Time Series Econometrics, Jan, 2016

Question 1. Given an AR(2) process $X_t + 0.2X_{t-1} - 0.48X_{t-2} = \varepsilon_t$, determine whether it is causal.

Question 2. If $\{X_t\}$ is a covariance stationary process with autocovariance function $\gamma(\cdot)$ and if $\sum |c_j| < \infty$, then, for each t, the series $Y_t = \sum_j c_j X_{t-j}$ converges absolutely with probability one and in mean square to the same limit. In addition, the process $\{Y_t\}$ is covariance stationary with autocovariance function

$$\gamma_Y(h) = \sum_{j,k} c_j c_k \gamma(h - j + k).$$

In the special case that $\{X_t\}$ are IID with mean zero and variance σ^2,

$$\gamma_Y(h) = \sigma^2 \sum_{j=0}^{\infty} c_j c_{j+|h|}.$$

Question 3. Suppose $y = X\beta + u$ where $X' = [x_1, ..., x_n]$ and $U' = [u_1, ..., u_n]$. If (i) (x_t, u_t) stationary and ergodic, (ii) $E|x_{it}u_t| < \infty$, $E(x_{it}u_t) = 0$, (iii) $E(x_{it}^2) < \infty$, $M = E(x_t x_t')$ positive definite, prove that

$$\hat{\beta} = (X'X)^{-1}X'y \to \beta \text{ a.s.}$$

Question 4. How to test Cointegration ? What is the Residual-Based Test for cointegration? What is the Johansen's test for cointegration?