BOSTON COLLEGE
Department of Economics

Econometrics Field Comprehensive Exam

January 29, 2016

YOU MUST COMPLETE 3 PARTS OF THIS EXAM

Part A – ECON7771 – Econometric Methods – Prof. Lewbel

Part B – ECON8821 – Time Series – Prof. Xiao

Part C – ECON8822 – Cross Section & Panel Econometrics – Profs. Hoderlein and Spindler

Please use a separate bluebook for each part.

Please write your alias, part and question number(s) on each bluebook.

Please read the entire exam before writing anything.

Field Examination in Econometrics
Jan 2016 PART A

1. Suppose we have a linear regression model in matrix form $Y = Xb + e$ where the Gauss-Markov assumptions hold, but the errors e are NOT normally distributed. Let $\widehat{b}$ be the ordinary least squares estimator of b, and let $\widetilde{b}$ be the correctly specified maximum likelihood estimator of b. Assume all of the regularity conditions required for standard maximum likelihood estimation are satisfied. For each of the following statements, say if it is definitely true, possibly true, or definitely false. Explain your answers (formal proofs are not needed).

a. $E(\widehat{b}) \neq E(\widetilde{b})$
b. $\widehat{b}$ is BLUE.
c. $\widehat{b}$ is at least as efficient as $\widetilde{b}$.
d. $\widehat{b}$ has an asymptotically smaller variance than $\widetilde{b}$.
e. $\widehat{b}$ is has the same or smaller mean squared error than $\widetilde{b}$.
f. Both $\widehat{b}$ and $\widetilde{b}$ are asymptotically normally distributed.

2. Assume iid observations of the vector Z_i, and assume moment conditions $E\left[G\left(Z_i, \theta_0\right)\right] = 0$ for a known vector valued function G, and an unknown parameter vector θThe formula for GMM estimation of the parameter vector θ is given by

$$\widehat{\theta} = \arg\min \left(\frac{1}{n}\sum_{i=1}^{n} G(Z_i, \theta)\right)' W \left(\frac{1}{n}\sum_{i=1}^{n} G(Z_i, \theta)\right)$$

Assume all the standard conditions for asymptotic root n normality of GMM hold. Let θ_0 denote the true value of θ. Different choices of the weighting matrix W will yield different $\widehat{\theta}$ estimators. Consider four possible choices for W:

$\widehat{\theta}_1$ is the estimate where W is the identity matrix.

$\widehat{\theta}_2$ is the estimate where $W = \left[\frac{1}{n}\sum_{i=1}^{n} G(Z_i, \widehat{\theta}_1) G(Z_i, \widehat{\theta}_1)'\right]^{-1}$

$\widehat{\theta}_3$ is the estimate where $W = \left(\frac{1}{n}\sum_{i=1}^{n} \widetilde{G}_i \widetilde{G}_i'\right)^{-1}$ where $\widetilde{G}_i = G(Z_i, \widehat{\theta}_1) - \frac{1}{n}\sum_{i=1}^{n} G(Z_i, \widehat{\theta}_1)$

$\widehat{\theta}_4$ is the estimate where $W = \left[var\left(G(Z, \theta_0)\right)\right]^{-1}$

a. Asymptotically, is $\widehat{\theta}_2$ more efficient, less efficient, or equally efficient compared to $\widehat{\theta}_1$? Explain your answer.

b. Answer the same question as a. for $\widehat{\theta}_3$ vs $\widehat{\theta}_2$, and for $\widehat{\theta}_4$ vs $\widehat{\theta}_3$.

d. What are the asymptotic distributions of $\widehat{\theta}_1$, $\widehat{\theta}_2$, $\widehat{\theta}_3$ and $\widehat{\theta}_4$?

3. Suppose we have n iid observations of X_i, Y_i. The variable Y_i equals one with probability p and equals zero with probability $1-p$. The variable X_i equals one with probability $2p$ and equals zero with probability $1-2p$.

a. If X_i and Y_i are independent random variables, what is the maximum likelihood estimator for p?

b. If X_i and Y_i are not independent random variables, what would be a good (relatively low variance) consistent estimator for p?

PART B

Time Series Econometrics, Jan, 2016

Question 1. Given an AR(2) process $X_t + 0.2X_{t-1} - 0.48X_{t-2} = \varepsilon_t$, determine whether it is causal.

Question 2. If $\{X_t\}$ is a covariance stationary process with autocovariance function $\gamma(\cdot)$ and if $\sum |c_j| < \infty$, then, for each t, the series $Y_t = \sum_j c_j X_{t-j}$ converges absolutely with probability one and in mean square to the same limit. In addition, the process $\{Y_t\}$ is covariance stationary with autocovariance function

$$\gamma_Y(h) = \sum_{j,k} c_j c_k \gamma(h - j + k).$$

In the special case that $\{X_t\}$ are IID with mean zero and variance σ^2,

$$\gamma_Y(h) = \sigma^2 \sum_{j=0}^{\infty} c_j c_{j+|h|}.$$

Question 3. Suppose $y = X\beta + u$ where $X' = [x_1, ..., x_n]$ and $U' = [u_1, ..., u_n]$. If (i) (x_t, u_t) stationary and ergodic, (ii) $E|x_{it}u_t| < \infty$, $E(x_{it}u_t) = 0$, (iii) $E(x_{it}^2) < \infty$, $M = E(x_t x_t')$ positive definite, prove that

$$\hat{\beta} = (X'X)^{-1}X'y \to \beta \text{ a.s.}$$

Question 4. How to test Cointegration ? What is the Residual-Based Test for cointegration? What is the Johansen's test for cointegration?

EC 8822

Stefan Hoderlein and Martin Spindler

January, 29 2016

Instructions: this part is an one hour, closed book exam. Attempt to answer formally and avoid lengthy verbal parts. There are 40 credits in total, and the individual credits for the questions are displayed as guidance. All questions may be answered, but the best grade is usually awarded with less than full credit. Please use two separate books for questions 1-7 and 8-10.

1. Consider the density of a d-dim random vector X. Why is this an interesting object from an economic perspective? Give two examples of economic applications (4 credits)

2. What is an univariate Kernel? Define formally ("a Kernel is a function K such that....") (3 credits)

3. Let $f_X(x)$ denote the density of the vector X at a point x. Define the (product) kernel based estimator for $f_X(x)$, and explain its building principle. In particular, explain the functioning of a kernel. (4 credits)

4. based on your answers in 1.- 3.., how would you estimate the mean regression of a random variable Y on a d-random vector X denoted $m(x)$? Hint: use the fact that in the population

$$m(x) = E[Y|X = x] = \int y f_{Y|X}(y; x) dy,$$

and derive a sample counterparts estimator. (5 credits)

5. Can you run a nonparametric regression if Y is binary? If yes, why, if not, why not? (2 credits)

6. Does the choice of bandwidth differ fundamentally between mean regression and density estimation? In particular, is there any fundamental difference in the way the dimensionality of X affects the choice? (3 credits)

7. What are compliers? Why is it a disadvantage that LATE is only defined on the compliers? (4 credits)

8. What is problematic about inference after model selection? [Poetscher-Leeb-Critique] What is the basic intuition? Use a simple example for your explanations! (6 credits)

9. Is valid inference after model selection possible? Describe a method to do valid inference after model selection! (Hint: Double Selection method) (5 credits)

10. Are 8. and 9. contradictions? How can they be reconciled? (4 credits)