All at Once! A Comprehensive and Tractable Semi-Parametric Method to Elicit Prospect Theory Components*

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Abstract

Eliciting all the components of prospect theory –curvature of the utility function, weighting function and loss aversion– remains an open empirical challenge. We develop a semi-parametric method that keeps the tractability of parametric methods while providing more precise estimates. Applying the new method to the datasets of Tversky and Kahneman (1992) and Bruhin et al. (2010), we reject the convexity of the utility function in the loss domain and show that the probability weighting function does not exhibit duality and equality across domains, in line with cumulative prospect theory and in contrast with original prospect and rank dependent utility theories. Furthermore, our method highlights that the overweighting of tail probabilities is more pronounced in the gain domain than in the loss domain. Overall, our results show that the utility function varies little across domains, thus suggesting that probability distortions are key to capture differences in risk attitudes in the gain and loss domains.

Keywords: Prospect theory; semi-parametric elicitation; risk attitudes; weighting function; loss aversion.

JEL codes : D81, C91

1 Introduction

There is now a large body of empirical evidence showing systematic violations of expected utility theory (EUT; see Starmer, 2000, for a review). The original version of prospect theory (OPT; see Kahneman and Tversky, 1979, henceforth KT79) and its subsequent refinements,

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most notably cumulative prospect theory (CPT; see Tversky and Kahneman, 1992, henceforth TK92), explain these empirical violations by introducing probability distortions (Bleichrodt and Pinto, 2000) and loss aversion (Wakker, 2010). Classical methods for eliciting risk attitudes (such as Holt and Laury, 2002) that are based on EUT may lead to biased estimates (Abdellaoui et al. (2011a) andAbdellaoui et al. (2008, Figure 1; henceforth ABL)) and produce incoherent results (Bleichrodt et al., 2001; Hershey and Schoemaker, 1985). New methods have thus been proposed that build on CPT instead of EUT (ABL). These methods fall into three broad categories: parametric (with parametric form of utility and probability weighting functions), semi-parametric (with parametric form of the utility function and parameter-free probability weighting function) and non-parametric (no parametric form for either function).

While all parametric methods rely on a parametric specification of both utility and probability weighting, they may differ in terms of the required data structures and the employed estimation procedures. The data used by parametric methods usually involve certainty equivalents (e.g. Fehr-Duda et al., 2006; Bruhin et al., 2010; l'Haridon and Vieider, 2019, TK92), binary choices (e.g. Harrison and Rutström, 2008, 2009; Harrison and Swarthout, 2020), as well as indifference between non-degenerate lotteries from the so-called tradeoff method (Booij et al., 2010). Then, commonplace estimation procedures range from simple arithmetic calculations (Tanaka et al., 2010) to more convoluted econometric techniques such as maximum likelihood estimation (MLE) (e.g. Fehr-Duda et al., 2006; Stott, 2006; Harrison and Rutström, 2008, 2009; Harrison and Swarthout, 2020), nonlinear least squares (NLS)(e.g.TK92, Gonzalez and Wu, 1999, henceforth GW99), OLS (Fox and Poldrack, 2009), and Bayesian methods (e.g. Nilsson et al., 2011; Toubia et al., 2013; Murphy and ten Brincke, 2018; Spiliopoulos and Hertwig, 2019; Baillon et al., 2020; Gao et al., 2020).

Parametric methods are more often used in applied research than non-parametric and semi-parametric methods (ABL) because they have four appealing properties: *tractable*, *data-efficiency*, *easy* and *error-robust* (see Section 2 for definitions), which are associated with minimal data requirements and simple estimation procedures.

These virtues, however, come at a cost. First, empirical estimates are sensitive to the specification of the utility and probability weighting functions (Abdellaoui, 2000). Second, parametric methods only provide an overall measure of the goodness of fit of the model, rather than separate measures for each of its components – one for the utility function and one for the probability weighting function (see GW99).

The aim of this paper is to establish a comprehensive semi-parametric method that satisfies the four desirable properties of parametric methods (see Section 2 for definitions). Doing so, we develop a method that increases the precision of parameter estimates of parametric methods while being easy to implement and estimate. Our method also comes with an additional advantage because it can be implemented not only under CPT but also under OPT and alternative theories such as rank dependent utility theory (henceforth RDU; see Quiggin, 1982; Gilboa, 1987; Schmeidler, 1989)

This paper proceeds as follows. Section 2 outlines the existing semi-parametric and non-parametric methods. Section 3 presents our elicitation method under cumulative prospect theory. Section 4 focuses on identification and misspecification. Section 5 provides two applications of the method. Section 6 concludes.

2 Existing semi-parametric and non-parametric methods

In this section, we review the existing semi- and non-parametric methods of eliciting risk attitudes under CPT. Table 1 provides an overview of these methods along the four desirable properties of parametric methods (*tractability*, *data-efficiency*, *easiness*, *error-robustness*) and *comprehensiveness*. We define these properties below.

Tractable: the method allows for estimating risk attitudes with standard tools, like OLS or NLS (Abdellaoui et al., 2007a, ABL).
Error-robust: the method accounts for the fact that subjects make response errors when answering questions (ABL).
Easy (not cognitively demanding): the method relies on *simple choices* involving the lowest possible number of outcomes, that is 3 (ABL). A useful benchmark comes from TK92 who ask their subjects to make *simple choices* between a sure outcome and binary lottery.
Data-efficient: the method requires few measurements (observations) to estimate the parameters of the utility function and the probability weights. As a rule of thumb, we take TK92 (with 28 measurements for 9 probability weights and one utility parameter per domain) as benchmark, and consider a method data-efficient if it requires no more than three measurements per estimate of a probability weight.[1]
Comprehensive: Comprehensive: the method allows for estimating utility function, loss aversion and weighting function under risk (i.e. known probability) as well as under uncertainty (i.e. unknown probability).

The existing methods typically use the tradeoff approach of Wakker and Deneffe (1996) in three steps.[2] In the first step, the tradeoff method estimates the utility function by eliciting a sequence of outcomes $(x_0, x_1, x_2, ..., x_n)$ which are equally spaced in terms of utility: $u(x_i) - u(x_{i-1}) = constant$ for $i = 1, 2, ..., n$. In the second step, the utility function obtained in the first step is used to estimate the weighting function. If the utility function in the gain and loss domains is elicited on the same scale (Abdellaoui et al., 2007b, 2016), the loss aversion can then be inferred in a third step. The tradeoff method is not error-robust because it assumes that the first-step values $(x_1, x_2, ..., x_n)$ are elicited without errors. This assumption is particularly restrictive because the elicitation of these values is subject to error propagation: any error in the first-stage elicitation of a given value (x_i) affects the subsequent estimates of values $(x_{i+1}, ..., x_n)$.[3] It follows that semi-parametric and non-parametric elicitation techniques based on the tradeoff method are not error-robust (see Table 1). Moreover, ABL note that the complexity of the elicitation procedure makes the tradeoff method cognitively demanding.[4]

The literature proposes three notable alternatives to the tradeoff method (GW99; ABL;

[1]ABL use the term "efficient" instead. Since this property only relates to the physical resources (such as time and money) used for data collection, we coin the term *data-efficiency* to avoid confusion with the (unrelated) statistical notion of the efficiency of an estimator.

[2]See, e.g. Abdellaoui (2000); Bleichrodt and Pinto (2000); Etchart-Vincent (2004); Abdellaoui et al. (2007b); Etchart-Vincent (2009a); Booij and Van de Kuilen (2009); Van De Kuilen and Wakker (2011); Abdellaoui et al. (2016); Attema et al. (2018); Bleichrodt et al. (2018); Blavatskyy (2021). The main reason of the popularity of the tradeoff method is that the elicitation of the utility function is robust to probability distortions.

[3]Another issue with the standard tradeoff method is strategic responding (Harrison and Rutström, 2008; Abdellaoui et al., 2020). For the sake of illustration, suppose that the researcher is looking for outcomes x_1 and x_2 by eliciting a pair of chained indifference values x_1 and x_2 such that $(x_1, \$1; 0.5, 0.5) \sim (\$10, \$5; 0.5, 0.5)$ and $(x_2, \$1; 0.5, 0.5) \sim (x_1, \$5; 0.5, 0.5)$. One of these four lotteries is picked at random for payoff at the end of the experiment. Hence, the decision-making problem boils down to calibrating the following lottery: $R(x_1, x_2) = (\$1, \$5, \$10, x_1, x_2; 1/4, 1/4, 1/8, 1/4, 1/8)$. This, in turn, provides incentives to overstate the values of x_1 and x_2, since $x_1^* > x_1$ and $x_2^* > x_2$ yields a lottery $R(x_1^*, x_2^*)$ that first-order stochastically dominates $R(x_1, x_2)$. Recently, Johnson et al. (2019) have developed a new mechanism, called PRINCE, that alleviates this issue by *ex ante* fixing the real choice situation that determines the final payment.

[4]This is because under the tradeoff method subjects need to compare two binary lotteries, while other methods based on certainty equivalent elicitation only ask subjects to compare a certain amount with a binary lottery. Thus, the tradeoff method requires processing more information which makes it relatively cognitively demanding.

Abdellaoui et al., 2011c).[5] However, GW99 is not a comprehensive method because it does not elicit loss aversion. Also, Van De Kuilen and Wakker (2011) point to the lack of tractability and efficiency of this method. Furthermore, their method tends to produce an extremely concave (resp. convex) utility function in the gain (resp. loss) domain (see footnote 13).

The semi-parametric method of ABL, in turn, satisfies the four appealing properties of parametric methods, and provides information on the goodness of fit of the functional form chosen for estimating the utility function. Yet, ABL is not a comprehensive method because it cannot estimate the weighting function. Achieving comprehensiveness by including an additional step to estimate the weighting function, as in Abdellaoui et al. (2011c), comes at the cost of potentially multiplying response errors (Etchart-Vincent, 2004, pp. 221).

In addition, the approach in ABL and Abdellaoui et al. (2011c) has two caveats when it comes to measuring loss aversion. First, estimating the utility function separately in the gain and loss domains makes it impossible to impose partial reflection (*i.e.*, identical utility curvature in both domains) which is often required to circumvent the arbitrary measurement of loss aversion (see Wakker, 2010, pp. 267-270).[6] By allowing for a joint estimation of the utility function in both domains, our method allows for testing and imposing partial refection whenever needed.[7] The second problem, as pointed out by Wakker and Deneffe (1996, pp. 1148), comes from the fact that the elicitation of loss aversion in ABL and Abdellaoui et al. (2011c) is based on asking subjects to provide a loss amount L on an unbounded interval $(-\infty, 0]$. This procedure is more cognitively demanding than stating L on a bounded interval (Abdellaoui et al., 2007b). Then, not knowing the lowest possible value of the loss amount L could lead to large response errors that potentially inflate the estimates of loss aversion, as reported in ABL (see Table 11, pp. 263-264).

[5]A method related to Abdellaoui et al. (2011c) is the *source method* of Abdellaoui et al. (2011a) that allows for eliciting the source function and the utility function under the biseparable preference model of Ghirardato and Marinacci (2001). An additional assumption is that decision makers can assign subjective probabilities (*i.e.*, beliefs) to events even when they do not maximize subjective expected utility. Another method proposed by Bertani et al. (2019) elicits the probability weighting function. However, this method is restrictive because it is only valid for the dual theory of Yaari (1987) in which the utility function is assumed to be linear.

[6]Note that this issue also applies to the parametric methods in TK92 and Fehr-Duda et al. (2006) which estimate the utility function in the gain and loss domains in two separate steps. Other parametric methods do not suffer from this problem (Harrison and Rutström, 2008; Post et al., 2008; Tanaka et al., 2010).

[7]With our one-step procedure, it is also possible to test whether the probability weighting functions are the same in the gain and loss domains. Outside the framework of CPT, our method also allows for testing the duality of the probability weighting function under RDU.

Table 1: Summary of literature on semi-parametric and non-parametric methods

Method or combination of methods	*Tractable*	*Data-efficient*	*Easy*	*Error-robust*	*Comprehensive*
Non-parametric methods based on tradeoff method					
Abdellaoui et al. (2007b) and Abdellaoui (2000)	Yes	Yes	No	No	No (only risk)
Abdellaoui et al. (2016) Bleichrodt et al. (2018) and Attema et al. (2018)	Yes	Yes	No	No	Yes
Van De Kuilen and Wakker (2011)	Yes	Yes	No	No	No (only w)
Blavatskyy (2021)	Yes	Yes	No	No	No (only u and λ)
Semi-parametric methods based on tradeoff method					
Etchart-Vincent (2004, 2009a)	Yes	Yes	No	No	No (only u and w)
Bleichrodt and Pinto (2000)	Yes	Yes	No	No	No (only u and w)
Chai and Ngai (2020)	Yes	Yes	No	No	No (only w)
Non-parametric method *not* based on tradeoff method					
GW99	No	No	Yes	Yes	No (only u and w)
Semi-parametric methods *not* based on tradeoff method					
ABL	Yes	Yes	Yes	Yes	No (only u and λ)
Abdellaoui et al. (2011c)	Yes	Yes	Yes	No	Yes
This paper	Yes	Yes	Yes	Yes	Yes

3 Method

3.1 Notations

Consider a binary lottery $L = (x, y; p, 1-p)$ yielding outcome x with probability p and outcome y with probability $1-p$, both outcomes being real numbers.[8] For notational convenience, let $x > y \geq 0$ ($x < y \leq 0$) for non-mixed prospects involving only gains (losses). For mixed prospects (*i.e.*, involving both gains and losses), let $y < 0 < x$. $\succsim$ is a preference relation over prospects with $\succ$ ($\sim$) denoting strict preference (indifference). Preferences are represented by CPT with a probability weighting function w^i and a value function v as defined in equation (1) for non-mixed prospects and in equation (2) for the mixed ones:

$$CPT(L) = (v(x) - v(y))\, w^i(p) + v(y) \tag{1}$$

$$CPT(L) = w^+(p)v(x) + w^-(1-p)v(y) \tag{2}$$

[8] This notation is related to decision under risk. In the case of decisions under uncertainty, one would simply replace p and $1-p$ by E and E^c respectively. E denotes an event in a state space Ω and E^c denotes its complement in Ω. In that case, $L = (x, y; E, E^c)$ is a binary prospect that gives outcome x if E occurs, and y otherwise.

where w^i and v are both continuous, strictly increasing and satisfying $v(0) = 0$, $w^i(0) = 0$ and $w^i(1) = 1$, and $i =$"+" ($i =$"−") stands for the gain (loss) domain.[9]

Following the seminal study by TK92, as well as the subsequent developments in Köbberling and Wakker (2005) and ABL, we assume that the value function $v(.)$ is composed of the loss aversion index $\lambda > 0$ which reflects the exchange rate between gain and loss utility units, and the utility function $u(.)$ that reflects the intrinsic value of outcomes:

$$v(x) = \begin{cases} u(x) & \text{if } x \geq 0 \\ \lambda u(x) & \text{if } x < 0 \end{cases} \tag{3}$$

However, without further assumptions, loss aversion (λ) as defined in (3) is not empirically identifiable. Indeed, we can rescale the utility function $u(.)$ in the gain and loss domains with two different linear transformations $u^*(x) = \rho u(x)$ for $x \geq 0$ and $u^*(x) = \tau u(x)$ for $x < 0$, so that we have a linear transformation $v^*(x) = \rho v(x)$ for the value function by defining $\lambda^* = \lambda\frac{\rho}{\tau}$ (Wakker, 2010, p. 248):

$$v^*(x) = \begin{cases} u^*(x) & \text{if } x \geq 0 \\ \lambda^* u^*(x) & \text{if } x < 0 \end{cases} \qquad \text{with} \qquad \lambda^* = \lambda\frac{\rho}{\tau}$$

As a result, as long as $\rho \neq \tau$, we have two different values for the loss aversion index (λ and λ^*) that represent the same underlying preferences. However, one can avoid this arbitrary measurement of loss aversion using utility functions that are differentiable at 0 with $u'(0) \neq 0$ (e.g. exponential utility in equation (5)). In the case of power utility functions (see equation (4)), the issue of identification of loss aversion is present unless *partial reflection* (i.e., identical utility curvature in the full domain) is imposed.

3.2 Elicitation

We start by considering two standard utility functions that have been previously shown to provide a good fit to experimental data: the power utility function (see, e.g. GW99; Stott, 2006) and the exponential utility function (see, e.g. Attema et al., 2013).[10] Next, we detail the steps for estimating the probability weighting function, the curvature of the utility function, and the loss aversion index.

3.2.1 Utility function

As in Booij et al. (2010), we use the following notation for the power (4) and exponential (5) utility functions:

$$u(x) = (\mathbf{1}_{(x\geq 0)} - \mathbf{1}_{(x<0)})\,|x|^{\alpha_p \mathbf{1}_{(x\geq 0)} + \beta_p \mathbf{1}_{(x<0)}} \tag{4}$$

$$u(x) = (\mathbf{1}_{(x\geq 0)} - \mathbf{1}_{(x<0)})\frac{1 - exp\left((\beta_e \mathbf{1}_{(x<0)} - \alpha_e \mathbf{1}_{(x\geq 0)})x\right)}{\alpha_e \mathbf{1}_{(x\geq 0)} + \beta_e \mathbf{1}_{(x<0)}} \tag{5}$$

where $\mathbf{1}_{(.)}$ refers to the indicator function. The important properties of these functions are related to domain-specific curvature, loss aversion, and partial reflection, and can be summarized as follows. For gains (losses), the power function in (4) is concave if $\alpha_p < 1$ ($\beta_p > 1$), linear if $\alpha_p = 1$ ($\beta_p = 1$), and convex if $\alpha_p > 1$ ($\beta_p < 1$). For gains (losses), the exponential function in (5) is concave if $\alpha_e > 0$ ($\beta_e < 0$), linear if $\alpha_e \longrightarrow 0$ ($\beta_e \longrightarrow 0$), and convex if $\alpha_e < 0$

[9]CPT makes no explicit link between weighting functions $w^+(.)$ and $w^-(.)$ which makes it more general than OPT in which $w^+(p) = w^-(p)$, or RDU that includes the duality condition $w^+(p) = 1 - w^-(1-p)$.

[10]Note, however, that our method is compatible with any utility function.

($\beta_e > 0$). Furthermore, the two functions imply two different definitions of loss aversion. For (4), the loss aversion index is $\lambda = -\frac{v(-\$1)}{v(\$1)}$ which corresponds to the standard definition in TK92. For (5), the loss aversion index is given by the definition in Köbberling and Wakker (2005), that is $\lambda = \frac{v'_{\uparrow}(0)}{v'_{\downarrow}(0)}$ with $v'_{\uparrow}(0)$ and $v'_{\downarrow}(0)$ representing the left and right derivatives of the value function at the reference point.[11] Finally, partial reflection corresponds to $\alpha_p = \beta_p$ and $\alpha_e = \beta_e$ in (4) and (5).

In addition, we use the following method for curvature comparisons of different utility functions for a given interval, such as $[0, \overline{x}]$ in the gain domain or $[\underline{x}, 0]$ in the loss domain. We compute the following measure of utility curvature (see also Abdellaoui et al., 2016):

$$\alpha = \frac{1}{\overline{x}u(\overline{x})} \int_0^{\overline{x}} u(t)dt \qquad \text{if} \qquad \overline{x} \geq 0 \tag{6}$$

$$\beta = \frac{1}{\underline{x}u(\underline{x})} \int_{\underline{x}}^{0} u(t)dt \qquad \text{if} \qquad \underline{x} < 0 \tag{7}$$

For power (exponential) utility function, this yields $\alpha = \frac{1}{1+\alpha_p}$ and $\beta = -\frac{1}{1+\beta_p}$ ($\alpha = \frac{1}{1-exp(-\alpha_e\overline{x})} - \frac{1}{\alpha_e\overline{x}}$ and $\beta = \frac{1}{exp(\beta_e\underline{x})-1} - \frac{1}{\beta_e\underline{x}}$.)

Then, $\alpha > 0.5$ / $\alpha = 0.5$ / $\alpha < 0.5$ correspond to concave / linear / convex utility functions in the gain domain. In the loss domain we have $\beta > -0.5$ / $\beta = -0.5$ / $\beta < -0.5$ that correspond to concave / linear / convex utility functions.

3.2.2 Estimating probability weighting functions and utility curvature

The first step of our estimation procedure consists of three parts. First, we select the set of probabilities $\{p_k : k = 1, 2, ..., K\}$ for which weights are estimated in the gain and loss domains, with $p_k < p_{k+1}$. For any p_k, its complement $1 - p_k$ must also be included in the set of probabilities, so that $p_{K-k+1} = 1 - p_k$ for $k = 1, 2, ..., K$. Then, in a given domain, one elicits (at least) two certainty equivalents for each probability p_k:

$$ce^i_{j,k} \sim (x^i_{j,k}, y^i_{j,k}; p_k, 1 - p_k) \ \ , \ \ j = 1, 2, ..., N^i_k \quad \text{and} \quad N^i_k \geq 2 \tag{8}$$

where N^i_k stands for the number of certainty equivalents for p_k in domain $i \in \{\text{“}+\text{”}; \text{“}-\text{”}\}$, and $x^i_{j,k}$ and $y^i_{j,k}$ are outcomes such that $x^+_{j,k} > y^+_{j,k} \geq 0$ and $x^-_{j,k} < y^-_{j,k} \leq 0$.[12]

Thus, in total one needs to elicit $N^+ = \sum_{k=1}^K N^+_k \geq 2 \times K$ certainty equivalents in the gain domain and $N^- = \sum_{k=1}^K N^-_k \geq 2 \times K$ certainty equivalents in the loss domain. For invertible u and using (1) and (3), these certainty equivalents satisfy the following condition:

$$ce^i_{j,k} = u^{-1}\left[\left(u(x^i_{j,k}) - u(y^i_{j,k})\right) w^i(p_k) + u(y^i_{j,k})\right] \tag{9}$$

Let $\boldsymbol{ce}, \boldsymbol{x}$ and $\boldsymbol{y}$ be column vectors containing all the realizations of $ce^i_{j,k}$, $x^i_{j,k}$ and $y^i_{j,k}$, respectively. Any column vector $\mathbf{z} \in \{\boldsymbol{ce}, \boldsymbol{x}, \boldsymbol{y}\}$ is constructed as follows:

[11] Regardless of the exact definition, it is always the case that loss aversion (loss seeking) corresponds to $\lambda > 1 (\lambda < 1)$, whereas $\lambda = 1$ captures loss neutrality.

[12] Note that having the same outcomes for each probability ($x^i_{j,k} = x^i_j$ and $y^i_{j,k} = y^i_j$ for all k) allows for an immediate test of the monotonicity of preferences by checking if certainty equivalents increase with probabilities for given pairs of outcomes (x^i_j, y^i_j). This choice of outcomes could also reduce the cognitive burden of the task.

$$\boldsymbol{z} = \begin{pmatrix} \boldsymbol{z}_1^+ \\ \boldsymbol{z}_2^+ \\ . \\ . \\ . \\ \boldsymbol{z}_K^+ \\ \boldsymbol{z}_1^- \\ \boldsymbol{z}_2^- \\ . \\ . \\ . \\ \boldsymbol{z}_K^- \end{pmatrix} \text{ with } \boldsymbol{z}_k^+ = \begin{pmatrix} z_{1,k}^i \\ z_{2,k}^i \\ . \\ . \\ . \\ z_{N_k^i,k}^i \end{pmatrix}, i \in \{\text{``}+\text{''}; \text{``}-\text{''}\} \text{ and } k = 1, 2, ..., K$$

As in the literature (e.g. ABL; Hey et al., 2009; Bruhin et al., 2010, henceforth BFE10), we assume that certainty equivalents are observed with additive response error with mean 0. Thus, the empirical counterpart of (9) is given by:[13]

$$\boldsymbol{ce}_l = u^{-1}\left[(u(\boldsymbol{x}_l) - u(\boldsymbol{y}_l)) \times \left(\sum_{k=1}^{K}(\delta_k^+ \boldsymbol{D}_l^+ + \delta_k^- \boldsymbol{D}_l^-)\boldsymbol{I}_l^k\right) + u(\boldsymbol{y}_l)\right] + \boldsymbol{e}_l^i \tag{10}$$

where $\boldsymbol{I}^k$ is a dummy variable set to 1 if the probability equals p_k and 0 otherwise, $\boldsymbol{D}^+$ ($\boldsymbol{D}^-$) is a dummy variable set to 1 for a positive (negative) certainty equivalent and 0 otherwise, $\boldsymbol{e}^i$ the response error term and l is the l^{th} line in $\boldsymbol{ce}$, $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{I}^k$ and $\boldsymbol{e}^i$. Probability weights correspond to:[14] [15]

$$w^+(p_k) = \delta_k^+ \text{ and } w^-(p_k) = \delta_k^- \text{ for } k = 1, ..., K \tag{11}$$

Note that u^{-1} in (10) should be written in the full domain. To do that, one can first write u in the full domain using indicator functions as in (4) and (5), and u^{-1} can be derived using

[13] An alternative approach would be to introduce the response error term at the utility level (GW99, eq. 7): $u(\boldsymbol{ce}_l) = (u(\boldsymbol{x}_l) - u(\boldsymbol{y}_l)) \times \left(\sum_{k=1}^{K}(\delta_k^+ \boldsymbol{D}_l^+ + \delta_k^- \boldsymbol{D}_l^-)\boldsymbol{I}_l^k\right) + u(\boldsymbol{y}_l) + \boldsymbol{e}_l^i$. However, defining the response error at the utility level is problematic when using certainty equivalents data because it produces solutions that are characterized by unrealistic concavity of the utility and probability weighting functions. To illustrate this point, suppose that we are interested in eliciting utility only over strictly positive outcomes with a power utility function $u(z) = z^\alpha$. For an extremely concave utility function (i.e., $\alpha > 0$ and $\alpha \longrightarrow 0$) and an extremely concave weighting function (i.e., $\delta_k^+ = 1$ for $k = 1, 2, .., K$) along with the PT assumptions $w^+(0) = 0$ and $w^+(1) = 1$, we have $\boldsymbol{e}_l^+ = 0$ for all $l = 1, 2, ..., N^+$. For the non-parametric method of GW99 which aims at estimating $u(z)$ for $z \in A(z) \equiv \{\$25, \$50, \$75, \$100, \$150, \$200, \$400, \$800\}$ and the probability weights $w(p)$ for $p \in B(p) \equiv \{0.01, 0.05, 0.10, 0.25, 0.40, 0.50, 0.60, 0.75, 0.90, 0.95, 0.99\}$, it follows that an extremely concave utility function (i.e., $u(z) = constant > 0$ for $z \in A(z)$ and $u(0) = 0$) and an extremely concave weighting function (i.e., $w(p) = 1$ for $p \in B(p)$ and $w(0) = 1 - w(p) = 0$) are solutions of the optimization problem. A similar issue can be found in Section 4.3 of Green and Silverman (1993).

[14] Take a probability p_s for $s \in 1, 2, ..., K$. $\boldsymbol{I}_l^s$ equals 1 for any observation l that involves p_s, and all the other probability dummy variables $\boldsymbol{I}_l^c$ for $c \neq s$ are set to 0. In that case, we have $\sum_{k=1}^{K}(\delta_k^+ \boldsymbol{D}_l^+ + \delta_k^- \boldsymbol{D}_l^-)\boldsymbol{I}_j^k = \delta_s^+ \boldsymbol{D}_l^+ + \delta_s^- \boldsymbol{D}_l^-$. In the gain domain, the dummy variable $\boldsymbol{D}_l^+$ equals 1 while $\boldsymbol{D}_l^-$ equals 0 so that we get $\sum_{k=1}^{K}(\delta_k^+ \boldsymbol{D}_l^+ + \delta_k^- \boldsymbol{D}_l^-)\boldsymbol{I}_j^k = \delta_s^+ \boldsymbol{D}_l^+ + \delta_s^- \boldsymbol{D}_l^- = \delta_s^+$. Hence, $w^+(p_k) = \delta_k^+$ in (10) for $k = 1, 2, ..., K$. Analogously, $w^-(p_k) = \delta_k^-$ in (10) for $k = 1, 2, ..., K$.

[15] Note that we do not require monotonicity of the weighting function as is commonly done in the literature (see e.g. GW99, p. 147). Requiring monotonicity in our method can be achieved by adding the following restriction on weights: $\delta_k^i = \delta_1^i + \sum_{j=2}^{k} exp(a_j^i)$ for $k \geq 2$. In this case, the estimated parameters are δ_1^i, $a_2^i, a_3^i, ..., a_K^i$.

standard algebra. Finally, the parameters in (10) can be estimated as long as one provides a functional form for u (and thus for u^{-1}), such as (4) or (5).[16]

The estimation of equation (10) can be done with either NLS or MLE. Under standard assumptions of normally distributed error terms and homoscedasticity, both methods provide identical point estimates for risk-attitude components, with MLE being a more efficient method (see, e.g. Wooldridge, 2010, p. 470).[17]

In addition, two sources of heteroscedasticity can be present at the level of individual data: the variance of the error term may vary (i) with respect to the range $|x_l - y_l|$ of a lottery, but also (ii) across domains $i = +, -$ (gains vs. losses). Both of them can be accounted for by MLE (l'Haridon and Vieider, 2019, BFE10). Herein, we adopt a more general form of heteroscedasticity than these two studies by assuming that $\sigma_{i,l} = \sigma_i |x_l - y_l|^\psi$ with $\psi \geq 0$ where $\psi \neq 0$ implies there is heteroscedasticity due to the range of outcomes.[18]

Our method allows us to account for these various sources of heteroscedasticity by applying MLE to equation (10). The log-likelihood function is:

$$\begin{aligned} logL(\theta^+, \theta^-, \delta^+, \delta^-, \sigma^+, \sigma^-, \psi) = & -(N^+ + N^-)log\left(\sqrt{2\pi}\right) - \sum_{l=1}^{N^+ + N^-} log(\sigma_i \left|\boldsymbol{x}_l - \boldsymbol{y}_l\right|^\psi) - \\ & \frac{1}{2}\sum_{l=1}^{N^+ + N^-}\left(\frac{\boldsymbol{e}_l^i}{\sigma_i \left|\boldsymbol{x}_l - \boldsymbol{y}_l\right|^\psi}\right)^2 \end{aligned} \tag{12}$$

where θ^i stands for the parameters associated with the utility function in domain i. Maximizing the log-likelihood with respect to all the parameters provides a simultaneous estimation of the utility function and probability weights for the gain and loss domains.

We use two criteria to assess the achieved goodness of fit: Akaike Information Criterion (AIC) and leave-one-out Cross Validation (CV). AIC is a standard measure in the literature (see, e.g. Fehr-Duda et al., 2006; Stott, 2006; Hey and Orme, 1994) and is given as $AIC = 2n_p - 2logL$, where $logL$ is the log-likelihood function given in equation (12) and n_p is the total number of estimated parameters in the utility function, the probability weights and the variance of response error. For a one-parameter and domain-specific utility function (as in equations 4 and 5), there are $n_p = 2K + 5$ parameters in the AIC computation: $2K$ probability weights (gain and loss domains), 2 utility parameters (gain and loss domains), and 3 variance parameters σ_+, σ_- and ψ. As a descriptive alternative to AIC, we also compute CV based on the following multi-step procedure (see, e.g. Baillon et al., 2020, for a similar approach). In each step, we estimate the model on $N^+ + N^- - 1$ non-mixed lotteries and predict the certainty equivalent for the remaining (excluded) lottery. The (absolute) difference between the predicted and actual certainty equivalents is the (absolute) prediction error for that lottery. This out-of-sample prediction procedure is repeated over $N^+ + N^-$ steps such that each lottery is left out of the sample once. Then, the value of the criterion is the mean absolute prediction error.

The certainty equivalent in equation (10) is given by equations (13) and (14) for power and exponential utility functions, respectively

[16]One can choose the functional form that best performs in terms of goodness of fit (e.g. Hey and Orme, 1994; Fehr-Duda et al., 2006; Stott, 2006).

[17]In Appendix A4, we also provide an illustration of how to apply Bayesian techniques with our semi-parametric method using data from l'Haridon and Vieider (2019).

[18]A third source of heteroscedasticity may arise at the aggregate level (pooled data) when the variance of response errors differs across individuals (e.g. Harrison and Rutström, 2008, 2009; l'Haridon and Vieider, 2019, BFE10)

$$\boldsymbol{ce}_l = (\boldsymbol{D}_l^+ - \boldsymbol{D}_l^-)\left[\left(|\boldsymbol{x}_l|^{\alpha_p \boldsymbol{D}_l^+ + \beta_p \boldsymbol{D}_l^-} - |\boldsymbol{y}_l|^{\alpha_p \boldsymbol{D}_l^+ + \beta_p \boldsymbol{D}_l^-}\right)\left(\sum_{k=1}^{K}(\delta_k^+ \boldsymbol{D}_l^+ + \delta_k^- \boldsymbol{D}_l^-)\boldsymbol{I}_l^k\right) + |\boldsymbol{y}_l|^{\alpha_p \boldsymbol{D}_l^+ + \beta_p \boldsymbol{D}_l^-}\right]^{\frac{1}{\alpha_p \boldsymbol{D}_l^+ + \beta_p \boldsymbol{D}_l^-}} + \mathbf{e}_l^i \tag{13}$$

$$\boldsymbol{ce}_l = ln\left[\left(exp((\beta_e \boldsymbol{D}_l^- - \alpha_e \boldsymbol{D}_l^+)\boldsymbol{x}_l) - exp((\beta_e \boldsymbol{D}_l^- - \alpha_e \boldsymbol{D}_l^+)\boldsymbol{y}_l)\right)\left(\sum_{k=1}^{K}(\delta_k^+ \boldsymbol{D}_l^+ + \delta_k^- \boldsymbol{D}_l^-)\boldsymbol{I}_l^k\right) + exp((\beta_e \boldsymbol{D}_l^- - \alpha_e \boldsymbol{D}_l^+)\boldsymbol{y}_l)\right]\frac{1}{\beta_e \boldsymbol{D}_l^- - \alpha_e \boldsymbol{D}_l^+} + \mathbf{e}_l^i \tag{14}$$

3.2.3 Estimating loss aversion

As a second step, we measure the loss aversion index λ as defined in (3) based on the estimates of the utility function and the probability weights outlined in subsection 3.2.2. Following Abdellaoui et al. (2007b), the estimation of the loss aversion index can be done using a set of K indifference relationships that involve mixed binary prospects:

$$ce_k \sim (x_k, y_k; p_k, 1-p_k)\ , \quad k = 1, 2, ..., K \tag{15}$$

where $y_k < 0 < x_k$. Under CPT, these indifferences imply that:

$$ce_k = v^{-1}\left[w^+(p_k)v(x_k) + w^-(1-p_k)v(y_k)\right] \tag{16}$$

Because ce_k belongs to the interval (y_k, x_k), it could either be a gain or a loss. Also, note that for each k both $w^+(p_k)$ and $w^-(1-p_k)$ are known since they have been elicited in the previous step. Echoing our previous notation, let $\boldsymbol{ce}$, $\boldsymbol{x}$ and $\boldsymbol{y}$ contain the realizations of ce_k, x_k and y_k. In addition, denote by $\boldsymbol{\delta}^+$ and $\boldsymbol{\delta}^-$ the column vectors such that $\boldsymbol{\delta}^{+'} \equiv (\delta_1^+, \delta_2^+, ..., \delta_K^+)$ and $\underline{\boldsymbol{\delta}}^{-'} \equiv (\underline{\delta}_1^-, \underline{\delta}_2^-, ..., \underline{\delta}_K^-) = (\delta_K^-, \delta_{K-1}^-, ..., \delta_1^-)$.[19] Assuming that certainty equivalents are observed with an additive and normally distributed response error term $(\boldsymbol{e}_k)$, the empirical counterpart of equation (16) then becomes:

$$\boldsymbol{ce}_k = v^{-1}\left[\boldsymbol{\delta}_k^+ v(\boldsymbol{x}_k) + \underline{\boldsymbol{\delta}}_k^- v(\boldsymbol{y}_k)\right] + \boldsymbol{e}_k \tag{17}$$

v^{-1} in (17) can be derived similarly to u^{-1} in (10). Then, the respective certainty equivalent equations for power and exponential utility functions become:

$$\boldsymbol{ce}_k = (\boldsymbol{D}_k^+ - \boldsymbol{D}_k^-)\left[\frac{\boldsymbol{\delta}_k^+(\boldsymbol{x}_k)^{\alpha_p} - \lambda \underline{\boldsymbol{\delta}}_k^-(-\boldsymbol{y}_k)^{\beta_p}}{\boldsymbol{D}_k^+ - \lambda \boldsymbol{D}_k^+}\right]^{\frac{1}{\alpha_p \boldsymbol{D}_k^+ + \beta_p \boldsymbol{D}_k^-}} + \boldsymbol{e}_k \tag{18}$$

$$\boldsymbol{ce}_k = \frac{ln\left[1 - \frac{\alpha_e \boldsymbol{D}_k^+ + \beta_e \boldsymbol{D}_k^-}{\boldsymbol{D}_k^+ - \lambda \boldsymbol{D}_k^-}\left(\boldsymbol{\delta}_k^+\left(\frac{1-exp(-\alpha_e \boldsymbol{x}_k)}{\alpha_e}\right) - \lambda \underline{\boldsymbol{\delta}}_k^-\left(\frac{1-exp(\beta_e \boldsymbol{y}_k)}{\beta_e}\right)\right)\right]}{\beta_e \boldsymbol{D}_k^- - \alpha_e \boldsymbol{D}_k^+} + \boldsymbol{e}_k \tag{19}$$

Using the values of the probability weights and the parameters of $u(.)$ from the first step, we can estimate (18) or (19) by NLS or MLE to obtain λ.

[19] For clarity of exposition, we use underscores to refer to the loss domain so that: $\underline{\delta}_k^- \equiv \delta_{K-k+1}^- = w^-(1-p_k)$ for $k = 1, 2, ..., K$.

3.2.4 Key properties of our method

Comparison with ABL. We refine and extend the method previously proposed by ABL in several ways.

First, unlike ABL, we can estimate multiple probability weights and thus elicit the shape of the probability weighting function.

Second, our method uses a single step to estimate the probability weights and the utility function in the full domain, whereas ABL propose a two-step procedure. This feature of our method allows for testing several important restrictions (partial reflection, identical probability weighting functions across domains, and duality) as well as imposing these restrictions whenever necessary.

Imposing partial reflection helps avoid the problem of arbitrary measurement of loss aversion with power utility functions (see Wakker, 2010). Testing for identical probability weighting functions across domains (*i.e.*, $w^+(p_k) = w^-(p_k)$ for all k) allows us to test a key assumption of OPT. In addition, this assumption must also be made under CPT whenever loss aversion is present and preferences are homogeneous (Al-Nowaihi et al., 2008).[20] Our method allows for testing and imposing duality (*i.e.*, $w^+(p_k) = 1 - w^-(1-p_k)$ for all k).[21] By allowing for testing and imposing duality as well as identical probability weighting across domains, our method can be applied under RDU (Quiggin, 1982; Gilboa, 1987; Schmeidler, 1989) and OPT (KT79). This is not the case for existing parametric, semi-parametric, or non-parametric methods.

Third, certainty equivalents for mixed prospects are obtained using a different procedure than the one proposed by ABL. In ABL, subjects are asked to provide a loss amount L for which they are indifferent between the status-quo (0) and a binary lottery $(G, L; p_g, 1-p_g)$ where G is a fixed gain and $L \in (-\infty, 0]$ is a loss. In this elicitation procedure, the researcher does not know the lower bound of the loss interval. By contrast, equation (15) keeps track of the upper and lower bounds of the loss interval because ce_k belongs to the interval (y_k, x_k). This is an appealing property of our method for two reasons. First, asking subjects to provide indifference values on unbounded intervals can be cognitively demanding (Wakker and Deneffe, 1996; Abdellaoui et al., 2007b). This may lead to errors that potentially inflate the estimates of loss aversion, as reported by ABL (see Table 11, pp. 263-264). Second, eliciting indifference values on bounded intervals allows us to use a standard switching outcome procedure (Booij and Van de Kuilen, 2009).

Comparison to standard parametric methods. Our method retains all the appealing properties of parametric methods. First, it is as data-efficient as parametric methods because, for K probability weights to be elicited, the smallest number of certainty equivalents required to measure all the three components of risk attitudes in the full domain is $5K$ (*i.e.*, $2K$ certainty equivalents in the gain domain, $2K$ in the loss domain and K for mixed lotteries). Second, we use simple choices (comparisons of certain outcomes and binary lotteries) so that the method is not cognitively demanding for subjects. Third, our method is tractable because we can measure risk attitudes using standard econometric tools. Fourth, our estimation method accounts for response errors.

In parametric methods, it is key to assess the validity of the functional forms used for the probability weighting and utility functions. However, parametric methods do not allow the researcher to separately assess the goodness of fit of each of these functions (GW99). In contrast, our semi-parametric method does not make any parametric assumption regarding the probability weighting function and allows the researcher to evaluate the goodness of fit

[20] Homogeneity of preferences holds whenever multiplying all the payoffs of a non-mixed lottery by a positive constant c also leads the certainty equivalent of the lottery to be multiplied by c.

[21] As pointed out by Abdellaoui (2000, pp. 1509-1510), testing for duality with parametric methods and non-parametric methods based on the tradeoff approach requires using the specific probability weighting function of Goldstein and Einhorn (1987) and Lattimore et al. (1992).

of the utility function separately. Thus, one can select the utility function with the best fit, further improving the accuracy of the elicitation of risk attitudes of the semi-parametric method compared to the parametric one.

Applicability to unknown probabilities. Our method is also directly applicable to cases of uncertainty where probabilities are unknown. It does not require setting any specific conditions on the event space, and hence can be applied to real-life uncertainty situations (Baillon et al., 2018).[22] Extending our method to the case of uncertainty can thus be done by replacing probability dummy variables by event dummy variables in equations (10) and (17).

Robustness to monotonicity problem. We further note that the monotonicity problem raised by Apesteguia and Ballester (2018), which could lead to identification issues, does not apply to our case. First, our method is not based on binary choices (as in random utility models), but on eliciting certainty equivalents. Second, we define the error term at the certainty equivalent level and not at the utility level. For power and exponential utility functions, the certainty equivalent is monotonic in the utility parameters and probability weights. This is also true for a broad range of utility functions once we use the Arrow-Pratt approximation for binary lotteries in the context of RDU and CPT. A related problem to Apesteguia and Ballester (2018) can arise with the certainty equivalent method when the error term is defined at the utility level, as in the non-parametric method of GW99 (see footnote 13 for a discussion of these issues).

Spline extension. Our semi-parametric method requires specifying a utility function which allows us to keep the data-efficiency property of parametric methods. However, in the case of an extensive dataset (as in GW99), our semi-parametric method can be extended to use spline approximation of the utility function. In appendix A3, we provide a linear spline extension for our semi-parametric method.

4 Parameter recovery and misspecification

Following Gao et al. (2020), Nilsson et al. (2011) and Murphy and ten Brincke (2018), we report in this section two types of simulation exercises: parameter recovery and robustness to misspecification. In the parameter recovery exercise, we estimate a model assuming that we know the specification of the utility and weighting functions used to simulate the data. Our aim is to assess the extent to which an estimation method can identify the parameters underlying the simulated data. In the robustness to misspecification exercise, we estimate a model assuming an incorrect specification of the utility and weighting functions and check for the extent to which an estimation method identifies the underlying parameters from simulated data. For the sake of comparison, the simulations are made for both our semi-parametric method and parametric methods. Subsections 4.1 and 4.2 explain the simulation exercises and results are presented in subsection 4.3.

4.1 Parameter recovery

The calibration of lotteries follows TK92. We consider 9 probabilities – 0.01, 0.05, 0.10, 0.25, 0.50, 0.75, 0.90, 0.95, and 0.99 – along with possible outcomes (x, y) in each domain: $(0, 100), (0, 400)$ and $(50, 150)$ for the gain domain, and $(-100, 0), (-400, 0)$ and $(-150, -50)$

[22]Non-parametric methods (e.g. Van De Kuilen and Wakker, 2011; Abdellaoui et al., 2020) require the event space to be rich (e.g. continuous). This means that in applied contexts, the universal event is an interval on a real scale (e.g. temperature in town). The semi-parametric method of Abdellaoui et al. (2011a) requires equally likely events either with (i) preset priors (like in Ellsberg's urn experiments), or (ii) a rich event space analogous to non-parametric methods.

for the loss domain. Each of the 9 probabilities is combined with each pair of outcomes, resulting in a lottery $(x, y; p, 1-p)$. This gives a total of 27 lotteries in each domain, and each probability occurs three times in the dataset.

4.1.1 Our method

Simulation 1: Power utility function. We simulate data for 1000 ($s = 1, 2, ..., 1000$) hypothetical subjects. For each subject s, we generate random parameters of the power utility functions in the gain and loss domains from a uniform distribution $U(0.1, 2.1)$ (Spiliopoulos and Hertwig, 2019, ABL).

For the 9 probabilities involved in the lotteries, we generate probability weights using a uniform distribution $U(0, 1)$, making sure that higher probabilities are assigned to higher weights.

Then, these simulated power utility parameters (one per domain) and probability weights (9 per domain) are plugged into CPT formulas to generate *noiseless* certainty equivalents. In the last step of the data generation process, we use two random variables from $U(0, 0.025)$ to simulate standard deviations $\sigma_{s,i}$ in each domain $i \in \{+, -\}$, and then draw 27 random values from $N(0, \sigma_{s,i}|x-y|)$ which we add to the previously generated 27 *noiseless* certainty equivalents to obtain the *noisy* ones.[23] Finally, we use the *noisy* certainty equivalents as input data and compute MLE outcomes for our semi-parametric method.

Simulation 2: Exponential utility function. This simulation exercise is based on the same principles as Simulation 1, the sole difference being the utility function. This time, we draw exponential utility parameters for each domain from $U(-0.01, 0.01)$.[24]

4.1.2 Parametric methods

We consider eight common parametric specifications. These parametric specifications arise from the combination of the two standard utility functions (power and exponential) (see Stott, 2006) and four popular weighting functions (see TK92; Goldstein and Einhorn, 1987; Lattimore et al., 1992; Prelec, 1998). These weighting functions are:

$$w^i(p) = \frac{p^{(a\mathbf{1}_{(i=+)}+c\mathbf{1}_{(i=-)})}}{\left[p^{(a\mathbf{1}_{(i=+)}+c\mathbf{1}_{(i=-)})} + (1-p)^{(a\mathbf{1}_{(i=+)}+c\mathbf{1}_{(i=-)})}\right]^{\frac{1}{a\mathbf{1}_{(i=+)}+c\mathbf{1}_{(i=-)}}}} \tag{20}$$

where $a, c \in (0, 1]$

$$w^i(p) = exp\left[-\left(-ln(p)\right)^{b\mathbf{1}_{(i=+)}+d\mathbf{1}_{(i=-)}}\right] \tag{21}$$

where $b > 0, d > 0$

$$w^i(p) = exp\left[-\left(b\mathbf{1}_{(i=+)} + d\mathbf{1}_{(i=-)}\right) \times \left(-ln(p)\right)^{a\mathbf{1}_{(i=+)}+c\mathbf{1}_{(i=-)}}\right] \tag{22}$$

where $a > 0, b > 0, c > 0, d > 0$

$$w^i(p) = \frac{(b\mathbf{1}_{(i=+)} + d\mathbf{1}_{(i=-)})p^{(a\mathbf{1}_{(i=+)}+c\mathbf{1}_{(i=-)})}}{(b\mathbf{1}_{(i=+)} + d\mathbf{1}_{(i=-)})p^{(a\mathbf{1}_{(i=+)}+c\mathbf{1}_{(i=-)})} + (1-p)^{(a\mathbf{1}_{(i=+)}+c\mathbf{1}_{(i=-)})}} \tag{23}$$

[23] A standard deviation of $0.025 \times |x-y|$ implies response errors of +/- \$20 around the true certainty equivalent value, which seems large given the range of lottery outcomes [0, \$400].

[24] Note that the scale of the exponential utility parameter depends on the scale of the outcomes used in the lotteries. Taking into account the midpoint of the outcome range $[0, 400]$, the index of absolute risk aversion generated by a power utility function with a parameter in the range $(0, 2)$ is approximately equal to the index generated by an exponential utility with a parameter in the range $\left(-\frac{1}{200}, \frac{1}{200}\right)$. In the simulation exercise, we allow for a wider range for that parameter: $(-0.01, 0.01)$.

where $a > 0, b > 0, c > 0, d > 0$. Equation (20) represents the one-parameter weighting function (per domain) of TK92. Equations (21) and (22) refer to the one- and two-parameter weighting functions of Prelec (1998), and (23) is the two-parameter weighting function of Goldstein and Einhorn (1987) and Lattimore et al. (1992). The vast majority of parametric estimations in the literature rely on one of these four weighting functions in combination with a standard utility function (Stott, 2006). Henceforth, we refer to (20), (21), (22) and (23) as TK92, P98-I, P98-II and GE87 respectively.[25]

Simulation 3 and 4. We then run two additional series of simulations (Simulations 3 and 4) that are based on the same principles as Simulations 1 and 2 (respectively), the sole difference being the weighting function. Depending on the simulation, we specify one of the four parametric weighting functions. For the weighting function of TK92, we draw the values for a and c from $U(0.2, 1)$ (e.g. Dhami, 2016, p. 122). For the remaining weighting functions, we draw a, b, c and d from $U(0.1, 1.5)$.

4.2 Parameter recovery under model misspecification

4.2.1 Our method

Simulation 5: Power utility function. We consider the data generated in Simulation 4 under an exponential utility function in combination with each of the four weighting functions in turn. We apply our estimation procedure to each of these four simulated datasets, while misspecifying the utility function which is assumed to be power instead of exponential.

Simulation 6: Exponential utility function. We follow the same procedure as in Simulation 5, this time using the data generated in Simulation 3 with power utility function. For the sake of model misspecification, we assume utility to be exponential.

4.2.2 Parametric methods

Simulation 7 and 8. As before, we run two additional series of simulations for parametric methods. In Simulation 7 (8), we rely on data previously generated in Simulation 2 (1) and misspecify the model in the same way as in Simulation 5 (6).

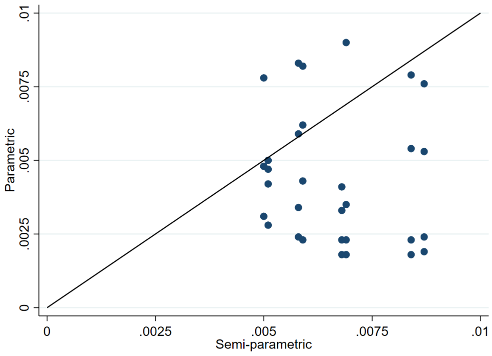
4.3 Result of parameter recovery and misspecifcation

Table 2 shows the average absolute error in parameter estimates, which is defined as the absolute difference between the true parameter value and its estimate. Figure 1 plots the average absolute error in parameter estimates across all parameters and all simulations. When a point is above the 45° line, this means that the average absolute error in the parametric specification is greater than the corresponding simulation in the semi-parametric specification. For parameter recovery exercises (Figure 1, panel (a)), parametric specifications tend to perform better than the semi-parametric method. However, for misspecification exercises, the semi-parametric method performs better than parametric methods (Figure 1, panel (b)). Taking both types of simulations into account (figure 1, panel (c)), we highlight that average absolute errors in parameter recovery exercises are substantially smaller than misspecification errors. Overall, we conclude that the semi-parametric method (i) is less sensitive to model misspecification and (ii) produces more reliable estimates when the model is misspecified than standard parametric methods.

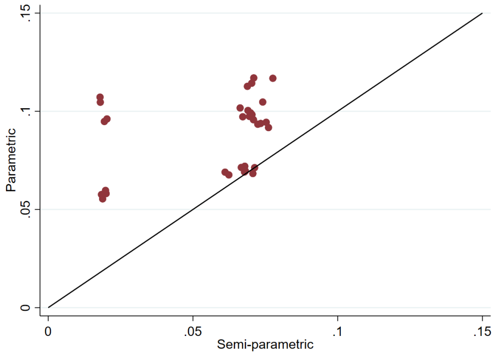
[25] For these specifications, identical probability weighting across domains corresponds to $a = c$ and $b = d$. Note that an appealing property of the GE87 specification is that it allows for a straightforward test of duality (Abdellaoui, 2000) by checking whether $a = c$ and $b = \frac{1}{d}$.

Figure 1: Absolute error in estimates: semi-parametric vs. parametric methods

(a) Parameter recovery



(b) Misspecification



(c) Parameter recovery and misspecification

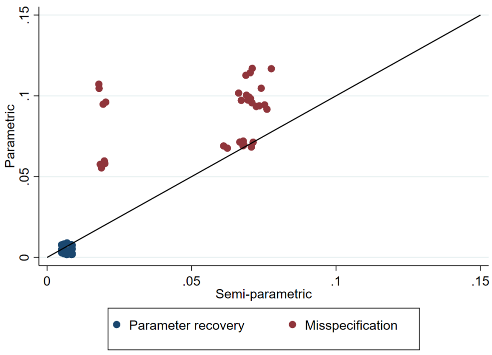


Table 2: Simulation results

Simulation n°	Estimation		Data generation		Average absolute value of error			
	$u()$	$w()$	$u()$	$w()$	α	w^+	β	w^-
Parameter recovery								
1	Power	our method	Power	our method	0.0059	0.0087	0.0058	0.0084
2	Expo	our method	Expo	our method	0.0050	0.0069	0.0051	0.0068
3	Power	TK92	Power	TK92	0.0043	0.0024	0.0034	0.0023
3	Power	P98-I	Power	P98-I	0.0023	0.0019	0.0024	0.0018
3	Power	P98-II	Power	P98-II	0.0062	0.0053	0.0059	0.0054
3	Power	GE87	Power	GE87	0.0082	0.0076	0.0083	0.0079
4	Expo	TK92	Expo	TK92	0.0048	0.0023	0.0042	0.0023
4	Expo	P98-I	Expo	P98-I	0.0031	0.0018	0.0028	0.0018
4	Expo	P98-II	Expo	P98-II	0.0078	0.0090	0.0050	0.0041
4	Expo	GE87	Expo	GE87	0.0048	0.0035	0.0047	0.0033
Misspecifcation								
5	Power	our method	Expo	TK92	0.0703	0.0180	0.0710	0.0179
5	Power	our method	Expo	P98-I	0.0672	0.0194	0.0690	0.0203
5	Power	our method	Expo	P98-II	0.0611	0.0198	0.0624	0.0200
5	Power	our method	Expo	GE87	0.0667	0.0184	0.0681	0.0188
6	Expo	our method	Power	TK92	0.0688	0.0776	0.0663	0.0741
6	Expo	our method	Power	P98-I	0.0695	0.0724	0.0699	0.0753
6	Expo	our method	Power	P98-II	0.0707	0.0709	0.0713	0.0704
6	Expo	our method	Power	GE87	0.0678	0.0761	0.0679	0.0734
7	Power	TK92	Expo	our method	0.1143	0.1046	0.1170	0.1072
7	Power	P98-I	Expo	our method	0.0972	0.0948	0.1004	0.0961
7	Power	P98-II	Expo	our method	0.0690	0.0597	0.0676	0.0581
7	Power	GE87	Expo	our method	0.0714	0.0576	0.0696	0.0554
8	Expo	TK92	Power	our method	0.1127	0.1168	0.1017	0.1047
8	Expo	P98-I	Power	our method	0.0974	0.0934	0.0992	0.0944
8	Expo	P98-II	Power	our method	0.0683	0.0957	0.0713	0.0983
8	Expo	GE87	Power	our method	0.0691	0.0917	0.0720	0.0938

5 Applications

In this section we use existing experimental data to compare our semi-parametric method and parametric methods. Because our method relies on the elicitation of certainty equivalents, we compare it to parametric methods that also make use of certainty equivalents. Since our exercise requires the use of datasets that allow for the elicitation of certainty equivalents, we start this section by detailing our choice of datasets before comparing the methods in terms of goodness of fit and providing estimation results.

5.1 Data

To apply our method, we need to elicit certainty equivalents in the gain and loss domains for two-outcome lotteries. These lotteries should vary each of the outcomes as well as the corresponding probabilities, and at least two certainty equivalents should be elicited for each probability.

We made an extensive search of the literature to identify available datasets from which we could make individual estimates. We reviewed the datasets from Harrison and Rutström (2009); Eisenberg et al. (2019); l'Haridon and Vieider (2019); Pedroni et al. (2017); Andersson et al. (2020); BFE10; GW99 and TK92. The data used in Harrison and Rutström (2009); Eisenberg et al. (2019); Pedroni et al. (2017); Andersson et al. (2020) rely on binary choices, which cannot be used as input in our method. The data of Bruhin et al. (2010); l'Haridon and Vieider (2019); GW99 and TK92 contain certainty equivalents for binary lotteries. However, the dataset in l'Haridon and Vieider (2019) does not match our criteria at the individual level because it only contains one certainty equivalent for some of the probability weights. The dataset of GW99 satisfies our criteria, with an important caveat that it contains observations from only 10 subjects and solely in the gain domain. By contrast, BFE10 [and more specifically, their "Zurich 03" experiment][26] collected data on 179 subjects in the gain and loss domains with several certainty equivalents per probability, thus fully matching our selection criteria. However, this dataset does not include mixed lotteries, thus not allowing us to estimate loss aversion. In addition, TK92 provide median data that contain several certainty equivalents per probability in the gain and loss domains, also in line with our selection criteria. They also include mixed lotteries so that we can elicit loss aversion. We thus apply our method to the median data of TK92 and to the individual data of BFE10.

5.2 Goodness of fit across models

We start by evaluating the goodness of fit of our method relative to parametric alternatives. The corresponding values of AIC and CV are reported in Tables 3 and 4.

For the data of TK92 (BFE10), power (exponential) utility function best fits the data under our semi-parametric method according to both AIC and CV. The best parametric specification is the combination of a power utility function and the one-parameter weighting function of TK92 (an exponential utility function and the two-parameter weighting function of GE87) for the data of TK92 (BFE10).

For each dataset and each criterion, the best-fitting specification under our semi-parametric method outperforms the best-fitting parametric specification. This implies that our semi-parametric method fits the data better than standard parametric methods.

Table 3: Goodness of fit across methods: data of TK92

	Parametric								Semi-parametric	
	AIC*				CV*				AIC	CV
$u(.)w(.)$	TK92†	P98-I†	P98-II†	LG92†	TK92†	P98-I†	P98-II†	LG92†	-	-
Power	320	327	321	318	3.41	3.57	3.52	3.87	284‡	3.15‡
Exponential	339	347	328	320	6.21	4.35	4.06	5.39	286	3.31

* AIC: Akaike information criterion, CV: Leave one out of sample cross-validation. The best specification is the one that minimizes a considered criterion (AIC or CV).

† TK92 (one-parameter weighting function of Tversky and Kahneman (1992)), P98-I (one-parameter weighting function of Prelec (1998)),P98-II (two-parameter weighting function of Prelec (1998), LG92 (two-paramete weighting function of Lattimore et al. (1992)))

‡ The semi-parametric method with power utility function provides smaller AIC and CV

[26]The authors also conducted two other experiments, but these datasets do not match our criteria at the individual level since they include only one certainty equivalent for some of the probability weights.

Table 4: Goodness of fit across methods: data of BFE10

	Parametric								Semi-parametric	
	AIC*				CV*				AIC	CV
$u(.)w(.)$	TK92†	P98-I†	P98-II†	LG92†	TK92†	P98-I†	P98-II†	LG92†	-	-
Power	51110	50980	50895	50828	10.005	4.381	4.383	4.375	50776	4.400
Exponential	51489	51066	50704	50636	4.842	4.512	4.304	4.253	50601‡	4.249‡

* AIC: Akaike information criterion, CV: Leave one out of sample cross-validation. The best specification is the one that minimizes a considered criterion (AIC or CV).
† TK92 (one-parameter weighting function of Tversky and Kahneman (1992)), P98-I (one-parameter weighting function of Prelec (1998)),P98-II (two-parameter weighting function of Prelec (1998), LG92 (two-paramete weighting function of Lattimore et al. (1992)))
‡ The semi-parametric method with exponential utility function provides smaller AIC and CV

5.3 Results

In this section, we focus our analysis on the best-fitting specifications under parametric and semi-parametric methods, as highlighted in the previous section.

5.3.1 Curvature of the utility function for gains and losses

Table 5 summarizes the semi-parametric and parametric estimates of the main components of the CPT value function (curvature in each domain and loss aversion) for the data of TK92 and BFE10.[27] In the remainder of the results section, we use z-tests to assess whether a coefficient is equal to a specific value and whether two coefficients are equal, and χ^2-tests for joint hypotheses. Tests are two-sided, unless stated otherwise.

Our semi-parametric estimations show that the utility function is concave in the gain domain: the estimated values of $\alpha = 0.525$ (TK92) and $\alpha = 0.587$ (BFE10) are significantly greater than 0.5 ($p-values < 0.0292$). In the loss domain, the estimated values also suggest the utility function is concave because $\beta = -0.483$ and $\beta = -0.425$ are greater than -0.5 ($p-value < 0.0757$ and $p-value < 0.0001$, respectively). Concavity in the loss domain is in line with the findings of ABL, Attema et al. (2013) and Etchart-Vincent and l'Haridon (2011). Furthermore, we reject partial reflection ($H_0 : \alpha + \beta = 0, p-values < 0.0058$).

5.3.2 Loss aversion

With our semi-parametric method, we replicate the standard finding of loss aversion, with $\lambda = 1.751$.[28] Our estimate of the loss aversion index is close to the estimated value of 1.6 that was elicited in both Booij et al. (2010) who use structural estimation techniques, and ABL for pooled data. It is also similar to the estimates reported by Tom et al. (2007), Pennings and Smidts (2003), and Booij and Van de Kuilen (2009): $\lambda = 1.93$, $\lambda = 1.8$, and $\lambda = 1.87$, respectively. Note that a large meta-analytical study by Brown et al. (2021) suggests that the

[27] The utility curvature is computed based on equations (6) and (7). Detailed results for the semi-parametric method are reported in Appendix A1.2 for TK92 and in Appendix A2 for BFE10.

[28] In section 3, we propose to estimate loss aversion λ in a second step after estimating utility and weighting functions in a first step. This two-step procedure we propose can be applied regardless of whether elicitation of loss aversion is of interest (which requires both steps) or not (which only requires the first step of estimation). In the former case, one could alternatively apply a one-step procedure (e.g. l'Haridon and Vieider, 2019) that simultaneously estimates equations (10) and (17). Table 17 in appendix A1.5 summarizes the maximum likelihood estimates of utility curvature, probability weights and loss aversion obtained through a one-step procedure. In the one-step procedure, estimated loss aversion is 1.688 compared to the median value of 1.751 in the two-step procedure.

mean loss aversion coefficient lies between 1.8 and 2.1. From that perspective, our estimates are on the conservative side and come close to the lower bound of that interval.

Table 5: Curvature of the utility function and loss aversion

	Data of TK92		Data of BFE10	
	Semi-parametric	Parametric	Semi-parametric	Parametric
Curvature Gain (α)	0.525	0.544	0.587	0.589
Curvature Loss (β)	-0.483	-0.525	-0.425	-0.428
Loss aversion (λ)	1.751	1.730	-	-

5.3.3 Probability weighting function

Data of TK92. Figure 2 presents the estimates of the probability weighting function across domains (labeled *Semi-para*) for the data of TK92. In the gain domain, probabilistic risk neutrality $w^+(p) = p$ is rejected for most probabilities (all $p - values < 0.0258$), except for 0.25 ($p - value = 0.0671$). Overall, we reject the joint hypothesis of linearity of the probability weighting function over the whole range of probabilities in the gain domain ($p - value < 0.0001$). The resulting weighting function is inverse S-shaped because there is overweighting for $p \in (0, 0.25]$, and underweighting for $p \in (0.25, 1)$. Similar patterns emerge in the loss domain, with overweighting starting for lower probabilities $p \in (0, 0.1]$, and then shifting to underweighting for $p \in (0.1, 1)$. We cannot reject $H_0 : w^-(0.1) = 0.1$ ($p - value = 0.7042$).

Over the 9 probabilities in the data of TK92, the hypothesis of identical probability weights across domains ($w^+(p_k) = w^-(p_k)$) is rejected for 4 probabilities (0.01, 0.05, 0.25 and 0.90; all $p - values < 0.0241$), but not for others (0.1, 0.5, 0.75, 0.95 and 0.99; all $p - values > 0.1192$). For tail probabilities ($p = 0.01$ and 0.05 in this dataset),[29] overweighting is more pronounced in the gain than in the loss domain ($p - values < 0.0121$). Using a joint test, the hypothesis of identical probability weights across domains is rejected ($p - value < 0.0001$).

The hypothesis of duality ($w^+(p_k) = 1 - w^-(1 - p_k)$) is rejected for 7 probabilities (all $p - values < 0.001$), with the exception of probabilities 0.01 and 0.05 (both $p - values > 0.2365$). Using a joint test, the duality hypothesis is rejected ($p - value < 0.0001$).

Data of BFE10. Figure 3 presents the estimates of the probability weighting function across domains (once again, labeled *Semi-para*) for the data of BFE10.

In the gain domain, probabilistic risk neutrality $w^+(p) = p$ is rejected for all probabilities (all $p - values < 0.0001$), except for 0.5 ($p - value = 0.9496$). Overall, we reject the hypothesis of linearity of the probability weighting function over the whole range of probabilities in the gain domain. The resulting weighting function is once again inverse S-shaped with overweighting for $p \in (0, 0.5]$ and underweighting for $p \in (0.5, 1)$.

Similar patterns emerge in the loss domain with overweighting for $p \in (0, 0.5]$, and underweighting for $p \in (0.5, 1)$. We cannot reject $H_0 : w^-(0.5) = 0.5$ ($p - value = 0.8881$).

Over the 7 probabilities in the data of BFE10, the hypothesis of identical probability weights across domains ($w^+(p_k) = w^-(p_k)$) is rejected for the probability $p = 0.05$ ($p - values < 0.0204$), but not for others (all $p - values > 0.0553$). For tail probability ($p = 0.05$ in this dataset), overweighting is more pronounced for gains that for losses. Using a joint test, the hypothesis of identical probability weights across domains is rejected ($p - value < 0.0001$).

Finally, the hypothesis of duality ($w^+(p_k) = 1 - w^-(1 - p_k)$) is rejected for $p = 0.9$ ($p - value < 0.0146$) but not for the remaining probabilities (all $p - values > 0.1159$). A joint test rejects the duality hypothesis ($p - value < 0.0467$).

[29] Tail probabilities are typically considered to be equal to 5% or less (see Barron and Erev, 2003; Erev, 2007; Erev et al., 2017; Hertwig et al., 2004; Corgnet et al., 2020).

Figure 2: Semi-parametric and parametric measurements of the probability weighting function (median data from TK92)

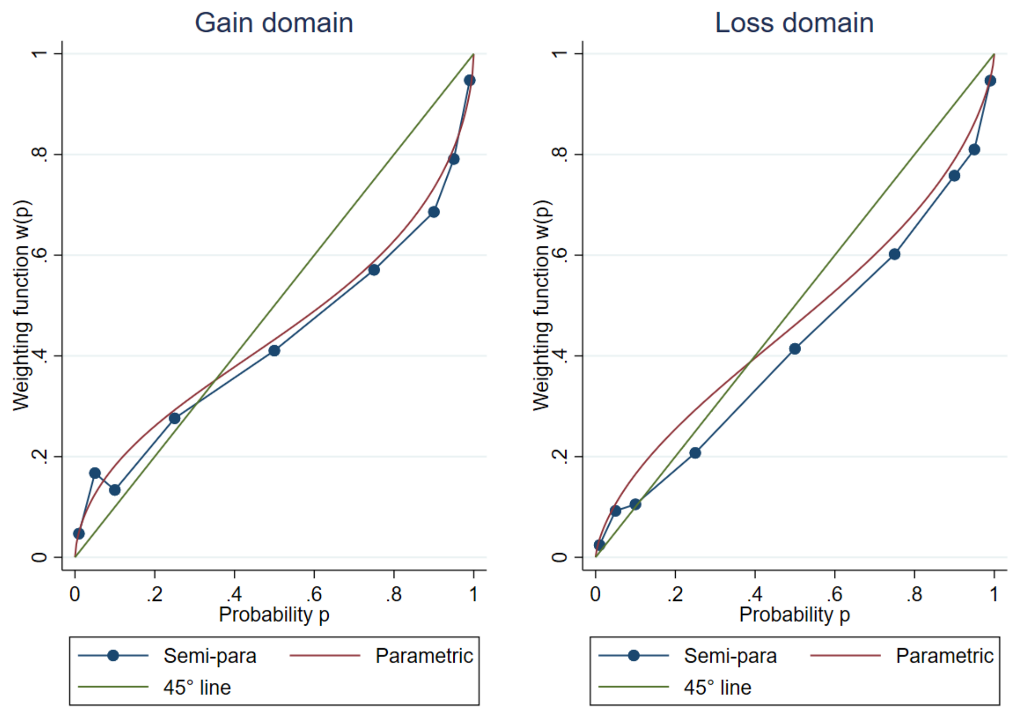
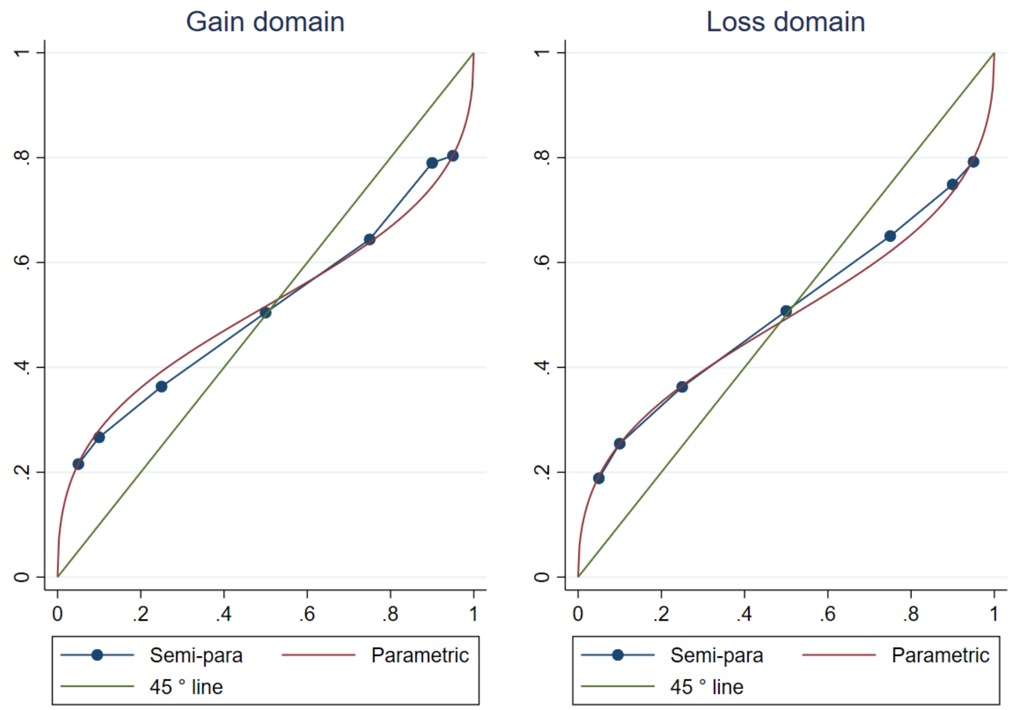


Figure 3: Semi-parametric and parametric measurements of the probability weighting function (results on pooled data of BFE10)



5.3.4 Discussion

Table 6 summarizes the discussion for both datasets. Echoing the seminal findings in TK92,[30] the parametric estimates reported in Table 5 imply concavity in the gain domain ($\alpha = 0.544$, statistically different from 0.5 with $p - value < 0.0001$) and convexity in the loss domain ($\beta = -0.525$, statistically different from -0.5 with $p - value = 0.001$). In addition, partial reflection cannot be rejected at the 5% significance level ($H_0 : \alpha + \beta = 0$, $p - value = 0.0936$). When considering the data of BFE10, the parametric estimates imply concavity in both gain ($\alpha = 0.589$, statistically different from 0.5 with $p-value < 0.0001$) and loss ($\beta = -0.428$, statistically different from -0.5 with $p - value < 0.0001$) domains. In addition, partial reflection is rejected ($H_0 : \alpha + \beta = 0$, $p - value < 0.0001$). We note that parametric estimates lead to inconsistent results across datasets for utility curvature in the loss domain and partial reflection. By contrast, semi-parametric estimates lead to different findings: (i) concave utility in both domains and (ii) rejection of partial reflection. Unlike parametric methods, these findings are consistent across datasets. The absence of convexity of the utility function in the loss domain is consistent with a number of previous studies (ABL; BFE10 Abdellaoui et al., 2011c; Attema et al., 2013, 2016; Kemel and Mun, 2020). ABL, Attema et al. (2013, 2016), Abdellaoui et al. (2011c) and Kemel and Mun (2020) use the semi-parametric method developed by ABL, whereas BFE10[31] use a parametric method with the two-parameter probability weighting function of GE87. In contrast, studies based on the tradeoff method find support for the convexity of the utility function in the loss domain (Abdellaoui, 2000; Etchart-Vincent, 2004, 2009b; Abdellaoui et al., 2007b, 2013, 2016; Booij and Van de Kuilen, 2009; Hajimoladarvish, 2017; Attema et al., 2018; Bleichrodt et al., 2018). From a theoretical standpoint, people who exhibit a concave utility in the loss domain can still be risk seeking (Chateauneuf and Cohen, 1994). Our semi-parametric method thus allows for such a possibility.

The empirical evidence on partial reflection in the literature is mixed. Our rejection of partial reflection is consistent with some studies (Abdellaoui et al., 2013, 2016; Attema et al., 2013, 2016, ABL) but not others (e.g. Abdellaoui, 2000; Andersen et al., 2006; Abdellaoui et al., 2007b; Booij and Van de Kuilen, 2009; Harrison and Rutström, 2009; Booij et al., 2010).

Parametric estimates on the data of TK92 lead to a rejection of equality of probability weighting function across domains ($p - value < 0.0001$). As observed by TK92, parametric estimates imply that both $w^+(0.5)$ and $w^-(0.5)$ are less than 0.5 so that the duality condition is rejected. When considering the data of BFE10, identical probability weighting cannot be rejected ($p - value < 0.0001$). Using data of BFE10, parametric estimates imply the equality of probability weighting functions across domains ($p - value = 0.9172$) and the rejection of duality ($p - value < 0.0001$). Again, parametric estimates lead to inconsistent results across datasets for the comparison of probability weighting functions across domains.

In contrast, our method provides consistent results for the comparison of probability weighting functions across domains. We reject duality (RDU, Quiggin, 1982; Gilboa, 1987; Schmeidler, 1989) and identical probability weights across domains across datasets(OPT, Kahneman and Tversky, 1979). Tests of duality and identical probability weightings that cover the whole range of probabilities are scarce in the literature. Our findings echo Abdellaoui (2000) who reject both duality and identical probability weighting functions across domains under risk. However, under uncertainty, Abdellaoui et al. (2005) do not reject duality, although they reject identical weighting functions across domains. Importantly, our rejection of both duality and identical probability weights provides support for CPT.[32]

Our method also reveals an interesting pattern in the probability weighting function – more

[30] See Appendix A1.4 for detailed information about parametric specifications and results.

[31] We also report similar results in Table 5.

[32] Even though we reject both duality and identical probability weights, in Appendix A1.3 we show how to impose such constraints in our method.

overweighting of tail probabilities in the gain domain than in the loss domain – which fully stands in line with CPT, but does not arise under the parametric approach.[33] Hence, the level of optimism for very small probabilities of gains is more pronounced than the level of pessimism for very small probabilities of losses.

Two important similarities with TK92 also emerge. First, our estimates of loss aversion (λ) are close to 2, in line with the estimate provided by TK92. Second, echoing the central tenets of CPT, we find that the probability weighting function is domain-specific. Furthermore, in both domains it is characterized by the overweighting of small probabilities and the underweighting of large ones.

Altogether, our results mesh well with CPT. In both domains, the inverse S-shaped probability weighting function affects risk preferences alongside the utility function.

Table 6: Summary of the discussion

	Data of TK92		Data of BFE10		Consistent	
	Semi-para	Para	Semi-para	Para	Semi-para	Para
Curvature Gain (α)	Concave	Concave	Concave	Concave	Yes	Yes
Curvature Loss (β)	Concave	Convex	Concave	Concave	Yes	No
Loss aversion (λ)	Loss aversion	Loss aversion	-	-	-	-
Partial reflection	No	Yes	No	No	Yes	No
OPT: $w^+() = w^-()$	No	No	No	Yes	Yes	No
RDU: $w^+(p) = 1 - w^-(1-p)$	No	No	No	No	Yes	Yes

6 Conclusion

ABL and Abdellaoui et al. (2011b) deploy a semi-parametric method to elicit the utility function and loss aversion. In this paper, we go one step further by developing a semi-parametric method that elicits all dimensions of risk attitudes, including the whole range of probability weights. Importantly, it retains the four appealing properties of the parametric methods that have been discussed at length in the literature.

Our method is also flexible because it can be applied to both risk and uncertainty. Furthermore, it can be used to extend the popular elicitation technique of Holt and Laury (2002) to the case in which probabilities are distorted, following the approach of Abdellaoui et al. (2011b). Finally, even though our method does not readily apply to the context envisioned by Kőszegi and Rabin (2007),[34] one can speculate on a possible procedure combining Köszegi and Rabin's approach and our semi-parametric method. This procedure could start by introducing probability weighting functions in Kőszegi and Rabin (2007) following the work of Baillon et al. (2020). We see this as a promising avenue for future research.

[33] In addition, this pattern also holds for the Bayesian estimations reported in Appendix A4.

[34] They assume a stochastic reference point and the absence of any probability distortions, whereas we assume a fixed reference point and probability distortions.

7 Appendix

A1-Appendix on TK92 data

A1.1—Data

a) Source

We use median data previously reported by TK92. They run a computerized experiment with 25 graduate students from Berkeley and Stanford with no particular training in decision theory. Each subject participated in three separate one-hour sessions organized over several days, and received \$25 for participation. We use all the median observations from non-mixed prospects (see their Table 3) as well as the first six median observations from mixed prospects (see their Table 6).

b) Procedure

The data are generated via the switching outcome procedure in which an indifference value is inferred through a list of equally spaced certain outcomes, ranging from the admissible maximum indifference value to the admissible minimum indifference value. Note that an alternative approach, the direct matching procedure in which subjects are directly asked to provide their indifference values, tends to produce more inconsistencies (Bostic et al., 1990; Booij and Van de Kuilen, 2009). Internal consistency of the responses to each prospect is monitored by the computer software to reduce response errors.

c) Data for the first step

All outcomes are expressed in US dollars. In Table 3 of TK92, there are 28 median values of certainty equivalents for binary lotteries that involve 7 pairs of positive monetary outcomes (0, 50), (0, 100), (0, 200), (0, 400), (50, 100), (50, 150) and (100, 200), and 9 probabilities of getting the higher outcome: 0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95 and 0.99.

Also, the same Table 3 in TK92 provides 28 median values of certainty equivalents for binary lotteries that involve 7 pairs of negative monetary outcomes (0, -50), (0, -100), (0, -200), (0, -400), (-50, -100), (-50, -150) and (-100, -200), and the same list of 9 probabilities as in the gain domain. These probabilities are now associated to losing the higher outcome.

As required by our method, at least two certainty equivalents for each of the nine probabilities are available per domain so as to perform a simultaneous measurement of the utility function and the probability weighting function in the full domain.

c) Data for the second step

All outcomes are expressed in US dollars. In Table 6 of TK92, there are 6 cases of indifferences involving mixed prospects. The first four items consist in eliciting the values of gains x to make subjects indifferent between the mixed prospects $(x, y; 0.5, 0.5)$ and 0. The values of y are -25, -50, -100 and -150. The two others cases consist in eliciting gains x that make subjects indifferent between two mixed prospects $(x, y; 0.5, 0.5)$ and $(z, w; 0.5, 0.5)$. The triplets (y, z, w) take the values of either (-50,50,-20) or (-125,150,-50). Note that here the experimenter has no control over the maximum level of x which may hinder the use of the switching outcome procedure for finding indifference value. For this reason, we make changes in the third step of the original method of ABL through equations (16) - (19). Also, our method is based on the comparisons of binary lotteries and sure outcomes. Hence, these data do not exactly fit our method. With the present data, we compute loss aversion for each of the six questions and take the median as estimated value to account for response error (following ABL). For the first four items, we compute loss aversion as follows:

$$\lambda = \frac{w^+(0.5)}{w^-(0.5)} \times \frac{x^{\alpha_p}}{(-y)^{\beta_p}} \tag{24}$$

$$\lambda = \frac{w^+(0.5)}{w^-(0.5)} \times \frac{1 - exp(-\alpha_e x)}{1 - exp(\beta_e y)} \times \frac{\beta_e}{\alpha_e} \tag{25}$$

Formulas (24) and (25) correspond to power and exponential specifications, respectively.

For the last two questions, we compute loss aversion as follows:

$$\lambda = \frac{w^+(0.5)}{w^-(0.5)} \times \frac{z^{\alpha_p} - x^{\alpha_p}}{(-w)^{\beta_p} - (-y)^{\beta_p}} \tag{26}$$

$$\lambda = \frac{w^+(0.5)}{w^-(0.5)} \times \frac{exp(-\alpha_e x) - exp(-\alpha_e z)}{exp(\beta_e y) - exp(\beta_e w)} \times \frac{\beta_e}{\alpha_e} \tag{27}$$

Formulas (26) and (27) correspond to the power and exponential specifications, respectively. Finally, following ABL, we compute the median loss aversion.

A1.2— Our main semi-parametric measurements

Our semi-parametric measurements are presented in Tables 7 and 8.

Table 7: Results of the first step

	Power utility				Exponential utility	
	No constraint		Constraint $\alpha_p = \beta_p$			
	Gain domain					
α_p or α_e	0.904***	(0.0366)	0.976***	(0.0314)	0.00158*	(0.000886)
α	0.525***	(0.0101)	0.506***	(0.00804)	0.552***	(0.0290)
$w(0.01)$	0.0471***	(0.00828)	0.0369***	(0.00659)	0.0445***	(0.00874)
$w(0.05)$	0.167***	(0.0138)	0.157***	(0.0138)	0.164***	(0.0143)
$w(0.10)$	0.134***	(0.0151)	0.115***	(0.0133)	0.124***	(0.0142)
$w(0.25)$	0.276***	(0.0142)	0.263***	(0.0142)	0.274***	(0.0155)
$w(0.50)$	0.410***	(0.0139)	0.388***	(0.0123)	0.409***	(0.0176)
$w(0.75)$	0.571***	(0.0137)	0.558***	(0.0140)	0.573***	(0.0164)
$w(0.90)$	0.686***	(0.0137)	0.666***	(0.0133)	0.692***	(0.0201)
$w(0.95)$	0.791***	(0.0124)	0.783***	(0.0133)	0.793***	(0.0139)
$w(0.99)$	0.947***	(0.00474)	0.943***	(0.00520)	0.957***	(0.00834)
	Loss domain					
β_p or β_e	1.069***	(0.0485)	0.976***	(0.0314)	-0.00154*	(0.000931)
β	-0.483***	(0.0113)	-0.506***	(0.00804)	-0.449***	(0.0305)
$w(0.01)$	0.0244***	(0.00570)	0.0337***	(0.00646)	0.0236***	(0.00551)
$w(0.05)$	0.0924***	(0.0127)	0.103***	(0.0136)	0.0932***	(0.0123)
$w(0.10)$	0.105***	(0.0139)	0.127***	(0.0135)	0.106***	(0.0126)
$w(0.25)$	0.207***	(0.0140)	0.222***	(0.0140)	0.205***	(0.0141)
$w(0.50)$	0.414***	(0.0152)	0.440***	(0.0123)	0.405***	(0.0186)
$w(0.75)$	0.602***	(0.0146)	0.617***	(0.0139)	0.595***	(0.0168)
$w(0.90)$	0.758***	(0.0139)	0.776***	(0.0121)	0.746***	(0.0190)
$w(0.95)$	0.810***	(0.0133)	0.819***	(0.0132)	0.805***	(0.0148)
$w(0.99)$	0.947***	(0.00552)	0.951***	(0.00512)	0.936***	(0.0107)
Log Likelihood	-120.9645		-124.5477		-121.9931	
N	56		56		56	
AIC criterion	283.9289		289.0954		285.9861	
CV	3.149		3.344		3.305	

Standard errors in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$ to test the significance of coefficients.

Table 8: Results of the second step (loss aversion)

Observations	Power utility		Exponential utility
	No constraint	Constraint $\alpha_p = \beta_p$	
1	1.304	2.106	2.306
2	0.981	1.751	1.815
3	0.875	1.751	1.616
4	0.762	1.622	1.355
5	0.892	1.786	1.741
6	0.739	1.735	1.247
Median	**0.884**	**1.751**	**1.679**

A1.3—Semi-parametric results under identical probability weighting function and duality assumptions

This appendix show results under the constraints of identical probability weighting (OPT) and duality assumption (RDU). Tables 9 and 10 present these results.

Table 9: Results under identical probability weighting assumption

	Power utility		Exponential utility	
α_p or α_e	0.904***	(0.0456)	0.00132	(0.000942)
α	0.525***	(0.0126)	0.544***	(0.0310)
β_p or β_e	1.053***	(0.0523)	-0.00127	(0.000961)
β	-0.487***	(0.0124)	-0.458***	(0.0316)
$w^+(0.01) = w^-(0.01)$	0.0326***	(0.00760)	0.0301***	(0.00700)
$w^+(0.05) = w^-(0.05)$	0.128***	(0.0148)	0.125***	(0.0137)
$w^+(0.10) = w^-(0.10)$	0.119***	(0.0161)	0.114***	(0.0137)
$w^+(0.25) = w^-(0.25)$	0.242***	(0.0156)	0.237***	(0.0153)
$w^+(0.50) = w^-(0.50)$	0.414***	(0.0159)	0.408***	(0.0184)
$w^+(0.75) = w^-(0.75)$	0.587***	(0.0155)	0.584***	(0.0169)
$w^+(0.90) = w^-(0.90)$	0.720***	(0.0151)	0.715***	(0.0200)
$w^+(0.95) = w^-(0.95)$	0.801***	(0.0141)	0.799***	(0.0147)
$w^+(0.99) = w^-(0.99)$	0.947***	(0.00557)	0.950***	(0.00962)
N	56		56	
Log Likelihood	-145.7		-143.2982	
AIC	315.400		310.596	
CV	3.456		3.518	

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 10: Results under duality assumption $w^{+}(p) = 1 - w^{-}(1-p)$

	Power utility		Exponential utility	
α_p or α_e	0.738***	(0.0476)	0.00771***	(0.00174)
α	0.575***	(0.0157)	0.724***	(0.0381)
β_p or β_e	0.754***	(0.0465)	0.00270*	(0.00155)
β	-0.570***	(0.0151)	-0.588***	(0.0488)
$w^{+}(0.01) = 1 - w^{-}(0.99)$	0.0431***	(0.00819)	0.0330***	(0.0125)
$w^{+}(0.05) = 1 - w^{-}0.95)$	0.170***	(0.0200)	0.178***	(0.0220)
$w^{+}(0.10) = 1 - w^{-}0.90)$	0.182***	(0.0190)	0.187***	(0.0267)
$w^{+}(0.25) = 1 - w^{-}0.75)$	0.327***	(0.0208)	0.347***	(0.0255)
$w^{+}(0.50) = 1 - w^{-}(0.50)$	0.478***	(0.0195)	0.516***	(0.0297)
$w^{+}(0.75) = 1 - w^{-}(0.25)$	0.662***	(0.0205)	0.693***	(0.0241)
$w^{+}(0.90) = 1 - w^{-}0.10)$	0.755***	(0.0203)	0.817***	(0.0272)
$w^{+}(0.95) = 1 - w^{-}(0.05)$	0.834***	(0.0196)	0.856***	(0.0189)
$w^{+}(0.99) = 1 - w^{-}(0.01)$	0.953***	(0.00812)	0.989***	(0.00555)
Log Likelihood	-165.9685		-161.7984	
N	56		56	
AIC	355.937		347.597	
CV	4.977		9.024	

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$ to test the significance of coefficients.

A1.4– Parametric measurements

We consider the following parametric specifications (28) and (29)

$$\begin{aligned}\boldsymbol{ce}_l =&(\boldsymbol{D}_l^{+} - \boldsymbol{D}_l^{-})\Bigg[\Big(|\boldsymbol{x}_l|^{\alpha_p \boldsymbol{D}_l^{+} + \beta_p \boldsymbol{D}_l^{-}} - |\boldsymbol{y}_l|^{\alpha_p \boldsymbol{D}_l^{+} + \beta_p \boldsymbol{D}_l^{-}}\Big) \times W_l + \\ &|\boldsymbol{y}_l|^{\alpha_p \boldsymbol{D}_l^{+} + \beta_p \boldsymbol{D}_l^{-}}\Bigg]^{\frac{1}{\alpha_p \boldsymbol{D}_l^{+} + \beta_p \boldsymbol{D}_l^{-}}} + \mathbf{e}_l^i\end{aligned} \tag{28}$$

$$\begin{aligned}\boldsymbol{ce}_l =&ln\Bigg[\Big(exp((\beta_e \boldsymbol{D}_l^{-} - \alpha_e \boldsymbol{D}_l^{+})\boldsymbol{x}_l) - exp((\beta_e \boldsymbol{D}_l^{-} - \alpha_e \boldsymbol{D}_l^{+})\boldsymbol{y}_l)\Big) \times W_l + \\ &exp((\beta_e \boldsymbol{D}_l^{-} - \alpha_e \boldsymbol{D}_l^{+})\boldsymbol{y}_l)\Bigg]\frac{1}{\beta_e \boldsymbol{D}_l^{-} - \alpha_e \boldsymbol{D}_l^{+}} + \mathbf{e}_l^i\end{aligned} \tag{29}$$

with W_l can be one of the four specifications of TK92, P98-I, P98-II and GE87:

$$W_l = \frac{p_l^{(a\boldsymbol{D}_l^{+} + c\boldsymbol{D}_l^{-})}}{\Big(p_l^{(a\boldsymbol{D}_l^{+} + c\boldsymbol{D}_l^{-})} + (1-p_l)^{(a\boldsymbol{D}_l^{+} + c\boldsymbol{D}_l^{-})}\Big)^{\frac{1}{a\boldsymbol{D}_l^{+} + c\boldsymbol{D}_l^{-}}}}$$

$$W_l = exp\Bigg[-\Big(-ln(p_l)\Big)^{a\boldsymbol{D}_l^{+} + c\boldsymbol{D}_l^{-}}\Bigg]$$

$$W_l = exp\Bigg[-\big(b\boldsymbol{D}_l^{+} + d\boldsymbol{D}_l^{-}\big) \times \Big(-ln(p_l)\Big)^{a\boldsymbol{D}_l^{+} + c\boldsymbol{D}_l^{-}}\Bigg]$$

$$W_l = \frac{(b\boldsymbol{D}_l^+ + d\boldsymbol{D}_l^-) \times p_l^{(a\boldsymbol{D}_l^+ + c\boldsymbol{D}_l^-)}}{(b\boldsymbol{D}_l^+ + d\boldsymbol{D}_l^-) \times p_l^{(a\boldsymbol{D}_l^+ + c\boldsymbol{D}_l^-)} + (1-p_l)^{(a\boldsymbol{D}_l^+ + c\boldsymbol{D}_l^-)}}$$

Equation (28) and (29) allow us to elicit the utility and probability weighting function parameters in the full domain.

Once the parameters of the utility function and the probability weighting function are obtained, we estimate loss aversion as described in equations (24) - (27).

Table 11 presents the results of the first step that simultaneously estimates the utility and probability weighting functions in the full domain. Table 12 summarizes the estimates of loss aversion using parameters of the utility and probability weighting functions from the first step as inputs.

Table 11: Results of the first step: power utility (without constraint)

	TK92		P98-I		P98-II		GE87	
				Gain domain				
α_p	0.839***	(0.0280)	0.772***	(0.0224)	0.834***	(0.0510)	0.907***	(0.0537)
α	0.544***	(0.00828)	0.564***	(0.00715)	0.545***	(0.0151)	0.524***	(0.0148)
a	0.643***	(0.0120)	0.589***	(0.0158)	0.590***	(0.0145)	0.620***	(0.0181)
b					1.076***	(0.0557)	0.693***	(0.0555)
				Loss domain				
β_p	0.906***	(0.0286)	0.867***	(0.0254)	1.042***	(0.0709)	1.058***	(0.0641)
β	-0.525***	(0.00786)	-0.536***	(0.00729)	-0.490***	(0.0170)	-0.486***	(0.0151)
a	0.704***	(0.0153)	0.676***	(0.0205)	0.680***	(0.0189)	0.708***	(0.0239)
b					1.186***	(0.0687)	0.673***	(0.0568)
N	56		56		56		56	
Log Likelihood	-155.124		-158.499		-153.315		-152.111	
AIC	320		327		321		318	
CV	3.41		3.57		3.52		3.87	

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 12: Results of the second step for Loss aversion: power utility (without constraint)

Observations	TK92	P98-I	P98-II	GE87
1	1.618	1.429	1.143	1.405
2	1.318	1.156	0.846	1.066
3	1.258	1.082	0.734	0.979
4	1.253	1.058	0.732	0.961
5	1.146	0.980	0.630	0.841
6	1.127	0.920	0.580	0.822
Median	**1.256**	**1.070**	**0.773**	**0.970**

Table 13: Results of the first step: power utility (with constraint)

	TK92		P98-I		P98-II		GE87	
				Gain domain				
α_p	0.872***	(0.0204)	0.814***	(0.0179)	0.926***	(0.0441)	0.978***	(0.0425)
α	0.534***	(0.00582)	0.551***	(0.00544)	0.519***	(0.0119)	0.506***	(0.0109)
a	0.635***	(0.0110)	0.583***	(0.0165)	0.590***	(0.0152)	0.629***	(0.0185)
b					1.166***	(0.0511)	0.632***	(0.0422)
				Loss domain				
β_p	0.872***	(0.0204)	0.814***	(0.0179)	0.926***	(0.0441)	0.978***	(0.0425)
β	-0.534***	(0.00582)	-0.551***	(0.00544)	-0.519***	(0.0119)	-0.506***	(0.0109)
a	0.713***	(0.0152)	0.681***	(0.0225)	0.678***	(0.0199)	0.695***	(0.0226)
b					1.083***	(0.0474)	0.741***	(0.0477)
N	56		56		56		56	
Log Likelihood	-156.499		-162.219		-156.147		-153.719	
AIC	321		332		324		319	
CV	3.313		3.586		3.426		3.589	

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 14: Results of the second step for Loss aversion: power utility (with constraint)

Observations	TK92	P98-I	P98-II	GE87
1	2.040	2.009	2.076	2.175
2	1.738	1.723	1.764	1.844
3	1.730	1.723	1.743	1.808
4	1.730	1.716	1.743	1.808
5	1.668	1.637	1.704	1.792
6	1.615	1.616	1.496	1.674
Median	**1.730**	**1.720**	**1.743**	1.808

Table 15: Results of the first step: exponential utility

	TK92		P98-I		P98-II		GE87	
	Gain domain							
α_e	0.00168***	(0.0006)	0.00314***	(0.0005)	0.0004	(0.0008)	-0.000550	(0.0007)
α	0.556***	(0.0193)	0.602***	(0.0157)	0.513***	(0.0257)	0.482***	(0.0243)
a	0.649***	(0.0189)	0.639***	(0.0243)	0.595***	(0.0190)	0.628***	(0.0179)
b					1.216***	(0.0540)	0.588***	(0.0405)
	Loss domain							
β_e	0.0016**	(0.0007)	0.00225***	(0.0006)	-0.0006	(0.0011)	-0.0006	(0.0007)
β	-0.551***	(0.0222)	-0.574***	(0.0189)	-0.479***	(0.0368)	-0.459***	(0.0326)
a	0.721***	(0.0238)	0.709***	(0.0281)	0.673***	(0.0221)	0.695***	(0.0213)
b					1.183***	(0.0670)	0.658***	(0.0552)
N	56		56		56		56	
Log Likelihood	-164.567		-168.725		-157.124		-152.808	
AIC	339		347		328		320	
CV	6.21		4.35		4.06		5.39	

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 16: Results of the second step for Loss aversion: exponential utility

Observations	TK92	P98-I	P98-II	GE87
1	2.234	2.238	2.328	2.300
2	1.824	1.794	1.906	2.028
3	1.800	1.701	1.897	1.979
4	1.747	1.637	1.832	1.940
5	1.577	1.434	1.700	1.912
6	1.496	1.192	1.641	1.807
Median	**1.774**	**1.669**	**1.865**	**1.960**

A1.5 - One step estimation of utility function, weighting function and loss aversion

Table 17: Simultaneous estimation of utility function, weighting function and loss aversion

	Power utility				Exponential utility	
	No constraint		Constraint $\alpha_p = \beta_p$			
			Gain domain			
α_p or α_e	0.896***	(0.0433)	0.991***	(0.0326)	-0.000209	(0.000922)
α	0.528***	(0.0121)	0.502***	(0.0082)	0.493***	(0.0307)
$w^+(0.01)$	0.0484***	(0.00990)	0.0351***	(0.00696)	0.0327***	(0.00798)
$w^+(0.05)$	0.169***	(0.0161)	0.155***	(0.0154)	0.152***	(0.0162)
$w^+(0.10)$	0.136***	(0.0179)	0.111***	(0.0143)	0.107***	(0.0149)
$w^+(0.25)$	0.278***	(0.0166)	0.260***	(0.0158)	0.256***	(0.0175)
$w^+(0.50)$	0.412***	(0.0165)	0.384***	(0.0134)	0.377***	(0.0185)
$w^+(0.75)$	0.572***	(0.0160)	0.555***	(0.0157)	0.551***	(0.0190)
$w^+(0.90)$	0.688***	(0.0161)	0.662***	(0.0147)	0.656***	(0.0227)
$w^+(0.95)$	0.792***	(0.0143)	0.781***	(0.0150)	0.778***	(0.0172)
$w^+(0.99)$	0.948***	(0.00547)	0.943***	(0.00590)	0.940***	(0.0111)
			Loss domain			
β_p or β_e	1.045***	(0.0383)	0.991***	(0.0326)	-0.000979	(0.000723)
β	-0.489	(0.0092)	0.502***	(0.0082)	-0.467***	(0.0239)
$w^+(0.01)$	0.0265***	(0.00496)	0.0320***	(0.00554)	0.0262***	(0.00470)
$w^+(0.05)$	0.0950***	(0.0107)	0.101***	(0.0113)	0.0957***	(0.0100)
$w^+(0.10)$	0.110***	(0.0117)	0.123***	(0.0120)	0.111***	(0.0103)
$w^+(0.25)$	0.211***	(0.0116)	0.220***	(0.0119)	0.210***	(0.0114)
$w^+(0.50)$	0.421***	(0.0123)	0.436***	(0.0115)	0.415***	(0.0146)
$w^+(0.75)$	0.606***	(0.0120)	0.615***	(0.0119)	0.602***	(0.0132)
$w^+(0.90)$	0.762***	(0.0112)	0.773***	(0.0107)	0.755***	(0.0147)
$w^+(0.95)$	0.812***	(0.0109)	0.818***	(0.0110)	0.809***	(0.0116)
$w^+(0.99)$	0.948***	(0.00448)	0.950***	(0.00436)	0.941***	(0.00800)
			Mixed Prospect			
λ	0.864***	(0.242)	1.688***	(0.0786)	1.684***	(0.135)
Log Likelihood	-131.289		-134.001		-133.181	
N	60		60		60	
AIC criterion	308.5782		312.0025		312.3615	

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

A2-Appendix for the application to the data of BFE10

In this appendix, we provide details on individual and pooled results based on the data of BFE10. We also allow for heteroscedastic errors, as discussed in Section 3.2.2.

A2-1- Pooled data

Tables 18 and 19 summarize our pooled data estimates that we present in the main text.

Table 18: Semi-parametric estimation results: pooled data

	Power		Exponential	
	Gain domain			
Utility				
parameter	1.031***	(0.0216)	0.00710***	(0.0007)
$w^+(0.05)$	0.191***	(0.0062)	0.220***	(0.0063)
$w^+(0.10)$	0.250***	(0.0083)	0.276***	(0.0068)
$w^+(0.25)$	0.330***	(0.0076)	0.359***	(0.0067)
$w^+(0.50)$	0.450***	(0.0067)	0.501***	(0.0060)
$w^+(0.75)$	0.606***	(0.0074)	0.635***	(0.0064)
$w^+(0.90)$	0.734***	(0.0075)	0.756***	(0.0063)
$w^+(0.95)$	0.779***	(0.0069)	0.799***	(0.0061)
	Loss domain			
Utility				
parameter	1.088***	(0.0259)	-0.00606***	(0.00072)
$w^-(0.05)$	0.193***	(0.0077)	0.195***	(0.0065)
$w^-(0.10)$	0.263***	(0.0091)	0.270***	(0.0068)
$w^-(0.25)$	0.374***	(0.0083)	0.373***	(0.0068)
$w^-(0.50)$	0.518***	(0.0073)	0.501***	(0.0063)
$w^-(0.75)$	0.659***	(0.0078)	0.655***	(0.0069)
$w^-(0.90)$	0.730***	(0.0082)	0.730***	(0.0071)
$w^-(0.95)$	0.798***	(0.0064)	0.787***	(0.0065)
N	8906		8906	
Log Likelihood	-25191.14		-25103.62	
AIC	50776.27		50601.23	
CV	4.400		4.2494	

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 19: Parametric estimation results with power utility: pooled data

	TK92		P98-I		P98-II		GE87	
	Gain domain							
α_p	1.219***	(0.0152)	1.102***	(0.0115)	1.043***	(0.0212)	1.040***	(0.0215)
α	0.451***	(0.00310)	0.476***	(0.00260)	0.490***	(0.00509)	0.490***	(0.00517)
a	0.591***	(0.00490)	0.484***	(0.00704)	0.483***	(0.00705)	0.870***	(0.0196)
b					0.955***	(0.0140)	0.475***	(0.00708)
	Loss domain							
β_p	1.442***	(0.0186)	1.330***	(0.0148)	1.091***	(0.0257)	1.078***	(0.0249)
β	-0.409***	(0.00312)	-0.429***	(0.00274)	-0.478***	(0.00587)	-0.481***	(0.00577)
a	0.616***	(0.00539)	0.508***	(0.00792)	0.506***	(0.00799)	1.049***	(0.0256)
b					0.853***	(0.0147)	0.477***	(0.00753)
N	8906		8906		8906		8906	
Log Likelihood	-25369		-25305		-25260		-25227	
AIC	51110		50980		50895		50828	
CV	10.005		4.381		4.383		4.375	

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 20: Parametric estimation results with exponential utility: pooled data

	TK92		P98-I		P98-II		GE87	
				Gain domain				
α_e	-0.0008	(0.0005)	0.0004	(0.0004)	0.0066***	(0.0007)	0.0072***	(0.0007)
α	0.490***	(0.0065)	0.505***	(0.0056)	0.581***	(0.0079)	0.589***	(0.0079)
a	0.345***	(0.0132)	0.502***	(0.0071)	0.492***	(0.0072)	1.025***	(0.0163)
b					0.862***	(0.0093)	0.467***	(0.0068)
			Loss domain					
β_e	-0.0171***	(0.0009)	-0.0144***	(0.0007)	-0.0068***	(0.0007)	-0.0059***	(0.00069)
β	-0.307***	(0.0087)	-0.333***	(0.0067)	-0.416***	(0.0083)	-0.428***	(0.0083)
a	0.319***	(0.0146)	0.492***	(0.0077)	0.490***	(0.0079)	1.015***	(0.0167)
b					0.871***	(0.0096)	0.468***	(0.0071)
N	8906		8906		8906		8906	
Log Likelihood	-25560		-25348		-25165		-25131	
AIC	51489		51066		50704		50636	
CV	4.842		4.512		4.304		4.253	

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

A2.2- Individual results

This appendix provides results based on individual estimates. We focus on the exponential utility function which is found to provide a better fit to the data than the power utility function under both our semi-parametric method and the parametric one. Analyses presented below are based on median comparisons of coefficients using Sign Rank tests. All tests are two-sided, unless stated otherwise.

Curvature of the utility function

The distributions of the curvature coefficients obtained under both methods are plotted in Figures 4 and 5, and their median values are summarized in Table 21.

Figures 4 and 5 indicate that, regardless of the estimation method at hand, the dominant pattern is the concavity of the utility function in both domains.

For gains, the estimated median exponential utility parameter under the semi-parametric method is 0.0068 and significantly greater than 0 ($p - value < 0.0001$, one-sided Sign Rank test). In the loss domain, the median exponential utility parameter is estimated at -0.0065 and significantly below 0 ($p - value < 0.0001$). Furthermore, partial reflection is rejected: comparing the median estimates of both coefficients yields $p - value < 0.0001$. The parametric method leads to very similar results.[35]

Probability weighting function

Table 22 and Figure 6 summarize the estimated probability weighting functions across domains. In addition, Figures 7 and 8 show the underlying distributions of individual estimates. Both

[35]The corresponding median values are 0.0081 and -0.0063. The respective statistical tests all yield $p - values < 0.0001$. In addition, the differences between the median estimates obtained through both methods are small and insignificant for gains ($p - value = 0.6539$) as well as for losses ($p - value = 0.8812$).

methods consistently point to an inverse S-shaped weighting function across domains, with a crossover point around $p = 0.5$. We cannot reject $H_0 : w^+(0.5) = 0.5$ or $H_0 : w^-(0.5) = 0.5$ with $p - values > 0.3698$.

Focusing on our semi-parametric method, we report that the median difference in probability weights between gains and losses is positive and significant ($p - values < 0.0361$) for the probabilities $p = 0.05$ and $p = 0.10$, and insignificant for the remaining probabilities (all $p-values > 0.1347$). This suggests that there is stronger overweighting of probabilities $p = 0.05$ and $p = 0.10$ in the gain domain than in the loss domain. Hence, subjects are more optimistic about very small probabilities of gaining money than they are pessimistic about very small probabilities of losing money. In contrast, identical probability weighting cannot be refuted with the parametric method because the median differences in the estimated probability weights are systematically insignificant (all $p - values > 0.7651$).[36]

Table 21: Median estimates for exponential utility function using individual estimates

	Semi-parametric		Parametric	
	Median	IQR†	Median	IQR†
Utility parameter (Gain)	0.0068	[-0.0001; 0.0152]	0.0081	[0.0007; 0.0136]
Utility parameter (Loss)	-0.0065	[-0.0172; 0.0001]	-0.0063	[-0.0158; 0.0002]
AIC*	50,601.23		50,636.95	
CV*	4.2494		4.2529	

† IQR stands for interquartile range

* AIC and CV are from pooled estimates

Table 22: Median values of probability weights based on individual estimates

	Semi-parametric		Parametric	
	Median	IQR$^+$	Median$^+$	IQR$^+$
	Gain			
$w^+(0.05)$	0.2156	0.1339; 0.3181	0.2033	0.1273; 0.3030
$w^+(0.10)$	0.2666	0.1826; 0.3696	0.2670	0.1865; 0.3502
$w^+(0.25)$	0.3635	0.2763; 0.4521	0.3741	0.2971; 0.4437
$w^+(0.50)$	0.5046	0.4196; 0.5660	0.5163	0.4302; 0.5685
$w^+(0.75)$	0.6441	0.5251; 0.7269	0.6323	0.5549; 0.7067
$w^+(0.90)$	0.7900	0.6524; 0.8720	0.7315	0.6420; 0.8135
$w^+(0.95)$	0.8038	0.6916; 0.8961	0.7991	0.6999; 0.8698
	Loss			
$w^-(0.05)$	0.1886	0.0908; 0.3026	0.1922	0.1184; 0.3000
$w^-(0.10)$	0.2545	0.1490; 0.3811	0.2521	0.1729; 0.3688
$w^-(0.25)$	0.3628	0.2750; 0.4907	0.3663	0.2906; 0.4568
$w^-(0.50)$	0.5076	0.4289; 0.5724	0.4928	0.4345; 0.5645
$w^-(0.75)$	0.6506	0.5770; 0.7392	0.6398	0.5463; 0.6967
$w^-(0.90)$	0.7488	0.6340; 0.8353	0.7560	0.6564; 0.8118
$w^-(0.95)$	0.7922	0.6867; 0.8821	0.8131	0.7123; 0.8721

IQR stands for interquartile range

$^+$ For parametric methods, the median weights and IQR are computed from the individual probability weighting function

[36]The same results hold on the median differences of the parameters of the probability weighting function (all $p - values > 0.6539$)

Figure 4: Curvature of the utility function across domains: semi-parametric estimates

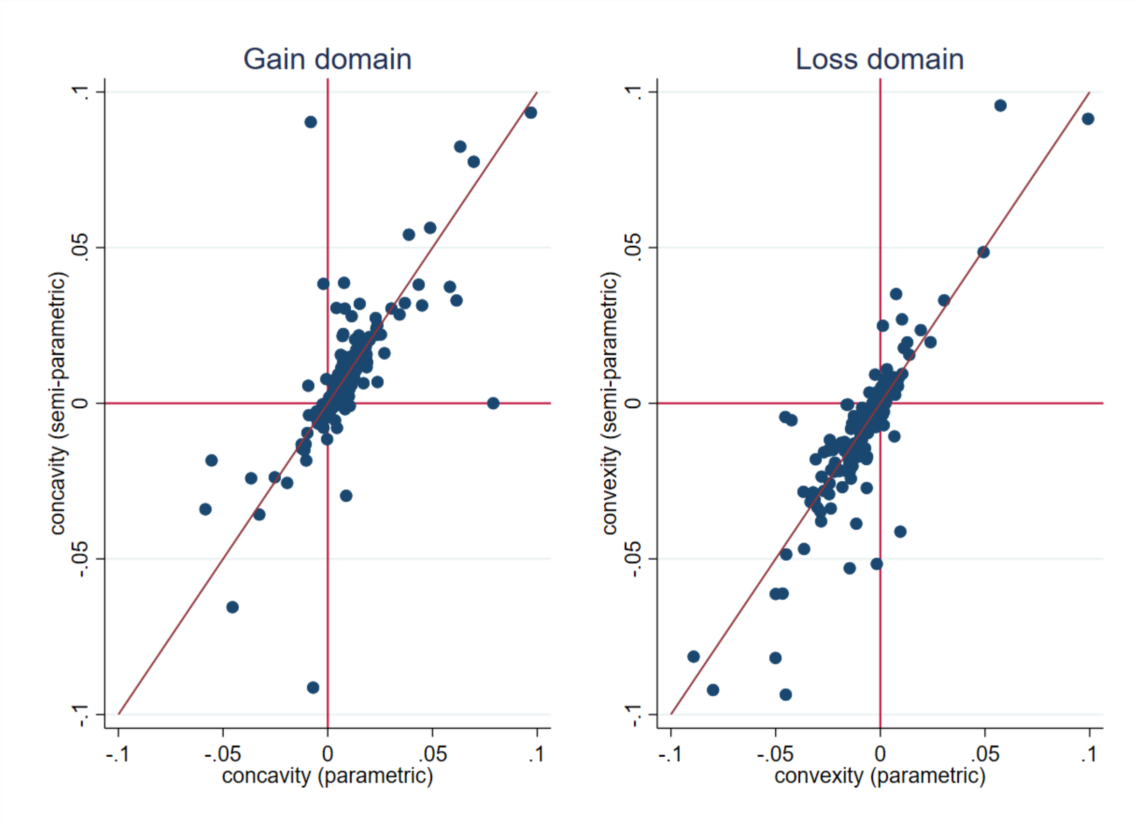


Figure 5: Curvature of the utility function under different estimation methods

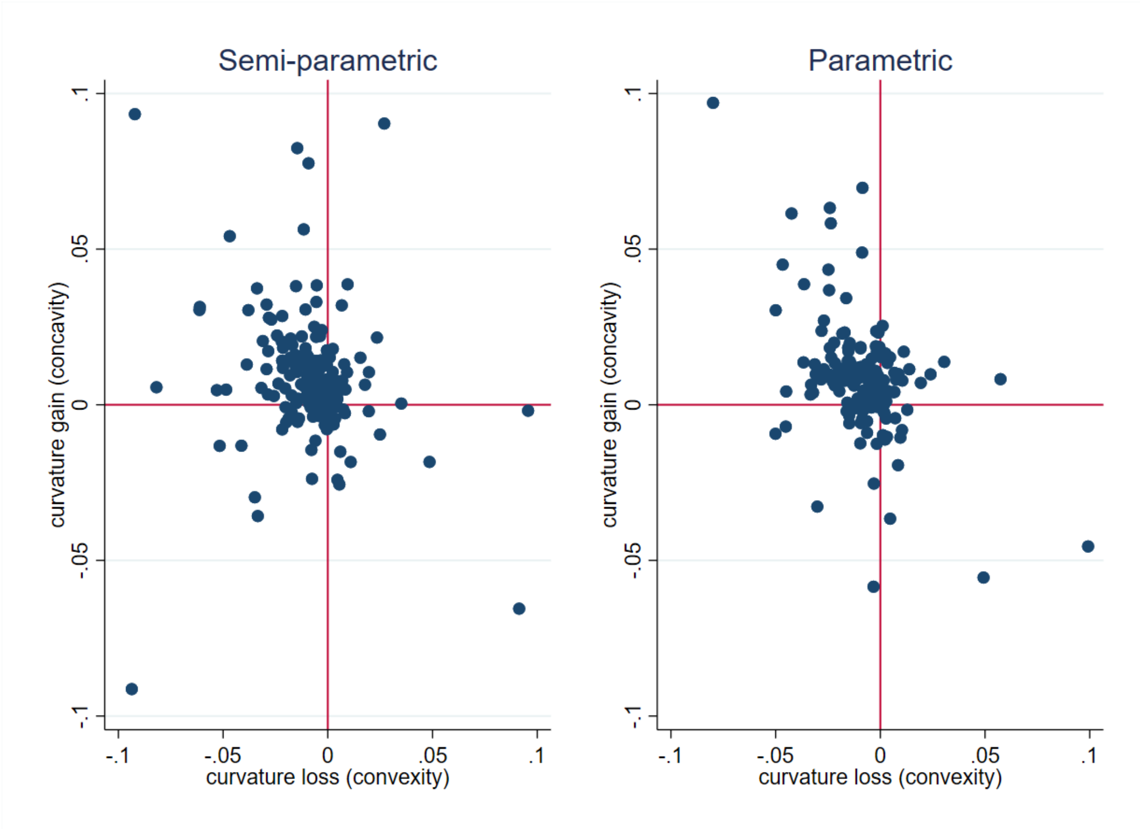


Figure 6: Median probability weights under different estimation methods

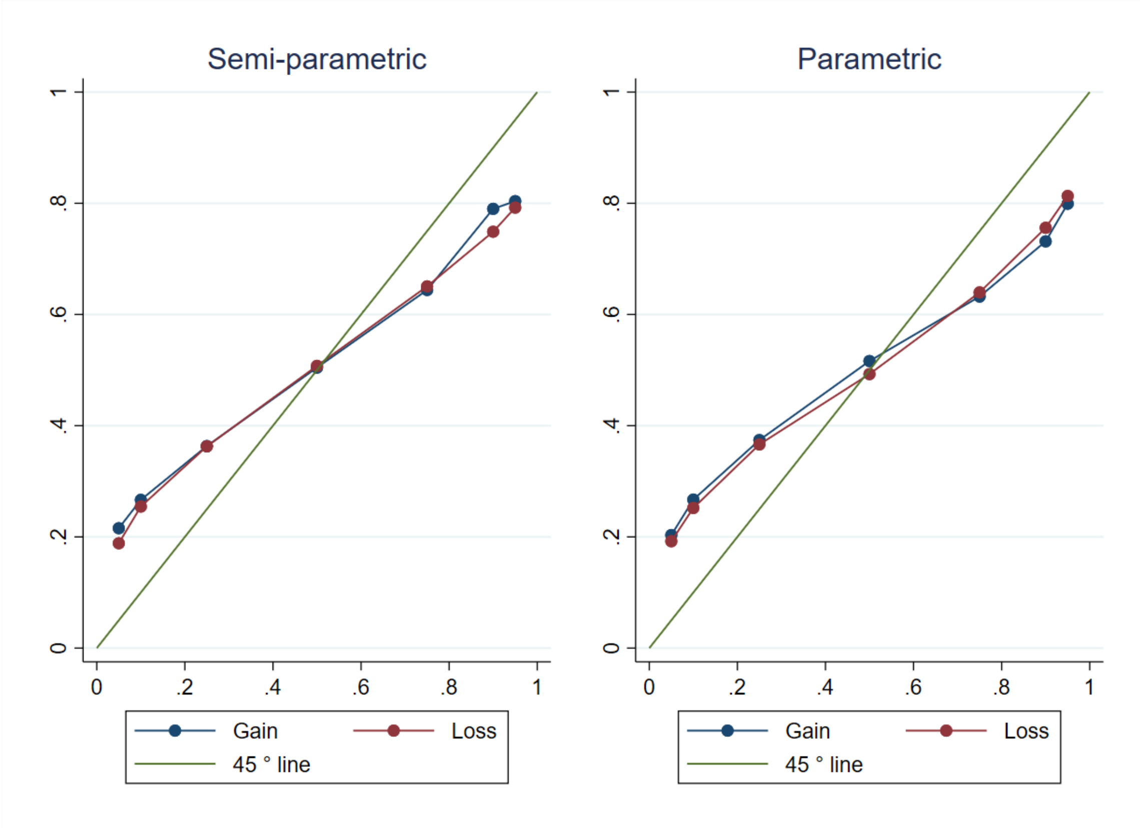


Figure 7: Distribution of probability weights for gains

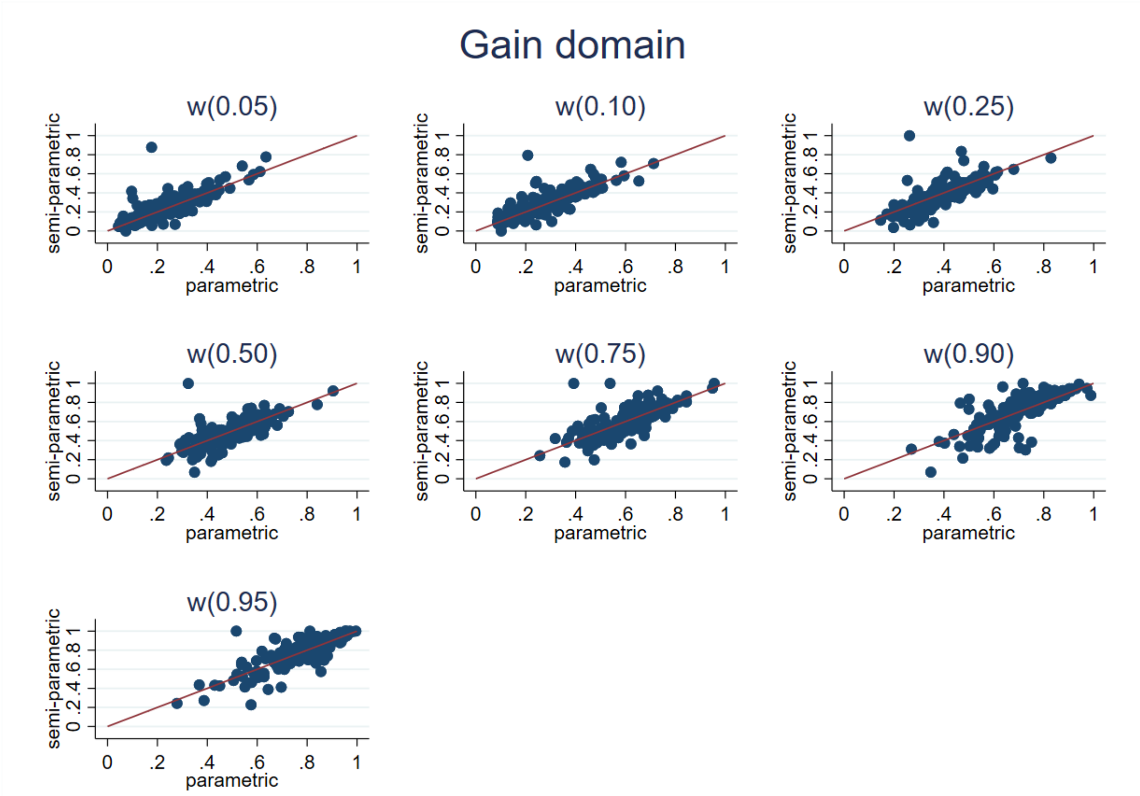
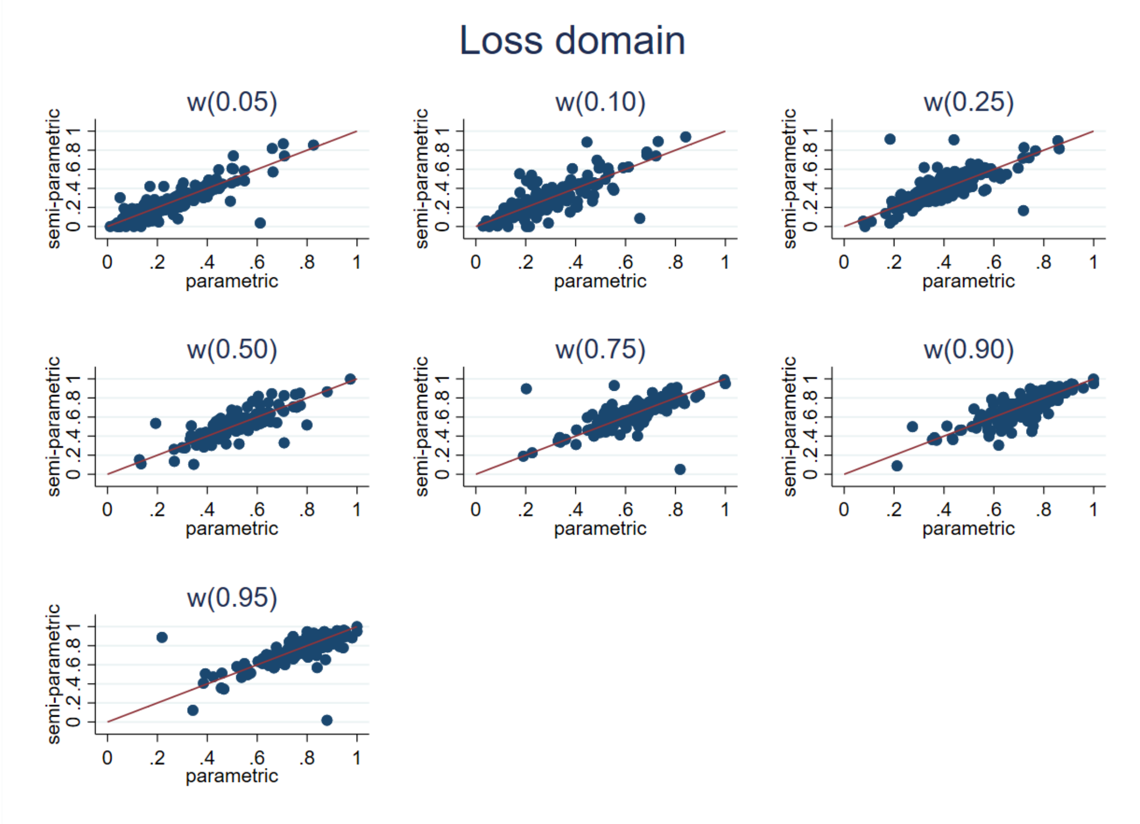


Figure 8: Distribution of probability weights for losses



A3- Spline extension

Our semi-parametric method requires specifying a utility function. Using spline instead may further serve to reduce or eliminate such parametric assumptions. In this section, we provide a linear spline extension for our semi-parametric method.[37]

A3.1-Estimating probability weighting functions and utility curvature

We consider utility function over a range $[\underline{q}, 0] \cup [0, \overline{q}]$. We divide this range in arbitrarily small intervals $[q_j, q_{j+1}]$ over which utility is assumed to be linear, with $\underline{q} \equiv q_{-\tau^-} < q_{-\tau^-+1} < ... < q_0 \equiv 0 < q_1 < ... < q_{\tau^+-1} < q_{\tau^+} \equiv \overline{q}$ and $j \in \{-\tau^-, -\tau^- + 1, ..., -1, 0, 1, ..., \tau^+ - 1, \tau^+\}$. Notations ($\boldsymbol{ce}_l$, $\boldsymbol{x}_l$, $\boldsymbol{y}_l$, $\boldsymbol{I}_l^k$, $\boldsymbol{D}_l^+$ and $\boldsymbol{D}_l^-$) are the same as in Sections 3.2.2 and 3.2.3. We assume that all the outcomes $\boldsymbol{x}_l$ and $\boldsymbol{y}_l$ in the binary lotteries are such that $\{\boldsymbol{x}_l, \boldsymbol{y}_l\} \in \{\underline{q} \equiv q_{-\tau^-}, q_{-\tau^-+1}, ..., q_0 \equiv 0, q_1, ..., q_{\tau^+-1}, q_{\tau^+} \equiv \overline{q}\}$. A linear spline approximation of the utility of any certainty equivalent $\boldsymbol{ce}_l \in [q_j, q_{j+1}]$ is thus given by:

$$u(\boldsymbol{ce}_l) = u(q_j) + \frac{u(q_{j+1}) - u(q_j)}{q_{j+1} - q_j}(\boldsymbol{ce}_l - q_j) \tag{30}$$

where $u(0) = 0$. The subsequent identification of loss aversion as in Köbberling and Wakker (2005) requires the utility function to be differentiable at 0 with $u'(0) = 1$. This implies that $u(q_{-1}) = q_{-1}$ and $u(q_1) = q_1$. We assume response errors at the level of the utility ratio and

[37] The use of linear spline to approximate the utility function meshes well with the observation that utility is quasi-linear over a small range of outcomes (see Wakker and Deneffe, 1996; Bleichrodt and Pinto, 2000; Rabin, 2000; Fehr-Duda et al., 2006).

get the equation:[38]

$$\frac{u(\boldsymbol{ce}_l)-u(\boldsymbol{y}_l)}{u(\boldsymbol{x}_l)-u(\boldsymbol{y}_l)}=\sum_{k=1}^{K}(\delta_k^+\boldsymbol{D}_l^+ + \delta_k^-\boldsymbol{D}_l^-)\boldsymbol{I}_l^k+\boldsymbol{e}_l^i \tag{31}$$

with the scaling $u(0)=0$, $u(q_{-1})=q_{-1}$ and $u(q_1)=q_1$. Equation (31) then allows us to estimate the probability weights and the utility evaluated at the knots: $q_{-\tau^-}, q_{-\tau^-+1}, ..., q_{-2}, q_2, ..., q_{\tau^+-1}, q_{\tau^+}$.

A3.2-Estimating loss aversion

With utility function and probability weights estimated in the previous steps, we can rewrite equation (16) as follows:

$$\left(1_{(ce_k\geq 0)}+\lambda 1_{(ce_k<0)}\right)u(ce_k)=w^+(p_k)u(x_k)+\lambda w^-(1-p_k)u(y_k) \tag{32}$$

where $\{\boldsymbol{x}_k,\boldsymbol{y}_k\}\in\{\underline{q}\equiv q_{-\tau^-},q_{-\tau^-+1},...,q_0\equiv 0,q_1,...,q_{\tau^+-1},q_{\tau^+}\equiv\overline{q}\}$ and $1_{(.)}$ refers to the indicator function. Using the same notation as in Section 3.2.3 and assuming an additive error at the basic utility scale ($\boldsymbol{e}_k$), the empirical counterpart of equation (32) then becomes:

$$\left(\boldsymbol{D}_k^+ +\lambda\boldsymbol{D}_k^-\right)\hat{u}(\boldsymbol{ce}_k)=\hat{\boldsymbol{\delta}}_k^+\hat{u}(\boldsymbol{x}_k)+\lambda\underline{\hat{\boldsymbol{\delta}}}_k^-\hat{u}(\boldsymbol{y}_k)+\boldsymbol{e}_k \tag{33}$$

We can then estimate the loss aversion index of Köbberling and Wakker (2005) from equation (33) by minimizing the sum of squared errors with respect to λ.

A3.3-Comparing our semi-parametric method with its non-parametric spline version

To conduct spline estimations, we need data with a high number of certainty equivalents per subject. Among the several existing datasets that we reviewed (e.g. GW99; BFE10; l'Haridon and Vieider, 2019; Harrison and Rutström, 2009; Andersson et al., 2020; Pedroni et al., 2017; Eisenberg et al., 2019), the one of GW99 is best suited for our analysis because it includes 165 certainty equivalents per subject.

The 165 values of certainty equivalents correspond to binary lotteries that involve 15 pairs of positive monetary outcomes (0, 25), (0, 50), (0, 75), (0, 100), (0, 150), (0, 200), (0, 400), (0, 800), (25, 50), (50, 75), (50, 100), (50, 150), (100, 150), (100, 200) and (150, 200) and 11 probabilities of obtaining the higher outcome: 0.01, 0.05, 0.1, 0.25, 0.4, 0.5, 0.6, 0.75, 0.9, 0.95 and 0.99. The subjects are 10 graduate students in psychology. GW99 use the switching outcome procedure for eliciting certainty equivalents, resulting in a total of 1650 certainty equivalents.

Results

We perform both semi-parametric and linear spline estimations using individual-level and median data. Figures 9 and 10 show the estimates for the utility and probability weighting functions.[39] For the median data, both methods lead to remarkably similar estimates for both the utility and probability weighting functions. Both methods estimate a concave utility function and an inverse S-shaped weighting function with crossover point around $p=0.4$. Even though we cannot reject the absence of differences in probability weights between the two methods (all $p-values>0.3438$), the concavity of the utility function is more pronounced in the spline

[38] We choose to define the error term at the level of the utility ratio rather than the utility itself so as to circumvent the problem of extreme utility curvature (see footnote 13 for further explanation).

[39] To compare the utility function over the range [\$0, \$800] from these two estimations, we convert them into a common scale, so that $u(\$800)=1$ and $u(0)=0$.

estimation ($p - value = 0.0010$).
At the individual level, the utility function is also found to be predominantly concave in both methods. The typical inverse S-shaped weighting function is also pervasive across methods, with the exception of Subject 6 whose weighting function is weakly concave.

Figure 9: Estimated utility functions

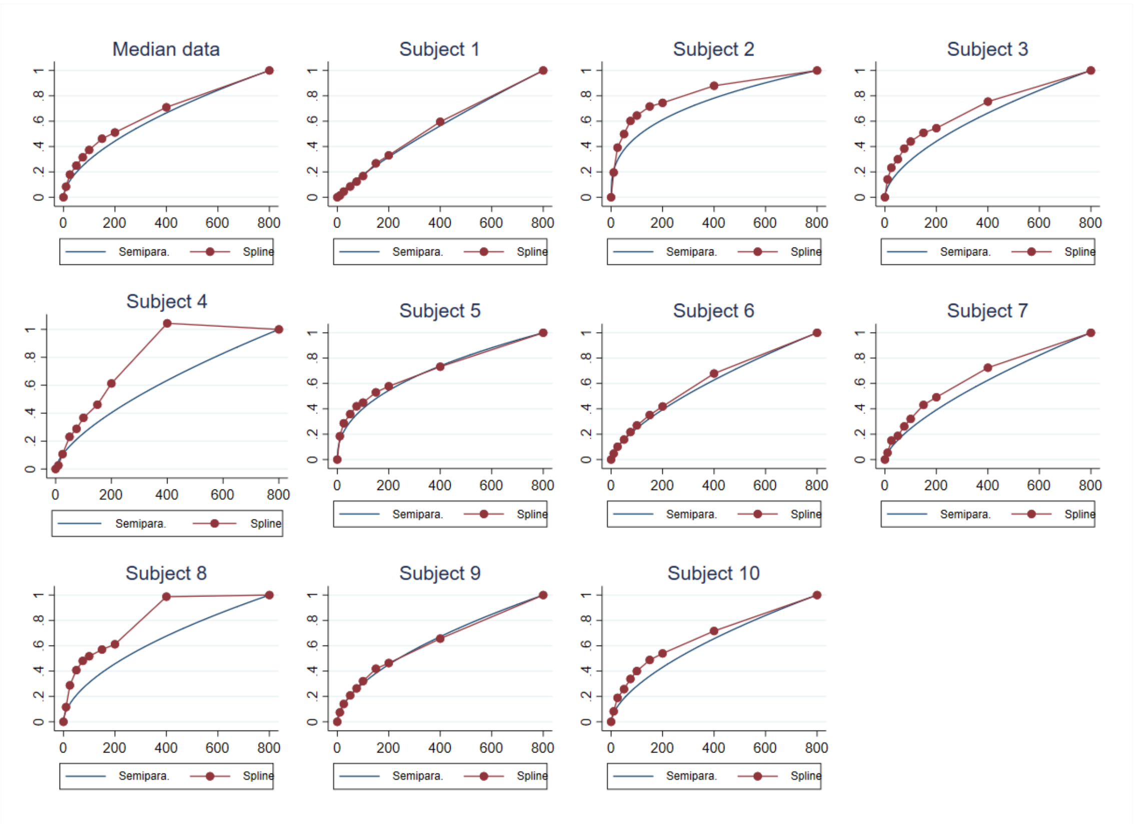
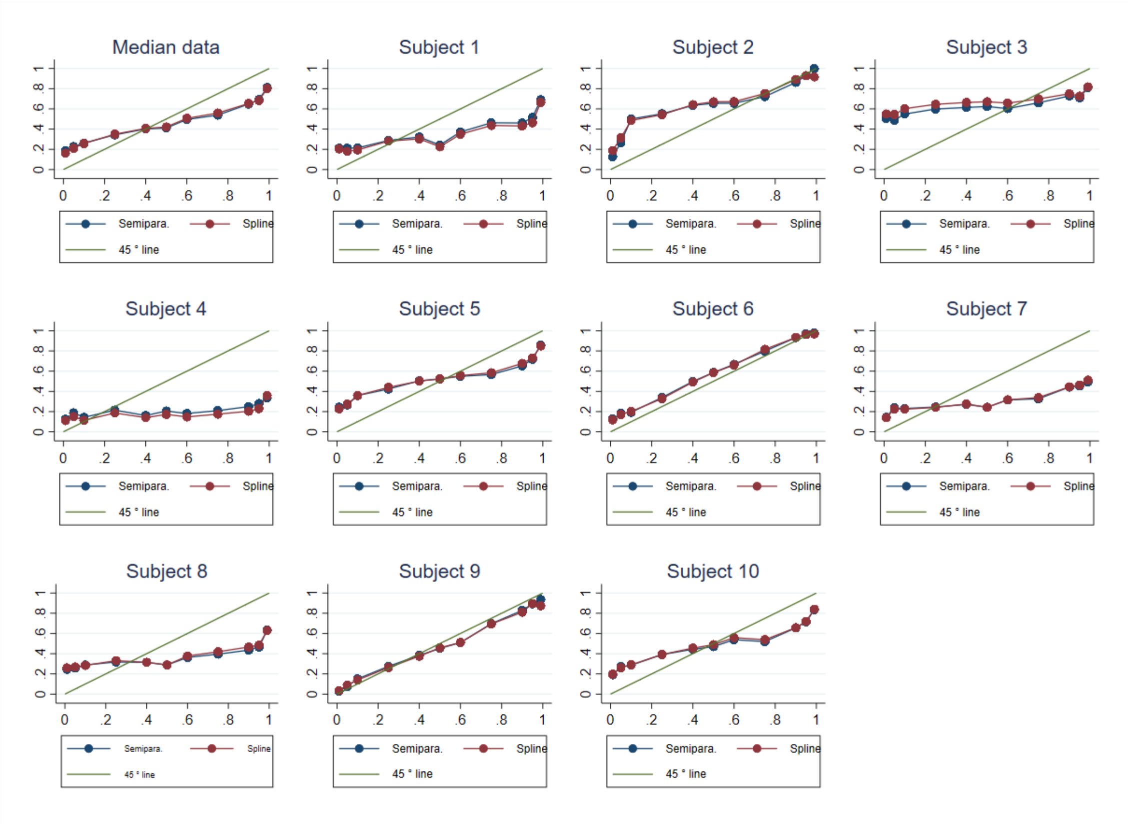


Figure 10: Estimated weighting functions



A4- Hierarchical Bayesian Parameter Estimation

The goal of this application is to illustrate how to deploy Bayesian techniques with our method. To that end, we use the data of l'Haridon and Vieider (2019) who elicit risk parameters from individual decisions of 2,939 subjects across 30 countries.

We use Hierarchical Bayesian Parameter Estimation (HBPE) (Nilsson et al., 2011; Murphy and ten Brincke, 2018; Baillon et al., 2020; Gao et al., 2020) to estimate world-level and country-level utility functions, probability weights and loss aversion. To perform HBPE, we rewrite equation (10) to account for the fact that risk preferences are elicited at the country level (c):

$$\boldsymbol{ce}_{s,c,l} = u_c^{-1}\left[(u_c(\boldsymbol{x}_l) - u_c(\boldsymbol{y}_l)) \times \left(\sum_{k=1}^{K}(\delta_k^{c,+}\boldsymbol{D}_l^{+} + \delta_k^{c,-}\boldsymbol{D}_l^{-})\boldsymbol{I}_l^{k}\right) + u_c(\boldsymbol{y}_l)\right] + \boldsymbol{e}_{s,c,l}^{i} \qquad (34)$$

with $\delta_k^{c,+} = F(z_k^{c,+})$, $\delta_k^{c,-} = F(z_k^{c,-})$, $F(.)$ is the normal cumulative distribution function, c refers to the country of residence and $e_{s,c,l}^{i}$ is a normally distributed error term $N(0, \sigma_i^2)$. As in l'Haridon and Vieider (2019), we consider a country-specific exponential utility function $u_c(.)$ characterized by α_e^c and β_e^c.

Following Rouder and Lu (2005), Nilsson et al. (2011) and Gao et al. (2020), we assume the following prior and hyperprior distributions. As priors we take: $\alpha_e^c \rightsquigarrow N(\alpha_e, \sigma_{\alpha_e^2})$, $\beta_e^c \rightsquigarrow N(\beta_e, \sigma_{\beta_e^2})$, $z_k^{c,i} \rightsquigarrow N(z_k^i, \sigma^2_{z_k^i})$, $\sigma_+^2 \rightsquigarrow IG(0.001, 0.001)$ and $\sigma_-^2 \rightsquigarrow IG(0.001, 0.001)$ where $IG(.)$ stands for inverse gamma distribution.

As hyperpriors, we take: $\alpha_e \rightsquigarrow N(0, 10)$, $\beta_e \rightsquigarrow N(0, 10)$, $z_k^i \rightsquigarrow N(0, 1)$, $\sigma^2_{\alpha_e} \rightsquigarrow IG(0.001, 0.001)$, $\sigma^2_{\beta_e} \rightsquigarrow IG(0.001, 0.001)$, $\sigma^2_{z_k^i} \rightsquigarrow IG(0.001, 0.001)$. We estimate the posterior distributions of world- and country- specific parameters by using Markov Chain Monte Carlo (MCMC) with blocked Gibbs sampling (Baillon et al., 2020; Gao et al., 2020). After discarding a burn-in of 10000 samples, we collect 40000 samples to approximate the posterior distributions of the parameters of interest. We confirm the convergence of the MCMC chain by visual inspection of the trace plots, the autocorrelation plots and kernel densities of parameters based on the first and second halves of the sample.

We then use mean estimates of utility curvature and probability weights as inputs to estimate loss aversion. Like Nilsson et al. (2011), Spiliopoulos and Hertwig (2019) and Gao et al. (2020), we assume that the country-specific loss aversion λ^c follows a log-normal distribution $LN(\lambda, \sigma^2_\lambda)$ with $\lambda \rightsquigarrow N(0, 10)$ and $\sigma^2_\lambda \rightsquigarrow IG(0.001, 0.001)$.

Results

Utility and probability weighting functions

Figures 11 and 12 show the posterior distributions of the world-level utility curvature in the gain and loss domains. The mean of the posterior distribution of the utility curvature in the gain (loss) domain is 0.0085 (-0.0055) suggesting that the utility function is generally concave. In the gain domain, the 95% credible interval is [0.0021, 0.0150] showing that the utility function deviates significantly from linearity. In the loss domain, linearity of the utility function cannot be rejected at the 5% significance level with the 95% credible interval of $[-0.0165, 0.0057]$.

Figure 11: Posterior distributions of curvature and probability weights in the gain domain (world-level)

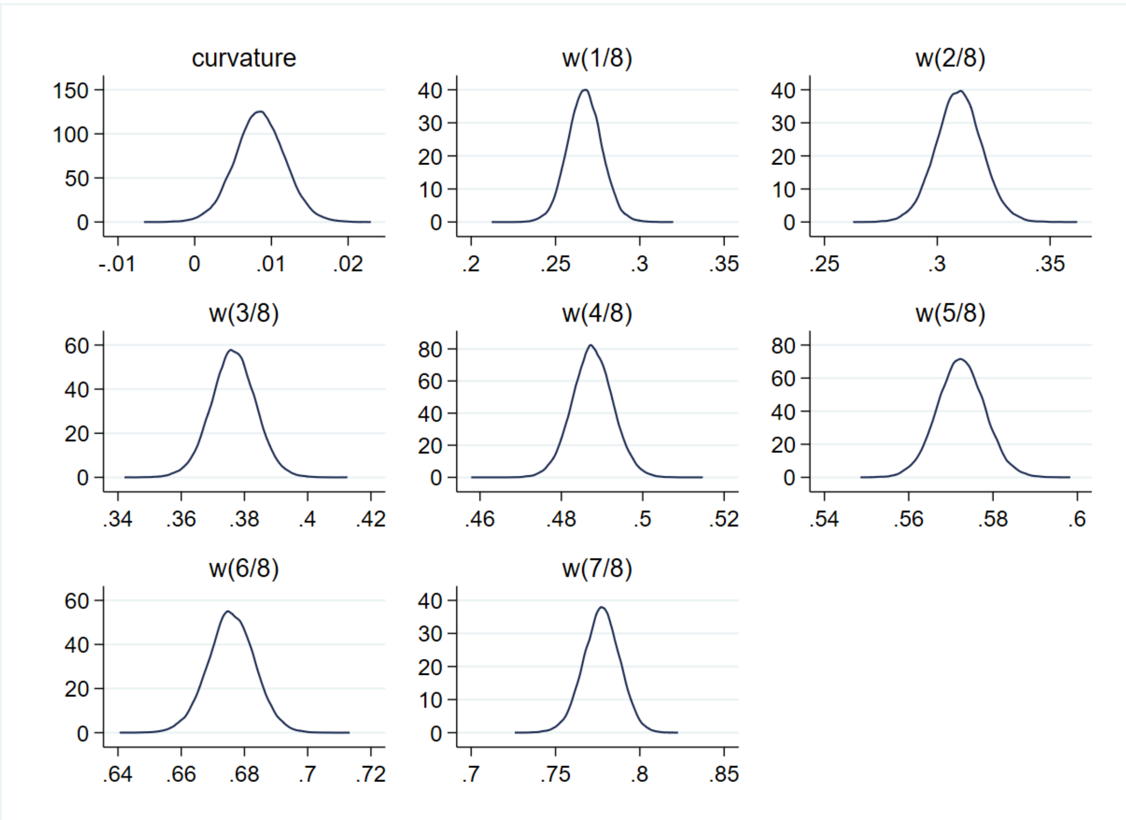
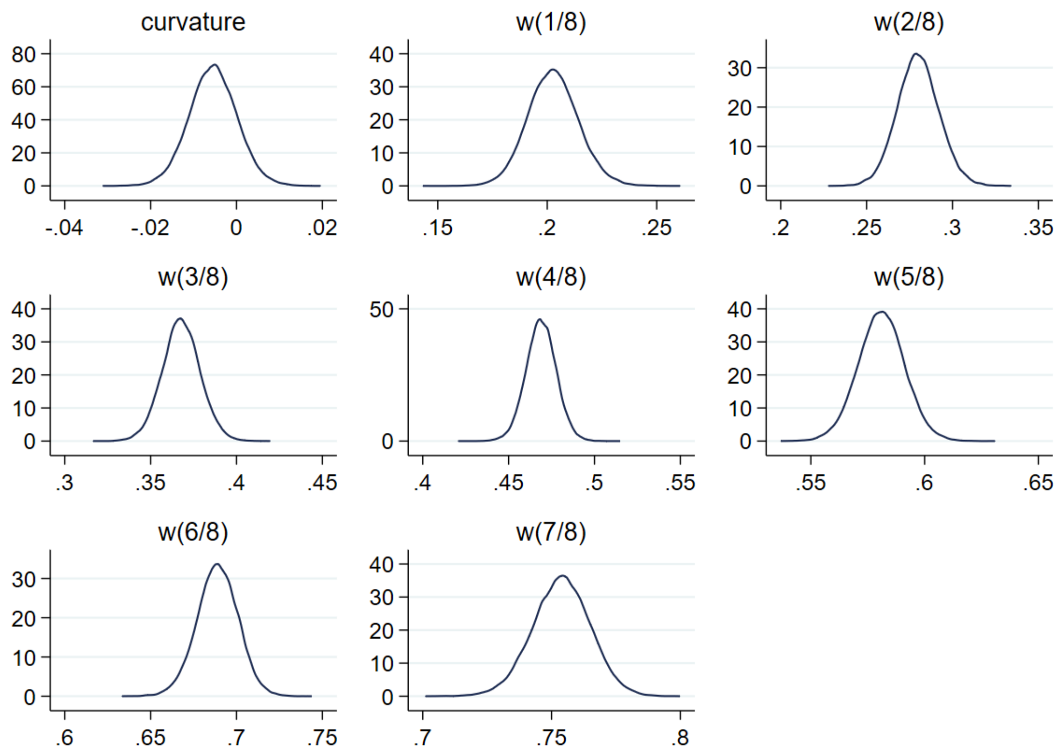


Figure 12: Posterior distributions of curvature and probability weights in the loss domain (world-level)



Turning to probability weights (see the posterior distributions in Figures 11 and 12 in the gain and loss domains for each weight), we present the mean of the posterior distribution for each weight in Figure 13. We observe the standard results of an underweighting of small probabilities and an overweighing of large probabilities in both domains (l'Haridon and Vieider, 2019). The crossover point is around 3/8 in both domains as the estimated mean lies within the 95% credible interval (see Table 23). We have overweighting for probabilities of 1/8 and 2/8, and underweighting for probabilities 4/8, 5/8, 6/8 and 7/8.

Figure 13: Estimated weighting functions

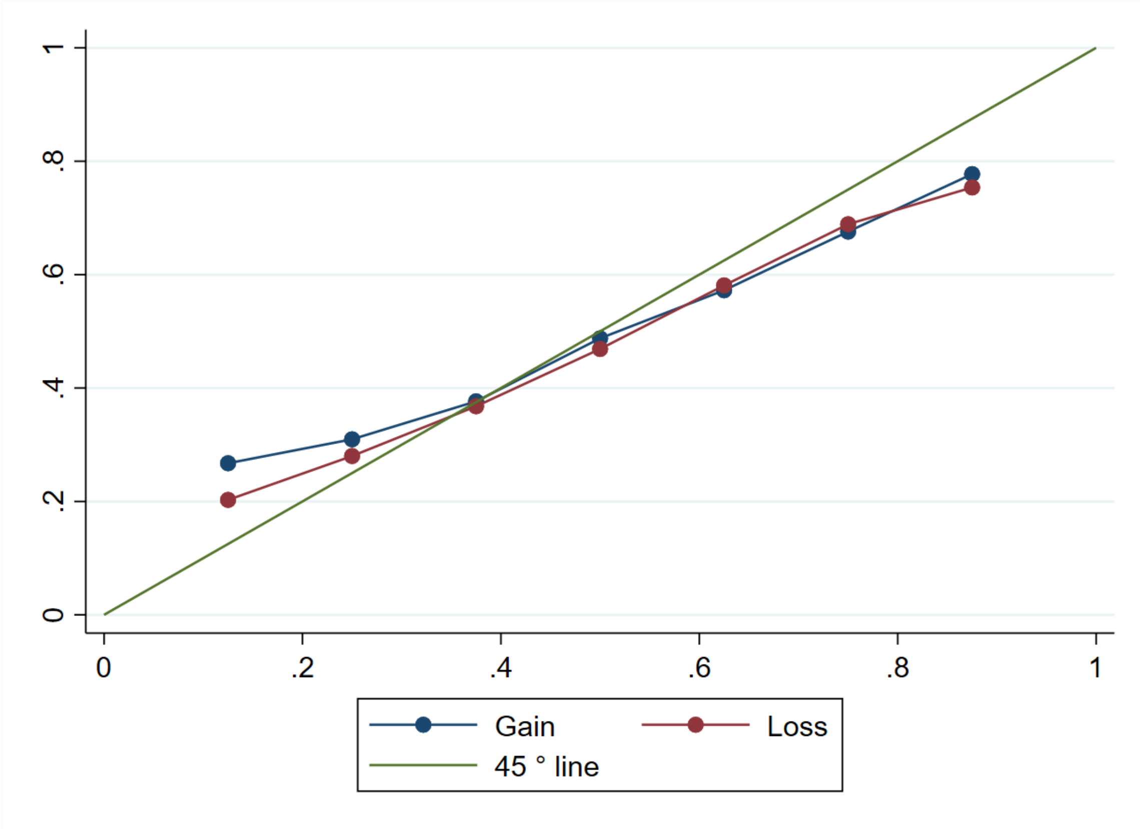


Table 23: Posterior statistics for utility and probability weights

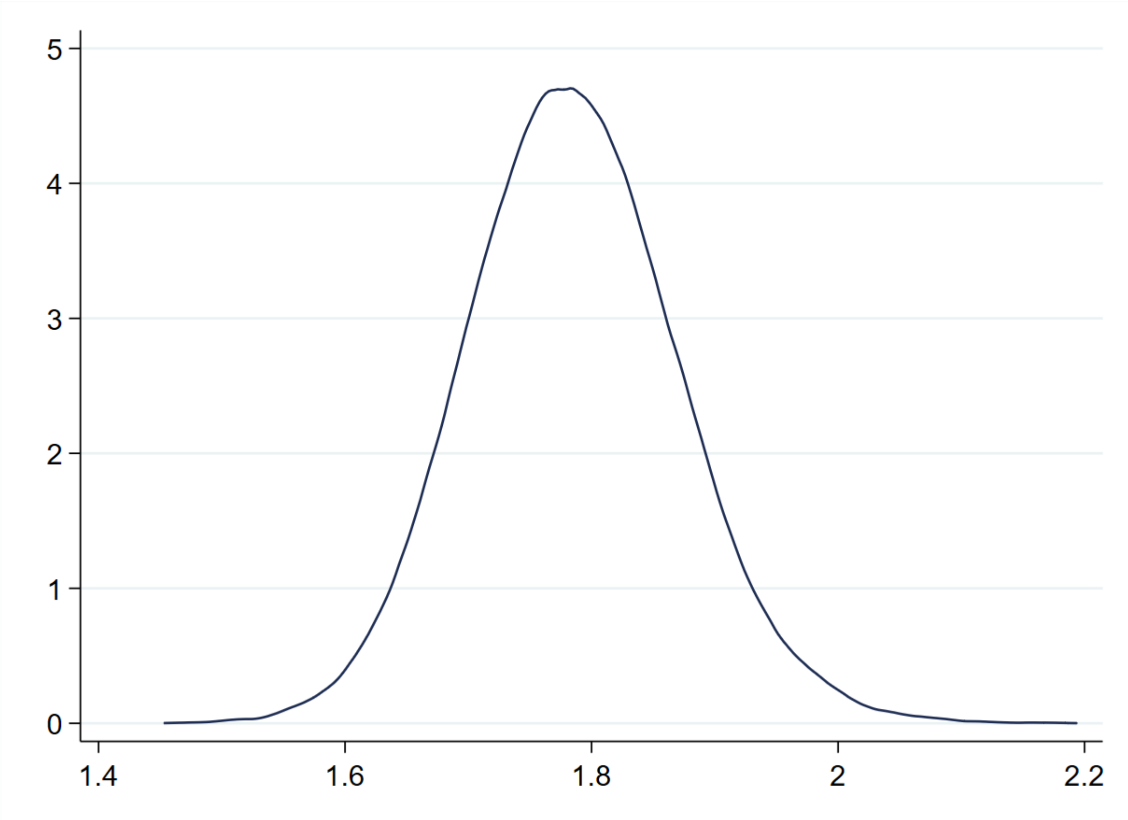
	Mean	Std. Dev.	MCSE	Median	95% credible interval
Gain domain					
α_e	0.0085	0.0033	0.0000	0.0085	0.0021 ; 0.0150
$w^+(1/8)$	0.2674	0.0102	0.0001	0.2673	0.2477 ; 0.2874
$w^+(2/8)$	0.3096	0.0102	0.0001	0.3096	0.2895 ; 0.3301
$w^+(3/8)$	0.3765	0.0071	0.0004	0.3765	0.3624 ; 0.3904
$w^+(4/8)$	0.4877	0.0050	0.0002	0.4876	0.4777 ; 0.4977
$w^+(5/8)$	0.5724	0.0056	0.0002	0.5724	0.5614 ; 0.5837
$w^+(6/8)$	0.6758	0.0074	0.0001	0.6758	0.6610 ; 0.6903
$w^+(7/8)$	0.7773	0.0108	0.0001	0.7774	0.7556 ; 0.7981
Loss domain					
β_e	-0.0055	0.0056	0.0000	-0.0055	-0.0165 ; 0.0057
$w^-(1/8)$	0.2027	0.0116	0.0001	0.2025	0.1805 ; 0.2263
$w^-(2/8)$	0.2801	0.0122	0.0001	0.2799	0.2565 ; 0.3047
$w^-(3/8)$	0.3678	0.0111	0.0003	0.3677	0.3461 ; 0.3898
$w^-(4/8)$	0.4689	0.0088	0.0003	0.4689	0.4516 ; 0.4862
$w^-(5/8)$	0.5810	0.0103	0.0003	0.5810	0.5606 ; 0.6013
$w^-(6/8)$	0.6889	0.0120	0.0001	0.6890	0.6648 ; 0.7121
$w^-(7/8)$	0.7538	0.0112	0.0001	0.7540	0.7312 ; 0.7755

* MCSE stands for Monte Carlo standard errors

Loss aversion

Figure 14 shows the posterior distribution of the world-level loss aversion coefficient. The mean of the posterior distribution is 1.785, pointing to a substantial degree of loss aversion. The 95% credible interval of $[1.6228, 1.9594]$ also rejects loss neutrality and points to $\lambda > 1$.

Figure 14: Posterior distribution of loss aversion (world-level)



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