Econ 8825 May 28, 2021

Summer 2021

1. (30) Consider a variation of the standard *censored* selection model, where we get to observe both hours worked and wages of individuals. Hours worked are censored at 0 and we only observe wages for those who are working, and hence their hours worked is positive.

$$\begin{aligned} y_{1i} &= \max(w_i'\delta_0 + \eta_i, 0) \\ d_i &= I[y_{1i} > 0] \\ y_i &= d_i(x_i'\beta_0 + \epsilon_i) \end{aligned}$$

Observed variables: $(y_{1i}, d_i, w_i, y_i, x_i)$ and we assume $(\eta_i, \epsilon_i) \sim N(\mathbf{0}, \mathbf{\Sigma})$, where the (1,1) element of $\mathbf{\Sigma}$ is σ_1^2, the other diagonal element is σ_2^2, and the off diagonal terms are γ_0. We wish to estimate $(\delta_0, \beta_0, \gamma_0, \sigma_1^2, \sigma_2^2)$.

   (a) Write the likelihood function for this model.

   (b) Propose a NLLS estimator for the parameters in this model.

   (c) Propose a computationally friendly Heckman type two-step estimator where the first step involves a convex optimization procedure and the second step is closed form.

2. (30) Consider the censored regression model in the generic form:

$$\begin{aligned} y^* &= x'\beta + \sigma\epsilon \\ E[\epsilon] &= 0 \\ y &= \max(0, y^*) \\ \epsilon &\perp x \quad P(\epsilon \leq a) = F(a) \\ f(\epsilon) &= F'(\epsilon) \end{aligned}$$

   (a) Assume $F(\cdot), f(\cdot)$ are known to the econometrician- e.g. standard normal. Evaluate δ, the conditional marginal effect:

   $$\delta = \partial E[y|x]/\partial x$$

(b) Evaluate the Average Marginal Effect:

$$\bar{\delta} = E_x[\delta]$$

(c) Propose an estimator for $\bar{\delta}$ as a function of an estimator for β_0. Establish the asymptotic properties of your estimator.

(d) Informally answer the above questions if $F(\cdot), f(\cdot)$ are unknown.