Field Examination in Econometrics
Jan 2016 PART A

1. Suppose we have a linear regression model in matrix form $Y = Xb + e$ where the Gauss-Markov assumptions hold, but the errors e are NOT normally distributed. Let $\widehat{b}$ be the ordinary least squares estimator of b, and let $\widetilde{b}$ be the correctly specified maximum likelihood estimator of b. Assume all of the regularity conditions required for standard maximum likelihood estimation are satisfied. For each of the following statements, say if it is definitely true, possibly true, or definitely false. Explain your answers (formal proofs are not needed).

a. $E(\widehat{b}) \neq E(\widetilde{b})$
b. $\widehat{b}$ is BLUE.
c. $\widehat{b}$ is at least as efficient as $\widetilde{b}$.
d. $\widehat{b}$ has an asymptotically smaller variance than $\widetilde{b}$.
e. $\widehat{b}$ is has the same or smaller mean squared error than $\widetilde{b}$.
f. Both $\widehat{b}$ and $\widetilde{b}$ are asymptotically normally distributed.

2. Assume iid observations of the vector Z_i, and assume moment conditions $E\left[G\left(Z_i, \theta_0\right)\right] = 0$ for a known vector valued function G, and an unknown parameter vector θThe formula for GMM estimation of the parameter vector θ is given by

$$\widehat{\theta} = \arg\min \left(\frac{1}{n}\sum_{i=1}^{n} G(Z_i, \theta)\right)' W \left(\frac{1}{n}\sum_{i=1}^{n} G(Z_i, \theta)\right)$$

Assume all the standard conditions for asymptotic root n normality of GMM hold. Let θ_0 denote the true value of θ. Different choices of the weighting matrix W will yield different $\widehat{\theta}$ estimators. Consider four possible choices for W:

$\widehat{\theta}_1$ is the estimate where W is the identity matrix.

$\widehat{\theta}_2$ is the estimate where $W = \left[\frac{1}{n}\sum_{i=1}^{n} G(Z_i, \widehat{\theta}_1) G(Z_i, \widehat{\theta}_1)'\right]^{-1}$

$\widehat{\theta}_3$ is the estimate where $W = \left(\frac{1}{n}\sum_{i=1}^{n} \widetilde{G}_i \widetilde{G}_i'\right)^{-1}$ where $\widetilde{G}_i = G(Z_i, \widehat{\theta}_1) - \frac{1}{n}\sum_{i=1}^{n} G(Z_i, \widehat{\theta}_1)$

$\widehat{\theta}_4$ is the estimate where $W = \left[var\left(G(Z, \theta_0)\right)\right]^{-1}$

a. Asymptotically, is $\widehat{\theta}_2$ more efficient, less efficient, or equally efficient compared to $\widehat{\theta}_1$? Explain your answer.

b. Answer the same question as a. for $\widehat{\theta}_3$ vs $\widehat{\theta}_2$, and for $\widehat{\theta}_4$ vs $\widehat{\theta}_3$.

d. What are the asymptotic distributions of $\widehat{\theta}_1$, $\widehat{\theta}_2$, $\widehat{\theta}_3$ and $\widehat{\theta}_4$?

3. Suppose we have n iid observations of X_i, Y_i. The variable Y_i equals one with probability p and equals zero with probability $1-p$. The variable X_i equals one with probability $2p$ and equals zero with probability $1-2p$.

a. If X_i and Y_i are independent random variables, what is the maximum likelihood estimator for p?

b. If X_i and Y_i are not independent random variables, what would be a good (relatively low variance) consistent estimator for p?