

## Questions

1. In class, we derived optimal nonlinear pricing by a monopolist. Consider a similar setting in which the buyer has monopsony power and the seller has private information. Let the buyer's valuation be  $V(q)$ , the seller's cost be  $C(q, \theta)$ , and the seller's type  $\theta$  have density  $f(\theta)$  on  $[\underline{\theta}, \bar{\theta}]$ . Assume  $V$  and  $C$  are continuously differentiable several times and that the seller's outside option is 0. Further, assume that (1)  $C_\theta > 0$ , (2)  $C_{q\theta} > 0$ , (3)  $\frac{d}{d\theta} [f(\theta)/F(\theta)] \leq 0$  (non-increasing reverse hazard rate), (4)  $V_{qq} \leq 0$ , and (5)  $C_{qq} \geq 0$ . Making an appropriate assumption on the signs of  $C_{qq\theta}$  and  $C_{q\theta\theta}$ , characterize the optimal monopsony purchase quantity  $q(\theta)$  and corresponding marginal price  $P'(q)$  offered by the monopsonist (your derivation should constitute a proof of optimality). How is quantity distorted from first best?
2. Suppose a car rental company has costs of renting a car for  $q$  miles of  $C(q) = (25 + 0.05q)$ . There are two customer segments:  $2/3$  are business travelers who have value  $V^B(q)$  and  $1/3$  are tourists who have value  $V^T(q)$ , where

$$\begin{aligned} V^B(q) &= \begin{cases} 30q - 1.5q^2 & 0 \leq q \leq 10 \\ 150 & 10 \leq q \end{cases} \\ V^T(q) &= \begin{cases} \frac{1}{2}q - \frac{1}{2} \frac{1}{1000}q^2 & 0 \leq q \leq 500 \\ 125 & 500 \leq q \end{cases} \end{aligned}$$

- (a) Assuming second-degree price discrimination, what is the optimal car rental contract to offer, i.e. the optimal price as a function of mileage  $P(q)$ ? How is consumption distorted away from first-best?
  - (b) Does this coincide with the analysis in class? Why or why not? [**Hint:** if you get stuck on (a), please still attempt (b).]
  - (c) How would your answer to (a) differ if  $V^B(q) = 30q - 1.5q^2$  and  $V^T(q) = \frac{1}{2}q - \frac{1}{2} \frac{1}{1000}q^2$ ?
3. Crawford and Shum (2007) and Attanasio and Pastorino (2020) both assume consumer preferences satisfy the single crossing property, as in the standard theory. Is this a reasonable assumption in the cable TV market? Is this a reasonable assumption in rural markets for food staples? Is it a reasonable assumption for any market?

In the Attanasio and Pastorino (2020) setting you may wish to think about how family size (number of children) might affect preferences recalling that Attanasio and Pastorino (2020) state that “poorer households tend to have more children”.

# Bibliography

- [1] Attanasio, Orazio, and Elena Pastorino. 2020. "Nonlinear Pricing in Village Economies." *Econometrica* 88 (1):207-263. doi: 10.3982/ECTA13918.
- [2] Crawford, Gregory S., and Matthew Shum. 2007. "Monopoly Quality Degradation and Regulation in Cable Television." *The Journal of Law and Economics* 50 (1):181-219. doi: 10.1086/508310.