The 2021 Econometrics comprehensive exam will work like this:

* The exam will be administered online, through zoom (not canvas)

* Zoom login ID is 714364847 Password: 026664
https://bccte.zoom.us/j/714364847?pwd=andveHJWOEhlaUZheERUakJ2YmM3UT09
(this is the ECON7772 meeting room)

* The exam is Friday, May 28, 9:00am to noon EDT (boston timezone)

* The goal is to roughly replicate an in class exam.

* You are expected to work alone. I will be checking answers for evidence of collusion.

* You are expected to log in, and stay logged in, with your video on, throughout the exam (I will understand if you run into connection issues - having it on most the time is good enough. Please keep trying to reconnect if we become disconnected).

* At the start time, 9AM, I will email the exam to you.

* At the end of the exam you'll email your completed exam to me in any form you can: (e.g. scans or photos if handwritten, pdf if written on a tablet, whatever).

* Important: After you complete the exam, please wait until I acknowledge I've received it and can read it before exiting the zoom meeting.

* Everyone must complete part A of the exam
(based on Econ7772 - Econometric methods - Lewbel)

* Everyone must complete any 2 of the 3 remaining parts, corresponding to
Econ8821 - Time Series - Xiao,
Econ8823 - Applied Econometrics - Baum, and
Econ8825 - Topics in Econometric Theory - Khan

Field Examination in Econometrics
June 2021 PART A ECON7772

1. Consider the model $Y_i = a + (X_i + e_i)^2 b$ where a and b are unknown constants to be estimated, with $1 \leq a \leq 10$ and $1 \leq b \leq 10$. The data are n independently, identically distributed observations of the bounded random vector (Y_i, X_i). The unobserved errors e_i are mean zero and independent of X_j for all observations i and j. We do NOT know the distribution of e_i. Consider the estimator

$$\left(\widehat{a}, \widehat{b}\right) = \arg\min \sum_{i=1}^{n} \left(Y_i - a - X_i^2 b\right)^2$$

a. Is $\widehat{a}$ consistent? Is $\widehat{b}$ consistent? Explain your answers (you don't need to prove it).

b. If $\widehat{b}$ is inconsistent, then what would be a consistent estimator for b?
If $\widehat{b}$ is consistent, then how would you calculate its standard error?

2 . Suppose we have an iid sample of observations of the bounded, two element random vector (Y, X). Assume $Y = a + X + U$ and $X = bY + V$ where U and V are unobserved mean zero random variables.
a. What is the reduced form for this set of equations?
b. Are the constants a and b identified? Explain your answer.

3. Suppose we observe n iid observations of a random variable Z_i. let F denote the distribution function of Z, and let $E(Z_i) = \mu$ where μ is finite, but the variance of Z_i is *infinite*. Let $\overline{Z} = \sum_{i=1}^{n} Z_i / n$ and let $\widehat{F}(z) = \sum_{i=1}^{n} I(Z_i \leq z)/n$.

Tell if each of the following statements is definitely true, possibly true, or definitely false, and explain why (stating a relevant theorem or theorems is sufficient for explaining why).
a. $\overline{Z}$ is an unbiased estimator of μ.
b. $\overline{Z}$ is a consistent estimator of μ.
c. $\widehat{F}(z)$ is an unbiased estimator of $F(z)$ for any finite z.
d. $\widehat{F}(z)$ is a consistent estimator of $F(z)$ for any finite z.

Time Series Econometrics, May, 2021

Question 1. Let $\{X_t\}$ be a covariance stationary process with autocovariance function $\gamma(\cdot)$ and if $\sum |c_j| < \infty$. Show that, for each t, the series $Y_t = \sum_j c_j X_{t-j}$ converges absolutely with probability one and in mean square. In addition, the process $\{Y_t\}$ is covariance stationary with autocovariance function

$$\gamma_Y(h) = \sum_{j,k} c_j c_k \gamma(h - j + k).$$

In the special case that $\{X_t\}$ are IID with mean zero and variance σ^2,

$$\gamma_Y(h) = \sigma^2 \sum_{j=0}^{\infty} c_j c_{j+|h|}.$$

Question 2. Consider a m-dimensional VAR process $\{Y_t\}$ of order p ($VAR(p)$):

$$Y_t = C + \sum_{i=1}^{p} A_i Y_{t-i} + \varepsilon_t \tag{1}$$

where

- C is a m-dimensional vector of constants (intercept)
- A_i are $m \times m$ coefficient-matrices of real numbers.

Suppose that ε_t are m-dimensional i.i.d. $N(0, \Omega)$,

(1) Conditional on the first p observations, write down the conditional log likelihood function for a sample of size n**:** $(Y_1, \cdots, Y_n)$.

(2) Denote

$$\begin{aligned} X_t &= \left[\begin{array}{ccccc} 1, & Y'_{t-1}, & Y'_{t-2}, & \cdots, & Y'_{t-p} \end{array} \right]' \\ &: \; (mp+1) \times 1 \text{ vector of regressors,} \end{aligned}$$

$$\begin{aligned} B' &= \left[\begin{array}{ccccc} C, & A_1, & A_2, & \cdots, & A_p \end{array} \right] \\ &: \; m \times (mp+1) \text{ matrix of coefficients,} \end{aligned}$$

then the m equations of the VAR(p) model can be re-written as

$$Y_t = B' X_t + \varepsilon_t.$$

If we denote the j-th row of B' as b'_j, then b'_j is an $1 \times (mp+1)$ vector which contains the parameters of the j-th equation of the VAR(p) model.

If the conditional MLE of B is $\widehat{B}$, In particular, the j-th row of $\widehat{B}'$ is $\widehat{b}'_j$, Show that $\widehat{b}'_j$ is simply the estimator from an OLS regression of Y_{jt} on X_t:

$$Y_{jt} = b'_j X_t + \varepsilon_{jt},\ j = 1, \cdots, m.$$

Question 3. Consider the Unit Root process

$$y_t = \alpha y_{t-1} + u_t,\ \alpha = 1$$

where u_t are stationary but weakly dependent process whose partial sum process satisfies a functional central limit theorem. Consider the OLS estimation of the AR coefficient

$$\widehat{\alpha} = \frac{\sum y_{t-1} y_t}{\sum y_{t-1}^2}$$

what's the limiting distribution of $\widehat{\alpha}$?

Question 4. Consider the following cointegrating regression

$$y_t = \beta x_t + u_t,\ \ x_t = x_{t-1} + v_t.$$

where v_t and u_t are stationary but weakly dependent process whose partial sum processes satisfy functional central limit theorems, and consider the OLS estimation of the β coefficient

$$\widehat{\beta} = \frac{\sum x_t y_t}{\sum x_t^2}$$

what's the limiting distribution of $\widehat{\beta}$?

ECON8823: Applied Econometrics Comprehensive, May 2021

Answer all questions. They are equally weighted.

1. Write an essay discussing the applicability of generalized linear models (GLMs) in economic and financial applied research, providing examples of their use.

2. Write an essay discussing the use of the correlated random effects estimator for panel data, and discuss its advantages over alternative methods.

3. Write an essay discussing the coarsened exact matching procedure and its advantages over alternative methods.

Econ 8825 May 28, 2021

Summer 2021

1. (30) Consider a variation of the standard *censored* selection model, where we get to observe both hours worked and wages of individuals. Hours worked are censored at 0 and we only observe wages for those who are working, and hence their hours worked is positive.

$$
\begin{aligned}
y_{1i} &= \max(w_i'\delta_0 + \eta_i, 0) \\
d_i &= I[y_{1i} > 0] \\
y_i &= d_i(x_i'\beta_0 + \epsilon_i)
\end{aligned}
$$

Observed variables: $(y_{1i}, d_i, w_i, y_i, x_i)$ and we assume $(\eta_i, \epsilon_i) \sim N(\mathbf{0}, \mathbf{\Sigma})$, where the (1,1) element of $\mathbf{\Sigma}$ is σ_1^2, the other diagonal element is σ_2^2, and the off diagonal terms are γ_0. We wish to estimate $(\delta_0, \beta_0, \gamma_0, \sigma_1^2, \sigma_2^2)$.

   (a) Write the likelihood function for this model.

   (b) Propose a NLLS estimator for the parameters in this model.

   (c) Propose a computationally friendly Heckman type two-step estimator where the first step involves a convex optimization procedure and the second step is closed form.

2. (30) Consider the censored regression model in the generic form:

$$
\begin{aligned}
y^* &= x'\beta + \sigma\epsilon \\
E[\epsilon] &= 0 \\
y &= \max(0, y^*) \\
\epsilon &\perp x \quad P(\epsilon \leq a) = F(a) \\
f(\epsilon) &= F'(\epsilon)
\end{aligned}
$$

   (a) Assume $F(\cdot), f(\cdot)$ are known to the econometrician- e.g. standard normal. Evaluate δ, the conditional marginal effect:

$$\delta = \partial E[y|x]/\partial x$$

(b) Evaluate the Average Marginal Effect:

$$\bar{\delta} = E_x[\delta]$$

(c) Propose an estimator for $\bar{\delta}$ as a function of an estimator for β_0. Establish the asymptotic properties of your estimator.

(d) Informally answer the above questions if $F(\cdot), f(\cdot)$ are unknown.