 1.994 | 1.874 | 2.473 | 2.399 | 2.465 | 1.832 | 1.768 | 1.814 |
| | (0.005) | (0.008) | (0.007) | (0.007) | (0.010) | (0.010) | (0.004) | (0.004) | (0.006) |
| Treatment | | −0.006 | −0.008 | | −0.007 | 0.010 | | 0.003 | 0.025 |
| | | (0.009) | (0.008) | | (0.012) | (0.013) | | (0.005) | (0.007) |
| $\log(q)$ | −0.320 | | −0.290 | −0.188 | | −0.161 | −0.198 | | −0.157 |
| | (0.007) | | (0.009) | (0.007) | | (0.009) | (0.009) | | (0.010) |
| $\log(q) \times$ Treatment | | | −0.038 | | | −0.035 | | | −0.053 |
| | | | (0.013) | | | (0.013) | | | (0.015) |
| $R^2$ | 0.352 | 0.136 | 0.353 | 0.222 | 0.146 | 0.223 | 0.168 | 0.045 | 0.170 |
| Observations | 69,543 | 69,543 | 69,543 | 93,375 | 93,375 | 93,375 | 103,930 | 103,930 | 103,930 |

[a]Note: Wave fixed effects are included. Standard errors are clustered at the locality level.

- Dependent variable: log average unit price
- Treatment is Progresa
- Interpretation?

# BC Problem (Attanasio and Pastorino, 2020)

$$\text{(BC problem)} \max_{\{t(\theta),q(\theta)\}} \left( \int_{\underline{\theta}}^{\overline{\theta}} t(\theta) f(\theta)\, d\theta - c(Q) \right) \quad \text{s.t.}$$

$$\text{(IC)} \quad v\big(\theta, q(\theta)\big) - t(\theta) \geq v\big(\theta, q\big(\theta'\big)\big) - t\big(\theta'\big) \quad \text{for any } \theta, \theta',$$

$$\text{(IR')} \quad v\big(\theta, q(\theta)\big) - t(\theta) \geq \overline{u} \quad \text{for any } \theta,$$

$$\text{(BC)} \quad t(\theta) \leq I\big(\theta, q(\theta), w\big) \quad \text{for any } \theta.$$

- Subsistence constraint (min calories) equivalent budget constraint
- Budget $I(\theta, q, w) =$ most can spend and have enough left over to buy additional calories to make up the shortfall given purchase $q$.
  - Weakly increasing in $q$ as $q$ decreases additional calories needed
  - May increase in $\theta$ as higher taste for $q$ may be due to deriving more calories from $q$. (really?)

# IR Problem (Jullien, 2000)

$$\text{(IR problem)} \max_{\{t(\theta),q(\theta)\}} \left( \int_{\underline{\theta}}^{\overline{\theta}} t(\theta) f(\theta)\, d\theta - c(Q) \right) \quad \text{s.t.}$$

$$\text{(IC)} \quad v\big(\theta, q(\theta)\big) - t(\theta) \geq v\big(\theta, q\big(\theta'\big)\big) - t\big(\theta'\big) \quad \text{for any } \theta,\ \theta',$$

$$\text{(IR)} \quad v\big(\theta, q(\theta)\big) - t(\theta) \geq \overline{u}(\theta) \quad \text{for any } \theta,$$

- Under conditions of Prop 1: Solutions to BC and IR problems coincide
- To convert problems, set

$$I(\theta, q(\theta)) = V(\theta, q(\theta)) - \underline{u}_{IR}(\theta)$$

# IR–BC problem equivalence (Proposition 1)

- BC: $V(\theta\, q(\theta)) - t(\theta) \geq V(\theta, q(\theta)) - I(\theta, q(\theta), w)$
- IR' + BC: $V(\theta, q(\theta)) - t(\theta) \geq \max\{\underline{u}, V(\theta, q(\theta)) - I(\theta, q(\theta), w)\}$
- Define: $\underline{u}(\theta, q(\theta)) = \max\{\underline{u}, V(\theta, q(\theta)) - I(\theta, q(\theta), w)\}$
- IR problem with $\underline{u}$ would be equivalent to Jullien (2000) but for dependence on $q(\theta)$.
- Impose $I_q(\theta, q(\theta), w) = V_q(\theta, q(\theta))$ and dependence on $q(\theta)$ goes away $\rightarrow$ equiv. to Jullien (2000).[1]
- Restriction need only be imposed where BC is binding. (See Proposition 1, p. 222)

[1] As long as $\underline{u}(\theta)$ satisfies the same assumptions that Jullien (2000) imposes.

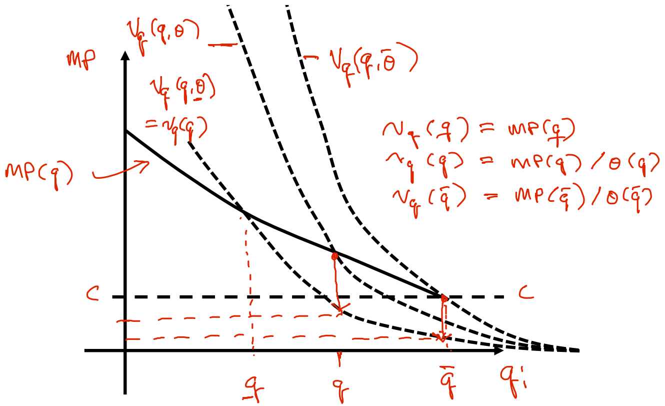
# Implications for Pricing

- Quantity discounts are consistent with allocations below **and above** FB
- Relative to standard model, as $\gamma \leq 1$ for each type
  - Higher quantity
  - Lower marginal price
  - Higher consumer surplus
- Both models, nonlinear pricing can increase surplus.
- Both models, consumer surplus is higher with linear pricing given full participation: nonlinear pricing better extracts surplus for firm.
- Het outside option model: If $q > q^{FB}$, then nonlinear contract may benefit otherwise excluded consumers.

# Implications for Income Transfers

- Type independent income transfer, $\tau(\theta) = \tau$
  - P(q) uniformly increase by $\tau$ (We should see $q$ constant and average $P(q)/q$ go up).
- Progresa transfer increasing in children and decreasing in income. Having more children is like being poorer: $\tau(\theta)$ weakly decreasing.
- Higher income $\rightarrow$ Can spend more $\rightarrow$ Like reduction in outside option
  - Reduction in outside option is largest for low types
- Proposition 4 / Corollary 1: Any progressive (weakly decreasing) transfer $\tau(\theta)$ will (for some interval of types)
  - Increase quantity and total price
  - Decrease marginal price
  - Increase concavity of $P(q)$ (if $v'''$ suff small)
- Increase in concavity is regressive. In kind transfers have the opposite effect (so may be preferable).
  - See recent JMP (Jiménez-Hernández and Seira, 2021) for benefits of in-kind government provision on seller pricing.

# Identification: Attanasio and Pastorino (2020)




- Firm FOC: $c = P_q(\bar{q})$ and
  $MP - c = \frac{\gamma(\theta) - F(\theta)}{f(\theta)} V_{q\theta}(q, \theta) = \frac{\gamma(\theta) - F(\theta)}{f(\theta)} v_q(q) = \frac{\gamma(\theta) - F(\theta)}{f(\theta)} \frac{MP}{\theta(q)}$
- Change of var: $\frac{\theta'(q)}{\theta(q)} = \frac{MP - c}{MP} \frac{g(q)}{\gamma(\theta(q)) - G(q)}$. Stuck without more ass.

# Identification: Attanasio and Pastorino (2020)

- Note, $\frac{\theta'(q)}{\theta(q)} = \frac{MP-c}{MP}\frac{g(q)}{\gamma(\theta(q))-G(q)}$, is equivalent to equation (12) on p. 229 where $\varphi(\theta) = \frac{\theta'(q)}{\theta(q)}$.
- Relative to Luo et al. (2018), have an additional function to identify, the shadow value $\gamma(\theta)$.
- To identify more (unless previously over identified) will need either additional data or additional assumptions.
- Attanasio and Pastorino (2020) use additional functional form assumptions

# Identification: Attanasio and Pastorino (2020)

- First, marginal cost and $\gamma$ are identified (Proposition 5) up to coefficient of absolute risk aversion $A(q) = -v''(q)/v'(q)$
  - But we don't know $A(q)$, so make parametric assumption on $\gamma$ and then ignore the problem. (I.e. functional form assumption.)
  - Page 236 describe $\gamma$ as being identified by difference in curvature of $p(q)$ and $G(q)$. Not entirely clear what extra is being used beyond Local IC and firm FOC that give (12)–(13) but still include $A(q)$ term. Seems to be assuming

$$x(q) \equiv c'(Q)g(q)\theta(q)/\theta'(q) = \chi_{vj0} + \chi_{vj1}q_{vji}$$

    in (17) is linear (p 232), and following parametric restriction on $\gamma(\theta)$.
- The rest follows similarly to Luo et al. (2018) (Propositions 6–7)
  - Assume[2] $V(\theta, q) = \theta v(q)$ and normalize $\underline{\theta} = 1$
  - Given MC and $\gamma$, can back out $\theta(q)$ hence $v(q)$ in the same way as Luo et al. (2018).

[2] Prior results already depend on this assumption.

# Comment

- Paper emphasizes that income transfer has no effect on purchasing (and hence pricing) in standard ($\underline{u} = 0$) model.
- But model is $U = \theta v(q) - P(q)$. Notice that we can re-normalize this as $\hat{U} = v(q) - \alpha P(q)$ for $\hat{U} = U/\theta$ and $\alpha = 1/\theta$.
  - In BLP's formulation, $\alpha$ follows the income distribution. (Difference to standard demand models is not $\theta$ it is $v(q)$)
  - Hence we could imagine $F(\theta)$ is endogenous to income without straying far from cannon.
  - Could model $\theta(w)$. Fn form ass would be more transparent. Observe $w$ and infer $\theta$ each household so should be able to identify...
- Any model: Can ask, is model reduced form or structural? Answer depends on the counter factual simulation planned. Parameters are structural if they should be constant in counter factual.
  - Perhaps type $\theta$ is endogenous to income...
  - Perhaps outside option $\underline{u}(\theta)$ is endogenous to other food sellers' response to Progresa...

# Results: Nonlinear Pricing (NLP) vs Linear Pricing (LP)

TABLE IV

LINEAR PRICING (LP) VERSUS NONLINEAR PRICING (NLP) BY PERCENTILE RANGES OF CONSUMER TYPES

| | Consumer Surplus under LP vs. NLP | | | | | Consumption under LP vs. NLP | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 5% | 25% | 50% | 75% | 100% | 5% | 25% | 50% | 75% | 100% |
| Rice | 79.6 | 88.2 | 81.1 | 87.8 | 96.4 | 46.9 | 51.5 | 46.3 | 65.8 | 89.1 |
| Kidney Beans | 30.1 | 23.7 | 26.6 | 23.2 | 54.7 | 17.9 | 3.7 | 3.6 | 3.6 | 49.1 |
| Sugar | 55.0 | 45.2 | 47.3 | 41.7 | 76.5 | 25.7 | 14.0 | 5.4 | 3.5 | 50.0 |

- Nonlinear pricing is actually benefiting the poorest consumers by increasing participation (except for Rice)
- Table shows % doing better (left panel) and consuming more (right panel) with LP by percentile consumer type.
- Harm of LP driven by exclusion: "The percentages of excluded households in the percentile ranges of Table IV are 20.4%, 11.8%, 18.3%, 10.7%, and 0.9% for rice; 69.9%, 74.8%, 70.8%, 71.4%, and 18.0% for kidney beans; and 45.0%, 54.8%, 51.5%, 52.3%, and 9.6% for sugar."

# Results: Effect of Progresa

- Despite previously reported stable average unit price (average $P(q)/q$) Progresa increases quantity discounting, meaning $P(q)/q$ increases for poor and decreases for wealthy.
- "For instance, the unit prices of the quantities in the bottom 25% of the distribution of quantities purchased across treated localities, paid by the households that purchase small quantities, on average are 13.2% higher for rice, 24.3% higher for kidney beans, and 29.8% higher for sugar than across control localities. On the contrary, the unit prices of the quantities in the top 25% of the distribution of quantities purchased across treated localities, paid by the households that purchase large quantities, on average are 12.3% lower for rice, 12.1% lower for kidney beans, and 5.6% lower for sugar than across control localities."
- "the transfer may have had a more limited beneficial impact than has commonly been inferred."
- Suggests we think harder about benefits of in-kind transfers that Economists normally hate.

# Comment

Nice paper shows role from development IO

- IO methods / models
- Adapted for development context
- But could also bring back to developed markets—income effects will matter e.g. in U.S. healthcare markets or other big-ticket items like housing.

# Competition

Literature on the effects of competition on non-linear pricing.

- E.g. Busse and Rysman (2005), Borzekowski, Thomadsen, and Taragin (2009), and Seim and Viard (2011)
- Borzekowski et al. (2009) have a particularly nice literature overview.

Competition may increase or decrease price discrimination

- competition may increase or decrease dispersion in customer WTP
- competition may increase or decrease the returns to paying the fixed cost of introducing an additional menu item

# Competition may increase or decrease dispersion in customer WTP

- Competition can decrease dispersion in WTP:
  - Suppose $U_{ij} = V_i - p_j$ and utility from the outside good is $U_{i0} = 0$.
  - Monopoly: Outside option = outside good.
    - $WTP_i = V_i \rightarrow$ can be very heterogeneous
  - Duopoly: outside option to firm 1 is firm 2 and vice-versa
    - WTP at firm 1 is $V_i - (V_i - p_2) = p_2 \rightarrow$ same for all
    - No scope for price discrimination
- Competition can increase dispersion in WTP
  - Two types of customers, loyal & non-loyal
  - Monopoly: $WTP_{\text{loyal}} = WTP_{\text{non-loyal}} = V$
  - Duopoly: $WTP_{\text{loyal}} = V$ but $WTP_{\text{non-loyal}} = p_2$.
  - Entry leads to two levels of WTP rather than only one.
  - See Chen, Narasimhan, and Zhang (2001)

# Competition may increase or decrease the returns to paying the fixed cost of introducing an additional menu item

- Competition may decrease the returns to paying the fixed cost of introducing an additional menu item
  - If dispersion in WTP decreases, the value of charging different prices may fall.
  - Given fixed costs to offering different prices, it may make sense to offer fewer menu items.
- Competition may increase the returns to paying the fixed cost of introducing an additional menu item
  - Offering additional menu items may be a good way of stealing customers from competitors,
  - Menu lengths may be strategic complements
  - Menu length may increase with competitive pressure

# Selected Empirical Findings

- Competition lowers all prices, but lowers high prices more than low prices, thereby increasing menu concavity and quantity discounting.
  - Documented in yellow pages advertising (Busse and Rysman, 2005), cellular phone service (Seim and Viard, 2011), and elsewhere (see Borzekowski et al. (2009) for other citations).
  - Borzekowski et al. (2009) point out that increased curvature may make markups more similar, so does not necessarily correspond to "more" price discrimination.
- Competition leads to larger menus of
  - mailing list subsets (Borzekowski et al., 2009)
  - cellular calling plans (Seim and Viard, 2011)

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