BOSTON COLLEGE
Department of Economics

Econometrics Field Comprehensive Exam

December 14, 2017

ALL MUST COMPLETE PART A

PART A – ECON7772 –PROF. LEWBEL

AND THEN CHOOSE 2 PARTS OF THIS EXAM:

PART B – ECON8821 –PROF. XIAO

PART C – ECON8822 –PROF. HODERLEIN

PART D – ECON8823 –PROF. BAUM

PART E – ECON8825 –PROF. KHAN

Please use a separate bluebook for each part.

Please write your alias, part and question number(s) on each bluebook.

Please read the entire exam before writing anything.

Field Examination in Econometrics
December 2017 PART A ECON 7772 - LEWBEL

1. Suppose $Y_i = a + bX_i + e_i$ satisfies the Gauss-Markov assumptions, including in the limit as n goes to infinity. Assume e_i is normal, and every element of X_i has an absolute value less than 10. Consider the following estimator: For any sample size $n > 100$, discard all but the last 100 observations, and let $\widehat{b}$ be the ordinary least squares estimate of b using just these last 100 observations. Answer each of the following questions, and explain why your answer is correct:

a. Is $\widehat{b}$ unbiased?
b. Is $\widehat{b}$ most efficient among linear estimators?
c. Is $\widehat{b}$ consistent?
d. Is $\widehat{b}$ asymptotically normal?
e. Suppose that instead of the last 100 observations, we now used the last $n/4$ observations to estimate $\widehat{b}$. Which of your answers above would change and which would stay the same?

2. Consider the model $Y_i = a + (X_iY_i + e_i)\,b$. The data are n independently, identically distributed observations of the random vector (Y_i, X_i) where $X_i > 0$. The unobserved errors e_i are mean zero and independent of X_j for all observations i and j. We do not know the distribution of e_i.

a. Describe any consistent estimator for a and b and explain why it is consistent (you do not need to provide its asymptotic distribution).

b. Describe the most asymptotically efficient estimator you can think of for a and b, and explain why your estimator is relatively efficient (you do not need to provide its asymptotic distribution).

3. Suppose Z_i for $i = 1, ..., n$ are i.i.d. random variables drawn from a distribution having some parameterized density function $f(Z_i \mid \theta)$. Let Ω_Z be the support of the distribution of Z_i, that is, Ω_Z is the set of values that each Z_i could take on. A technical assumption we made concerning maximum likelihood estimation of θ was that either Ω_Z does not depend on θ or that $f(z \mid \theta) = 0$ for all values of z on the boundary of Ω_Z. Suppose that this technical assumption does NOT hold, but all of the other assumptions we gave for maximum likelihood estimation do hold.

a. Show either that all the conditions needed for consistency of maximum likelihood will still hold, or show what condition or conditions we used to prove consistency might now no longer hold.

b. Suppose that, for a given problem you have, maximum likelihood is still consistent, despite the violation of this assumption. What step or steps in the derivation of the ML asymptotic distribution may no longer hold as a result of this violation?

PART B ECON 8821 - XIAO

Time Series Econometrics, December, 2017

Question 1. Calculate the auto-covariance function and the autocorrelation functions for an AR(1) process:

$$X_t = \alpha X_{t-1} + \varepsilon_t, |\alpha| < 1.$$

Question 2. Let random variables or vectors X and $Z \in L_2 =$ space of square integrable variables, we are interested in forecasting X based on information in Z. Denote the σ-field generated from Z by $\mathcal{F}_Z$, we look for $\mathcal{F}_Z$-measurable function X^* such that

$$E\left(X - X^*\right)^2 = \min_{g(Z)} E\left(X - g(Z)\right)^2,$$

Show that X^* is the expectation of X conditional on Z :

$$X^* = E(X|Z).$$

Question 3. Consider OLS estimation of the autoregression

$$y_t = \alpha y_{t-1} + u_t;\ \{u_t\} \equiv \text{iid}(0, \sigma^2),\ |\alpha| < 1,$$

$$\hat{\alpha} = \sum y_t y_{t-1} / \sum y_{t-1}^2.$$

Is this OLS estimator $\hat{\alpha}$ the Best Linear Unbiased Estimator for α? Is this OLS estimator $\hat{\alpha}$ a consistent for α? Prove your results.

Question 4. Cointegrating Regression:

$$y_t = \beta x_t + u_t,\ x_t = x_{t-1} + v_t.$$

where v_t and u_t are stationary but weakly dependent process, say, a mixing process, satisfying a functional central limit theorem, and consider the OLS estimation of the β coefficient

$$\widehat{\beta} = \frac{\sum x_t y_t}{\sum x_t^2}$$

what's the limiting distribution of $\widehat{\beta}$?

PART C

EC 8822

Field Exam

14-December-2017

Stefan Hoderlein

Instructions. Please follow the hints and suggestions given. In particular, observe the credits assigned to each questions. **The time is 80 minutes, and there are 45 credits to be earned.** You should try to answer all questions, but do not worry if you can not complete the exam in time. If you cannot answer a question, move one. Answer short, and as formal as you possibly can. The top grade will be given with significantly less than 80 credits. This is a closed book exam, and no calculator is required or admitted.

1. Nonparametrics (14 credits)

   a. Let $f_X(x)$ denote the density of the K-vector X at a point x. Define the (product) kernel based estimator for $f_X(x)$, and explain its building principle. In particular, define a kernel formally and explain its' functioning (3 credits).

   b. Let $\hat{f}_X(x)$ denote this estimator. Sketch its bias formally in the case of $K = 1$, and f_X being two times continuously differentiable with Lipschitz continuous second derivatives. Emphasize: formally (4 credits).

   c. How would the variance of the estimator change in the general case with $K > 1$. Explain the so-called curse of dimensionality (3 credits).

   d. Given the results in b. and c., explain the bias variance trade-off, and how it helps in selecting a bandwidth (4 credits).

2. Semiparametric Modeling (6 credits)

a. Consider the semiparametric binary choice model, i.e.,

$$Y = 1\{X'\beta - U > 0\},$$

where X is a K-vector of observable random variables, β is a fixed K vector of coefficients, and U is a scalar unobservable. Assume that U is fully independent of X. Derive a way to identify β up to scale (i.e., give a formal sketch of your argument, 4 credits).

b. Why is nonparametrics useful in the binary choice model? Specifically, what disadvantage of maximum likelihood does it allow to avoid, and how? (2 credits).

3. Treatment Effects (14 credits)

a. Suppose there is a binary treatment D, and an outcome Y. Formulate Y in "counterfactual" notation involving Y_0 and Y_1. (1 credit)

b. In addition, assume that there are covariates X. Explain how the conditional independence assumption, $D \perp (Y_0, Y_1)|X$ reflects selection on observables. (2 credits)

c. In this setup, derive the conditional ATE (3 credits)

d. The matching approach to estimating treatment effects exploits a conditional independence condition, and also identifies the ATE. The same is true of the regression discontinuity approach. What are parallels and differences in the identified object, and how do they reflect the precise nature of the identifying assumption (4 credits).

e. Consider the local average treatment effects (LATE) when there is a binary treatment D and a binary instrument Z, such that Z is independent of all unobservables in the system. Define this quantity, and explain how it can be identified (just the identification result, no proof). Why is this quantity instrument dependent, and what are associated disadvantages? (4 credits)

4. Machine Learning (11 credits)

a. Define the Lasso and the Ridge estimator and state the optimization problem on which both estimators are based (4 credits).

b. What is the difference between the two approaches? Focus on variable selection, and explain how this feature relates to the difference in the optimization problem you described in 4.a. Hint: use a detailed graphical explanation (4 credits).

c. Is a regression tree a nonparametric regression estimator? Describe the basic principle involved in building a tree, and explain your answer (3 credits).

PART D - BAUM

ECON8823: Applied Econometrics Comprehensive, December 2017

Answer all questions. They are equally weighted.

1. Write an essay describing Generalized Linear Models, giving examples of where these models might be employed and their advantages.

2. Write an essay discussing the fundamental problem of causal inference from observational data, and describe several techniques that you could employ to derive consistent estimates of the effect of a policy.

3. Write an essay describing the relative merits of the 2SLS and IV-GMM estimators of an instrumental variables model, and the diagnostic tests you would apply after estimation to validate the model.

PART E - ECON 8825 - KHAN

Field Exam

65 mins, 65 points. Points in parentheses.

1. (40) Consider a random variable y_i that is censored by an independently distributed random variable c_i. The econometrician observes a random sample of the pair (v_i, c_i) where $v_i = \max(y_i, c_i)$. Note the econometrician always observes the censoring variable, but does not always observe the censored variable y_i.

   Propose an estimator for the c.d.f of the random variable y_i, at a point, say t. State explicit conditions for your estimator to be root-n consistent and asymptotically normal.

2. (25) Consider the following partially linear model:

$$y_i = x_i'\beta_0 + f(z_i) + \epsilon_i$$

   where y_i, x_i, z_i are observed, β_0 is an unknown vector, and $f(\cdot)$ is an unknown function. ϵ_i is an unobserved random variable, whose mean is 0 conditional on x_i, z_i.

   (a) Assume z_i is a dummy variable, taking the values 1 an 0. Propose an OLS estimator for β_0 and $f(\cdot)$, and informally discuss finite sample and asymptotic properties.

   (b) Now suppose z_i is continuously distributed, and one knows the functions $E[x_i|z_i]$ and $E[y_i|z_i]$. Propose an OLS estimator for β_0 and informally discuss its asymptotic properties.