Field Examination in Econometrics
December 2017 PART A

1. Suppose $Y_i = a + bX_i + e_i$ satisfies the Gauss-Markov assumptions, including in the limit as n goes to infinity. Assume e_i is normal, and every element of X_i has an absolute value less than 10. Consider the following estimator: For any sample size $n > 100$, discard all but the last 100 observations, and let $\widehat{b}$ be the ordinary least squares estimate of b using just these last 100 observations. Answer each of the following questions, and explain why your answer is correct:

a. Is $\widehat{b}$ unbiased?
b. Is $\widehat{b}$ most efficient among linear estimators?
c. Is $\widehat{b}$ consistent?
d. Is $\widehat{b}$ asymptotically normal?
e. Suppose that instead of the last 100 observations, we now used the last $n/4$ observations to estimate $\widehat{b}$. Which of your answers above would change and which would stay the same?

2. Consider the model $Y_i = a + (X_i Y_i + e_i) b$. The data are n independently, identically distributed observations of the random vector (Y_i, X_i) where $X_i > 0$. The unobserved errors e_i are mean zero and independent of X_j for all observations i and j. We do not know the distribution of e_i.

a. Describe any consistent estimator for a and b and explain why it is consistent (you do not need to provide its asymptotic distribution).

b. Describe the most asymptotically efficient estimator you can think of for a and b, and explain why your estimator is relatively efficient (you do not need to provide its asymptotic distribution).

3. Suppose Z_i for $i = 1, ..., n$ are i.i.d. random variables drawn from a distribution having some parameterized density function $f(Z_i \mid \theta)$. Let Ω_Z be the support of the distribution of Z_i, that is, Ω_Z is the set of values that each Z_i could take on. A technical assumption we made concerning maximum likelihood estimation of θ was that either Ω_Z does not depend on θ or that $f(z \mid \theta) = 0$ for all values of z on the boundary of Ω_Z. Suppose that this technical assumption does NOT hold, but all of the other assumptions we gave for maximum likelihood estimation do hold.

a. Show either that all the conditions needed for consistency of maximum likelihood will still hold, or show what condition or conditions we used to prove consistency might now no longer hold.

b. Suppose that, for a given problem you have, maximum likelihood is still consistent, despite the violation of this assumption. What step or steps in the derivation of the ML asymptotic distribution may no longer hold as a result of this violation?