BOSTON COLLEGE
Department of Economics

Econometrics Field Comprehensive Exam

May 22, 2015

YOU MUST CHOOSE AND COMPLETE 2 PARTS OF THIS EXAM

Part I – ECON8821 – Time Series (Prof. Xiao)

Part II – ECON8822 – Cross Section & Panel Econometrics (Prof. Hoderlein)

Part III– ECON8823 - Applied Econometrics (Prof. Baum)

Please use a separate bluebook for each part.

Please write your alias, part and question number(s) on each bluebook.

Please read the entire exam before writing anything.

PART I

Time Series Econometrics (Comp. 2015)

Department of Economics
Boston College

1. Given an AR(2) process $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + u_t$, (i) solve for α_1 and α_2 as a function of the autocorrelation coefficients, and (ii) calculate the autocorrelation function $\rho(k)$ in terms of α_1 and α_2.

2. Given a MA(∞) process

$$X_t = \sum_{i=0}^{\infty} \theta_i \varepsilon_{t-i}, \text{ where } \theta_0 = 1,\ \varepsilon_t \sim WN(0, \sigma^2),$$

and information upto time n, find the optimal linear forecast of X_{n+h}, $h > 0$, forecast errors, and forecast error variance.

3. Consider a linear regression with correlated errors

$$y_t = \beta' x_t + u_t,$$

Assuming that (x_t, u_t) are weakly dependent time series satisfies LLN and CLT. Derive the limiting distribution of the OLS estimator of β.

4. Consider a unit root process:

$$y_t = \alpha y_{t-1} + u_t,\ \alpha = 1$$

where u_t are stationary AR(1) process

$$u_t = \rho u_{t-1} + \varepsilon_t,\ |\rho| < 1$$

and ε_t are iid$(0, \sigma^2)$. Consider the OLS estimation of α

$$\widehat{\alpha} = \frac{\sum y_{t-1} y_t}{\sum y_{t-1}^2}$$

(1) What's the limiting distribution of $\widehat{\alpha}$? (2) What would happen if $|\alpha| < 1$?

PART II

EC 822

Field Exam
May–22–2015

Stefan Hoderlein

Instructions. Please follow the hints and suggestions given. In particular, observe the credits assigned to each questions. **The time is 90 minutes, and there are 90 credits to be earned.** You should try to answer all questions, but do not worry if you can not complete the exam in time. If you cannot answer a question, move one, but make sure that you catch all questions/problems. Answer short, and as formal as you possibly can. The top grade will be given with significantly less than 90 credits. This is a closed book exam, and no calculator is required or admitted.

1. Kernel Estimation (60 points)

a. What is a standard Kernel, as used in nonparametric estimation? Give a formal definition, and explain all elements in detail.

b. Let X be a scalar continuous random variable. Given the kernel that you defined in section a, how would you estimate the *pdf* $f_X(x)$ at a point $X = x$. Define a nonparametric estimator formally, including all its components.

c. Based on your answers, how would you estimate the mean regression of a random variable Y on a d-random vector X denoted $m(x)$. Hint: use the fact that in the population

$$m(x) = E[Y|X = x] = \int y f_{Y|X}(y; x) dy,$$

and derive a sample counterparts estimator.

d. Derive formally the bias of an univariate standard Kernel density estimator $\hat{f}_X$ under the condition that the true density f_X is three times continuously differentiable with bounded third derivatives, and the data are *iid*. You may argue informally when it comes to third order terms. Discuss the structure of the bias term, including all elements.

e. The variance conditional on the data of an d-dimensional local polynomial estimator under homoskedasticity is given by

$$Var\left(\widehat{m}_{Y|X}(x)|\mathbb{X}\right) = \frac{1}{nh^d}\kappa_2\sigma_U^2 + o_p((nh^d)^{-1}),$$

where $\widehat{m}_{Y|X}(x)$ denotes the LLS estimator of $m_{Y|X}(x)$, h is the bandwidth, $\mathbb{X} = \sigma(X_1, .., X_n)$, i.e., the sigma algebra spanned by the $X_j, j = 1, .., n$, σ_U^2 is the residual variance, and κ_2 is a Kernel constant. Explain the curse of dimensionality. If this variance depends on $X_1, .., X_n$, why is it still an useful quantity?

f. The bias conditional on the data of the same local polynomial estimator is given by

$$Bias\left(\widehat{m}_{Y|X}(x)|\mathbb{X}\right) = \frac{h^2}{2}\mu_2 \sum_k \partial_k^2 m_{Y|X}(x) + o_p(h^2)$$

where μ_2 is the second moment of the kernel, and ∂_k^2 denotes the k-th partial second derivative. Explain the bias-variance trade-off, and how an optimal choice of bandwidth h resolves it.

2. Treatment Effects (30 points)

1. Suppose there is a binary treatment D, and an outcome Y. Formulate Y in "counterfactual" notation involving Y_0 and Y_1.

2. In addition, assume that there are covariates X. Explain how the conditional independence assumption, $D \perp (Y_0, Y_1)|X$ reflect selection on observables.

3. In this setup, derive how the average treatment effect is identified. Hint: derive CATE first.

4. Consider the (conditional) local average treatment effects (LATE) when there is a binary treatment D, but also a binary instrument Z, and covarariates X, such that Z is independent of all unobservables in the system, conditional on $X = x$. Define this quantity, and explain how it can be identified.

5. Why is this quantity instrument dependent, and what are associated disadvantages?

6. Propose an estimator, and explain why this proposal is sensible.

PART III

ECON8823: Applied Econometrics Comprehensive, Spring 2015

Answer all questions. They are equally weighted.

1. Write an essay discussing the diagnostic tests you would use to establish the validity of instrumental variables estimates computed via 2SLS and IV-GMM techniques.

2. Write an essay discussing the multivariate ARCH model, describing its structure and how it might be applied in empirical research.

3. Write an essay discussing the Tobit and Heckman models, indicating where you would employ them and how they differ in their underlying assumptions.