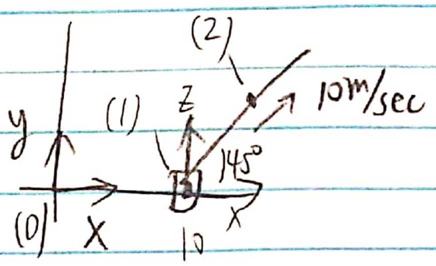
ICP6:



1.1 for $V_1=0$, find ${}^0({}^0V_2) = \begin{bmatrix} 10\cos 45^\circ \\ 10\sin 45^\circ \\ 0 \end{bmatrix}$

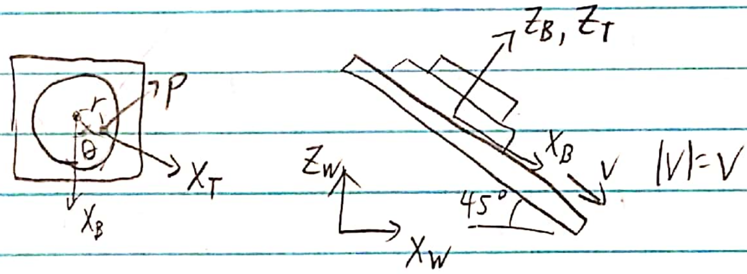
1.2 ${}^1({}^1V_2) = \begin{bmatrix} 10\cos 45^\circ \\ 0 \\ -10\sin 45^\circ \end{bmatrix}$

1.3 ${}^0({}^0V_1) = \begin{bmatrix} 16 \\ 42 \\ -10 \end{bmatrix}$ ${}^0({}^1V_2) = \begin{bmatrix} 10\cos 45^\circ - 16 \\ 10\sin 45^\circ - 42 \\ 10 \end{bmatrix}$

1.4

$${}^0({}^2V_2) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

6.2



6.2.1 ${}^T({}^TV_p) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

6.2.2 ${}^T({}^WV_p) = {}^TV_p + {}^TW_p \otimes {}^TP = Rot(\hat{z},\theta)\cdot \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ w \end{bmatrix} \times \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix}$

$$= \begin{bmatrix} c(-\theta) & -s(-\theta) & 0 \\ s(-\theta) & c(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ wr \\ 0 \end{bmatrix} = \begin{bmatrix} V\cos\theta \\ wr - V\sin\theta \\ 0 \end{bmatrix}$$

6.2.3 ${}^W({}^WV_p) = {}^W_TR\,{}^T({}^WV_p) = Rot(\hat{y}, 45^\circ)\,{}^T({}^WV_p)$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} V\cos\theta \\ wr - V\sin\theta \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} V\cos\theta \\ wr - V\sin\theta \\ -\frac{\sqrt{2}}{2} V\cos\theta \end{bmatrix}$$

$${}^{N-1}_{N}T = Rot(x,\alpha_{N-1})\, Trans(\hat{x}, a_{N-1})\, Rot(z,\theta_N)\, Trans(z, d_N)$$

$$= \begin{bmatrix} c_N & -s_N & 0 & a_{N-1} \\ s_N c\alpha_{N-1} & c_N c\alpha_{N-1} & -s\alpha_{N-1} & -s\alpha_{N-1} d_N \\ s_N s\alpha_{N-1} & c_N s\alpha_{N-1} & c\alpha_{N-1} & c\alpha_{N-1} d_N \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6.3

N	α_{N-1}	a_{N-1}	d_N	θ_N
1	90	0	l_1	θ_1
2	-90	l_2	0	θ_2
3	90	0	l_3	θ_3

assume ${}^0V_0 = {}^0W_0 = 0$

$${}^{N}V_P = V_0 + W_0 \otimes P, \quad {}^{N+1}W_{N+1} = {}^{N+1}_{N}R\, {}^{N}W_N + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{N+1} \end{bmatrix}$$

$${}^0_1T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ 0 & 0 & -1 & -l_1 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_2 & -S_2 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ -S_2 & -C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ 0 & 0 & -1 & -l_3 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1W_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \qquad {}^2W_2 = {}^2_1R\, {}^1W_1 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$$

$$= \begin{bmatrix} C_2 & 0 & -S_2 \\ -S_2 & 0 & -C_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -S_2\dot{\theta}_1 \\ -C_2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$${}^3W_3 = {}^3_2R\, {}^2W_2 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} C_3 & 0 & S_3 \\ -S_3 & 0 & C_3 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -S_2\dot{\theta}_1 \\ -C_2\dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} -S_2C_3\dot{\theta}_1 + S_3\dot{\theta}_2 \\ S_3S_2\dot{\theta}_1 + C_3\dot{\theta}_2 \\ C_2\dot{\theta}_1 + \dot{\theta}_3 \end{bmatrix}$$

$${}^0V_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad {}^1V_1 = {}^1_0R\, ({}^0V_0 + {}^0W_0 \times {}^0P_1) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$${}^2V_2 = {}^2_1R\, ({}^1V_1 + {}^1W_1 \times {}^1P_2) = \begin{bmatrix} C_2 & 0 & -S_2 \\ -S_2 & 0 & -C_2 \\ 0 & 1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \times \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \\ l_2\dot{\theta}_1 \end{bmatrix}$$

$${}^3V_3 = {}^3_2R\, ({}^2V_2 + {}^2W_2 \times {}^2P_3) = \begin{bmatrix} C_3 & 0 & S_3 \\ -S_3 & 0 & C_3 \\ 0 & -1 & 0 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ l_2\dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} \times \begin{bmatrix} 0 \\ -l_3 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} C_3 l_3\dot{\theta}_2 + S_3 l_2\dot{\theta}_1 \\ -S_3 l_3\dot{\theta}_2 + C_3 l_2\dot{\theta}_1 \\ 0 \end{bmatrix}$$

6.4

Jacobian Matrix of 6.3

$$\underline{V} = \underline{\underline{J}}(\theta)\, \underline{\dot{\theta}}$$

$$\begin{bmatrix} {}^3V_{3x} \\ {}^3V_{3y} \\ {}^3V_{3z} \\ {}^3W_{3x} \\ {}^3W_{3y} \\ {}^3W_{3z} \end{bmatrix} = \begin{bmatrix} S_3 l_2 & C_3 l_3 & 0 \\ C_3 l_2 & -S_3 l_3 & 0 \\ 0 & 0 & 0 \\ -S_2 C_3 & S_3 & 0 \\ S_3 S_2 & C_3 & 0 \\ C_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

6.5

N	α_{N-1}	a_{N-1}	d_N	θ_N
1	0	0	0	θ_1
2	0	l_1	d_2	0
3	90°	0	l_2	θ_3
4	−90°	l_3	0	θ_4

$${}^0_1T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2_3T = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ 0 & 0 & -1 & -l_2 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3_4T = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_4 & -C_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1W_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \quad {}^2W_2 = {}^2_1R\, {}^1W_1 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = I_{3\times3} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

$${}^3W_3 = {}^3_2R\, {}^2W_2 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} C_3 & 0 & S_3 \\ -S_3 & 0 & C_3 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} S_3(\dot{\theta}_1 + \dot{\theta}_2) \\ C_3(\dot{\theta}_1 + \dot{\theta}_2) \\ \dot{\theta}_3 \end{bmatrix}$$

$${}^4W_4 = {}^4_3R\, {}^3W_3 + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} C_4 & 0 & -S_4 \\ -S_4 & 0 & -C_4 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} S_3(\dot{\theta}_1 + \dot{\theta}_2) \\ C_3(\dot{\theta}_1 + \dot{\theta}_2) \\ \dot{\theta}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} C_4 S_3(\dot{\theta}_1 + \dot{\theta}_2) - S_4 \dot{\theta}_3 \\ -S_4 S_3(\dot{\theta}_1 + \dot{\theta}_2) - C_4 \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix}$$

$${}^0V_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad {}^1V_1 = {}^1_0R({}^0V_0 + {}^0W_0 \times {}^0P_1) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$${}^2V_2 = {}^2_1R({}^1V_1 + {}^1W_1 \times {}^1P_2) = I_{3\times3} \cdot \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \times \begin{bmatrix} l_1 \\ 0 \\ d_2 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ l_1 \dot{\theta}_1 \\ 0 \end{bmatrix}$$

$$
{}^3V_3 = {}^3_2R\left({}^2V_2 + {}^2W_2 \times {}^2P_3\right) = \begin{bmatrix} C_3 & 0 & S_3 \\ -S_3 & 0 & C_3 \\ 0 & -1 & 0 \end{bmatrix}\left(\begin{bmatrix} 0 \\ l_1\dot\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1+\dot\theta_2 \end{bmatrix} \times \begin{bmatrix} 0 \\ -l_2 \\ 0 \end{bmatrix}\right)
$$

$$
= \begin{bmatrix} C_3 & 0 & S_3 \\ -S_3 & 0 & C_3 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} l_2(\dot\theta_1+\dot\theta_2) \\ l_1\dot\theta_1 \\ 0 \end{bmatrix}
$$

$$
= \begin{bmatrix} C_3 l_2(\dot\theta_1+\dot\theta_2) \\ -S_3 l_2(\dot\theta_1+\dot\theta_2) \\ -l_1\dot\theta_1 \end{bmatrix}
$$

$$
{}^4V_4 = {}^4_3R\left({}^3V_3 + {}^3W_3 \times {}^3P_4\right) = \begin{bmatrix} C_4 & 0 & -S_4 \\ -S_4 & 0 & -C_4 \\ 0 & 1 & 0 \end{bmatrix}\left(\begin{bmatrix} C_3 l_2(\dot\theta_1+\dot\theta_2) \\ -S_3 l_2(\dot\theta_1+\dot\theta_2) \\ -l_1\dot\theta_1 \end{bmatrix} + \begin{bmatrix} S_3(\dot\theta_1+\dot\theta_2) \\ C_3(\dot\theta_1+\dot\theta_2) \\ \dot\theta_3 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right)
$$

$$
= \begin{bmatrix} C_4C_3l_2(\dot\theta_1+\dot\theta_2) + S_4l_1\dot\theta_1 \\ -S_4C_3l_2(\dot\theta_1+\dot\theta_2) + C_4l_1\dot\theta_1 \\ -S_3l_2(\dot\theta_1+\dot\theta_2) \end{bmatrix}
$$

$$
\begin{bmatrix} {}^4V_{4x} \\ {}^4V_{4y} \\ {}^4V_{4z} \\ {}^4W_{4x} \\ {}^4W_{4y} \\ {}^4W_{4z} \end{bmatrix} = \underbrace{\begin{bmatrix} (C_4C_3l_2+S_4l_1) & C_4C_3l_2 & 0 & 0 \\ (-S_4C_3l_2+C_4l_1) & -S_4C_3l_2 & 0 & 0 \\ -S_3l_2 & -S_3l_2 & 0 & 0 \\ C_4S_3 & C_4S_3 & -S_4 & 0 \\ -S_4S_3 & -S_4S_3 & -C_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\hookrightarrow\ {}^4J_4(\theta)} \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \\ \dot\theta_4 \end{bmatrix}
$$