Assignment #4

1. **Problem: How come 1-D closest pair of points problem can't apply to 2D?**

Let's say if there are n points on the plane

I will show the graph

If fig.1 is sorted by X

and there are four points

line on the boundary of the points graph

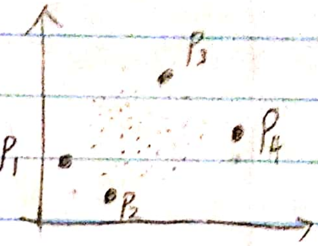

Fig.1

So $P_1.x < P_2.x < P_3.x < P_4.x$

index ① ② ③ ④

Howerer, if we sort them by y. Fig 2

we will find

$\Rightarrow P_2.y < P_1.y < P_4.y < P_3.y$

index ① ② ③ ④

$\Rightarrow$ the order of points in x, y are different

Conclusion1: so, we cannot match the point's x, y order by sorting x, y seperately

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Let's say if in X direction, From Fig.2,

P_1 and P_2 has the closest distance

Let's denote them the distance by $d_x(P_1, P_2)$

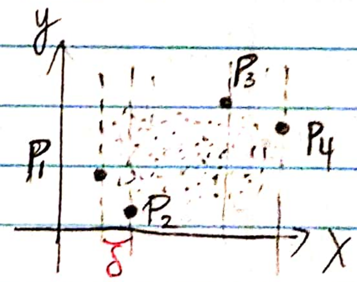

Fig.2

Howerer, if in y direction, from Fig.3

P_3 and P_4 has the closest distance

Let's denote them the distance by $d_y(P_3, P_4)$

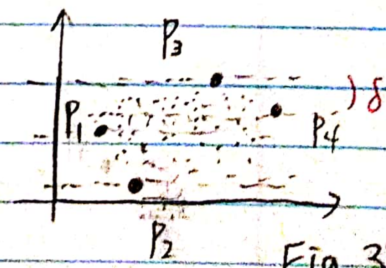

Fig.3

we'll find that we cannot tell which points are the closest pair of points only by order of x, y

and from Conclusion1, we cannot even match the points' distance by their index

Thus, 1-D closest pair of points problem can't apply to 2D

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2.

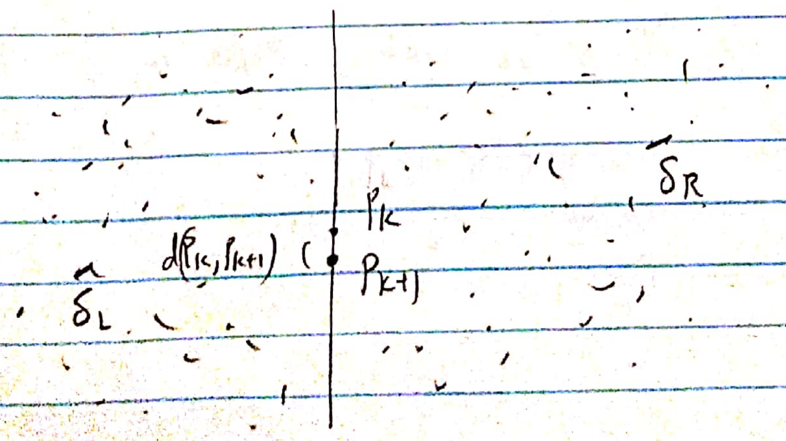
(a) If 2 points have the same x-coordinate, say, P_k, P_{k+1}
and P_k, P_{k+1} have the distance $d(P_k, P_{k+1})$,

During the initial seperation of the plane,
if the two points happen to be on the seperation line exactly,
then, the distance $d(P_k, P_{k+1})$ won't be compared with
the closest points, neither from left plane, nor from righ plane

Let's say if the shortest distance between points from left plane
is δ_L, and in the right plane is δ_R

When we want to construct the $\frac{\delta}{2}$ by $\delta/2$ squares,
we need to know what is the shortest distance δ,
we'll say $\delta = \min(\delta_L, \delta_R)$

but if the $d(P_k, P_{k+1}) < \delta_L$ and $d(P_k, P_{k+1}) < \delta_R$
then $d(P_k, P_{k+1}) < \delta$
which results in the situation that two points
will be in the same $\delta/2$ by $\delta/2$ square.
, so this is false.



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2.
(b) the claim holds true because when we find multiple points with the same x-coordinate and there is a distance δ_m which is smaller than δ (where we find from $\delta = \min(\delta_L, \delta_R)$)

then we can replace the δ with δ_m and reconstruct the $\frac{\delta}{2}$ by $\frac{\delta}{2}$ squares,
so all the points S_j when $j - i \geq 8$, then the distance between S_i and S_j will be $> \delta$

So when they find another shorter distance δ_a they will replace the δ with δ_a and reconstruct $\delta/2$ by $\delta/2$ squares

Thus, the claim holds true
QED.

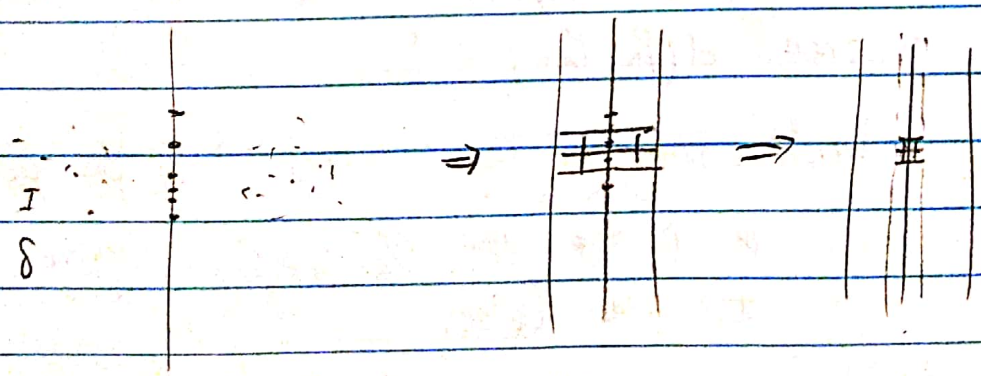

Fig.1. the $\delta/2$ by $\delta/2$ squares shrink as it finds another $\delta_d < \delta$ and replace it