Field Examination in Econometrics
May 2013 PART A

1. Suppose $Y = XB + e$ satisfies the Gauss-Markov assumptions, e is normal, and every element of X has an absolute value less than 5. Consider the following estimator: For any sample size $n > 20$, discard all but the last 20 observations, and let $\widehat{B}$ be the ordinary least squares estimate of B using just these last 20 observations. Answer each of the following questions, and explain why your answer is correct:

a. Is $\widehat{B}$ unbiased?

b. Is $\widehat{B}$ most efficient among linear estimators?

c. Is $\widehat{B}$ consistent?

d. Is $\widehat{B}$ asymptotically normal?

e. Suppose that instead of the last 20 observations, we now used the last $n/3$ observations to estimate $\widehat{B}$. Which of your answers above would change and which would stay the same?

2. Suppose we have n iid observations of X_i, Z_i. The variable X_i equals one with probability p and equals zero with probability $1-p$. The variable Z_i equals one with probability $2p$ and equals zero with probability $1-2p$. Assume that X_i and Z_i are not independent of each other, and we don't know how they are related. Describe the best estimator for p you can think of (best meaning consistent and smallest asymptotic variance).

3. Let X_i for $i = 1, ..., n$ be iid random variables with $E(X_i) = 3$ and finite variance σ^2. Let $\overline{X}$ equal the sample average of X_1,...,X_n. Let Z_i for $i = 1, ..., n$ be a sequence of random variables having $plim\left(n^{1/2} Z_n\right) = 2$. Let $Y_n = n\overline{X} Z_n$ and let $W_n = (Y_n - 6)^2$.

a. What is the asymptotic distribution of Y_n? Prove your answer.

b. What is the asymptotic distribution of W_n? Prove your answer.

4. Let (X_i, Y_i) be an iid random sample $i = 1, ..., n$. Assume X_i and Y_i are normal with $E(X_i) = E(Y_i) = \mu_i$ (note that the mean μ_i is different for each observation i), $Cov(X_i, Y_i) = 0$, and $var(X_i) = var(Y_i) = \sigma^2$. Note that this problem is nonstandard, because the parameters are $\mu_1, ..., \mu_n, \sigma^2$, so the number of parameters is $n+1$, which increases with the sample size.

a. What is the maximum likelihood estimate $\widehat{\mu}_i$ for each μ_i from $i = 1, ..., n$?

b. Is each $\widehat{\mu}_i$ unbiased?

c. Is each $\widehat{\mu}_i$ consistent?

d. What is the maximum likelihood estimate $\widehat{\sigma}^2$ for σ^2 (hint, you'll need to substitute $\widehat{\mu}_i$ into the score function)?

e. What does $plim\left(\widehat{\sigma}^2\right)$ equal?

f. Using your answer to part e., construct a consistent estimator for σ^2.