Econometric Theory Comprehensive Exam
May 2013

Let X and $W \equiv (W_1, W_2)$ be random vectors with supports $\mathbb{R}^d$ and $\mathbb{R}^\ell$ respectively and let U and Z be random variables with support $\mathbb{R}$ such that $X \perp U | W$. Suppose that Y is structurally generated by

$$Y = r(X, U),$$

with r a real-valued function. A sample $\{(Y_i, X_i, W_i, Z_i) : i = 1, ..., n\}$ of $i.i.d.$ observations is available. For any $x \in \mathbb{R}^d$, let $Y(x) \equiv r(x, U)$.

(a) How do you interpret $\Delta_1(x, x^*|w_1) \equiv E[Y(x^*) - Y(x)|W_1 = w_1]$? How do you interpret $R_1(x, x^*, w_1) \equiv E(Y|X = x^*, W_1 = w_1) - E(Y|X = x, W_1 = w_1)$? Does $R_1(x, x^*, w_1)$ identify $\Delta_1(x, x^*|w_1)$? Explain your reasoning.

(b) How do you interpret $\Delta_X(x, x^*|x) \equiv E[Y(x^*) - Y(x)|X = x]$? Is $\Delta_X(x, x^*|x)$ identified? If yes, provide an expression identifying $\Delta_X(x, x^*|x)$. If not, explain your reasoning.

Consider the parametric model $g(X; \theta)$ for $m(X) \equiv E(Y|X)$, where $\theta \in \Theta$, a compact subset of $\mathbb{R}^p$, is a vector of unknown parameters and $g(x; \cdot)$ is a known function that is twice continuously differentiable on $int(\Theta)$ for all $x \in \mathbb{R}^d$. Suppose that there exists a unique $\theta^* \in \Theta$ such that $g(X; \theta^*) = m(X)$ and assume that $\theta^* \in int(\Theta)$.

(c) Which minimization problem does the nonlinear least squares estimator $\hat{\theta}$ for θ^* solve?

Suppose that

$$\sqrt{n}(\hat{\theta} - \theta^*) \xrightarrow{d} N(0, \mathbf{A}^{*-1}).$$

(d) For $x \in \mathbb{R}^d$ and $\nabla_\theta g(x; \theta^*)$ finite and nonzero, derive the asymptotic distribution of the estimator $g(x; \hat{\theta})$ for $m(x)$.

Consider the nonparametric estimator $\hat{m}(x)$ for $m(x)$. Under regularity conditions, with $h = h_n \to 0$ and $nh^d \to \infty$ as $n \to \infty$ where h denotes a bandwidth, the approximate bias and variance of $\hat{m}(x)$ are given by

$$Bias(\hat{m}(x)) \approx h^2 B(x) \quad \text{and} \quad Var(\hat{m}(x)) \approx \frac{1}{nh^d} V(x),$$

where $B(x)$ and $V(x)$ are finite and do not depend on n.

(e) Does $\hat{m}(x) \xrightarrow{p} m(x)$? Demonstrate your answer.

Suppose that with $nh^d \to \infty$ and $nh^{d+4} \to M \in [0, \infty)$ as $n \to \infty$,

$$\sqrt{nh^d}(\hat{m}(x) - m(x) - h^2 B(x)) \xrightarrow{d} N(0, V(x)).$$

(f) For a constant c, discuss the consequences of the bandwidth choices $h^* = cn^{\frac{-1}{2+d}}$, $h_o = cn^{\frac{-1}{4+d}}$, and $h^\dagger = cn^{\frac{-1}{8+d}}$ on this asymptotic normality result.

(g) Is $g(x; \hat{\theta})$ a consistent estimator for $E[Y(x)]$? Is $\hat{m}(x)$ a consistent estimator for $E[Y(x)]$? Explain your reasoning.

Let β_o be a vector of constant coefficients and let δ_o and γ_o be nonzero constant scalars. Let $E(XX')$ be nonsingular and V be a random variable such that $E(XV) = 0$. Suppose that

$$r(X, U) = X'\beta_o + U\delta_o \quad \text{and} \quad Z = U\gamma_o + V.$$

(h) Can you provide a restriction on δ_o and γ_o such that $\beta^* \equiv [E(XX')]^{-1} E[X(Y - Z)] = \beta_o$? Explain your reasoning.

Let $\epsilon \equiv Y - Z - X'\beta^*$ and $\hat{\beta} \equiv (\frac{1}{n}\sum_{i=1}^n X_i X_i')^{-1}[\frac{1}{n}\sum_{i=1}^n X_i(Y_i - Z_i)]$ and suppose that $E(|X_{ji}|^2) < \infty$ and $E(|X_{ji}\epsilon_i|) < \infty$ for $j = 1, ..., d$.

(i) Does $\hat{\beta} \xrightarrow{p} \beta^*$? If yes, verify the conditions of any law of large numbers you use. If not, explain your reasoning.