

The 2020 Econometrics comprehensive exam will work like this:

- \* The exam will be administered online, through zoom (not canvas)

- \* Zoom login ID is 714364847 Password: 026664

<https://bccte.zoom.us/j/714364847?pwd=andveHJWOEhlaUZheERUakJ2YmM3UT09>

- \* The exam is Thursday Aug 27 9:00am to noon EST

- \* The goal is to roughly replicate an in class exam.

- \* You are expected to work alone. I will be checking answers for evidence of collusion.

- \* You are expected to log in, and stay logged in, with your video on, throughout the exam (I will understand if you run into connection issues - having it on most the time is good enough. Please keep trying to reconnect if we become disconnected).

- \* At the start time, 9AM, I will email the exam it to you.

- \* At the end of the exam you'll email to me your completed exam to me in any form you can: (e.g. scans or photos if handwritten, or pdf if written on a tablet, whatever).

- \* Important: After you complete the exam, please wait until I acknowledge I've received it and can read it before exiting the zoom meeting.

- \* Everyone must complete part A of the exam  
(based on Econ7772 - Econometric methods - Lewbel)

- \* Everyone must complete any 2 of the 3 remaining parts, corresponding to  
Econ8821 - Time Series - Xiao,  
Econ8823 - Applied Econometrics - Baum, and  
Econ8825 - Topics in Econometric Theory - Khan

Field Examination in Econometrics  
August 2020    PART A    ECON7772

1. Suppose we have  $n$  i.n.i.d (independent, not identically distributed) observations of the random variable  $Z_i$ , for  $i = 1, \dots, n$ . Assume for all  $i$  that  $E(Z_i) = 1/i$ ,  $var(Z_i) = \sigma^2$ , and that  $|Z_i| \leq c$  for some finite positive constants  $c$  and  $\sigma^2$ . Let  $\bar{Z}_n = \frac{1}{n} \sum_{i=1}^n Z_i$ . A useful fact is that  $\gamma = \lim_{n \rightarrow \infty} (\sum_{i=1}^n 1/i) - \ln n$  is a finite number (called the Euler–Mascheroni constant).

- a. Does  $\bar{Z}_n = o_p(1)$ ? Prove your answer.
- b. Does  $n^{1/2} (\bar{Z}_n - E(\bar{Z}_n))$  have an asymptotic distribution? Prove your answer.

2. Assume that  $(\varepsilon_i, X_i)$  for  $i = 1, \dots, n$  are **bounded iid** random vectors,  $X_i > 0$ , and  $E(\varepsilon_i | X_i) = 0$ . Assume  $Y_i = e^{\alpha_0 X_i} + \varepsilon_i$  with  $|\alpha_0| < 1$ . Consider the estimator

$$\hat{\alpha} = \arg \min \sum_{i=1}^n (Y_i - e^{\alpha X_i})^2$$

You can assume the minimizing value  $\hat{\alpha}$  always lies between -1 and 1. Is  $\hat{\alpha}$  a consistent estimator of  $\alpha_0$ ?

3. Consider the following system of equations:

$$\begin{aligned} Y_i &= aX_i + bZ_i + e_i \\ X_i &= bZ_i + U_i \end{aligned}$$

Assume our data are independent, identically distributed observations of  $(Z_i, X_i, Y_i)$  for  $i = 1, \dots, n$ . Assume the errors  $(e_i, U_i)$  are mean zero, but they *are* correlated with  $(Z_i, X_i, Y_i)$ . Assume  $E[(Z_i, X_i, Y_i)] \neq 0$  and that all the elements of the matrix  $E[(Z_i, X_i, Y_i)(Z_i, X_i, Y_i)']$  are finite and nonzero.

- a. Find a consistent estimator for the finite constants  $a$  and  $b$  (hint: use what you know about the errors).
- b. Explain how you could calculate standard errors for your estimates.

## Time Series Econometrics, August, 2020

**Question 1.** Determine whether the following process is causal and/or invertible:

$$X_t + 1.6X_{t-1} = \varepsilon_t - 0.4\varepsilon_{t-1} + 0.04\varepsilon_{t-2}, \text{ where } \varepsilon_t \sim WN(0, \sigma^2).$$

**Question 2.** Suppose that the calculation of sample autocovariances based on a time series of sample size  $T = 100$  are:  $\hat{\gamma}(0) = 1382.2$ ,  $\hat{\gamma}(1) = 1114.4$ ,  $\hat{\gamma}(2) = 591.73$ , and  $\hat{\gamma}(3) = 96.216$ . Use these values to find the Yule-Walker estimates of  $\alpha_1$ ,  $\alpha_2$ , and  $\sigma^2$  in the model

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \varepsilon_t, \text{ where } \varepsilon_t \sim WN(0, \sigma^2).$$

**Question 3.** Given two observations  $x_1$  and  $x_2$  from the causal AR(1) process satisfying

$$X_t = \alpha_1 X_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t \sim WN(0, \sigma^2).$$

and assuming that  $|x_1| \neq |x_2|$ , find the maximum likelihood estimates of  $\alpha_1$  and  $\sigma^2$ .

**Question 4** Consider the AR process

$$y_t = \alpha y_{t-1} + u_t, \alpha = 1$$

where  $u_t$  are stationary but weakly dependent process, say, a mixing process whose partial sums satisfies a functional central limiting theorem, and consider the OLS estimation of the AR coefficient

$$\hat{\alpha} = \frac{\sum y_{t-1} y_t}{\sum y_{t-1}^2}$$

(i) what's the limiting distribution of  $\hat{\alpha}$ ? (ii) How to construct a unit root test based on the OLS estimator?

ECON8823: Applied Econometrics Comprehensive, August 2020

Answer all questions. They are equally weighted.

1. Write an essay discussing the Vector Error Correction Model (VECM), explaining the tests you would use to justify its use, and the interpretation of the results of estimation.
2. Write an essay discussing the use of propensity score matching as a method of computing treatment effects from observational data.
3. Write an essay discussing the ‘fractional logit’ model of Papke and Wooldridge, explaining the context where you would employ this technique and its advantages in that empirical context.

1. (30) Recall the standard *censored* selection model:

$$\begin{aligned}d_i &= I[w_i'\delta_0 + \eta_i > 0] \\ y_i &= d_i(x_i'\beta_0 + \epsilon_i)\end{aligned}$$

Observed variables:  $(d_i, w_i, y_i, x_i)$  and we assume  $(\eta_i, \epsilon_i) \sim N(\mathbf{0}, \Sigma)$ , where the (1,1) element of  $\Sigma$  is 1 (scale normalization), the other diagonal element is  $\sigma^2$ , and the off diagonal terms are  $\gamma_0$ . We estimated  $(\delta_0, \beta_0, \gamma_0, \sigma^2)$ .

Now we will consider the truncated selection model, where we only get to see wages (or hours worked) for those in the sample who are working. So now the econometrician only observes  $(y_i, x_i, w_i)$  for the subset of population for which  $d_i = 1$ .

- (a) Write the likelihood function for this model.
  - (b) Propose a NLLS estimator for the parameters in this model.
2. (15) Consider the following nonparametric censored regression model:

$$y_i = \max(\mu(x_i) + \epsilon_i, 0)$$

where  $y_i, x_i$  are scalar random variables;  $\epsilon_i$  is an unobserved disturbance term;

- (a) Suppose  $\epsilon_i$  satisfies  $Med(\epsilon_i|x_i) = 0$ . Propose an estimator for  $\mu(x_i)$ .
  - (b) Now suppose that  $\epsilon_i \perp x_i$ , and  $\epsilon_i$  satisfies the location normalization that  $Med(\epsilon_i) = 0$ . Propose an estimator for  $\mu(x_i)$ . What are the possible advantages of estimating  $\mu(x_i)$  under this condition compared to the first part of this question?
3. (15) Consider the following conditional moment inequality model:

$$E[y_i|x_i] \leq 0 \quad \forall x_i$$

where  $y_i$  is a scalar dependent variable, and  $x_i$  is a scalar independent variable that is continuously distributed on its support  $\mathcal{X}$ . Consider the following statement:

$$E[y_i|x_i] \leq 0 \quad \forall x_i \iff E[y_i I[t_1 \leq x_i \leq t_2]] \leq 0 \forall t_1, t_2 \in \mathcal{X}, t_1 \leq t_2$$

(a) Is the above statement true? Prove your answer either way.

Next, consider the statement:

$$E[y_i|x_i] \leq 0 \quad \forall x_i \iff E[y_i I[x_i \leq t_1]] \leq 0 \forall t_1 \in \mathcal{X}$$

(b) Is the above statement true? Prove your answer either way.

Finally, consider the statement pertaining to moment equalities:

$$E[y_i|x_i] = 0 \quad \forall x_i \iff E[y_i I[x_i \leq t_2]] = 0 \forall t_2 \in \mathcal{X}$$

(c) Is the above statement true? Prove your answer either way.

4. (15) Frechet(1951) was interested in knowing whether knowing the marginal distributions of continuous random variables  $X$  and  $Y$  tells us anything about their joint distribution.

He showed that, given knowledge of the distribution functions  $F(\cdot)$  and  $G(\cdot)$  of  $X$  and  $Y$ , respectively, their joint distribution  $K(\cdot, \cdot)$  is such that for all  $a, b$ :

$$\max(F(a) + G(b) - 1, 0) \leq K(a, b) \leq \min(F(a), G(b))$$

Prove this result.