BOSTON COLLEGE
Department of Economics

Econometrics Field Comprehensive Exam

May 25, 2018

YOU MUST COMPLETE PART A

PART A – ECON7772 –PROF. LEWBEL

AND THEN CHOOSE 2 PARTS OF THIS EXAM:

PART B – ECON8821 –PROF. XIAO

PART C – ECON8822 –PROF. HODERLEIN

PART D – ECON 8825 –PROF. KHAN

Please use a separate bluebook for each part.

Please write your alias, part and question number(s) on each bluebook.

Please read the entire exam before writing anything.

Field Examination in Econometrics
May 2018 PART A

1. In the regression model $Y = XB + e$, assume the Gauss Markov assumptions hold. Let $\widehat{B} = CY$ be an estimator of B for some matrix C that is a function of X.

a. What are the minimum properties that C must satisfy to ensure $\widehat{B}$ is unbiased?

b. Assume the properties you gave in part a. hold. Give additional conditions on C that are sufficient to make $\widehat{B}$ consistent.

2. Suppose $Y_i = X_i Z_i b + e_i$ and $X_i = Z_i a + u_i$ where X_i, Z_i, e_i, and u_i are bounded, mean zero random scalars. Assume Z_i is independent of (e_i, u_i), that $E(e_i u_i) \neq 0$, and that a and b are finite, nonzero scalar constants. Suppose we have n iid observations of the vector (Y_i, X_i, Z_i) for $i = 1, ..., n$.

Give a consistent estimator (the best one you can think of) for b, and provide its asymptotic distribution (you don't need to derive the distribution, just say what it is).

3. Suppose we have n iid observations of a continuously distributed random scalar X_i for $i = 0, ..., n$. Let $f(x)$ be the probability density function of X_i, which can take on any real value. The function $f(x)$ is bounded, differentiable, and has a bounded derivative. Let $I(\cdot)$ be the indicator function that equals one if the argument $\cdot$ is true and zero otherwise. For some x in the support of X_i, consider the estimator

$$\widehat{\theta}_h = \frac{1}{n} \sum_{i=1}^{n} \frac{X_i}{x} K\left(\frac{X_i - x}{h}\right)$$

Where K is a mean zero, bounded, symmetric probability density function. What does $\lim_{h \to 0} E\left(\widehat{\theta}_h\right)$ equal?

Time Series Econometrics, May, 2018

Question 1. Suppose that $\{X_t\}$ is a covariance stationary process with autocovariance function $\gamma(\cdot)$ and $\sum |c_j| < \infty$, show that (i) the series $Y_t = \sum_j c_j X_{t-j}$ converges absolutely with probability one and in mean square to the same limit. (ii) In addition, the process $\{Y_t\}$ is covariance stationary with autocovariance function

$$\gamma_Y(h) = \sum_{j,k} c_j c_k \gamma(h - j + k).$$

(iii) What is the autocovariance function of $\{Y_t\}$ in the special case that $\{X_t\}$ are IID with mean zero and variance σ^2?

Question 2 Given a covariance stationary AR(p) process $X_t - \sum_{i=1}^{p} \alpha_i X_{t-i} = \varepsilon_t$, show that

$$\rho_X(h) - \sum_{i=1}^{p} \alpha_i \rho_X(h - i) = 0, \tag{1}$$

where $\rho_X(h)$ is the autocorrelation function of X_t.

Question 3 Consider an GARCH(1,1) model

$$\begin{aligned} u_t &= \sigma_t \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma_1 \sigma_{t-1}^2, \end{aligned}$$

(i) What is the condition for u_t to be stationary? (2) What is the definition of an IGARCH(1,1) ? (3) How to estimate this GARCH model?

Question 4 Consider the Trending regression

$$y_t = \beta x_t + u_t;$$

where

$$x_t = (1, t, \cdots, t^p)',$$

and consider the OLS estimation of the β coefficient

$$\widehat{\beta} = \left(\sum x_t^2\right)^{-1} \left(\sum x_t y_t\right)$$

what's the limiting distribution of $\widehat{\beta}$ in the following two cases: (i) Case 1: where u_t is stationary but weakly dependent process, say, a mixing process, that satisfies LLN and CLT. (ii) Case 2: where $u_t = u_{t-1} + v_t$, v_t is a stationary but weakly dependent process, say, a mixing process, so that a partial sum process of v_t satisfies FCLT.

EC 8822

Stefan Hoderlein

May, 25 2018

Instructions: this part is an one hour, closed book exam. Attempt to answer formally and avoid lengthy verbal parts. There are 35 credits in total, and the individual credits for the questions are displayed as guidance. Try to answer all questions. However, the best grade is usually awarded with less than full credit.

1. State the Kernel based estimator for m_X, the nonparametric regression of a scalar random variable Y on d-dim random vector X. Define all elements formally, including the Kernel ("a Kernel is a function K such that...."). Explain how the estimator differs from a Kernel based estimator, if X had only one dimension. (6 credits)

2. What is the bias-variance trade-off? Explain it by using the bias and variance formulas for the Kernel density estimator you have suggested in question 1, and focus in particular on the bandwidth h. Hint: you do not need derive bias and variance, just state the two formulas and discuss. (4 credits)

3. Consider the semiparametric binary choice model, i.e.,

$$Y = 1\{X'\beta - U > 0\},$$

where X is a K-vector of observable random variables, β is a fixed K vector of coefficients, and U is a scalar unobservable. Assume that U is fully independent of X. Derive a way to identify β up to scale (i.e., give a formal sketch of your argument, 4 credits).

4. Why is nonparametrics useful in the binary choice model? Specifically, what disadvantage of maximum likelihood does it allow to avoid, and how? (2 credits)

The following two questions (i.e., question 5 and question 6) apply to the linear random coefficients model

$$Y = B_0 + X'B_1,$$

where Y is an observed random scalar, X is an observed random K-vector, $B = (B_0, B_1)'$ is an unobserved random $K+1$ vector, and X is fully independent of B. The goal is to obtain an estimator for the density f_B. To this end, define the conditional characteristic function of $Y|X = x$ as

$$\varphi_{Y|X}(t; x) = E\left[\exp(itY)|X = x\right],$$

where $t, x \in R^{K+1}$, and $i = \sqrt{-1}$.

5. Show that there is a one-to-one relation between $\varphi_{Y|X}(t; x)$ and $\varphi_B(s)$ for any value of $(t, x, s) \in R^{2K+2}$ (5 credits)

6. Why is it sufficient to show the identification of $\varphi_B(s)$, if the ultimate goal is to estimate $f_B(b)$ for any value of b. (2 credits)

7. What is the main identifying assumption in the DiD approach? Explain with an example (Hint: Mariel), and explain how this is reflected in the identification principle (4 credits)

8. Define the Lasso and the Ridge estimator and state the optimization problem on which both estimators are based (4 credits).

9. What is the difference between the two approaches? Focus on variable selection, and explain how this feature relates to the difference in the optimization problem you described in 8. Hint: use a detailed graphical explanation (4 credits)

May 23, 2018

Qualifying Exam Questions

75 total points for this section.

1. (Total 15 points) Suppose that

$$
\begin{aligned}
y_i &= x_i^*\beta_0 + \epsilon_i \\
x_i &= x_i^* \cdot \nu_i \\
z_i &= x_i^* \cdot \eta_i
\end{aligned}
$$

where $x_i^*, \epsilon_i, \nu_i, \eta_i$ are mutually independent random variables. You observe a random sample of the triple (y_i, x_i, z_i) and are interested in estimating β_0. You may assume all relevant moments of the random variables exist.

Under what conditions is the estimator:

$$\hat{\beta} = \frac{\sum_{i=1}^n y_i x_i}{\sum_{i=1}^n x_i^2}$$

consistent?

2. (Total 50 points) Consider the following censored regression model:

$$y_i = \max(x_i'\beta_0 + \epsilon_i, c_i) \tag{1}$$

where in this model c_i is an unknown function of the regressors- i.e. $c_i = f(x_i)$. Furthermore, c_i is only observed for censored observations. As we did in class, we will let d_i denote a censoring indicator- i.e.

$$d_i = I[\text{observation } i \ \text{ is uncensored}]$$

The econometrician observes a random sample of the triple:

$$(d_i, x_i, \tilde{y}_i)$$

where

$$\tilde{y}_i = d_i(x_i'\beta_0 + \epsilon_i) + (1 - d_i)c_i$$

In this question we will be interested in estimating β_0 under the assumption that ϵ_i satisfies a conditional median restriction.

(a) (15) In class we discussed Powell's CLAD estimator:

$$\hat{\beta} = \arg\min_{\beta} \frac{1}{n} \sum_{i=1}^{n} |y_i - \max(x_i'\beta, c_i)| \tag{2}$$

Describe the properties of CLAD in the model described above. Is it consistent? If not, what extra assumption(s) are needed for consistency to hold?

(b) (10) Propose an estimator for the following function of the regressors:

$$p_0(x_i) \equiv P(\text{observation } i \text{ uncensored}|x_i)$$

Prove consistency of your estimator.

(c) (10) Recall we are imposing a conditional median condition on ϵ_i- i.e. its median is 0 for all values of x_i. What are the range of values of $p_0(x_i)$ for which we know $x_i'\beta_0 \geq c_i$?

(d) (15) Based on the above answer, propose an estimator for β_0 assuming you know the values of $p_0(x_i)$. Then propose a feasible estimator for β_0 where you replace the true values of $p_0(x_i)$ with an estimator of $p_0(x_i)$, where you explicitly state the form of your estimator of $p_0(x_i)$.

3. (Total 10 points) Let $Q_n(\beta)$ be a random function of β; that is, it depends on random variables as well as β. Let $\hat{\beta}$ be a maximizer of $Q_n(\beta)$ over a compact set from a random sample of n observations. State the three conditions for $\hat{\beta}$ to converge in probability to some number β_0.