



ECON 8873: EMPIRICAL METHODS IN MACROECONOMICS
SYLLABUS

Basic Information

- **Schedule:** MW 10:30PM - 11:45AM, Room: Maloney Hall 330
- **Contact Information:** Pablo A. Guerron-Quintana, Office: Maloney 325, Email: guerron@bc.edu
- **Office Hours:** by appointment only.

Course Description

In this course, we will learn numerical methods to solve and estimate nonlinear dynamic general equilibrium models (DSGE). Although the target audience is macro/international economics students, the class will be taught in a very general format so students from other fields may find beneficial to take the course. The material is mostly based on lecture notes jointly developed with Jesus Fernandez-Villaverde (U. Penn). They will be available before each class.

In the first part of the course, the student will be introduced to tools in software engineering and numerical analysis. The second part of the course is devoted to global methods to solve DSGE models. Some of these methods may be familiar to you because of economic examples in other Ph.D. courses. We will go into the details of why these methods work and how to apply them to a variety of situations.

In the final part, you will be introduced to techniques to estimate models displaying nonlinear dynamics. Most of this part will concentrate on the particle filter and estimation using likelihood-based methods.

Because of its nature, this course is highly applied. It means that you will have to spend a lot of time implementing algorithms and learning new software. This is error/trial so keep doing it! By this point, you should be proficient in Matlab (and Julia and Python and R). If you don't know a lower level language such as C/C++, it is a great moment to start doing so. Two excellent references are (Pitt-Francis and Whiteley, 2012) and (Lippman, Lajoie, and Moo, 2013). Fortran is a good option but is falling behind. Ron Gallant and Grey Gordon have excellent online material for C/C++ and Fortran, respectively. The site <https://www.cplusplus.com/> offers excellent tutorials for C++. QuanEcon offers nice tutorials for Julia and Python. Sergei Maliar and Lilia Maliar have also code for different tasks and languages.

Grade Composition

The final grade will be based on homework assignments (30% of the grade), very proactive class participation including a presentation (20%) and a final project (50%). Homework is individual but you may cooperate with one of your classmates. You need to acknowledge the cooperation on the first page of your submitted solution. The final project will involve replicating a recent paper in the literature. You need to look for options and discuss them with me.

Topics

Some Preliminaries

In this section, the student will be introduced to selected topics in software engineering and numerical methods needed in the second and third parts of the class. For the first and second parts, we will rely heavily on the lecture notes joint with Jesus Fernandez-Villaverde.

- Intro to high-performance computing.
- Basic software utilities: Operating systems, Shell, Editors, IDEs, Version control.
- Scientific computing languages: C++, Java, Matlab, Julia, CUDA, Thrust, Mathematica, Python (Aruoba and Fernandez-Villaverde, 2015), (Deng, Guerron-Quintana, and Tseng, 2021), (Guerron-Quintana, 2016), (Coleman, Lyon, Maliar, and Maliar, 2018).
- Numerical/automatic differentiation and integration (Judd, 1998), (Robert and Casella, 2005), (Judd, Maliar, and Maliar, 2016), (Martin, Frazier, and Robert, 2021), (Neidinger, 2010), (Yoon, 2021), (Kochenderfer and Wheeler, 2019), (Dick, Kuo, and Sloan, 2013) <https://alexey.radul.name/ideas/2013/introduction-to-automatic-differentiation/>.
- Optimization: derivative, non-derivative, simulation based (Judd, 1998), (Bartholomew-Biggs, 2008), (Arnoud, Guvenen, and Kleineberg, 2019), (Kochenderfer and Wheeler, 2019).
- Parallel programming: OpenMP, MPI, GPU-CUDA/Thrust, OpenACC, std++ (Kirk and Hwu, 2017).

Solving Nonlinear Models

The student will be exposed to methods to solve models that cannot be solved with traditional linearization methods. Examples of such cases are models with occasionally binding constraints (zero lower bound, non-negative investment, collateral constraints), default models, models with time varying risk. In addition, you will be exposed to a formal exposition of dynamic programming, which is a critical tool to work on models of search and matching or default.

- Introduction to Dynamic Programming — Contraction mapping theorem, envelope theorem, Benveniste-Scheikman theorem Chapters 3 - 5 in (Sargent and Ljungqvist, 2018), chapters 3 - 6 in (Stokey and Lucas, 1999), chapter 3 in (Powell, 2011) provides a tractable exposition, chapter 5 in (Aguilar and Amador, 2021).
- Value function iteration (Judd, 1998), (Sargent and Ljungqvist, 2018), (Heer and Maussner, 2009).
- Alternatives/extensions to value function iteration (Arellano, Maliar, Maliar, and Tsyrennikov, 2016), (Gordon and Qiu), (Cao, Luo, and Nie, 2022), chapter 12 in (Judd, 1998), chapter 4 in (Heer and Maussner, 2009).
- Heterogeneous agent models – OLG (Gordon, 2018), (Krusell and Smith, 1998), (Heer and Maussner, 2009), (Rios-Rull, 1997), (Nakajima, 2007), (Maliar, Maliar, and Winant, 2019).
- Perturbation methods, pruning, generalized impulse responses (Schmidt-Grohe and Uribe, 2016), (Judd, 1998), (Koop, Pesaran, and Potter, 1996), (Andreasen, Fernandez-Villaverde, and Rubio-Ramirez, 2016), (Fernandez-Villaverde and Guerron-Quintana, 2020), (de Groot, Durdu, and Mendoza, 2020), (Bianchi, Kung, and Tirsikh, 2022), (Dew-Becker, 2012), (Goncalves, Herrera, Kilian, and Pesavento, 2021), (Coeurdacier, Rey, and Winant, 2011), (de Groot, 2013), (Lopez, Lopez-Salido, and Vazquez-Grande, 2022).
- Projection Methods (Judd, 1998), (Heer and Maussner, 2009), (Fernandez-Villaverde, Rubio-Ramirez, and Schorfheide, 2016), (Maliar and Maliar, 2015), (Levintal, 2018), (Fernandez-Villaverde, Gordon, Guerron-Quintana, and Rubio-Ramirez, 2015a), (Kruger and Kubler, 2004), (Cai and Guerron-Quintana, 2023).

Estimation of Nonlinear Models

Now that you know how to solve nonlinear models, we turn the issue of how we take these models to the data.

- Refresher Kalman filtering.
- Extended and Unscented Kalman filtering (Särkkä and Svensson, 2023).
- Particle filtering (Herbst and Schorfheide, 2016), (Doucet, Freitas, and Gordon, 2001), (Lindsten, Jordan, and Schon, 2014), (Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez, 2015c), (Fernandez-Villaverde and Guerron-Quintana, 2021), (Chopin and Papaspiliopoulos, 2020), (Tulsyan, Gopaluni, and Khare, 2016).
- Other filtering approaches: Partial information filtering; Simulated method of moments and particle filtering (Drautzburg, Fernandez-Villaverde, and Guerron-Quintana, 2021), (Gordon and Guerron-Quintana, 2021).

- Metropolis Hasting meets Particle Filter (P-MCMC) (Herbst and Schorfheide, 2016), (Fernandez-Villaverde et al., 2015c), (Fernandez-Villaverde et al., 2016), (Fernandez-Villaverde and Rubio-Ramirez, 2007), (Robert and Casella, 2005), (Chopin and Papaspiliopoulos, 2020).¹
- Impulse-Response Matching (Christiano, Eichenbaum, and Trabandt, 2016), (Christiano, Trabandt, and Walentin, 2010), (Guerron-Quintana, Inoue, and Kilian, 2017), (Forneron and Ng, 2018).
- Recent advances: Variational inference, Hamiltonian Monte Carlo, Approximate Bayesian Computation, Bayesian Synthetic Likelihood, Machine learning in macro (Fernandez-Villaverde and Guerron-Quintana, 2021), (Koop and Korobilis, 2020), (Smidl and Quinn, 2006), (Forneron and Ng, 2018), (Maliar et al., 2019), (Maliar, Maliar, and Winant, 2021), (Fernandez-Villaverde, 2021), (Martin et al., 2021), (Goodfellow, Bengio, and Courville, 2017), (Childers, Fernandez-Villaverde, Perla, Rackauckas, and Wu, 2022), (Kahou, Fernandez-Villaverde, Perla, and Sood, 2021), (Kase, Melosi, and Rottner, 2022), (Azinovic, Gaegauf, and Scheidegger, 2022), (Chen, Didisheim, and Scheidegger, 2023), (Friedl, Kubler, Scheidegger, and Usui, 2023).
- Topics on VARs: Bayesian Estimation and Gibbs Sampling (Canova, 2007), (Kilian and Lutkepohl, 2017), (Chan, Koop, Poirier, and Tobias, 2019) Identification (Antolin-Diaz and Rubio-Ramirez, 2018), (Ludvigson, Ma, and Ng, Forthcoming), (Miranda-Agrippino and Ricco, Forthcoming), Heterogeneity (Chang, Chen, and Schorfheide, 2021), Nonlinear (Guerron-Quintana, Khazanov, and Zhong, 2021), (Aruoba, Mlikota, Schorfheide, and Villalvazo, 2022).²
- Applications: stochastic volatility — interest rate and fiscal uncertainty, time-varying parameters — good luck or good policy, smooth transition AR — productivity in Detroit, zero lower bound (Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez, 2015b), (Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe, 2011), (Herbst and Schorfheide, 2016), (Fernandez-Villaverde et al., 2015c), (Fernandez-Villaverde et al., 2015a), (van Dijk, Terasvirta, and Franses, 2002)

Potential papers for presentations. Consult with me before choosing one:

- (Saijo, 2020)
- (Childers, 2018)
- (Mlikota and Schorfheide, 2022)
- (Kahou et al., 2021)
- (Childers et al., 2022)

¹Excellent resource website on SMC and particle filters: https://www.stats.ox.ac.uk/~doucet/smc_resources.html

²Gary Koop has lots of Bayesian code for VARs: <https://sites.google.com/site/garykoop/home/computer-code-2>.

- (Azinovic et al., 2022)
- (Eftekhari and Scheidegger, 2022)
- (Kase et al., 2022)
- (Maliar et al., 2021)
- (Chang and Schorfheide, 2021)
- (Fernandez-Villaverde, Hurtado, and Nuno, 2020)
- (Lui and Plagborg-Moller, 2020)
- (Aruoba, Cuba-Borda, Higa-Flores, Schorfheide, and Villalvazo, 2021)
- (Bilbiie, Primiceri, and Tambalotti, 2023)

References

- M. Aguiar and M. Amador. *The Economics of Sovereign Debt and Default*. CREI Lectures in Macroeconomics. Princeton University Press, 2021.
- M. Andreasen, J. Fernandez-Villaverde, and J. Rubio-Ramirez. The pruned state-space system for non-linear models: Theory and empirical applications. Working paper, Emory University, 2016.
- J. Antolin-Diaz and J. Rubio-Ramirez. Narrative sign restrictions. *American Economic Review*, 108:2802–29, 2018.
- C. Arellano, L. Maliar, S. Maliar, and V. Tsyrennikov. Envelope condition method with an application to default risk models. *Journal of Economics Dynamics and Control*, 69: 436–459, 2016.
- A. Arnoud, G. Guvenen, and T. Kleineberg. Benchmarking global optimizer. Working paper, 2019.
- B. Aruoba and J. Fernandez-Villaverde. A comparison of programming languages in economics. *Journal of Economic Dynamics and Control*, 58:265–273, 2015.
- B. Aruoba, P. Cuba-Borda, K. Higa-Flores, F. Schorfheide, and S. Villalvazo. Piecewise-linear approximations and filtering for dsge models with occasionally-binding constraints. *Review of Economic Dynamics*, 41:96–120, 2021. ISSN 1094-2025. doi: <https://doi.org/10.1016/j.red.2020.12.003>. Special Issue in Memory of Alejandro Justiniano.
- B. Aruoba, M. Mlikota, F. Schorfheide, and S. Villalvazo. Vars with occasionally-binding constraints. *Journal of Econometrics*, 2022.
- M. Azinovic, L. Gaegauf, and S. Scheidegger. Deep equilibrium nets. Unpublished Manuscript, 2022.
- M. Bartholomew-Biggs. *Nonlinear Optimization with engineering applications*. Springer Optimization and its Applications. Springer, 2008.
- F. Bianchi, H. Kung, and M. Tirsikh. The origins and effects of macroeconomic uncertainty. Working paper, 2022.
- F. Bilbiie, G. Primiceri, and A. Tambalotti. Inequality and business cycles. Working paper, 2023.
- Y. Cai and P. Guerron-Quintana. Lopsided interest rates in international borrowing markets. Unpublished Manuscript, 2023.
- F. Canova. *Macroeconomics for Applied Macroeconomic Research*. Princeton University Press, 2007.
- D. Cao, W. Luo, and G. Nie. Global dsge models. Working paper, 2022.

- J. Chan, G. Koop, D. Poirier, and J. Tobias. *Bayesian Econometric Methods*. Econometric Exercises. Cambridge University Press, 2019.
- M. Chang and F. Schorfheide. Earnings heterogeneity and monetary policy shocks. Working paper, 2021.
- M. Chang, X. Chen, and F. Schorfheide. Heterogeneity and aggregate fluctuations. Working paper, 2021.
- H. Chen, A. Didisheim, and S. Scheidegger. Deep surrogates for finance: With an application to option pricing. Unpublished Manuscript, 2023.
- D. Childers. Solution of rational expectations models with function valued states. Working paper, 2018.
- D. Childers, J. Fernandez-Villaverde, J. Perla, C. Rackauckas, and P. Wu. Differentiable state-space models and hamiltonian monte carlo estimation. Working paper, 2022.
- N. Chopin and O. Papaspiliopoulos. *An Introduction to Sequential Monte Carlo*. Springer, 1 edition, 2020.
- L. J. Christiano, M. Trabandt, and K. Walentin. DSGE Models for Monetary Policy Analysis. In B. M. Friedman and M. Woodford, editors, *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics*, chapter 7, pages 285–367. Elsevier, 2010.
- L. J. Christiano, M. S. Eichenbaum, and M. Trabandt. Unemployment and business cycles. *Econometrica*, 84(4):1523–1569, 2016.
- N. Coeurdacier, H. Rey, and P. Winant. The risky steady state. *American Economic Review*, 101(3):398–401, May 2011.
- C. Coleman, S. Lyon, L. Maliar, and S. Maliar. Matlab, python, julia: What to choose in economics? *Computational Economics*, forthcoming, 2018.
- O. de Groot. Computing the risky steady state of dsge models. *Economics Letters*, 120(3): 566–569, 2013.
- O. de Groot, B. Durdu, and E. Mendoza. Approximately right?: Global v. local methods for open-economy models. Working paper, 2020.
- M. Deng, P. Guerron-Quintana, and L. Tseng. Parallel computation of sovereign default models. *Working Paper Boston College*, 2021.
- I. Dew-Becker. Essentially affine approximations for economic models. Working paper, 2012.
- J. Dick, F. Kuo, and I. Sloan. High-dimensional integration:. *Acta Numerica*, pages 133–288, 2013.
- A. Doucet, N. Freitas, and N. Gordon. Sequential monte carlo methods in practice. Springer, 2001.

- T. Drautzburg, J. Fernandez-Villaverde, and P. Guerron-Quintana. Filtering with limited information. Unpublished manuscript, 2021.
- A. Eftekhari and S. Scheidegger. High-dimensional dynamic stochastic model representation. *SIAM J. Sci. Computation*, 44, 2022.
- J. Fernandez-Villaverde. Deep learning for macroeconomists. Lecture Notes, 2021.
- J. Fernandez-Villaverde and P. Guerron-Quintana. Uncertainty shocks and business cycle research. *Review of Economic Dynamics*, 37:S118–S146, 2020.
- J. Fernandez-Villaverde and P. Guerron-Quintana. Estimating dsge models: Recent advances and future challenges. forthcoming, 2021.
- J. Fernandez-Villaverde and J. Rubio-Ramirez. Estimating macroeconomic models: A likelihood approach. *Review of Economic Studies*, 74:1059–1087, 2007.
- J. Fernandez-Villaverde, P. Guerron-Quintana, J. Rubio-Ramirez, and M. Uribe. Risk matters: The real effects of volatility shocks. *American Economic Review*, 101(6):2530–2561, 2011.
- J. Fernandez-Villaverde, G. Gordon, P. Guerron-Quintana, and J. Rubio-Ramirez. Nonlinear adventures at the zero lower bound. *Journal of Economic Dynamics and Control*, 57:182–204, 2015a.
- J. Fernandez-Villaverde, P. Guerron-Quintana, K. Kuester, and J. Rubio-Ramirez. Fiscal volatility shocks and economic activity. *American Economic Review*, 105(11):3352–84, 2015b.
- J. Fernandez-Villaverde, P. Guerron-Quintana, and J. Rubio-Ramirez. Estimating dynamic equilibrium models with stochastic volatility. *Journal of Econometrics*, 185:216–229, 2015c.
- J. Fernandez-Villaverde, J. Rubio-Ramirez, and F. Schorfheide. Solutions and estimation for dsge models. *Handbook of macroeconomics* vol. 2, 2016.
- J. Fernandez-Villaverde, S. Hurtado, and G. Nuno. Financial frictions and the wealth distribution. Working paper, 2020.
- J.-J. Forneron and S. Ng. The abc of simulation estimation with auxiliary statistics. *Journal of Econometrics*, 205(1):112–139, 2018.
- A. Friedl, F. Kubler, S. Scheidegger, and T. Usui. Deep uncertainty quantification: With an application to integrated assessment models. Forthcoming *Review of Economic Studies*, 2023.
- S. Goncalves, A. Herrera, L. Kilian, and E. Pesavento. Impulse response analysis in structural dynamic models with nonlinear regressors. *Journal of Econometrics*, forthcoming, 2021.
- I. Goodfellow, Y. Bengio, and A. Courville. *Deep Learning*. MIT Press, 1 edition, 2017.

- G. Gordon. Minicourse on heterogeneous agent models. Working Paper, 2018.
- G. Gordon and P. Guerron-Quintana. A quantitative theory of hard and soft sovereign defaults. Unpublished manuscript, 2021.
- G. Gordon and S. Qiu. A divide and conquer algorithm for exploiting policy function monotonicity. *Quantitative Economics*.
- P. Guerron-Quintana. Sovereign default on gpus. *Unpublished Manuscript, Boston College*, 2016.
- P. Guerron-Quintana, A. Inoue, and L. Kilian. Impulse response matching estimators for DSGE models. *Journal of Econometrics*, 196(1):144–155, 2017. doi: 10.1016/j.jeconom.2016.09.
- P. Guerron-Quintana, A. Khazanov, and M. Zhong. Analyzing financial and macroeconomics series with nonlinear factor models. Working paper, 2021.
- B. Heer and A. Maussner. Dynamic general equilibrium modeling. Springer, 2009.
- E. Herbst and F. Schorfheide. Stochastic volatility. Princeton university press, 2016.
- K. Judd. Numerical methods in economics. Mit press, 1998.
- K. Judd, L. Maliar, and S. Maliar. How to solve dynamic stochastic models computing expectations just once. *Quantitative Economics*, forthcoming:1–47, 2016.
- M. E. Kahou, J. Fernandez-Villaverde, J. Perla, and A. Sood. Exploiting symmetry in high-dimensional dynamic programming. Working paper, 2021.
- H. Kase, L. Melosi, and M. Rottner. Estimating nonlinear heterogeneous agents models with neural networks. Working paper, 2022.
- L. Kilian and H. Lutkepohl. *Structural Vector Autoregressive Analysis*. Themes in Modern Econometrics. Cambridge University Press, 2017.
- D. Kirk and W. Hwu. *Programming Massively Parallel Processors*. Parallel Programming. Morgan Kaufmann, 2017.
- M. Kochenderfer and T. Wheeler. *Algorithms for Optimization*. Parallel Programming. MIT Press, 2019.
- G. Koop and D. Korobilis. Bayesian dynamic variable selection in high dimensions. Working paper, 2020.
- G. Koop, H. Pesaran, and S. Potter. Impulse response analysis in non-linear models. *Journal of Econometrics*, 74(1):119–147, 1996.
- D. Kruger and F. Kubler. Computing equilibrium in olg models with stochastic production. *Journal of Economic Dynamics and Control*, 28:1411–1436, 2004.

- P. Krusell and A. Smith. Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5), 1998.
- O. Levintal. Taylor projection: A new solution method for dsge models. *International Economic Review*, 59:1345–1373, 2018.
- F. Lindsten, M. Jordan, and T. Schon. Particle gibbs with ancestor sampling. Working paper, 2014.
- S. Lippman, J. Lajoie, and B. Moo. *C++ Primer*. Addison-Wesley, 5 edition, 2013.
- P. Lopez, D. Lopez-Salido, and F. Vazquez-Grande. Accounting for risk in a linearized solution. Working paper, 2022.
- S. Ludvigson, S. Ma, and S. Ng. Uncertainty and business cycles: Impulse of endogenous response. *American Economic Journal: Macro*, Forthcoming.
- L. Lui and M. Plagborg-Moller. Full-information estimation of heterogenous agent models using macro and micro data. Working paper, 2020.
- L. Maliar and S. Maliar. Merging simulation and projection approaches to solve high-dimensional problems with an application to a new keynesian model. *Quantitative Economics*, 6:1–47, 2015.
- L. Maliar, S. Maliar, and P. Winant. Will artificial intelligence replace computational economists any time soon? Working paper, 2019.
- L. Maliar, S. Maliar, and P. Winant. Deep learning for solving dynamic economic models. Forthcoming, 2021.
- G. Martin, D. Frazier, and C. Robert. Approximating bayes in the 21st century. Working paper 24/21, 2021.
- S. Miranda-Agrippino and G. Ricco. The transmission of monetary policy shocks. *American Economic Journal: Macro*, Forthcoming.
- M. Mlikota and F. Schorfheide. Sequential monte carlo with model tempering. Working paper, 2022.
- M. Nakajima. Note on type distribution of heterogenous agents. Working Paper, 2007.
- R. Neidinger. Introduction to automatic differentiation and matlab object-oriented programming. *SIAM Review*, 52(3):545–563, 2010.
- J. Pitt-Francis and J. Whiteley. *Guide to Scientific Computing in C++*. Undergraduate Topics in Computer Science. Springer, 2012.
- W. Powell. *Approximate Dynamic Programming: Solving the Curses of Dimensionality*. Wiley, 2 edition, 2011.

- J.-V. Rios-Rull. Computation of equilibria in heterogeneous agent models. Working Paper, 1997.
- C. Robert and G. Casella. Monte carlo statistical methods. Springer, 2005.
- H. Saijo. Uncertainty shocks in networks. Working paper, 2020.
- T. Sargent and L. Ljungqvist. Recursive macroeconomic theory. Fourth edition 2018, MIT Press, 2018.
- S. Särkkä and L. Svensson. *Bayesian filtering and smoothing*. Cambridge University Press, 2023.
- S. Schmidt-Grohe and M. Uribe. Open economy macroeconomics. Textbook, Columbia University, 2016.
- V. Smidl and A. Quinn. *The Variational Bayes Method in Signal Processing*. Springer, 1 edition, 2006.
- N. Stokey and R. Lucas. Recursive methods in economic dynamics. Harvard university press, 1999.
- A. Tulsyan, R. B. Gopaluni, and S. Khare. Particle filtering without tears: A primer for beginners. *Computers and Chemical Engineering*, 95:130–145, 2016.
- D. van Dijk, T. Terasvirta., and P. Franses. Smooth Transition Autoregressive Models — A Survey of Recent Developments. *Econometric Reviews*, 21(1):1–47, 2002.
- B. Yoon. A machine learning approach for efficient multi-dimensional integration. *Scientific Reports*, 11(1):18965, 2021. URL <https://doi.org/10.1038/s41598-021-98392-z>.