Time Series Econometrics, May, 2019

Question 1. **(i)** Given a MA(2) process

$$X_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}, \text{ where } \varepsilon_t \sim WN(0, \sigma^2),$$

and information upto time n, find the optimal linear forecast of X_{n+h}, $h > 0$, forecast errors, and forecast error variance. (ii) Given a MA(∞) process

$$X_t = \sum_{i=0}^{\infty} \theta_i \varepsilon_{t-i}, \text{ where } \theta_0 = 1, \; \varepsilon_t \sim WN(0, \sigma^2),$$

and information upto time n, find the optimal linear forecast of X_{n+h}, $h > 0$, forecast errors, and forecast error variance..

Question 2. Consider a linear Regression with correlated errors:

$$y_t = \beta' x_t + u_t,$$

where (i) $\{x_t\}$ and $\{u_t\}$ are independent with each other (ii) (x_t, u_t) is α-mixing with size $-r/r-1$ ($r > 1$), (iii) $\sup_{it} E|x_{it}u_t|^{r+\delta} < \infty$, (iv) $\sup_{it} Ex_{it}^{2(r+\delta)} < \infty$, $M = \frac{1}{n}\sum_1^n E(x_t x_t')$ is uniformly positive definite (i.e., $M_n \geq \varepsilon I > 0$ for all large n).

Prove that the OLS estimator for β is consistent and asymptotic normal.

Question 3. Consider the AR process

$$y_t = \alpha y_{t-1} + u_t, \; \alpha = 1$$

where u_t are stationary but weakly dependent process, say, a mixing process whose partial sums satisfies a functional central limiting theorem, and consider the OLS estimation of the AR coefficient

$$\widehat{\alpha} = \frac{\sum y_{t-1} y_t}{\sum y_{t-1}^2}$$

what's the limiting distribution of $\widehat{\alpha}$? (ii) How to construct an unit root test based on the OLS estimator?