Time Series Comp. Exam

2013 May

1. Is a linear combination of two jointly covariance stationary processes still covariance stationary? Prove it if it is true. Give a counterexample if not true.

2. Given a stationary AR(1) process

$$X_t = \alpha X_{t-1} + \varepsilon_t, t = 1, ..., n,$$

with ε_t being IID N(0,σ^2).

(1) Find out the log-likelihood function for estimating α and σ^2.

(2) If ε_t are IID (0, σ^2), but actually not normally distributed, what's the limiting distribution of the estimator of α based on maximizing the Gaussian likelihood function?

3. Consider a time series model given by the following equation system where the observed time series Y_t is described by

$$Y_t = h(\beta, X_t, Z_t, u_t)$$

X_t are observed exogenous variables, Z_t is a vector of state variables (some of which may be observable and others of which are not) , and u_t is the error term. The dynamics of Z_t is given by

$$Z_t = g(\gamma, Z_{t-1}, w_t) \quad (t = 1, 2, ...)$$

where w_t is a noise vector. The combined error vector (w_t', u_t') is usually assumed to be iid$(0, V)$. Describe how to estimate the unknown parameters β and γ under the assumption that (w_t', u_t') is iid.N$(0, V)$