ECON 8854: Costly Capacity & Price Dispersion (Dana 1999)

Michael D. Grubb

Boston College

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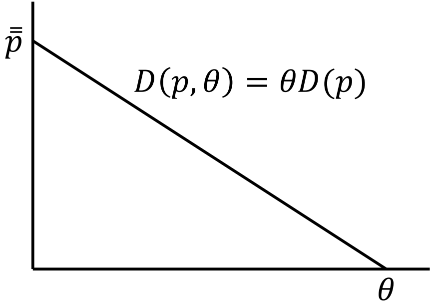
Equilibrium price dispersion under demand uncertainty: the roles of costly capacity and market structure

James D. Dana, Jr.*

Model—Demand

- Measure $\theta \sim F(\theta)$ consumers with $\theta \in [0, \bar{\theta}]$, each have unit demand with value $v \sim^{iid} G(v)$.
- Let $D(p) = 1 - G(p)$ be the fraction with value above p.
- Demand at a single price p is

$$D(p, \theta) = \theta D(p).$$



Note that $D(\bar{\bar{p}}) = 0$.

Model—Supply

- Marginal capacity cost λ and marginal production cost c.
- Before learning θ, firms choose a quantity to sell at each price.
  - Like choosing a pdf that integrates to total capacity.
  - Each firm chooses $q^i(p)$ which is positive on $[\underline{p}, \bar{p}]$, and integrates to $Q^i(p) = \int_{\underline{p}}^{p} q^i(x)dx$. Total capacity is $K^i = Q^i(\bar{p})$.
- Firm choices aggregate to $q(p) = \sum_i q^i(p)$, and $Q(p) = \sum_i Q^i(p)$.
- Proportional rationing.
- Note: Were θ a known constant, its value would not affect price.
  - *functional form assumption* $D(p, \theta) = \theta D(p)$ and constant MC.
  - Variation in θ would not be enough to create price dispersion.
  - Instead, price dispersion will follow costly capacity $\lambda > 0$ and uncertainty about θ at the time prices and capacity are chosen.

Residual Demand

Residual demand at price p given a price distribution $q(p)$ is:

$$RD(p,\theta) = D(p,\theta) - \int_{\underline{p}}^{p} \frac{D(p,\theta)}{D(x,\theta)} q(x)\, dx.$$

This is extremely elegant!

Residual Demand—Intuition Part 1

Imagine sales at one price $\tilde{p} < p$. Then

$$RD(p,\theta) = D(p,\theta) - \frac{D(p,\theta)}{D(\tilde{p},\theta)} q(\tilde{p})$$

- Demand would have been $D(p,\theta)$ had no sales taken place at price $\tilde{p}$.
- But $q(\tilde{p})$ units were purchased at the lower price, and hence there are $q(\tilde{p})$ fewer consumers in the market place.
- Residual demand is not simply $D(p,\theta) - q(\tilde{p})$
  - some of those who bought at price $\tilde{p}$ had values $v \in [\tilde{p}, p)$ so were already excluded from $D(p,\theta)$.
- How many sales at price $\tilde{p}$ went to individuals with $v \geq p$?
  - Proportional sales: Individuals with $v \geq p$ made up fraction $\frac{D(p,\theta)}{D(\tilde{p},\theta)}$ of those trying to buy $\rightarrow$ also made up fraction $\frac{D(p,\theta)}{D(\tilde{p},\theta)}$ of the sales.
- Thus we subtract $\frac{D(p,\theta)}{D(\tilde{p},\theta)} q(\tilde{p})$ from demand.

Residual Demand—Intuition Part 2

Imagine sales two lower prices $\tilde{p}_1 < \tilde{p}_2 < p$:

- RD at price p given sales at all prices $\tilde{p}_1$ and lower have taken place:

$$RD\left(p,\theta;\tilde{p}_1\right) = D\left(p,\theta\right) - \frac{D\left(p,\theta\right)}{D\left(\tilde{p}_1,\theta\right)} q\left(\tilde{p}_1\right).$$

- RD at price p given sales at all prices $\tilde{p}_2$ and lower have taken place?
- Want to subtract $q(\tilde{p}_2)$ units discounted by the fraction of those sales that went to consumers with values $v \geq p$. Similar logic, fraction $=$

$$\frac{RD\left(p,\theta;\tilde{p}_1\right)}{RD\left(\tilde{p}_2,\theta;\tilde{p}_1\right)} = \frac{D\left(p,\theta\right)\left(1-\frac{1}{D(\tilde{p}_1,\theta)}q\left(\tilde{p}_1\right)\right)}{D\left(\tilde{p}_2,\theta\right)\left(1-\frac{1}{D(\tilde{p}_1,\theta)}q\left(\tilde{p}_1\right)\right)} = \frac{D\left(p,\theta\right)}{D\left(\tilde{p}_2,\theta\right)}.$$

Simplification happily saves us from a recursive formula!

Residual Demand

$$\text{Discrete case: } RD(p_i,\theta) = D(p_i,\theta) - \sum_{j<i} \frac{D(p_i,\theta)}{D(p_j,\theta)} q(p_j)$$

$$\text{Continuous Case: } RD(p,\theta) = D(p,\theta) - \int_{\underline{p}}^{p} \frac{D(p,\theta)}{D(x,\theta)} q(x)\, dx.$$

Definitions:

- $\rho(\theta) =$ max sales price given θ: $RD(\rho(\theta),\theta) = 0$
- $\theta(p) =$ demand state at which $p =$ max sales price: $RD(p,\theta(p)) = 0$

$$\text{Re-writing: } RD(p,\theta) = \theta D(p) \left(1 - \int_{\underline{p}}^{p} \frac{q(x)}{\theta D(x)} dx\right)$$

$$\text{We have: } \theta(p) = \int_{\underline{p}}^{p} \frac{q(x)}{D(x)} dx$$

Perfect Competition[1]

Definitions:

- $q^*(p) =$ equilibrium market price distribution
- $y^*(p) =$ equilibrium probability that items with price p are sold

$$y(p) = 1 - F(\theta(p)) = 1 - F\left(\int_{\underline{p}}^{p} \frac{q(x)}{D(x)} dx\right)$$

Competitive equilibrium condition—zero profit on each unit of capacity:

$$(p - c)\, y(p) - \lambda = 0$$

or solving for p:

$$p = c + \frac{\lambda}{y(p)} = c + \frac{\lambda}{1 - F(\theta(p))} = c + \frac{\lambda}{1 - F\left(\int_{\underline{p}}^{p} \frac{q(x)}{D(x)} dx\right)}$$

$\rightarrow$ Price of each unit of capacity related to probability of sale.

[1] "First described by Prescott (1975)... developed more formally by Eden (1990)"

Perfect Competition—Equilibrium Price Distribution

- Zero-profit condition: $(p-c)\,y\,(p)-\lambda=0$
- Rearranging terms: $1-y(p)=1-\frac{\lambda}{p-c}$
- Substituting $y(p)=1-F(\theta(p))$ and applying F^{-1}:

$$\int_{\underline{p}}^{p}\frac{q\,(x)}{D\,(x)}dx=F^{-1}\left(1-\frac{\lambda}{p-c}\right)$$

- Differentiating w.r.t p on both sides:

$$\frac{q\,(p)}{D\,(p)}=\frac{1}{f\left(F^{-1}\left(1-\frac{\lambda}{p-c}\right)\right)}\frac{d}{dp}\left(1-\frac{\lambda}{p-c}\right)=\frac{\lambda\,(p-c)^{-2}}{f\left(F^{-1}\left(1-\frac{\lambda}{p-c}\right)\right)}$$

and therefore: $q\,(p)=D\,(p)\dfrac{\lambda\,(p-c)^{-2}}{f\left(F^{-1}\left(1-\frac{\lambda}{p-c}\right)\right)}$

Perfect Competition—Equilibrium Price Support $\left[c + \lambda, \bar{p}\right]$

Minimum price solves $y\left(\underline{p}\right) = 1$.

- As $y\left(p\right) = 1 - F\left(\int_{\underline{p}}^{p} \frac{q(x)}{D(x)} dx\right)$

$$y\left(\underline{p}\right) = 1 - F\left(\int_{\underline{p}}^{\underline{p}} \frac{q\left(x\right)}{D\left(x\right)} dx\right) = 1 - F\left(0\right) = 1$$

- As $p = c + \frac{\lambda}{y(p)}$, $y(\underline{p}) = 1$ yields

$$\underline{p} = c + \lambda$$

Perfect Competition—Equilibrium Price Support $\left[c+\lambda, \overline{p}\right]$

The maximum price is $\overline{p}$ (the price at which $D\left(\overline{p}\right)=0$)

- Why? If max price were $p^{\max} < \overline{p}$, could charge $p^{\max}+\varepsilon$ & profit.
- Sales little lower, still strictly positive in same states of θ:

$$RD\left(p^{\max}+\varepsilon,\theta\right)=\theta D\left(p^{\max}+\varepsilon\right)\left(1-\int_{\underline{p}}^{p^{\max}}\frac{q\left(x\right)}{D\left(x,\theta\right)}dx\right)$$
$$=\frac{D\left(p^{\max}+\varepsilon\right)}{D\left(p^{\max}\right)}RD\left(p^{\max},\theta\right)$$

  Essentially same probability of sale at higher price.
- Only when $p^{\max}=\overline{p}$, does this argument not apply.
- Note, this relies on $y\left(p^{\max}\right)>0$, which must be true for $p^{\max}$ to be finite and equal $c+\lambda/y\left(p^{\max}\right)$.[2]

[2] In fact, $y\left(\overline{p}\right)=\frac{\lambda}{\overline{p}-c}\in(0,1)$. True if $\overline{p}>c+\lambda$, as required for market operation.

Monopoly Profit Function

If a monopolist chooses $q(p)$, then profits are:

$$\pi(q) = \int_{\underline{\theta}}^{\bar{\theta}} \left(\int_{\underline{p}}^{\rho(\theta)} (p-c)\, q(p)\, dp - \lambda \int_{\underline{p}}^{\bar{p}} q(p)\, dp \right) f(\theta)\, d\theta$$

Integrating by parts:

$$\pi(q) = \int_{\underline{p}}^{\bar{p}} \left(\left(1 - F(\theta(p))\right)(p-c) - \lambda \right) q(p)\, dp$$

Monopoly FOC

Equation (7):

$$\left(1 - F\left(\theta\left(p\right)\right)\right)\left(p - c\right) - \lambda - \int_{\theta(p)}^{\theta(\bar{p})} \left(\rho\left(\theta\right) - c\right) \frac{D\left(\rho\left(\theta\right)\right)}{D\left(p\right)} f\left(\theta\right) d\theta = 0$$

Equivalent to:

$$p = c + \frac{\lambda}{1 - F\left(\theta\left(p\right)\right)} + E\left[\left(\rho\left(\theta\right) - c\right) \frac{D\left(\rho\left(\theta\right)\right)}{D\left(p\right)} \middle| \theta \geq \theta\left(p\right)\right]$$

Monopoly FOC—Intuition

- Standard Monopoly FOC:

$$MR = MC \leftrightarrow P + P'Q = MC \leftrightarrow P = MC - P'Q$$

- Dana's (1999) Monopoly FOC:

$$p = c + \frac{\lambda}{1 - F(\theta(p))} + E\left[(\rho(\theta) - c) \frac{D(\rho(\theta))}{D(p)} \middle| \theta \geq \theta(p) \right]$$

Like standard formula with different MC and MR

- Effective MC is $c + \lambda / (1 - F(\theta(p)))$
  - Inflates cost of capacity by $1/\Pr(sale)$ rather than deflating margin $(p - c)$ by $\Pr(sale)$.
- MR is $p - E\left[(\rho(\theta) - c) \frac{D(\rho(\theta))}{D(p)} \middle| \theta \geq \theta(p) \right]$
  - In state $\theta > \theta(p)$, the max sales price is $\rho(\theta) > p$. Selling one more unit at price p, means $D(\rho(\theta)) / D(p)$ units are sold to folks with value $\rho(\theta)$, so that many fewer units sold at price $\rho(\theta)$.
  - $E\left[(\rho(\theta) - c) \frac{D(\rho(\theta))}{D(p)} \middle| \theta \geq \theta(p) \right]$ is expected loss given $\theta \geq \theta(p)$.

Derivation $y(p)$ (eq 8) and $q(p)$ (eq 9)

To derive (8) and (9), recall $y(p) = 1 - F\left(\theta\left(p\right)\right)$, multiply both sides by $D\left(p\right)$ and write FOC as

$$y\left(p\right)\left(p-c\right)D\left(p\right) = \lambda D\left(p\right) + \int_{\theta(p)}^{\theta(\bar{p})}\left(\rho\left(\theta\right)-c\right)D\left(\rho\left(\theta\right)\right)f\left(\theta\right)d\theta$$

Also, $y'\left(p\right) = -f\left(\theta\left(p\right)\right)\theta'\left(p\right)$, so we can do a change of variables andwrite FOC as

$$y\left(p\right)\left(p-c\right)D\left(p\right) = \lambda D\left(p\right) + \int_{p}^{\bar{p}}\left(x-c\right)D\left(x\right)f\left(\theta\left(x\right)\right)\theta'\left(x\right)dx$$

$$y\left(p\right)\left(p-c\right)D\left(p\right) = \lambda D\left(p\right) - \int_{p}^{\bar{p}}\left(x-c\right)D\left(x\right)y'\left(x\right)dx$$

Now, differentiate both sides wrt p

$$\begin{aligned} y'\left(p\right)D\left(p\right)\left(p-c\right) + y\left(p\right)\left(D'\left(p\right)\left(p-c\right)+D\left(p\right)\right) \\ = \lambda D'\left(p\right) + \left(p-c\right)D\left(p\right)y'\left(p\right) \end{aligned}$$

Derivation $y(p)$ (eq 8) and $q(p)$ (eq 9) continued

y' terms cancel:

$$y(p)\left(D'(p)(p-c)+D(p)\right)=\lambda D'(p)$$

Solve for $y(p)$ to get eq(8):

$$y(p)=\frac{\lambda D'(p)}{D'(p)(p-c)+D(p)}$$

Now, recall that $\theta(p)=\int_{\underline{p}}^{p}\frac{q(x)}{D(x)}dx$, so $\theta'(p)=\frac{q(p)}{D(p)}$ and $q(p)=D(p)\theta'(p)$. Moreover, from $y'(p)=-f(\theta(p))\theta'(p)$ we have $\theta'(p)=-y'(p)/f(\theta(p))$. Therefore:

$$q(p)=D(p)\theta'(p)=-D(p)\frac{y'(p)}{f(\theta(p))}$$

Substituting in $\theta(p)=F^{-1}(1-y(p))$ we get eq(9).

Monopoly Markup Equation Revisited

Now we can re-write eq (8):

$$D'(p)(p-c)+D(p)=\frac{\lambda D'(p)}{y(p)}$$

$$p=c+\frac{\lambda}{y(p)}-\frac{D(p)}{D'(p)}=c+\frac{\lambda}{y(p)}-\frac{p}{\varepsilon_d}$$

- Same as in the standard monopoly problem
  - except MC adjusted for cost of capacity, inflated by $1/\Pr(sale)$.
- $\lambda=0$: unique price charged = standard monopoly price
- $\lambda>0$: $y(p)$ is determined so that this holds at all p in $[\underline{p},\bar{p}]$.

Homework Hint

See Dana (1999) Section 3 "A two-demand-state example" for help with the homework. Under Dana's (1999) assumptions

- P_L and Q_L Maximize profits in the low state
- P_H and Q_H Maximize expected "residual" profits from the high state

Oligopoly

- Dana (1999) extends results to oligopoly, for λ sufficiently large
- Monopoly and competition results are limiting cases
- Timing: Oligopoly model assumes firms choose prices and capacity at the same time given a capacity cost λ.
  - The results are the same as if there were a **binding** capacity constraint that led to shadow cost of capacity λ.
- This is not the same as a sequential game where capacity is chosen at cost λ before price.
  - That game may be more natural, and would correspond more closely to Cournot competition.
  - Model in Dana (1999) closer to Bertrand.

Price Dispersion and Market Structure

Dana (1999): Price dispersion increases with competition

- Prop 5: support of prices widens with competition
- Prop 6: variance of price higher with perfect competition than monopoly given linear demand.

Price Dispersion and Market Structure—Proposition 5

Monopoly Equilibrium Price Support: $\left[c + \lambda - \frac{D(\underline{p})}{D'(\underline{p})}, \overline{\overline{p}}\right]$

- By same logic as competitive case
  - Maximum price is $\overline{p} = \overline{\overline{p}}$
  - Minimum price set at $y\left(\underline{p}\right) = 1$
  - So $\underline{p} = c + \lambda - \frac{D(\underline{p})}{D'(\underline{p})}$
- Prop 5: Narrower support of prices:
  - top of support is the same
  - bottom is shifted up due to monopoly markup
- This result is misleading.
  - Support of prices is not a good measure of dispersion.
    A narrow support can be consistent with a higher variance.
  - If the firm must offer a finite number k prices, then maximum prices can vary with market structure.
  - See homework.

Price Dispersion and Market Structure—Proposition 6

Prop 6: variance of price higher with perfect competition than monopoly given linear demand.

- Intuition:
  - Constant MC + linear demand $\rightarrow$ $\text{PTR}_{\text{monopoly}} = \frac{1}{2}$.
  - Constant MC (perfectly elastic supply) $\rightarrow$ $\text{PTR}_{\text{competitive}} = 1$.
  - Constant MC + linear demand $\rightarrow$ competition increases the PTR
  - Increase in PTR $\rightarrow$ changes in $\lambda/y\,(p)$ get passed through more $\rightarrow$ more price dispersion.
- General result: price dispersion increases with PTR.
  - May or many not coincide with increasing competition.
- Glen Weyl has pointed out that in IO we often inadvertently assume our conclusions by specifying a demand system with PTR<1. See homework for an example with $PTR > 1$.
- Bottom line: Price dispersion likely increases with PTR, which often (but not always) increases with competition.

Uncertain Demand but Costless Capacity

- Suppose $\lambda = 0$
  - Capacity costs are sunk at the time prices are chosen
  - Always excess capacity (it is all used with probability 0)
- Dana (1999) predicts no price dispersion for monopoly or perfect competition.
- For monopoly, result relies on the assumption $D(\theta, p) = \theta D(p)$.
  - Relaxing this assumption can lead to price dispersion under monopoly but not perfect competition.
  - See homework.

References I

Dana, James D., J. (1999). Equilibrium Price Dispersion under Demand Uncertainty: The Roles of Costly Capacity and Market Structure. *RAND Journal of Economics 30*(4), 632–660.

Eden, B. (1990). Marginal Cost Pricing When Spot Markets Are Complete. *Journal of Political Economy 98*(6), 1293–1306. doi:10.1086/261735.

Prescott, E. C. (1975). Efficiency of the Natural Rate. *Journal of Political Economy 83*(6), 1229–1236. doi:10.1086/260391.