A flexible approach to parametric inference in nonlinear and time varying time series models

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December 5, 2023

Motivation

- Monetary policy rules have changed over time?
- For researchers working with macroeconomic and financial data, there is great interest in investigating whether structural breaks and regime-switching behavior occurs in the conditional mean, $E(y_t|T_{t-1})$, and the conditional variance, $var(y_t|T_{t-1})$.
- Models that are nonlinear or exhibit structural breaks or time variation in parameters.
- The set of possible models is huge. Date mining issue.
- Can we develop a flexible parametric model which nests almost all of these specifications?

Literature Review

- Structural break or time varying parameter models:
  Cogley and Sargent (2001, 2005), Boivin and Giannoni (2006), Primiceri (2005)
- Regime-switching models:
  Sims and Zha (2006), Koop and Potter (2006)
- Flexible parametric modeling:
  Hamilton (2001, 2003), Lundbergh et al. (2003), Bec et al. (2008), Giordani et al. (2007)

Model

- TVP written in state space form:

$$y_t = \theta_t x_t + \varepsilon_t \tag{1}$$
$$\theta_t = \theta_{t-1} + \nu_t \tag{2}$$

  ϵ_t is $i.i.d.\ N(0, \sigma_\epsilon^2)$and ν_t is $i.i.d.\ N(0, \sigma_\nu^2)$ - assume homoskedasticity in this simple case, include volatility issues in the general model.

- Add stochastic volatility to the measurement equation:

$$\varepsilon_t = \xi_t exp\left(\frac{1}{2}\alpha_t\right)$$
$$\xi_t \sim N(0,1)$$
$$\alpha_t = \alpha_{t-1} + \eta_t$$
$$\eta_t \sim N(0, \sigma_\eta^2)$$

Model

- The role of the distance function

$$y_t = \theta_t y_{t-1} + \varepsilon_t \tag{3}$$
$$\theta_t = \theta_{t-1} + d(t, t-1)\nu_t \tag{4}$$

- links bwtween this framework with other time series models
  - If $d(t, t-1) = 0 \rightarrow$ standard linear AR model
  - If $d(t, t-1) = 1 \rightarrow$ TVP model as in Koop and Potter (2001)
  - If $d(t, t-1) = 1$ if $t = \tau$ and $d(t, t-1) = 0$ otherwise, then $\theta_1 = \cdots = \theta_{\tau-1}$ and $\theta_\tau = \cdots = \theta_T \rightarrow$ model with a single structural break at τ
  - Add a second breakpoint $\rightarrow$ model with two structural breaks
  - $\cdots$

Model

- The role of hypothetical data reordering

$$y_s = \theta_s x_s + \varepsilon_s \tag{5}$$
$$\theta_s = \theta_{s-1} + d(z_s, z_{s-1})\nu_s \tag{6}$$

  where z_t is an exogenous index variable, γ define the ordering of the data according to z_t and s is the index under the new ordering

- Links between this framework with other time series models
  - If $z_t = y_{t-1}$ (then γ orders the data based on last period's y), define $d(z_s, z_{s-1}) = 1$ if $z_{s-1} < \tau$ and $z_s \geq \tau$ and $d(z_s, z_{s-1}) = 0$ otherwise $\rightarrow$ two-regime TAR model

$$y_t = \theta_1 x_t + \varepsilon_t \; if \; y_{t-1} < \tau$$
$$y_t = \theta_2 x_t + \varepsilon_t \; if \; y_{t-1} \geq \tau$$

Model

Table 1
Links between our framework and popular nonlinear time series models.

Model	Distance function	Index variable
AR(p)	0	$z_t = t$
TVP	1	$z_t = t$
Structural Break 1 Break	$=1$ at time τ $=0$ otherwise	$z_t = t$
Structural Break K Breaks	$=1$ at $\tau_1, \ldots, \tau_K$ $=0$ otherwise	$z_t = t$
Structural Break Unknown # Breaks	$=1$ with prob p $=0$ otherwise	$z_t = t$
Chib (1998) Structural K Breaks Model	$=1$ with restricted Markov transition probs. $=0$ otherwise	$z_t = t$
Various nonparametric TVP models	Smooth function (e.g. kernel)	$z_t = t$
Standard TAR	$=1$ if $z_{s-1} < \tau$ and $z_s \geq \tau$ $=0$ otherwise	$z_t = y_{t-d}$
Other TARs	$=1$ if $z_{s-1} < \tau$ and $z_s \geq \tau$ $=0$ otherwise	z_t exogenous var. or functions of lags
Multiple Regime TARs	$=1$ if $z_{s-1} < \tau_1$ and $z_s \geq \tau_1$ $=1$ if $z_{s-1} < \tau_2$ and $z_s \geq \tau_2$ etc.	z_t exogenous var. or functions of lags
STAR[a]	Smooth function	$z_t = y_{t-d}$
Multiple Regime STAR	Smooth function with multiple modes	$z_t = y_{t-d}$
Markov switching model	$=1$ with restricted Markov transition probs. $=0$ otherwise	$z_t = t$
Various nonparametric time series models	Smooth function (e.g. kernel)	z_t exogenous var. or functions of lags

[a] This relationship is approximate and is illustrated in the artificial data section.

Empirical work

- Artificial data
- Empirical illustrations using real GDP growth
- The oil price and GDP growth

Discussion

- This model nests virtually every popular model in the regime-switching and structural break literatures, including everything from abrupt change models (e.g. threshold autoregressive models or structural break models such as Bai and Perron (1998)) to those which allow gradual evolution of parameters (e.g. smooth transition autoregressive models or TVP models such as Primiceri (2005)).
- This model adds two simple concepts, hypothetical reordering and distance, to a standard state space framework.
- Retain the state space framework, bayesian econometric methods are relatively straightforward drawing on the existing literature.