ICP7:

7.1

N	α_{N-1}	a_{N-1}	d_N	θ_N
1	0	0	3	45°
2	$\pi/2$	0.1	4	0
3	$-\pi/2$	0.8	0	30°

N	m_N	$^N C_N$
1	30 kg	$[0,0,-1.5]^T$
2	20 kg	$[0.1,0,-2]^T$
3	10 kg	$[0.1,0.1,0.1]^T$

$$g_0=\begin{bmatrix}0\\0\\-9.8\end{bmatrix}$$

7.1.1

$$^1_2T=\begin{bmatrix}1&0&0&0.1\\0&0&-1&-4\\0&1&0&0\\0&0&0&1\end{bmatrix}\qquad ^2_1T=\begin{bmatrix}1&0&0&-0.1\\0&0&1&0\\0&-1&0&-4\\0&0&0&1\end{bmatrix}$$

$$^2_3T=\begin{bmatrix}\sqrt{3}/2&-\frac{1}{2}&0&0.8\\0&0&1&0\\-\frac{1}{2}&-\frac{\sqrt{3}}{2}&0&0\\0&0&0&1\end{bmatrix}\qquad ^0_1T=\begin{bmatrix}\sqrt{2}/2&-\sqrt{2}/2&0&0\\\sqrt{2}/2&\sqrt{2}/2&0&0\\0&0&1&3\\0&0&0&0\end{bmatrix}\qquad ^1_0T=\begin{bmatrix}\frac{\sqrt{2}}{2}&\frac{\sqrt{2}}{2}&0&0\\-\frac{\sqrt{2}}{2}&\frac{\sqrt{2}}{2}&0&0\\0&0&1&-3\\0&0&0&1\end{bmatrix}$$

$$^2_0T={}^2_1T\,{}^1_0T=\begin{bmatrix}1&0&0&-0.1\\0&0&1&0\\0&-1&0&-4\\0&0&0&0\end{bmatrix}\begin{bmatrix}\frac{\sqrt{2}}{2}&\frac{\sqrt{2}}{2}&0&0\\-\frac{\sqrt{2}}{2}&\frac{\sqrt{2}}{2}&0&0\\0&0&1&-3\\0&0&0&1\end{bmatrix}=\begin{bmatrix}\frac{\sqrt{2}}{2}&\frac{\sqrt{2}}{2}&0&-0.1\\0&0&1&-3\\\sqrt{2}/2&-\frac{\sqrt{2}}{2}&0&-4\\0&0&0&1\end{bmatrix}$$

$$^2\tau={}^2\tau_2+{}^2\tau_3=\left(m_2\,{}^2C_2+m_3\,{}^2_3T\,{}^3C_3\right)\otimes\left({}^2_0T\,{}^0g\right)$$

$$=\left(20\times\begin{bmatrix}0.1\\0\\-2\\1\end{bmatrix}+10\times\begin{bmatrix}\frac{\sqrt{3}}{2}&-\frac{1}{2}&0&0.8\\0&0&1&0\\-\frac{1}{2}&-\frac{\sqrt{3}}{2}&0&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}0.1\\0.1\\0.1\\1\end{bmatrix}_{4\times1}\right)\otimes\left(\begin{bmatrix}\frac{\sqrt{2}}{2}&\frac{\sqrt{2}}{2}&0&-0.1\\0&0&1&-3\\\frac{\sqrt{2}}{2}&-\frac{\sqrt{2}}{2}&0&-4\\0&0&0&1\end{bmatrix}\begin{bmatrix}0\\0\\-9.8\\0\end{bmatrix}_{4\times1}\right)$$

$$=(V_1)_{4\times1}\otimes(V_2)_{4\times1}$$

$\because$ dimension 4 cannot cross product directly, need to resize to 3×1, last row is redundant, so we can cut it of[f]

$$=(V_1)_{3\times1}\otimes(V_2)_{3\times1}$$

$$=\begin{bmatrix}-405.387\\0\\-101.587\end{bmatrix}$$

7.1.2

$$^1\tau = {}^1\tau_1 + {}^1\tau_2 + {}^1\tau_3$$

$$= \left(m_1\, {}^1C_1 + m_2\, {}^1_2T\, {}^2C_2 + m_3\, {}^1_2T\, {}^2_3T\, {}^3C_3\right) \otimes \left({}^1_0T\, {}^0g\right)$$

$$= \left(30 \times \begin{bmatrix} 0 \\ 0 \\ -1.5 \\ 1 \end{bmatrix} + 20 \times \begin{bmatrix} 1 & 0 & 0 & 0.1 \\ 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0 \\ -2 \\ 1 \end{bmatrix} + 10 \begin{bmatrix} 1 & 0 & 0 & 0.1 \\ 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} & 0 & 0.8 \\ 0 & 0 & 1 & 0 \\ \frac{-1}{2} & \frac{-\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)_{4\times1}$$

$$\otimes \left(\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -9.8 \\ 0 \end{bmatrix}\right)_{4\times1}$$

$$= (V_1)_{4\times1} \otimes (V_2)_{4\times1}$$

need resize, ∵ dimension 4 cannot cross product

$$= (V_1)_{3\times1} \otimes (V_2)_{3\times1}$$

$$= \begin{bmatrix} 770.613 \\ 130.987 \\ 0 \end{bmatrix}$$

⇒ torque on z-direction in frame 1 is zero

7.2

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

B.Cs: $\begin{cases} \theta(0)=10 \\ \dot\theta(0)=0 \end{cases}$ $\begin{cases} \theta(25)=85 \\ \dot\theta(25)=3.0 \end{cases}$

$\begin{cases} \theta(0)=10=a_0 \\ \dot\theta(0)=a_1=0 \end{cases}$ $\begin{cases} \theta(25)=10+a_2(25)^2+a_3(25)^3=85 \\ \dot\theta(25)=2a_2\times(25)+3a_3\times(625)=3 \end{cases}$

$\begin{cases} 50a_2+1875a_3=3 & —① \\ 625a_2+15625a_3=75 & —② \end{cases}$

②÷25 ⇒ $25a_2+625a_3=3$ —③

①-③ ⇒ $25a_2+1250a_3=0$

→ $a_2=-50a_3$ —④

put ④ into ③

$-625a_3=3$

∴ $\begin{cases} a_3=\frac{-3}{625} \\ a_2=\frac{6}{25} \end{cases}$ $\begin{cases} a_0=10 \\ a_1=0 \end{cases}$

7.3 $\theta(t)=\theta_a+A\sin(wt+\phi)$

B.C.S: $\begin{cases} \theta(0)=\theta_1 \\ \dot\theta(0)=0 \end{cases}$ $\begin{cases} \theta(t_f)=\theta_2 \\ \dot\theta(t_f)=0 \end{cases}$

$\begin{cases} \theta_1=\theta_a+A\sin\phi & —① \\ \theta_2=\theta_a+A\sin(wt_f+\phi) & —② \\ 0=wA\cos(\phi) & —③ \\ 0=wA\cos(wt_f+\phi) & —④ \end{cases}$

∵ $wA\cos(\phi)=0$

⇒ $\phi=\frac{\pi}{2}$ or $\frac{-\pi}{2}$

or

$w=0$ ⇒ $\begin{cases} \text{if } w=0 \\ \Rightarrow \theta(t) \text{ is not a function of } t, \Rightarrow\Leftarrow \end{cases}$

or

$A=0$ ⇒ if $\begin{cases} A=0 \\ \Rightarrow \theta(t) \text{ is not a function of } t, \Rightarrow\Leftarrow \end{cases}$

⇒ ∴ $\phi=\frac{\pi}{2}$ or $\frac{-\pi}{2}$

if $\phi=\frac{\pi}{2}$

from ④: $wt_f=0$ or π

⇒ if $wt_f=0$

∵ $w\neq0$ ∴ $t_f=0$

⇒ however, if $t_f=0$, then we will only have 2 independent B.Cs, and we cannot solve all 4 variables, thus ⇒⇐

⇒ ∴ $wt_f\neq0$ and $wt_f=\pi$ ⇒ $w=\frac{\pi}{t_f}$

$\begin{cases} \theta_2=\theta_a-A \\ \theta_1=\theta_a+A \end{cases}$ ⇒ $\begin{cases} A=\frac{\theta_1-\theta_2}{2} \\ \theta_a=\frac{\theta_1+\theta_2}{2} \end{cases}$

if $\phi=\frac{-\pi}{2}$, $w=\frac{\pi}{t_f}$

$\begin{cases} \theta_1=\theta_a-A \\ \theta_2=\theta_a+A \end{cases}$ ⇒ $\begin{cases} A=\frac{\theta_2-\theta_1}{2} \\ \theta_a=\frac{\theta_2+\theta_1}{2} \end{cases}$