origin $\boxed{s_i}$ destination $\boxed{t_i}$ size $\boxed{d_i}$ of each group $\boxed{C_i}$

in which, i is the group name, $i \in [1, k]$, in total there are k different groups.

① valid path p : start at $s_i(0) \in N^T$, end at $t_i(\tau)$ $\quad \forall p \in P$

② f_p : the num of agents using p / l_p: the cost (traversal time of p)

$F(e) \in A^T$: forward arc emerging from e / $B(e) \in A^T$ backward arc.

③ Formulation : minimization (averg arrival time) / (sum (travel of all agents))

$$\min \sum_{p \in P} f_p \cdot l_p \qquad (1)$$

subject to :

$$\sum_{p \in P_i} f_p \geq d_i \qquad \forall\, i = 1, \dots, k \quad (2)$$

$$\sum_{p \in P_a} f_p - u_e \leq 0 \qquad \forall\, e \in E,\ a \in F(e) \quad (3)$$

$$\sum_{p \in P_a} f_p + u_e \leq c_e^* \qquad \forall\, e \in E,\ a \in B(e) \quad (4)$$

$$\sum_{p \in P_v} f_p \leq c_v^* \qquad \forall\, v \in V^T \quad (5)$$

$$f_p \geq 0 \text{ and integral} \quad \forall\, p \in P \quad (6)$$

$$\cancel{f_e}\, u_e \geq 0 \text{ and integral} \quad \forall\, e \in E \quad (7)$$

Back to $G(T)$ explicity not possible generated question : bcs there are infinite paths in G^T, therefore, we have to select some of them which could improve the value of objective function$^{(1)}$, and these set are considered as "possibly useful" paths.

$\Rightarrow$ LP - relaxation (by column generation)

12.27. 周一.

周一 ~ 周五 Algoritm toolbox.
Data structure

① 15M 32 pts.

② 25m. 32 pts.

EP0 / EP1

MA (medial axis) $\xRightarrow{\text{sampling [4]}}$ MG = (V, E)
MA ∪ clearness from obstacles ∪ all cycles in Graph.

MG ⟹ capacitated graph G(N, E)

① drop the dead end from MG

② Each edge, traversal time $l_e = \frac{len(e)}{v_{max}|_{walk}}$

capacity of $C_e = \frac{\text{minimum clearance}}{\text{max personal space}, \rho_{max}}$

capacity of C_n = maximum person in node.

+ time (wait info)
Condensed time-expanded graph $G^T = (N^T, A^T)$ in which, T is time horizon
round up MG ⟹ with variable no. 1: time step Δt, therefore,

$$l_e^* = \lceil l_e / \Delta t \rceil$$
$$C_e^* = \lceil C_e \cdot \Delta t \rceil$$
$$C_n^* = \lceil C_n \cdot \Delta t \rceil$$

① N^T cope of each node at time step 0, 1, ..., T-1 (τ)

⊕ bidirection link ~~between~~ node {n, m} are created. for 32 pts

$n(\tau) \to m(\tau + l_e^*)$

$m(\tau) \to n(\tau + l_e^*)$

points.

⊕ waitting arc. A^T

$n(\tau+1)$ ∃ τ < T.

★ Attention: G^T is not explicited generated! need check 4.3 Column generation column generation the basic idea: ~~slove the LP problem → add variables (may~~

solve the Lp problem;
while no addition variables can be found:
    if variable can improve objective value $\overset{i.e.}{\Longrightarrow}$ reduced cost < 0:
        add variable

in which, the reduced cost of $p \in P_i$ for C_i is equal to:

$$l_p + \sum_{a\in\alpha(P)} \mu_a + \sum_{v\in\lambda(P)} \phi_v - \psi_i \qquad (8) \qquad (<0) \qquad (9)$$

① $\alpha(P)$, $\lambda(P)$ are the paths / nodes in G^T used by path p.

② ψ_i, μ_a, $\phi_v \geq 0$ are dual variables.

→ rewrite (8) can get more insight of ~~its~~ meaning

l_p: the sum of arc length contained in p $= \sum_{a\in\alpha(p)} l_a^*$, however, l_a^* is computed based on U_{max} latgroup, modify to single current C_i, i.e.

① $l'^*_a = [l_a^* \cdot U^{max} / V_i^{des}]$

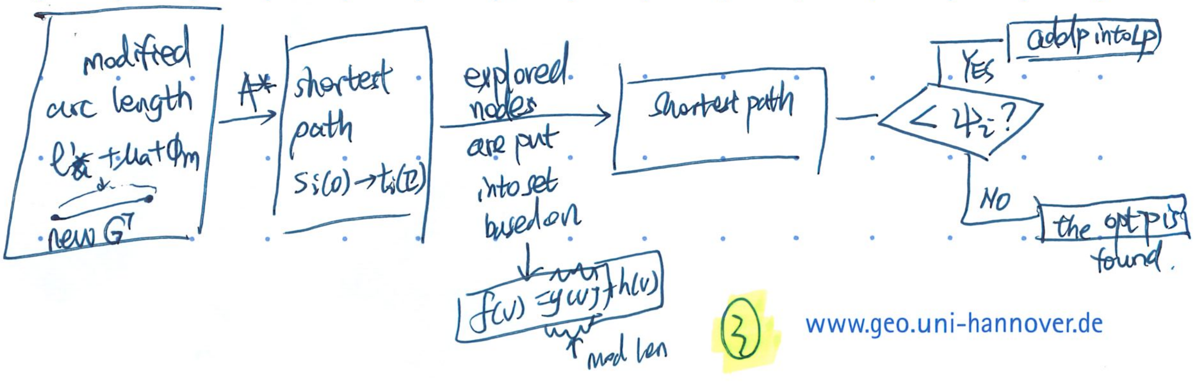
ϕ_v: when arc (n,m) exists, the node m will contribute to node,

② $\sum_{v\in\lambda(p)} \phi_v = \sum_{a\in\alpha(p)} \phi_m$

because of ① and ②, (9) became:

$$\underbrace{\sum_{a\in\alpha(p)} (l'^*_a + \mu_a + \phi_m) < \psi_i}_{} \qquad (10)$$

pricing problem ⟹ solved for each C_i by:



总结 column generation 算法：

$ Find initial (random/any) feasible solution and add them as columns to the LP.
$ repeat
  for each C_i in C ;
    find the shortest $(s_i(0) - t_i(T))$-path in G^T w.r.t. the mod-len.
    if mod-len $< \varphi$:
      add p as a new column of LP
  end for end if
$ until → nothing to be added :

* the result of LP could be fractional value (%)
  ① round down ⟹ some are left.
  ② Use Cooperative A* construct additional paths

PHASE A END HERE.

DETAILS BESIDE. $\boxed{\text{graph}}$ & $\boxed{\text{LP}}$