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ICP1

1. 1.1 $\begin{bmatrix} \cos 60^\circ & 0 & \sin 60^\circ \\ 0 & 1 & 0 \\ -\sin 60^\circ & 0 & \cos 60^\circ \end{bmatrix} = Rot(\hat{Y}, 60^\circ)$

1.2 $\begin{bmatrix} {}^1p \\ 1 \end{bmatrix} = {}^1_0T \begin{bmatrix} {}^0p \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 0^\circ & -\sin 0^\circ & -5 \\ \sin 0^\circ & \cos 0^\circ & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 1.5 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0.5 \\ 1 \end{bmatrix}$

$\therefore \; {}^1p = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix}$

1.3 ${}^0p = {}^0_2T \; {}^2p \qquad ({}^0_2T)^{-1} \; {}^0p = {}^2p$

$\begin{bmatrix} \cos(-135^\circ) & -\sin(-135^\circ) & 5 \\ \sin(-135^\circ) & \cos(-135^\circ) & 1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 1.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.3536 \\ -1.0607 \\ 1 \end{bmatrix}$

${}^2p = \begin{bmatrix} 0.3536 \\ -1.0607 \end{bmatrix}$

1.4 (A) ${}^A_BR = Rot(\hat{X}_A, 30^\circ) \, Rot(\hat{y}_B, 60^\circ)$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ \\ 0 & \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} \cos 60^\circ & 0 & \sin 60^\circ \\ 0 & 1 & 0 \\ -\sin 60^\circ & 0 & \cos 60^\circ \end{bmatrix}$

(B) ${}^Bp = \begin{bmatrix} 17 \\ -10 \\ 3 \end{bmatrix} \qquad {}^Bp = {}^B_AT \; {}^Ap \Rightarrow {}^Ap = ({}^B_AT)^{-1} \; {}^Bp$

${}^Ap = {}^A_BR \; {}^Bp = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ \\ 0 & \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} \cos 60^\circ & 0 & \sin 60^\circ \\ 0 & 1 & 0 \\ -\sin 60^\circ & 0 & \cos 60^\circ \end{bmatrix} \begin{bmatrix} 17 \\ -10 \\ 3 \end{bmatrix} = \begin{bmatrix} 11.0981 \\ -2.0490 \\ -11.4510 \end{bmatrix}$

1.5

$$\cancel{{}^0P = {}^0_1T\,{}^1P}$$

$$\cancel{{}^2P = {}^2_1T\,{}^1_0T\,{}^0P}$$

$$\cancel{{}^2_1T = Rot(\hat{Y},-20^\circ)\,Rot(\hat{Z},-30^\circ)\,T\left(\begin{bmatrix}-2\\4\\0\end{bmatrix}\right)\,Rot(\hat{Z},30^\circ)}$$

$$= \begin{bmatrix}\cos -20^\circ & 0 & \sin -20^\circ & 0\\ 0 & 1 & 0 & 0\\ -\sin -20^\circ & 0 & \cos -20^\circ & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos -30^\circ & -\sin -30^\circ & 0 & 0\\ \sin -30^\circ & \cos -30^\circ & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & -2\\ 0 & 1 & 0 & 4\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos 30^\circ & -\sin 30^\circ & 0 & 0\\ \sin 30^\circ & \cos 30^\circ & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Rewrite ${}^0_1T\,{}^1_2T\,{}^2P_1 = {}^0P_1^2$

$${}^2P_1 = ({}^0_1T)^{-1}({}^1_2T)^{-1}({}^0P_1^2) = \begin{bmatrix}\cos 30^\circ & -\sin 30^\circ & 0\\ \sin 30^\circ & \cos 30^\circ & 0\\ 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}\cos 20^\circ & 0 & \sin 20^\circ\\ 0 & 1 & 0\\ -\sin 20^\circ & 0 & \cos 20^\circ\end{bmatrix}^{-1}\begin{bmatrix}-2\\4\\0\end{bmatrix} = \begin{bmatrix}0.25\\4.4641\\0.0916\end{bmatrix} \#$$

1.6

$$Rot(\hat{X},\theta_1)\,Rot(\hat{Y},\theta_2)\,Rot(\hat{Z},\theta_3)$$

$$= \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta_1 & -\sin\theta_1\\ 0 & \sin\theta_1 & \cos\theta_1\end{bmatrix}\begin{bmatrix}\cos\theta_2 & 0 & \sin\theta_2\\ 0 & 1 & 0\\ -\sin\theta_2 & 0 & \cos\theta_2\end{bmatrix}\begin{bmatrix}\cos\theta_3 & -\sin\theta_3 & 0\\ \sin\theta_3 & \cos\theta_3 & 0\\ 0 & 0 & 1\end{bmatrix} \#$$

1.7

$$K_1 = \begin{bmatrix}5\\4\\4\end{bmatrix} \xrightarrow{\text{Normalize}} K_{1-no} = \frac{1}{\sqrt{57}}\begin{bmatrix}5\\4\\4\end{bmatrix}$$

$$R_{K\theta} = \begin{bmatrix}k_x^2 v\theta + c\theta & k_x k_y v\theta - k_z s\theta & k_x k_z v\theta + k_y s\theta\\ k_x k_y v\theta + k_z s\theta & k_y^2 v\theta + c\theta & k_y k_z v\theta - k_x s\theta\\ k_x k_z v\theta - k_y s\theta & k_y k_z v\theta + k_x s\theta & k_z^2 v\theta + c\theta\end{bmatrix}$$

$$= \begin{bmatrix}0.8356 & -0.2719 & 0.4774\\ 0.4774 & 0.7893 & -0.3861\\ -0.2719 & 0.5505 & 0.7893\end{bmatrix}$$