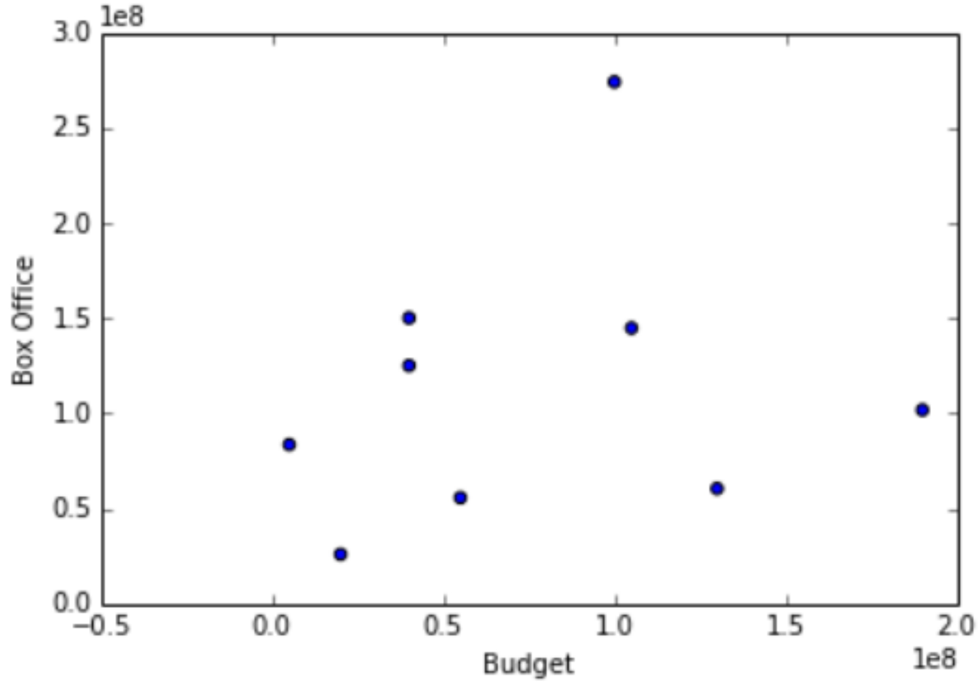
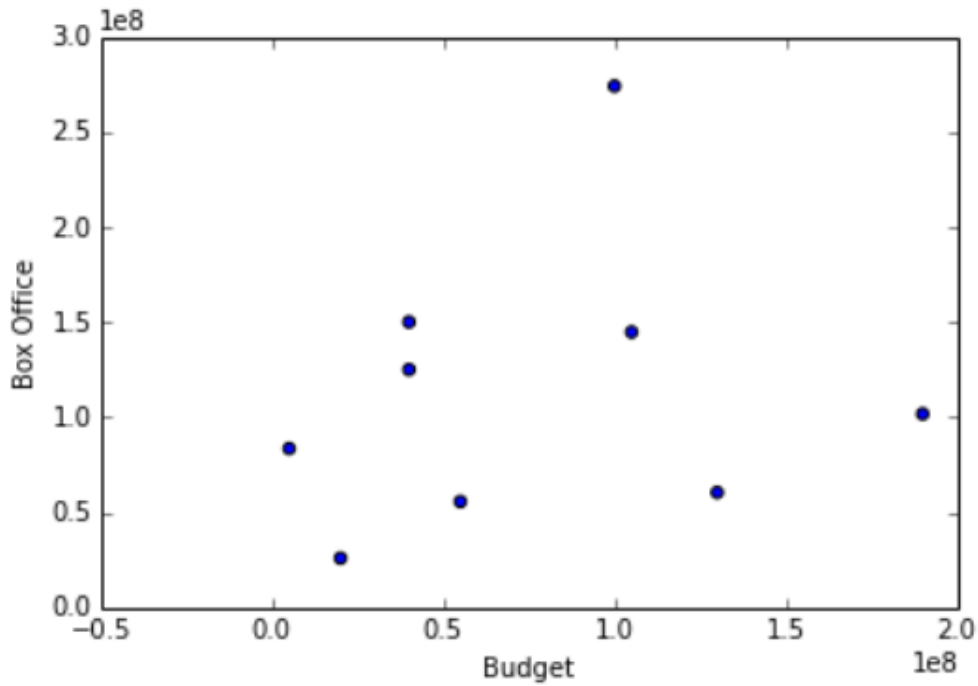
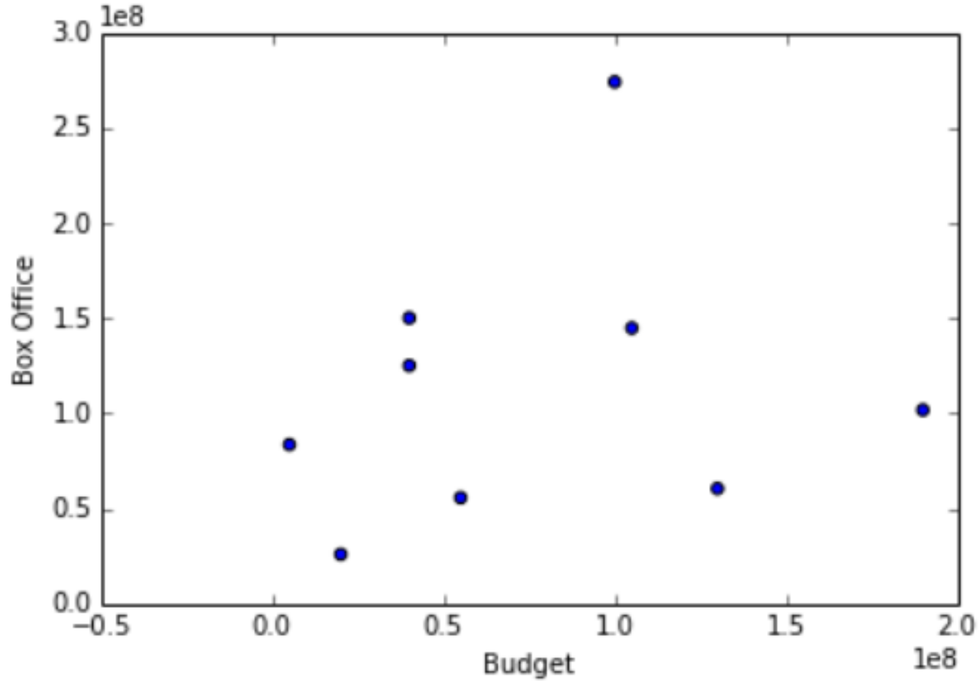
Linear Regression


1e8
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Box Office
−0.5
0.0
0.5
1.0
1.5
2.0
Budget
1e8



$$y_{\beta}(x) = \beta_0 + \beta_1 x + \varepsilon$$



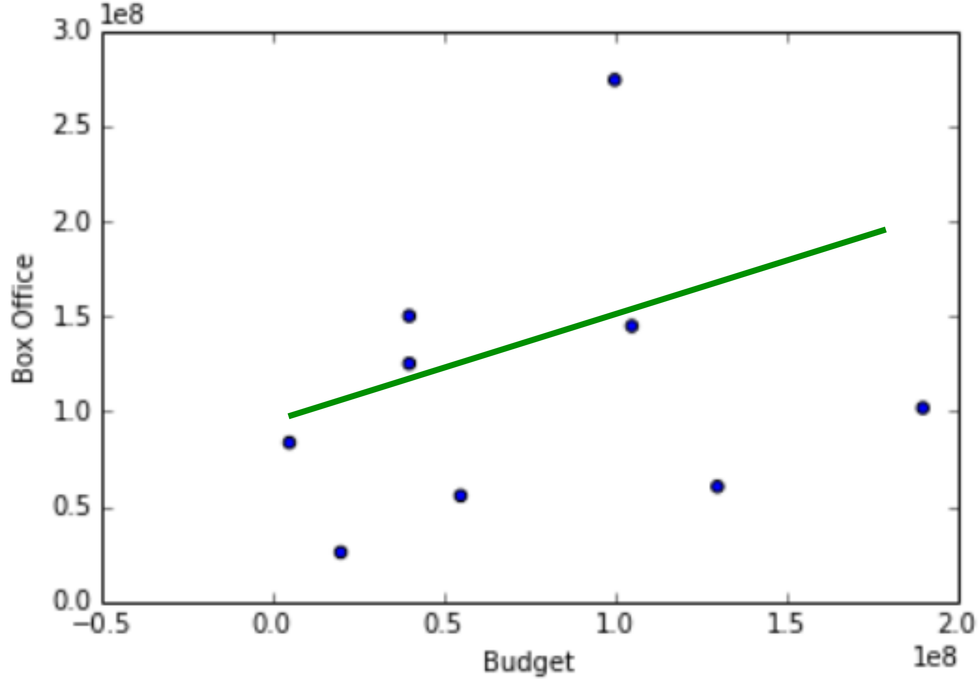
coef 0 coef 1

$$y_\beta(x) = \beta_0 + \beta_1 x + \varepsilon$$

Gross of movie

Budget of movie

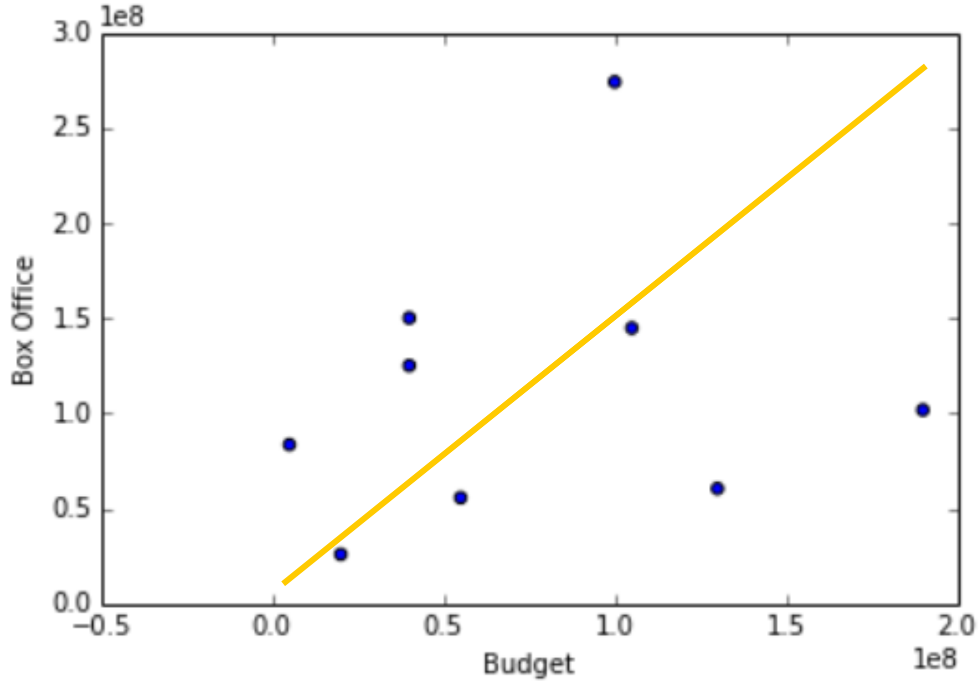
Noise (random for each movie)



$$y_\beta(x) = \beta_0 + \beta_1 x + \varepsilon$$

$\beta_0 = 80 million$

$\beta_1 = 0.5$



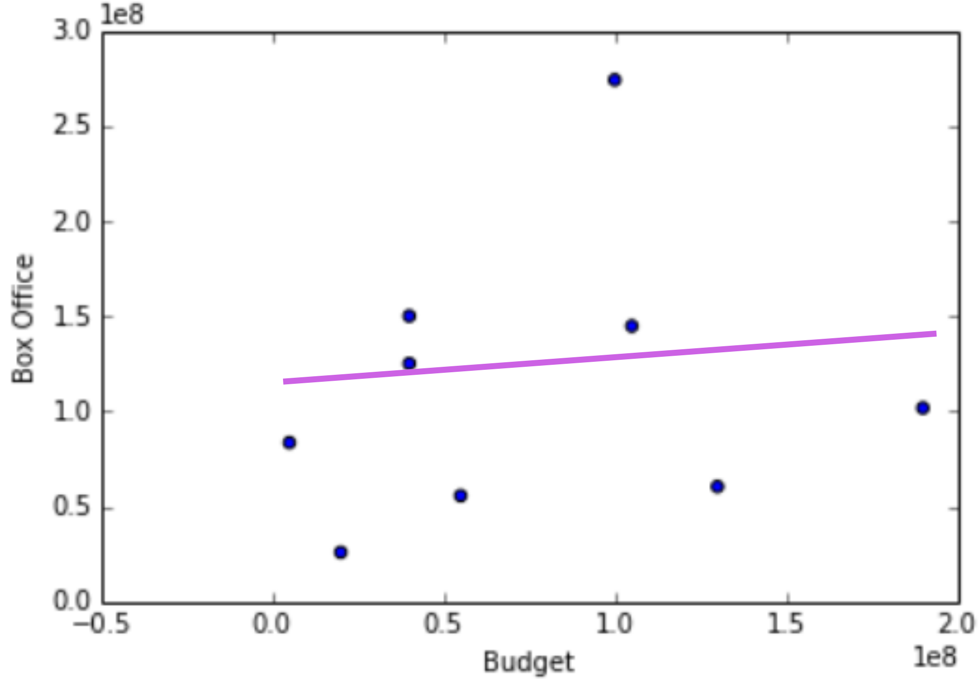
$$y_\beta(x) = \beta_0 + \beta_1 x + \varepsilon$$

$\beta_0 = 80 million$

$\beta_1 = 0.5$

$\beta_0 = 0$

$\beta_1 = 1.5$

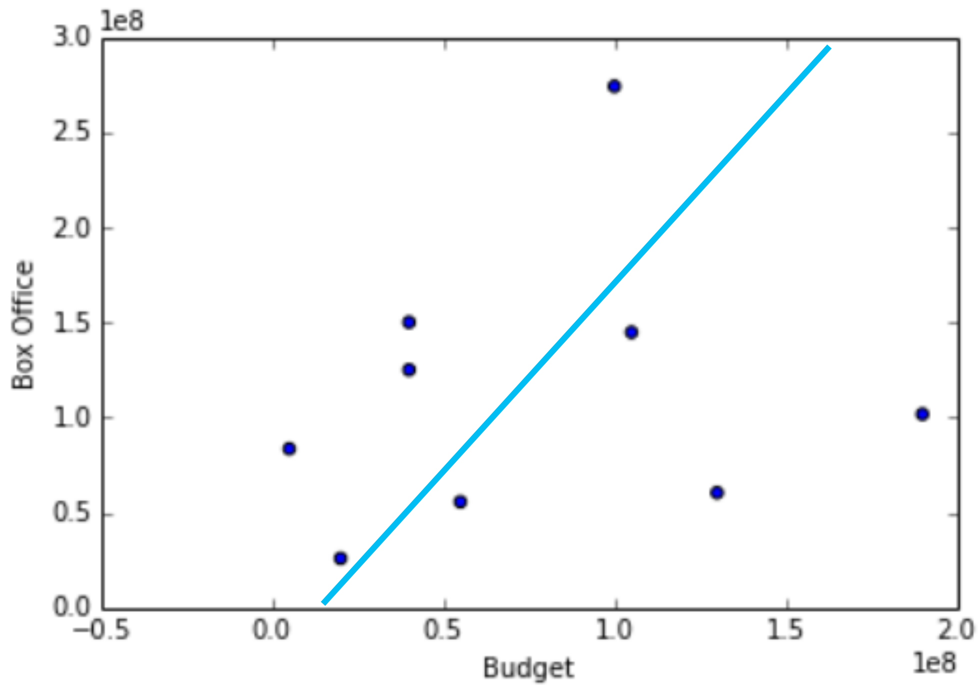


$$y_\beta(x) = \beta_0 + \beta_1 x + \varepsilon$$

$\beta_0 = 80 million$
$\beta_1 = 0.5$

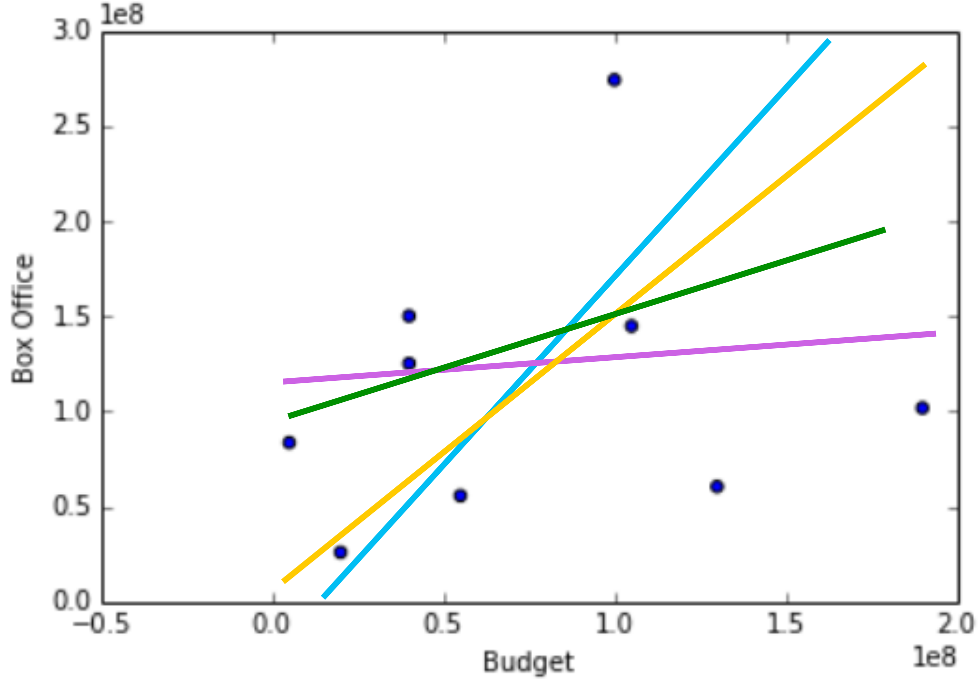
$\beta_0 = 0$
$\beta_1 = 1.5$

$\beta_0 = 120 million$
$\beta_1 = 0.1$



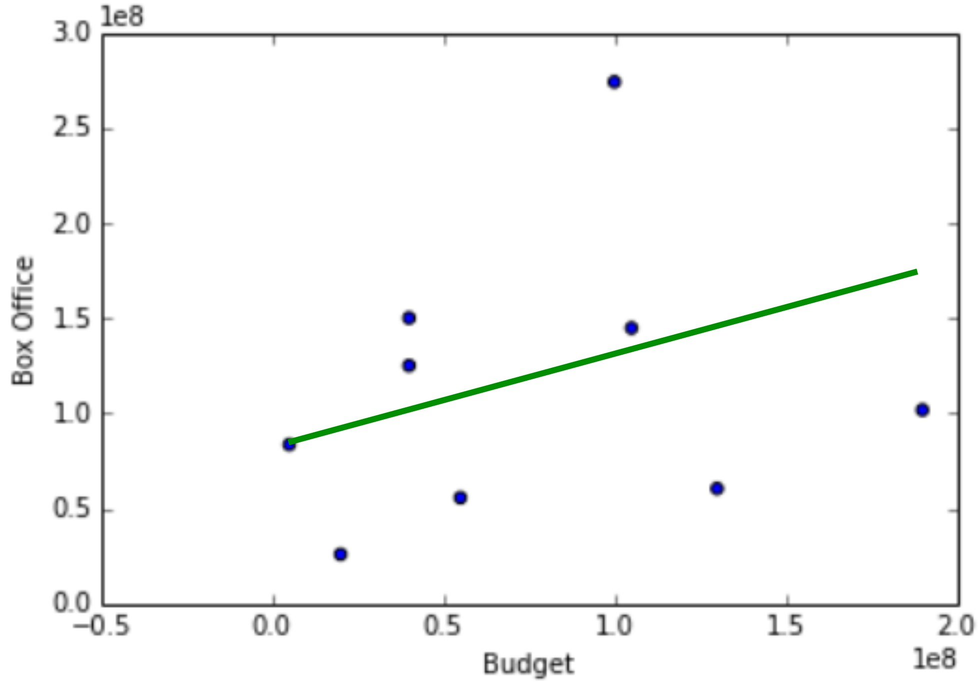
$$y_\beta(x) = \beta_0 + \beta_1 x + \varepsilon$$

$\beta_0 = 80 million$	$\beta_0 = 0$	$\beta_0 = 120 million$	$\beta_0 = 30 million$
$\beta_1 = 0.5$	$\beta_1 = 1.5$	$\beta_1 = 0.1$	$\beta_1 = 2$



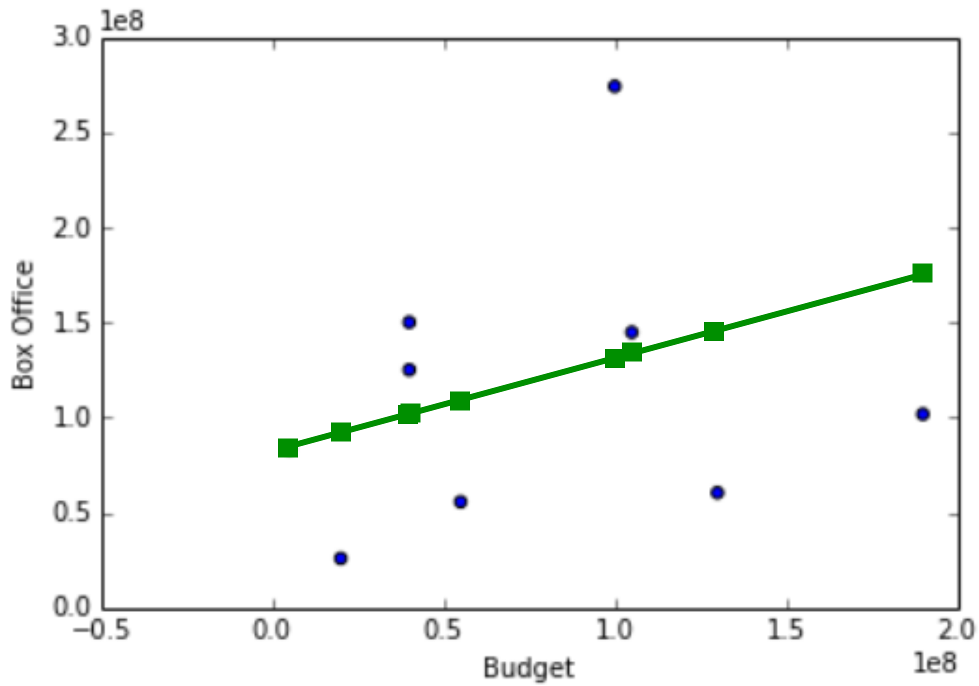
$$y_\beta(x) = \beta_0 + \beta_1 x + \varepsilon$$

$\beta_0 = 80 million$ $\beta_1 = 0.5$	$\beta_0 = 0$ $\beta_1 = 1.5$	$\beta_0 = 120 million$ $\beta_1 = 0.1$	$\beta_0 = 30 million$ $\beta_1 = 2$



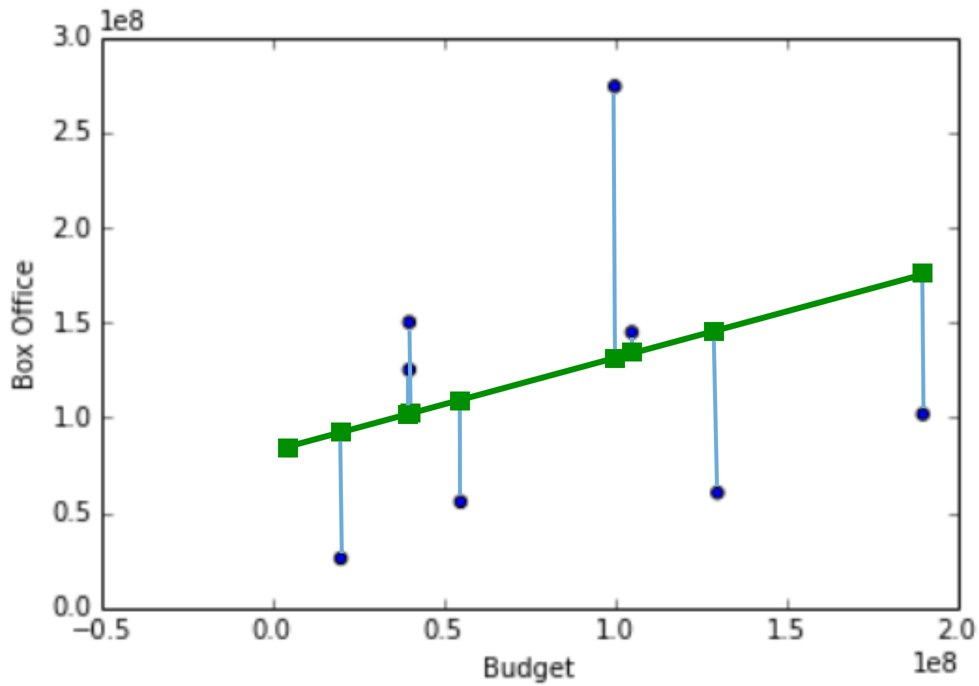
$$y_\beta(x) = \beta_0 + \beta_1 x + \varepsilon$$

$\beta_0 = 80 million$	$\beta_0 = 0$	$\beta_0 = 120 million$	$\beta_0 = 30 million$
$\beta_1 = 0.5$	$\beta_1 = 1.5$	$\beta_1 = 0.1$	$\beta_1 = 2$



$$y_{\beta}(x) = \beta_0 + \beta_1 x + \varepsilon$$

$\beta_0 = 80 million$	$\beta_0 = 0$	$\beta_0 = 120 million$	$\beta_0 = 30 million$
$\beta_1 = 0.5$	$\beta_1 = 1.5$	$\beta_1 = 0.1$	$\beta_1 = 2$



$$y_\beta(x_{obs}^{(0)}) - y_{obs}^{(0)}$$

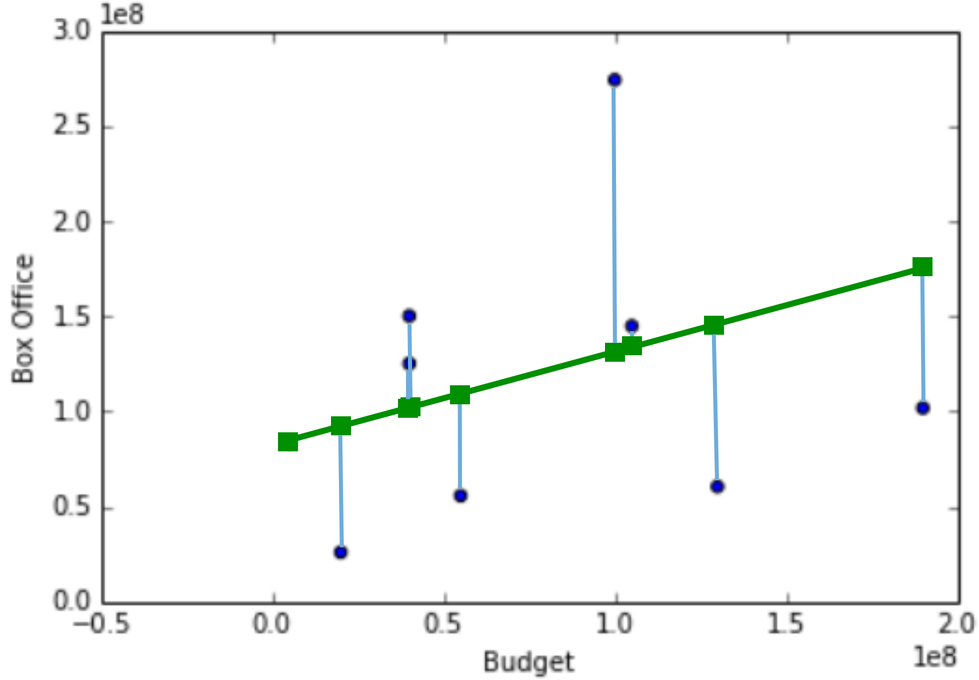
$$y_\beta(x_{obs}^{(1)}) - y_{obs}^{(1)}$$

$$y_\beta(x_{obs}^{(2)}) - y_{obs}^{(2)}$$

$$y_\beta(x_{obs}^{(3)}) - y_{obs}^{(3)}$$

Predicted value by model – Observed value

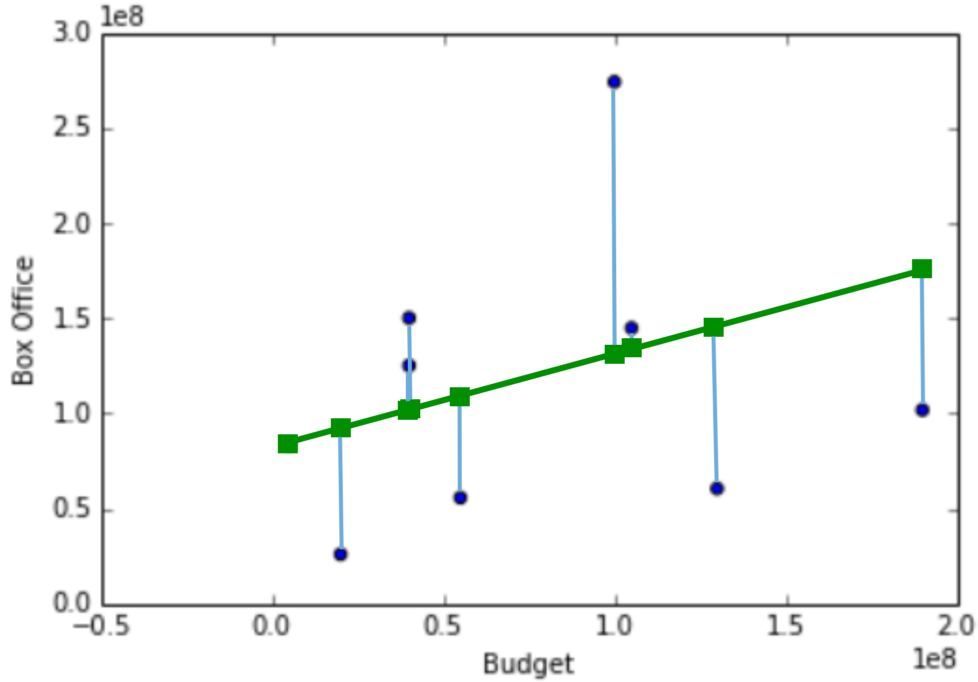
β0 =80M, β1=0.5



Predicted value by model – Observed value

β0 =80M, β1=0.5

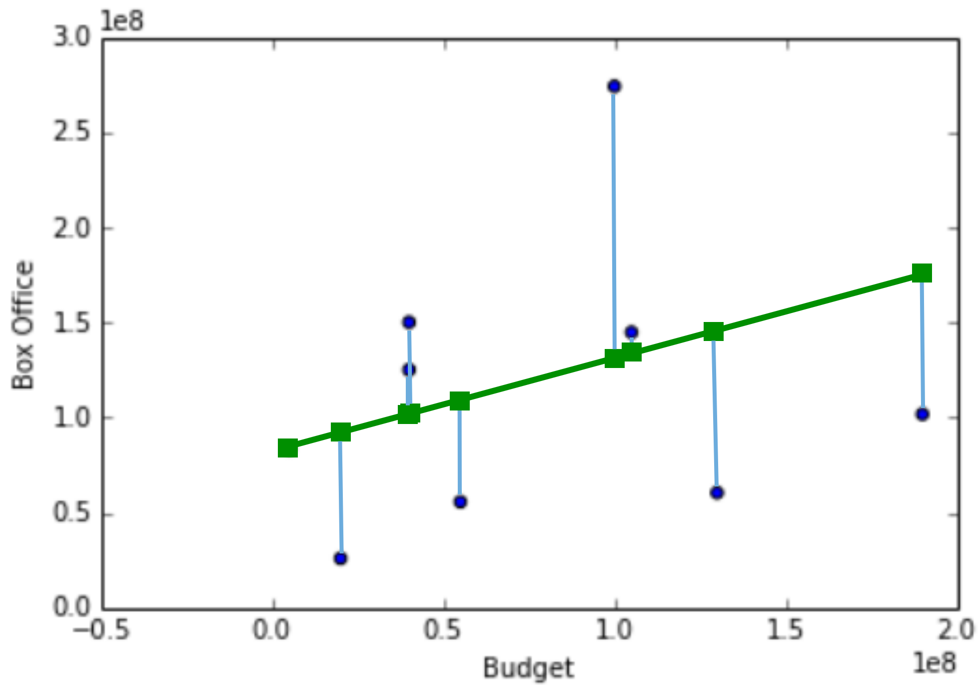
$$y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)}$$



Predicted value by model – Observed value

β0 =80M, β1=0.5

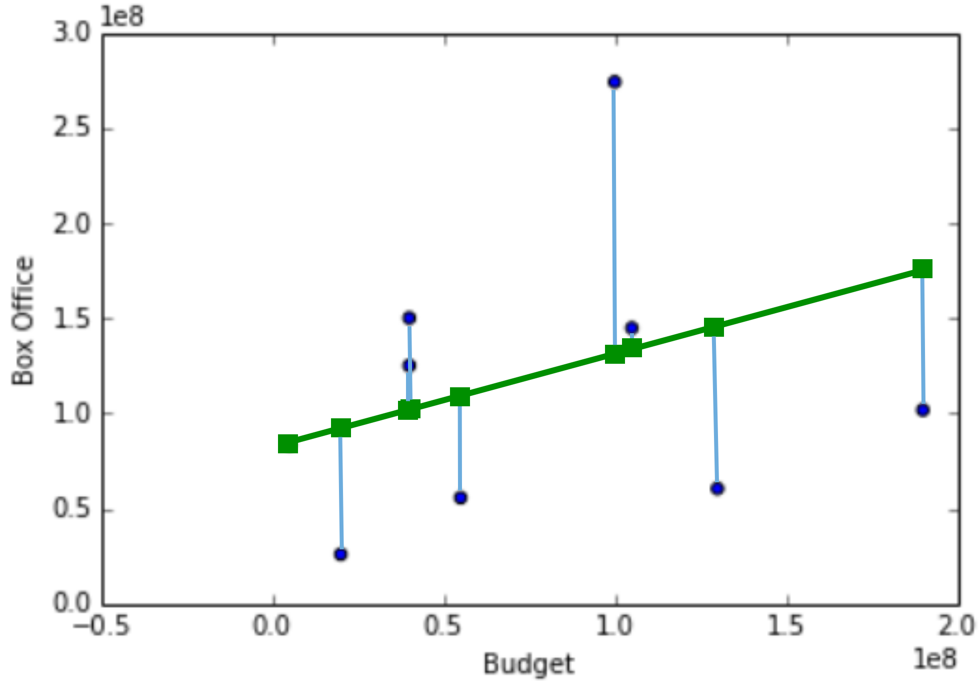
$$(\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)}$$



Predicted value by model – Observed value

β0 =80M, β1=0.5

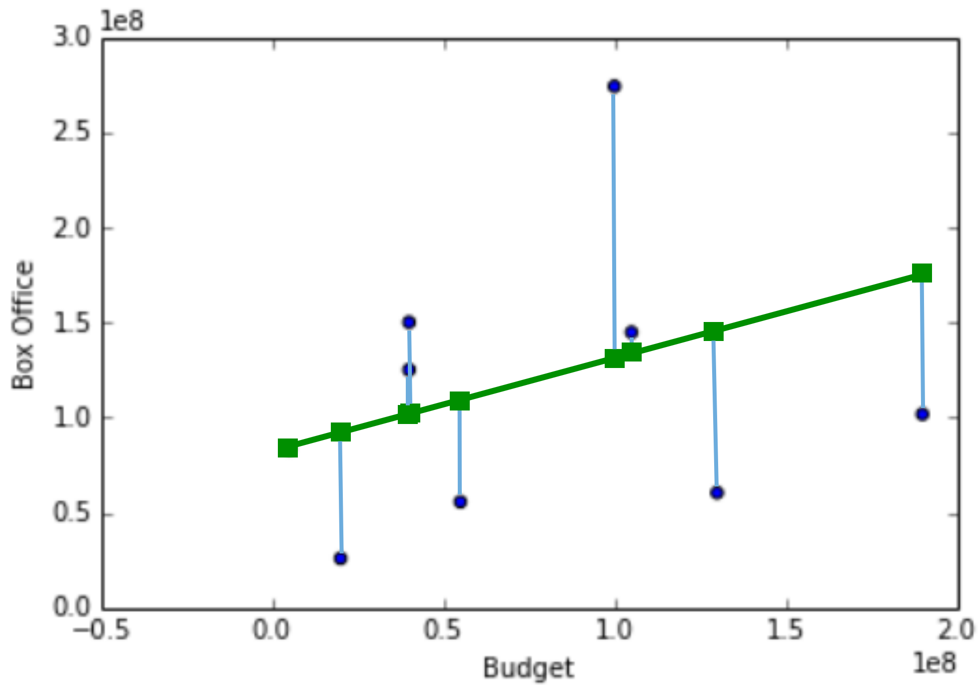
$$\sum_{i=1}^{m}\left((\beta_0+\beta_1 x_{obs}^{(i)})-y_{obs}^{(i)}\right)^2$$



Predicted value by model – Observed value

β0 =80M, β1=0.5

$$\min_{\beta_0,\beta_1} \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

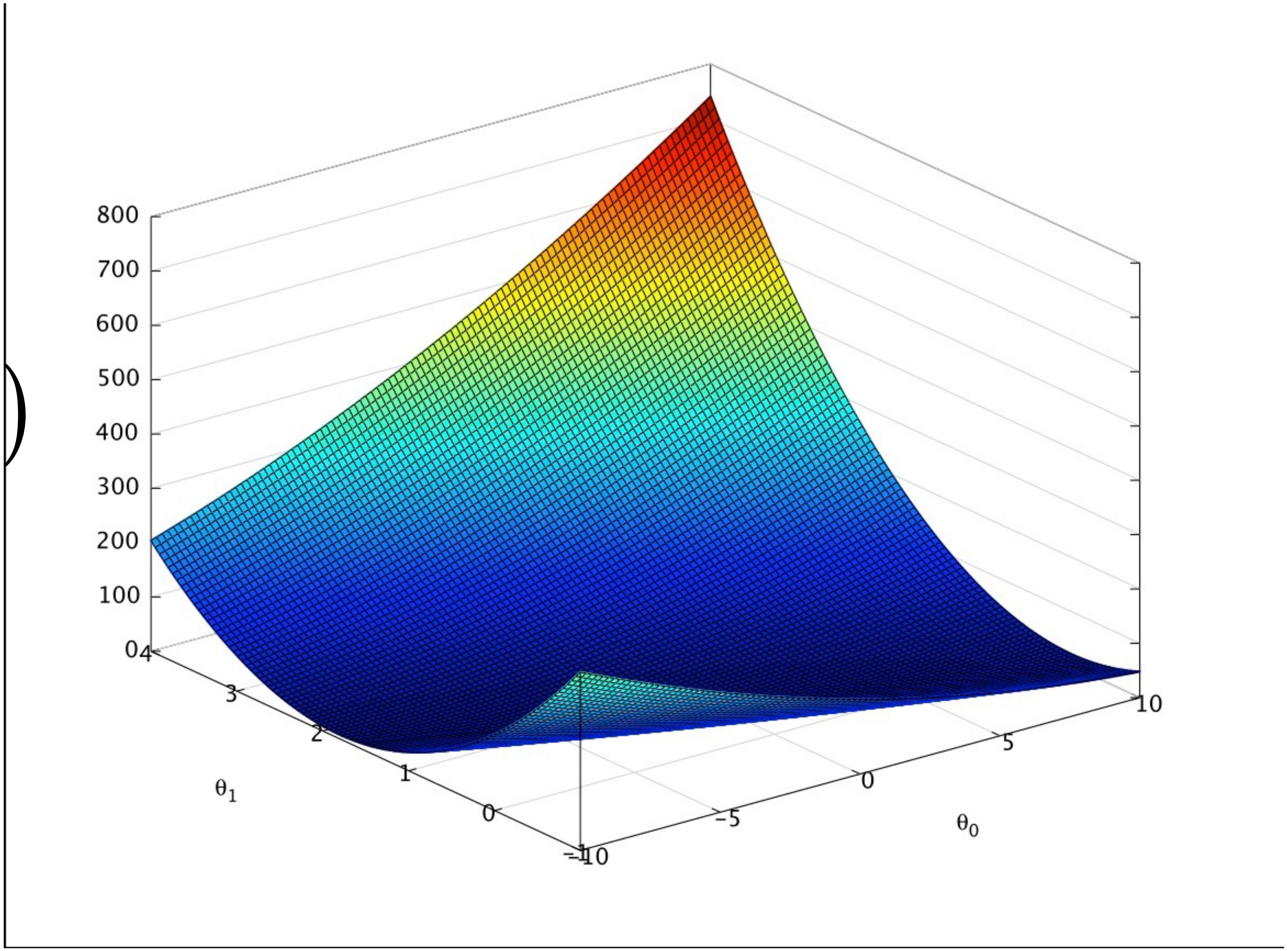


Cost function

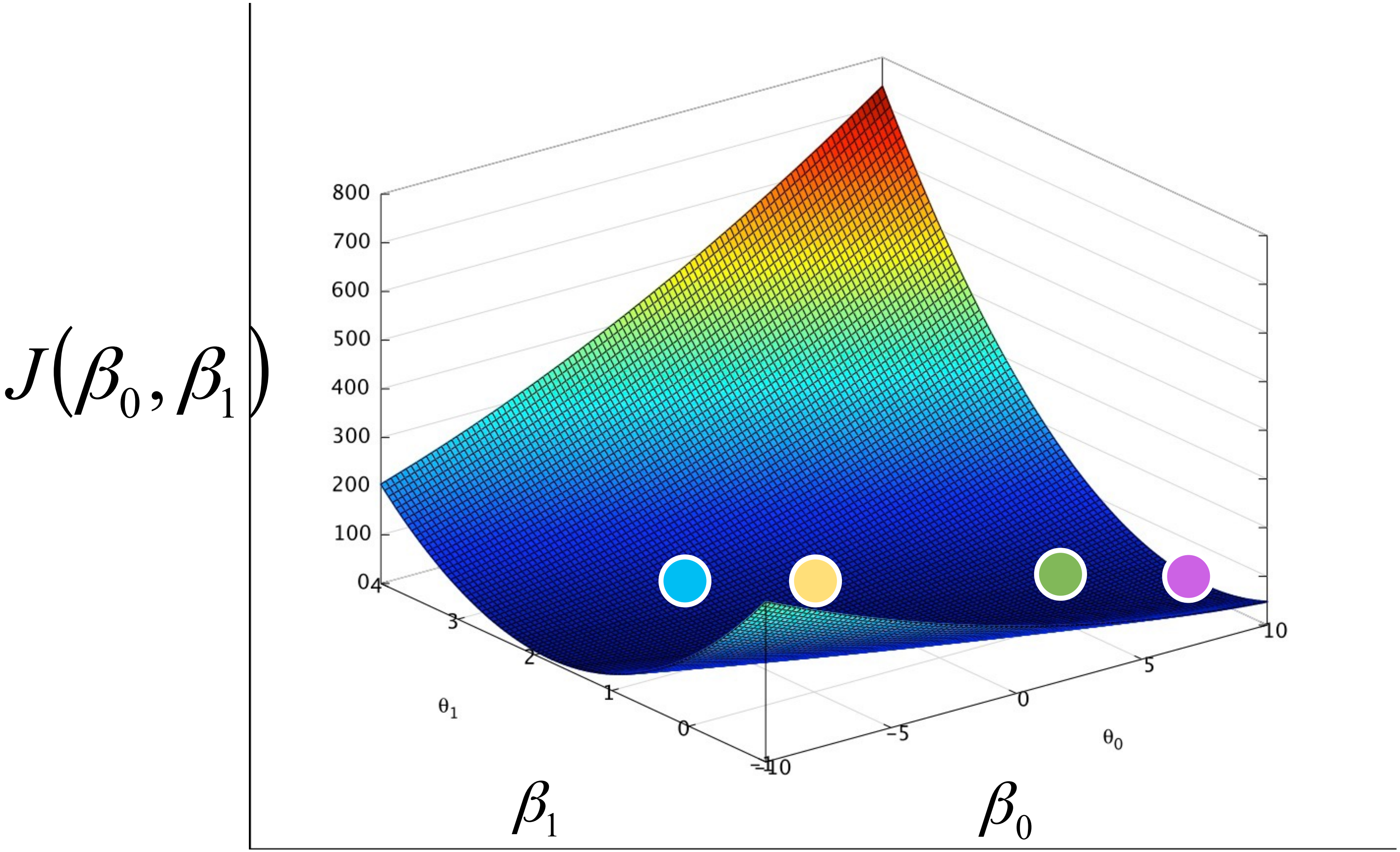
Takes a model (specific parameter values), returns score

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

$J(\beta_0, \beta_1)$



$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

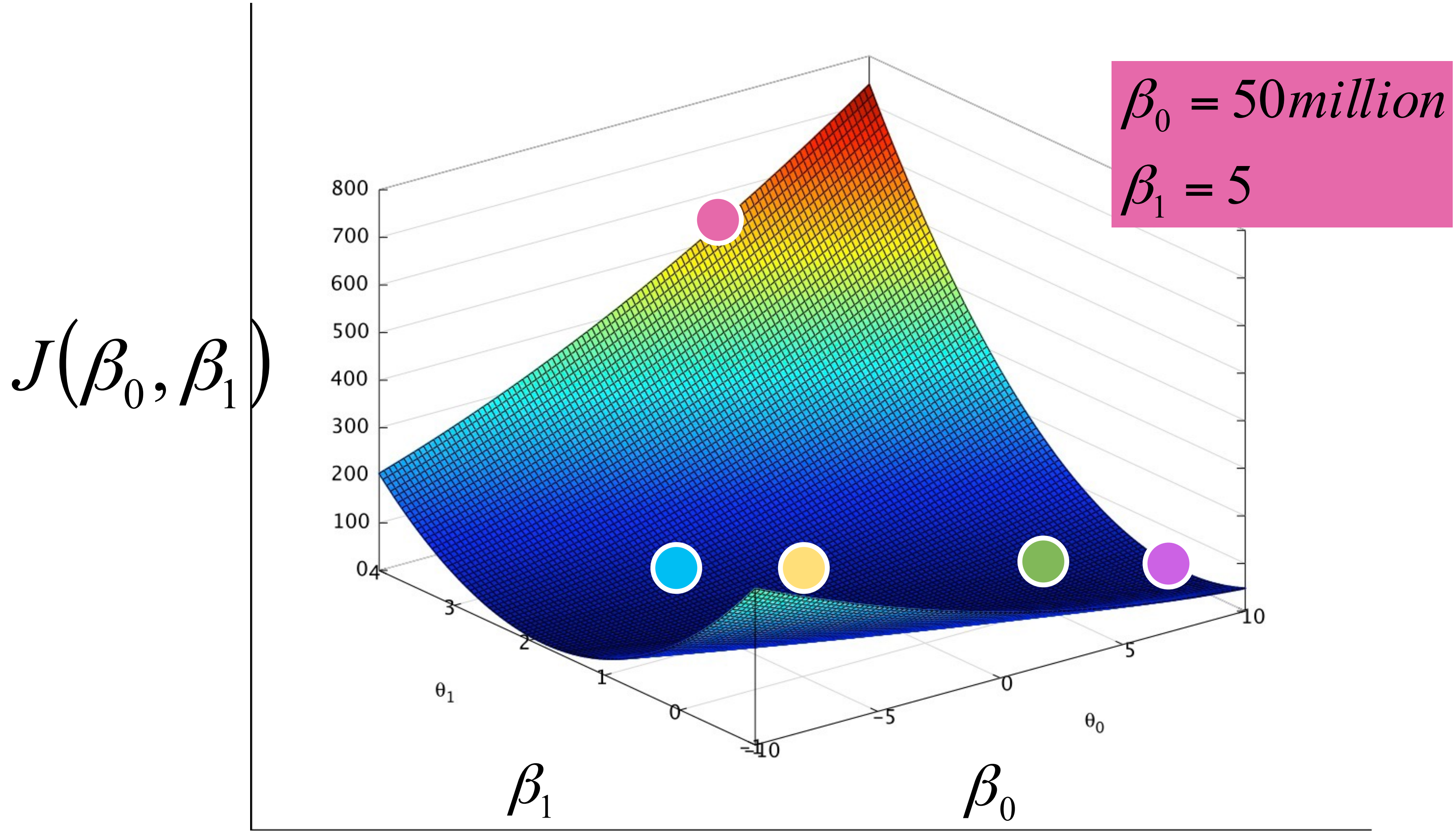


$\beta_0 = 80 million$
$\beta_1 = 0.5$

$\beta_0 = 0$
$\beta_1 = 1.5$

$\beta_0 = 120 million$
$\beta_1 = 0.1$

$\beta_0 = 30 million$
$\beta_1 = 2$

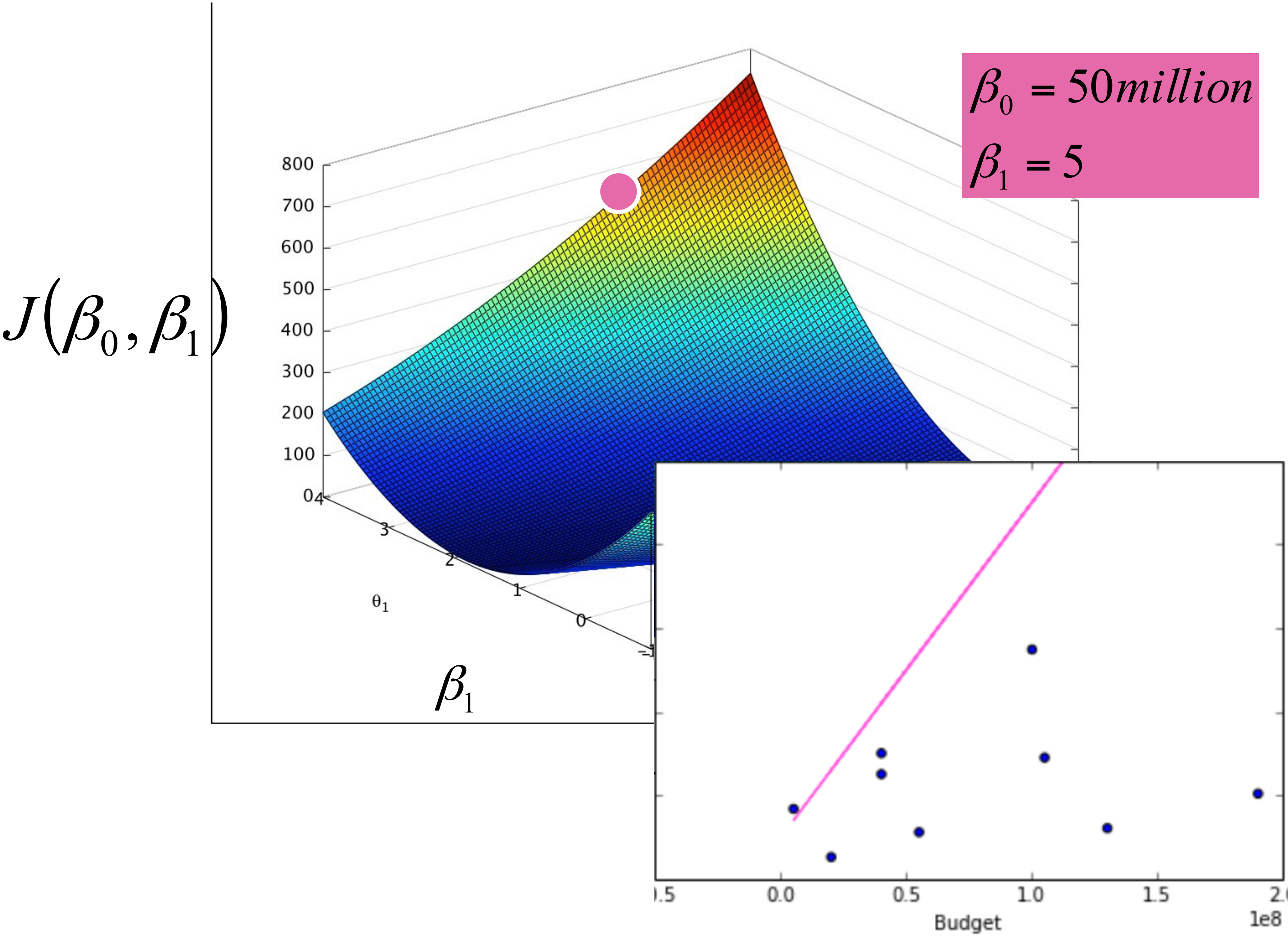


$\beta_0 = 80million$
$\beta_1 = 0.5$

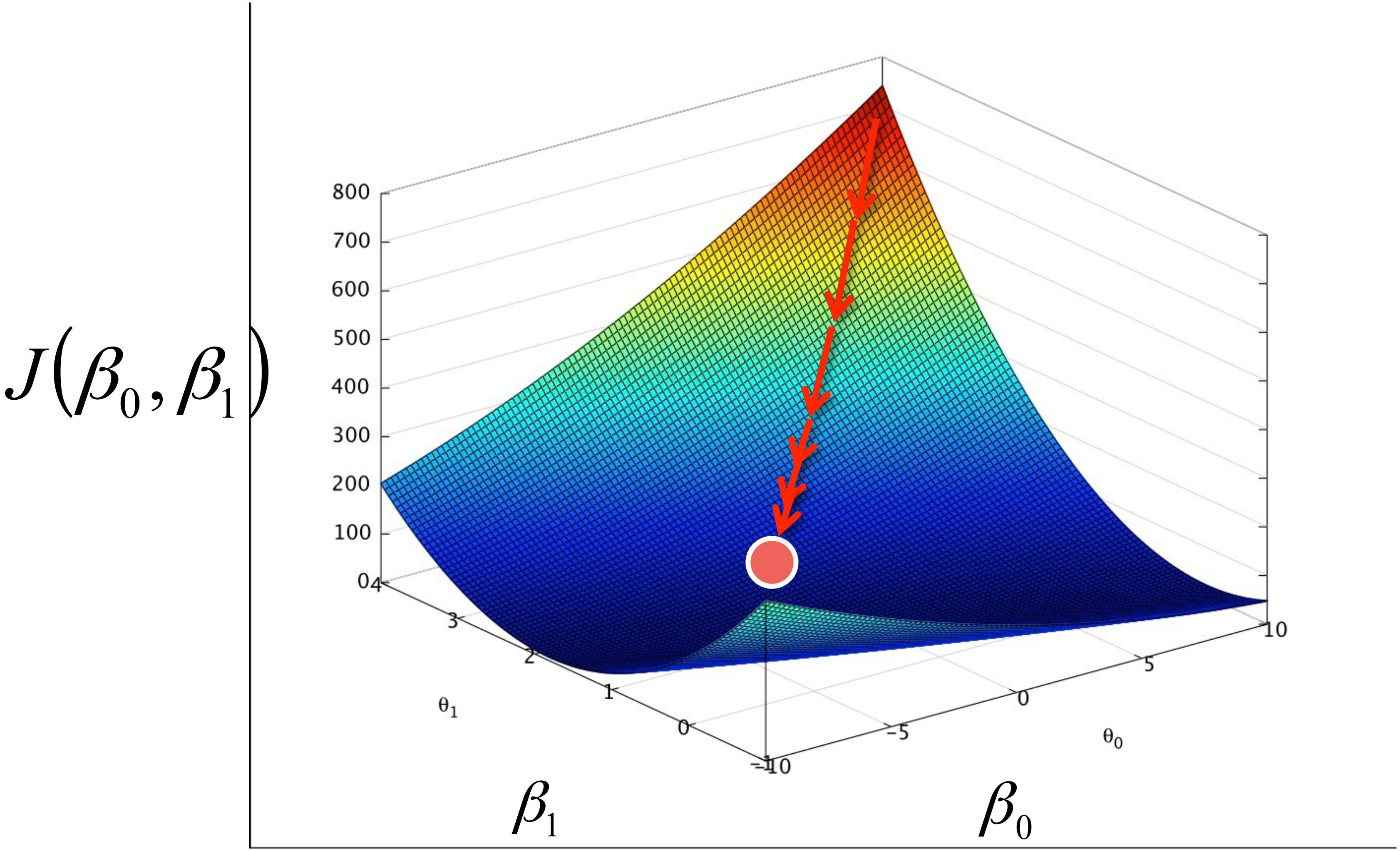
$\beta_0 = 0$
$\beta_1 = 1.5$

$\beta_0 = 120million$
$\beta_1 = 0.1$

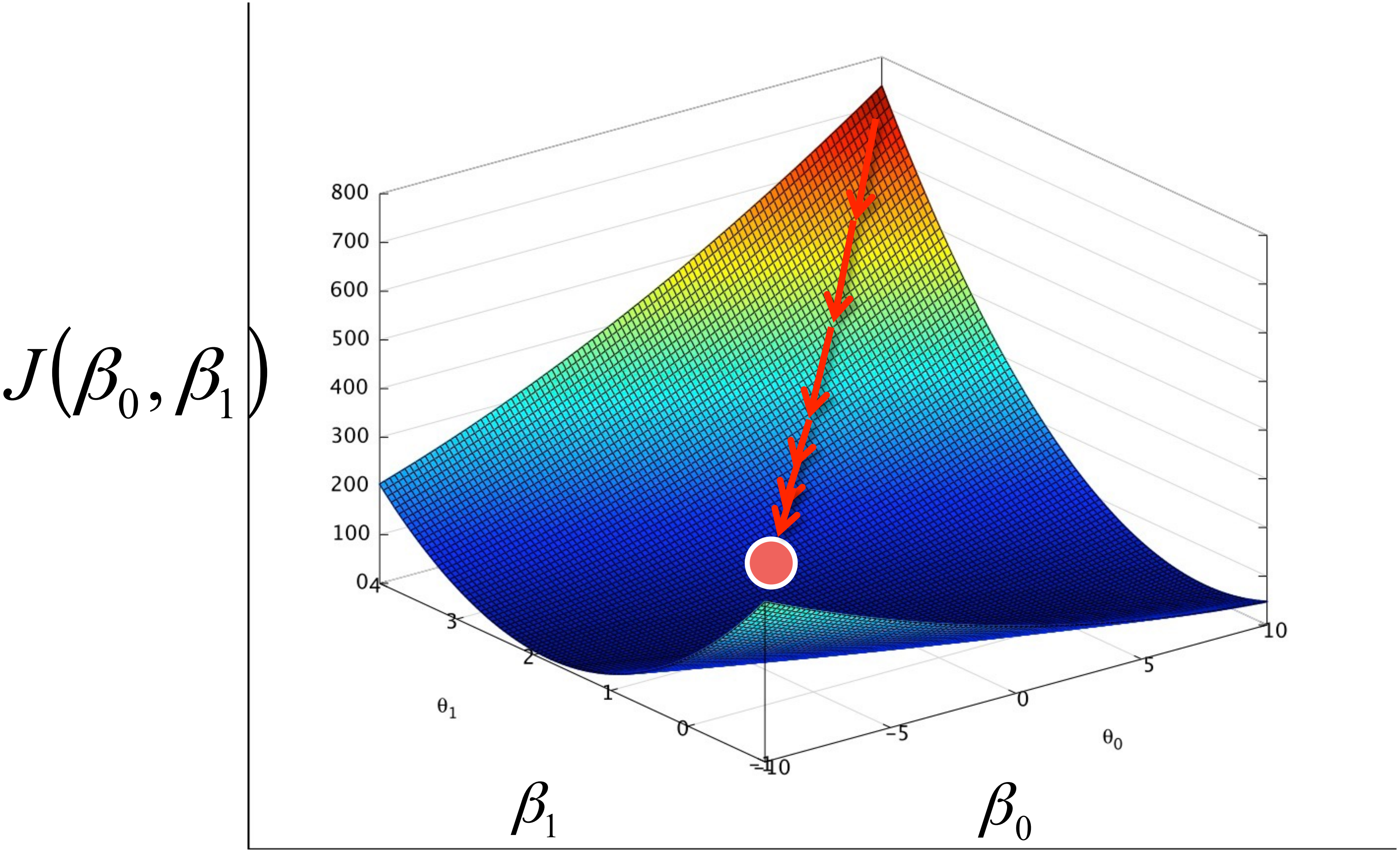
$\beta_0 = 30million$
$\beta_1 = 2$



$\beta_0 = 50million$
$\beta_1 = 5$
$J(\beta_0, \beta_1)$
800
700
600
500
400
300
200
100
0
4
3
2
1
0
θ_1
β_1
0.0
0.5
1.0
1.5
2.0
Budget
1e8

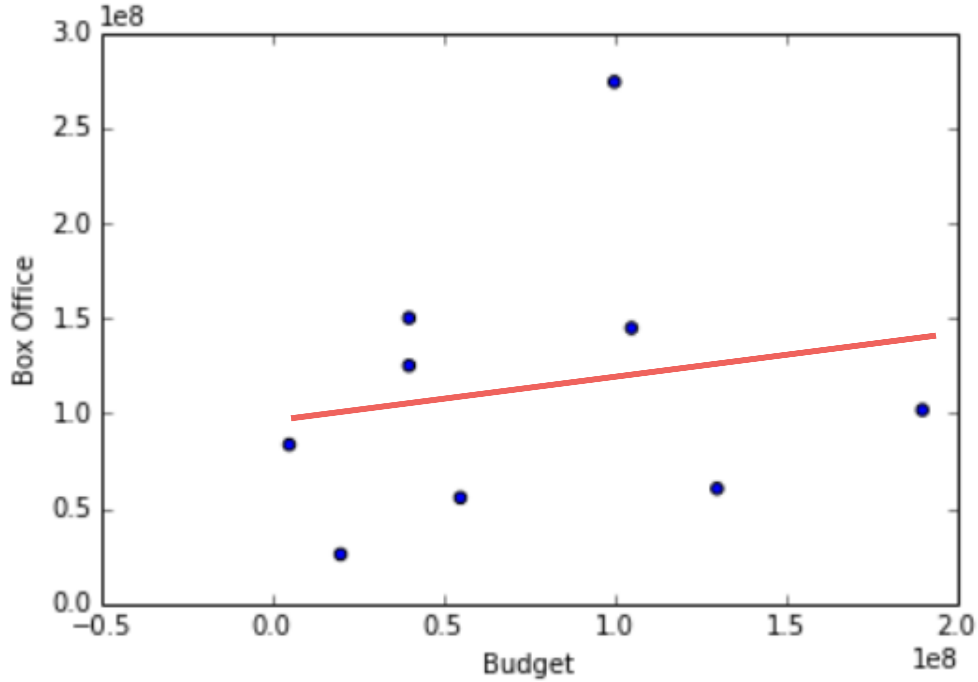



```
import statsmodels.formula.api as sm
linmodel = sm.OLS(Y, X).fit()
```



$$\beta_0 = 94.68 million$$

$$\beta_1 = 0.1$$



$$y_\beta(x) = \beta_0 + \beta_1 x + \varepsilon$$

$$\beta_0 = 94.68 million$$
$$\beta_1 = 0.1$$

Multiple Linear Regression

DATA SCIENCE BOOTCAMP

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

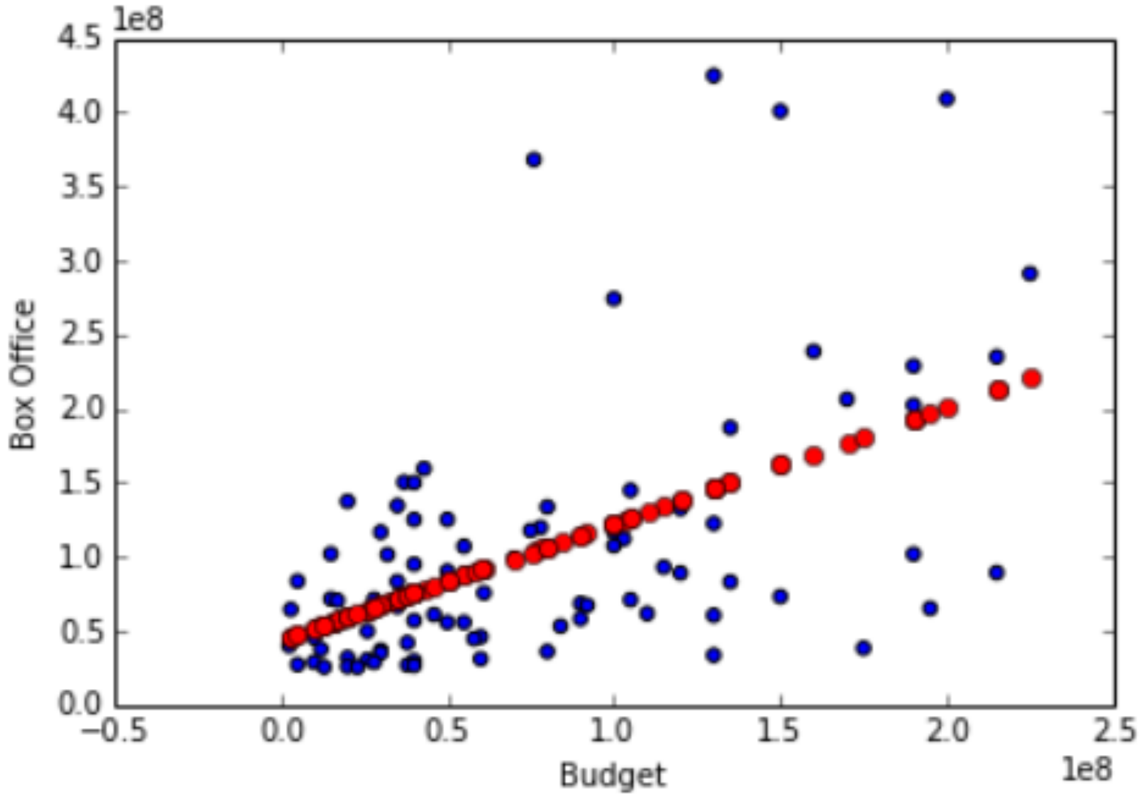
$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

$$\min J(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)$$

to find the best fitting model

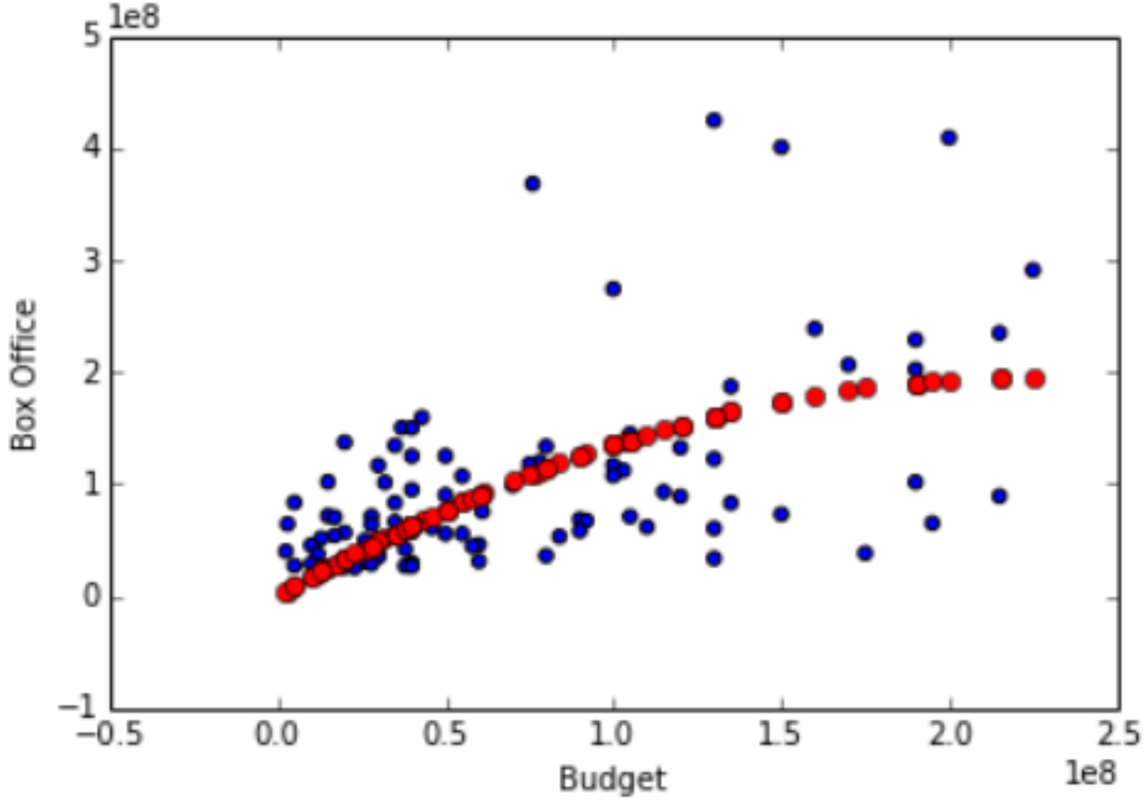
Polynomial regression

$$y_\beta(x) = \beta_0 + \beta_1 x + \varepsilon$$



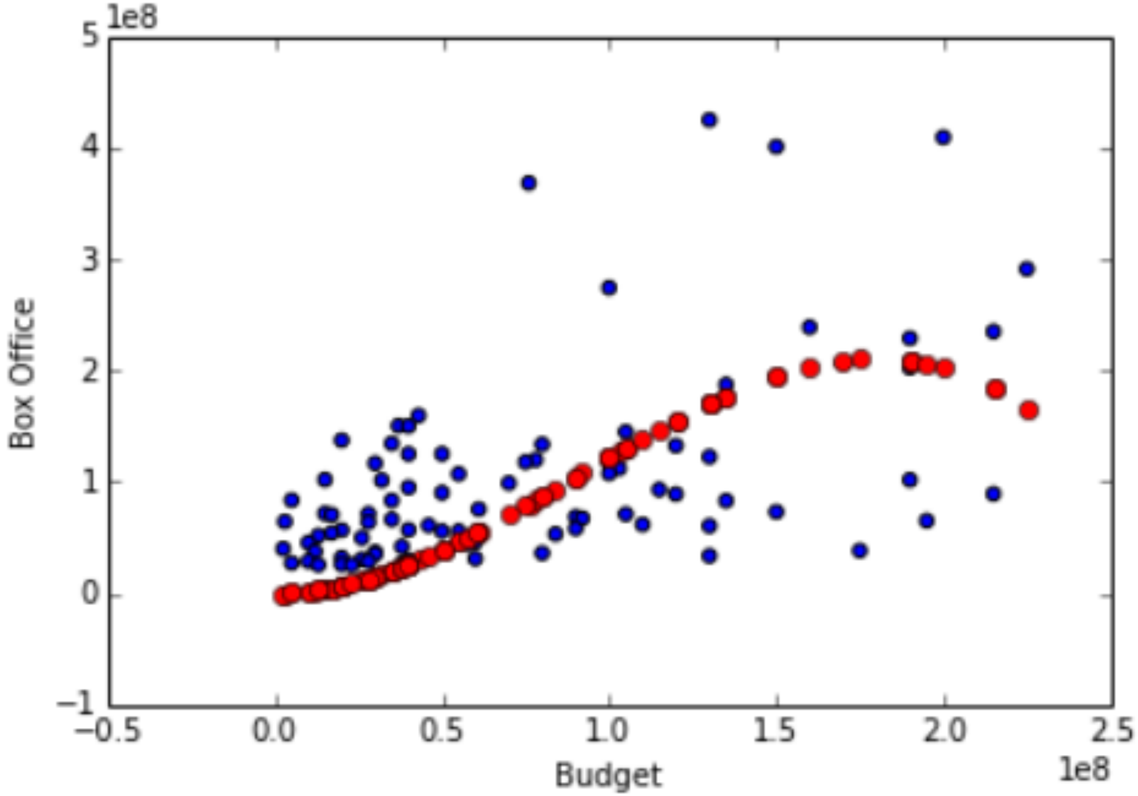
Polynomial regression

$$y_\beta(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \varepsilon$$



Polynomial regression

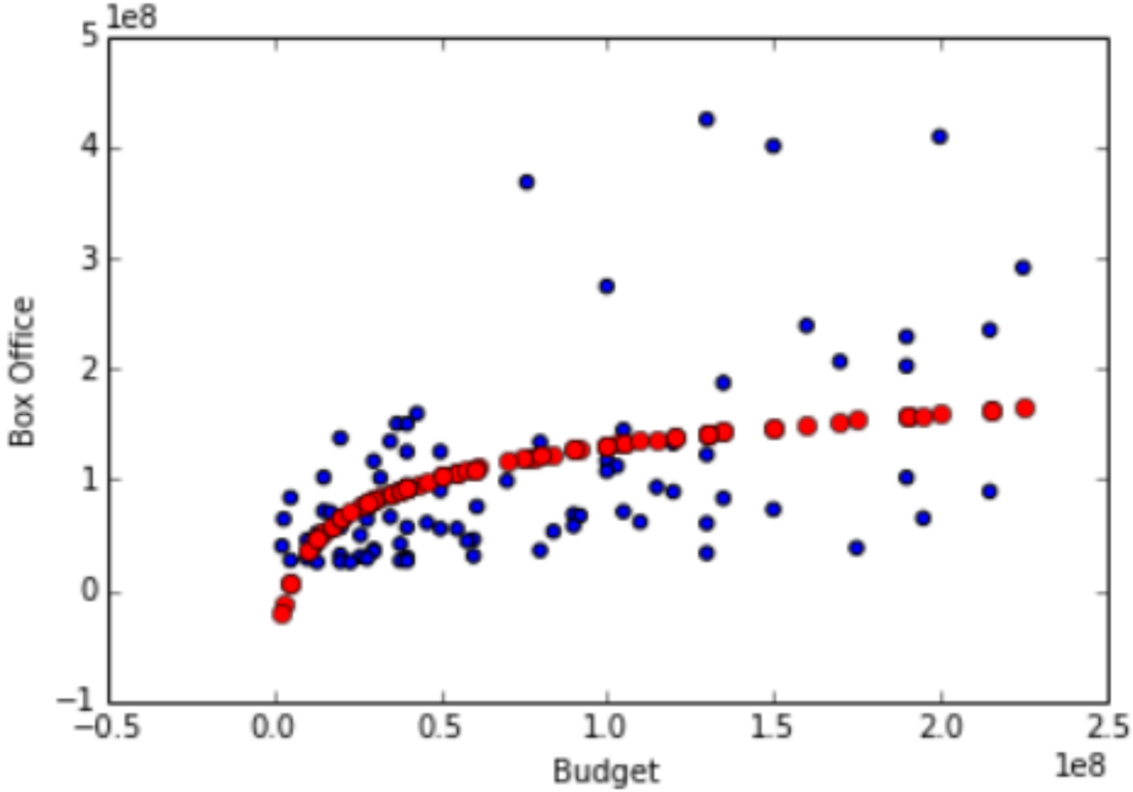
$$y_\beta(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \varepsilon$$



Other functional forms

log

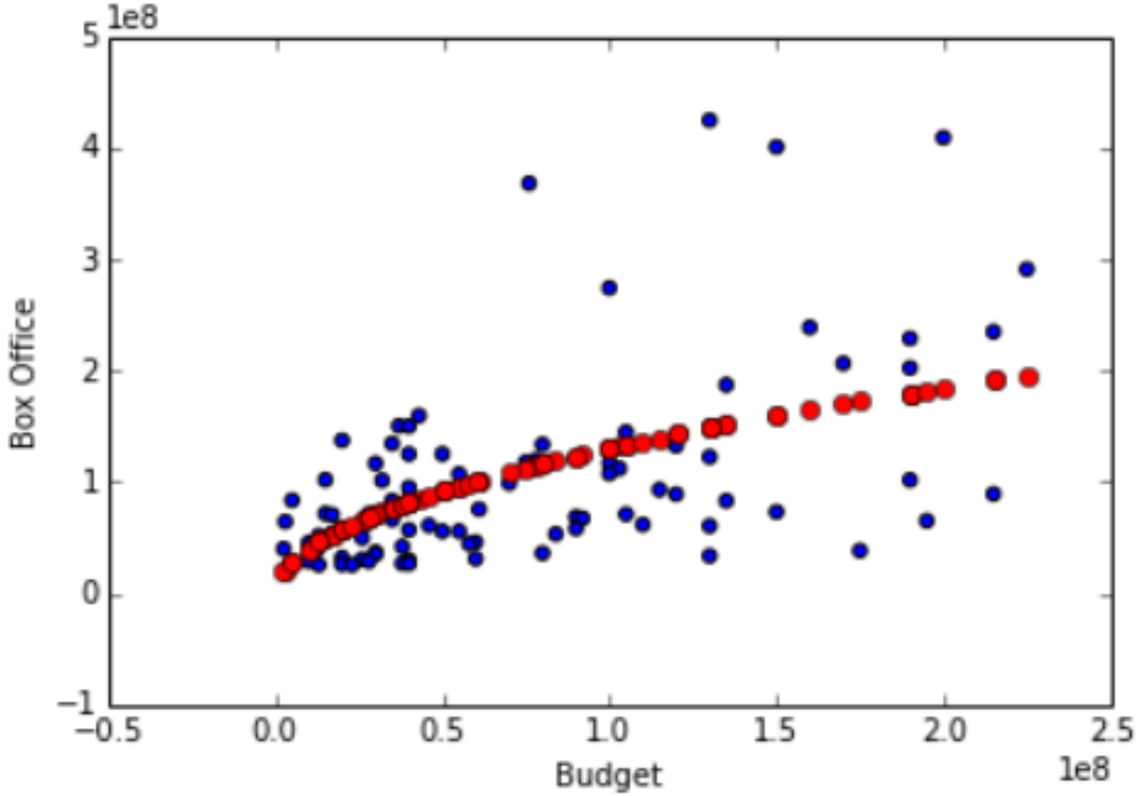
$$y_\beta(x) = \beta_0 + \beta_1 \log(x) + \varepsilon$$



Other functional forms

square root

$$y_{\beta}(x) = \beta_0 + \beta_1 \sqrt{x} + \varepsilon$$



Possible to combine variables

$$y_\beta(x) = \beta_0 + \beta_1 \exp(x_1) + \beta_2 x_2 + \beta_3 x_2^2 + \beta_4 \log(x_3) + \varepsilon$$

Possible to combine variables

$$y_\beta(x) = \beta_0 + \beta_1 \exp(x_1) + \beta_2 x_2 + \beta_3 x_2^2 + \beta_4 \log(x_3) + \varepsilon$$

Interactions

(example: existence of both genres has an each extra effect, different than the sum of each)

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \varepsilon$$

$$y_\beta(x) = \beta_0 + \beta_1 \exp(x_1) + \beta_2 x_2 + \beta_3 x_2^2 + \beta_4 \log(x_3) + \varepsilon$$

Linear Regression is not “linear” because we’re fitting “a line.”

We also fit many other forms.

It’s “linear” because the features are combined in a linear fashion ($\Sigma \beta_i f(x_i)$).

Linear

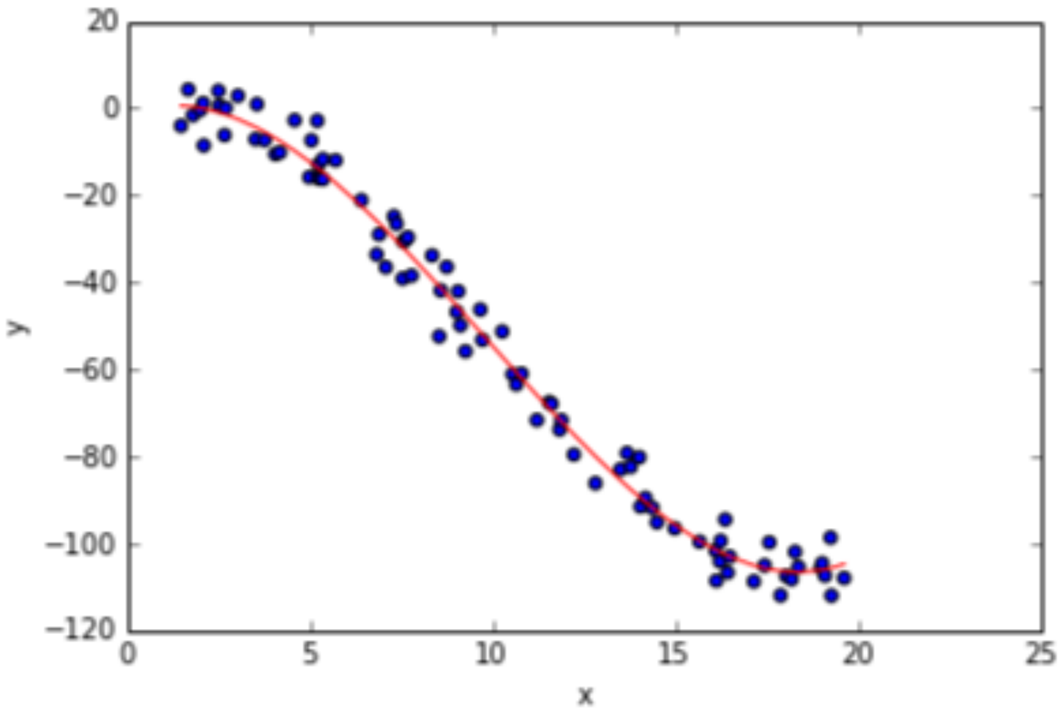
$$y_\beta(x) = \beta_0 + \beta_1 \exp(x_1) + \beta_2 x_2^{-1} + \varepsilon$$

Nonlinear

$$y_\beta(x) = \beta_0 + \beta_1 e^{\beta_2 x_1} + \frac{\beta_3 x_2}{(1 + \beta_4 x_2)} + \varepsilon$$

How to choose functional forms to try?

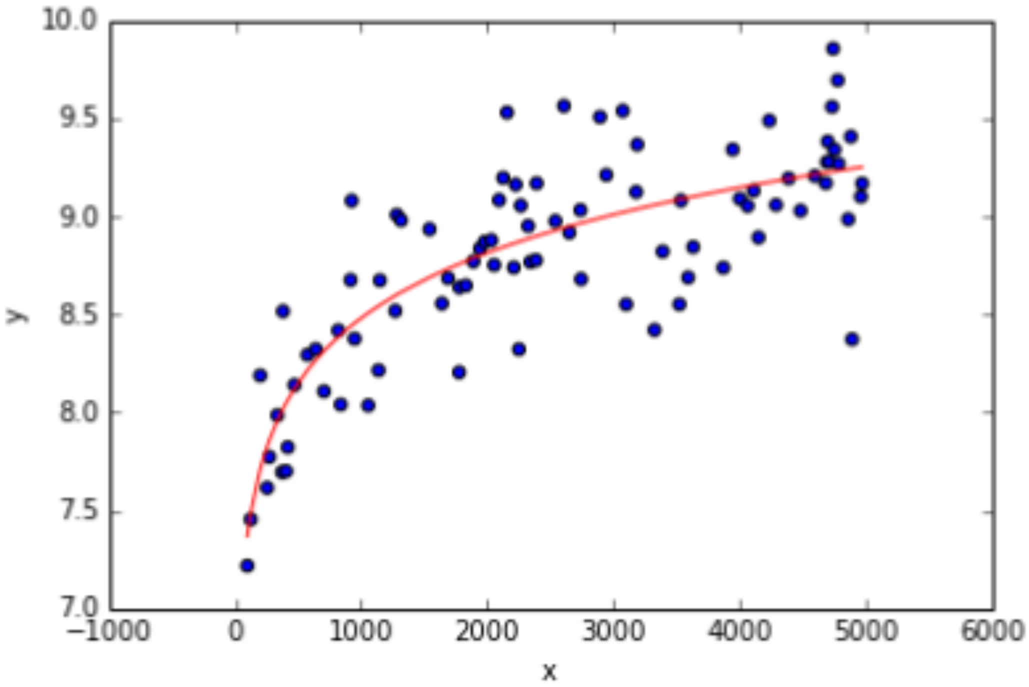
Check one on one relationship of variable with outcome



$$y_\beta(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \varepsilon$$

How to choose functional forms to try?

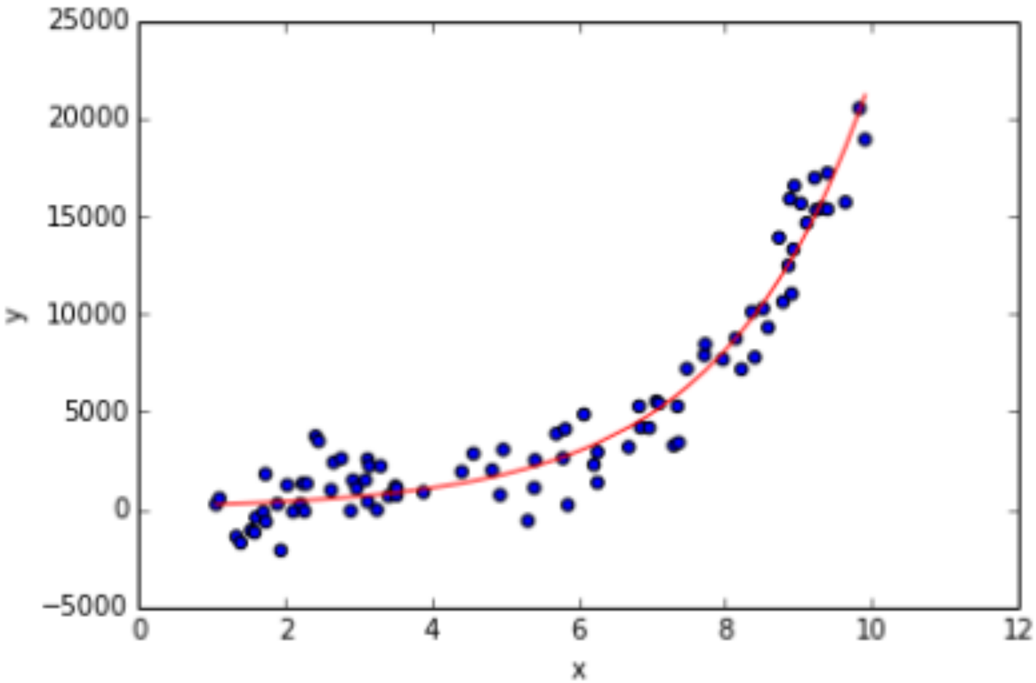
Check one on one relationship of variable with outcome



$$y_{\beta}(x) = \beta_0 + \beta_1 \log(x) + \varepsilon$$

How to choose functional forms to try?

Check one on one relationship of variable with outcome



$$\log(y_\beta(x)) = \beta_0 + \beta_1 x + \varepsilon$$

Data Science Killer #1: Overfitting

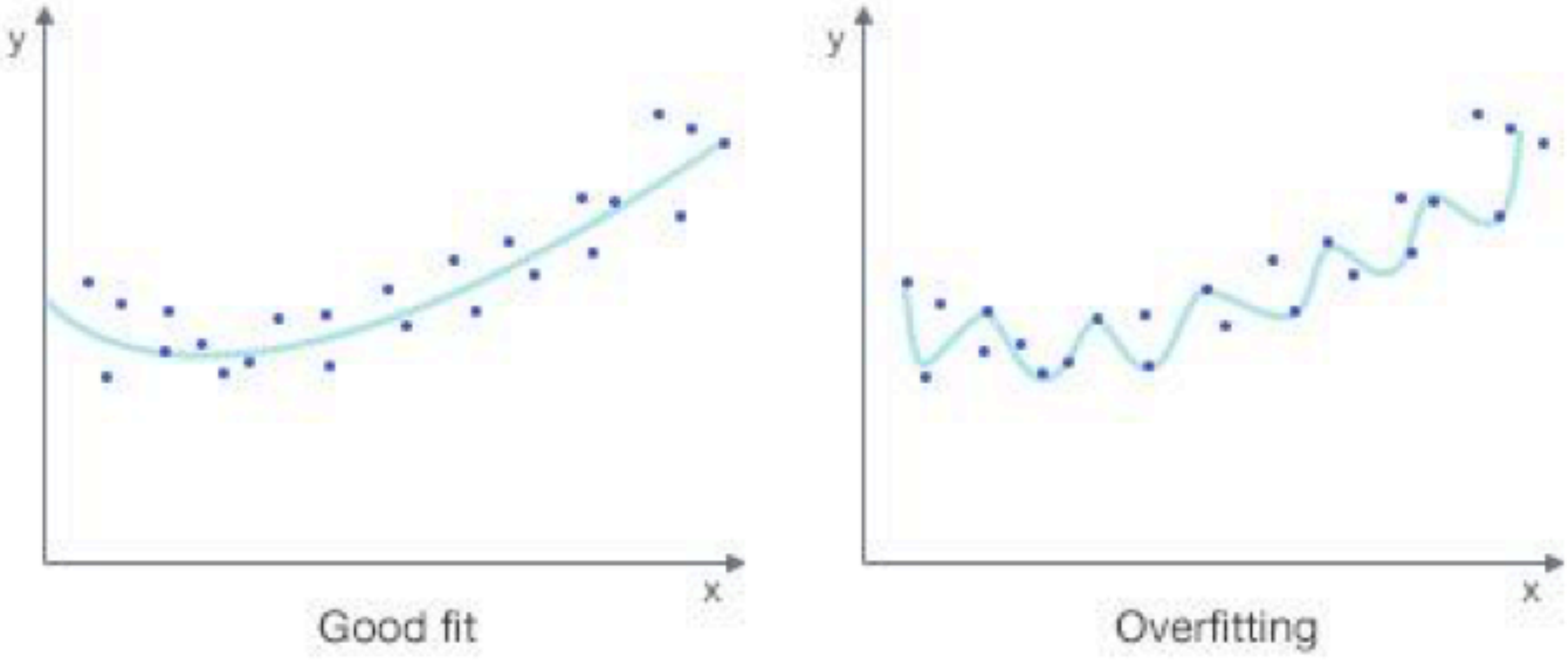
What is Overfitting?

When I fit too closely to my training set

Why is this bad?

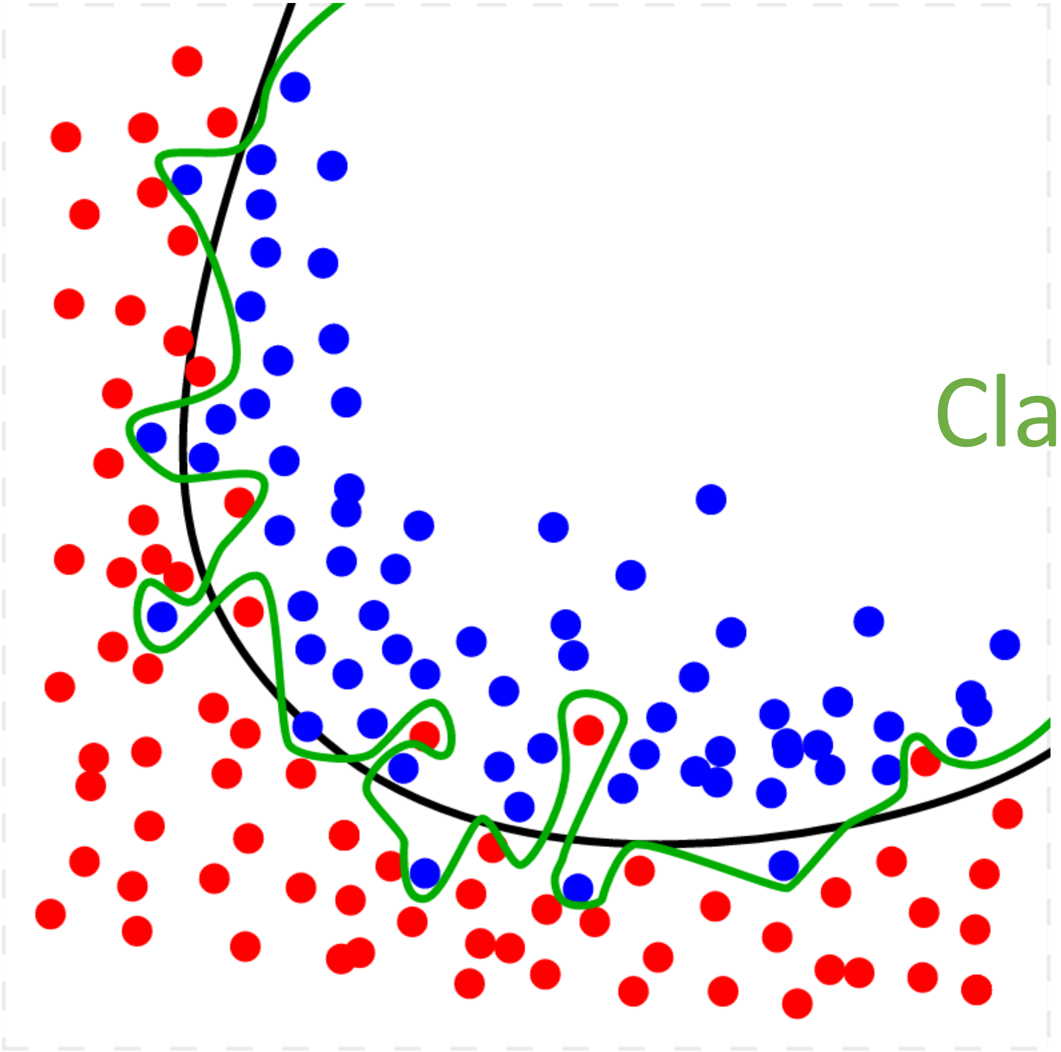
Because my model won't **generalize** well to **future data**!

What is Overfitting?



Regression

What is Overfitting?



Classification

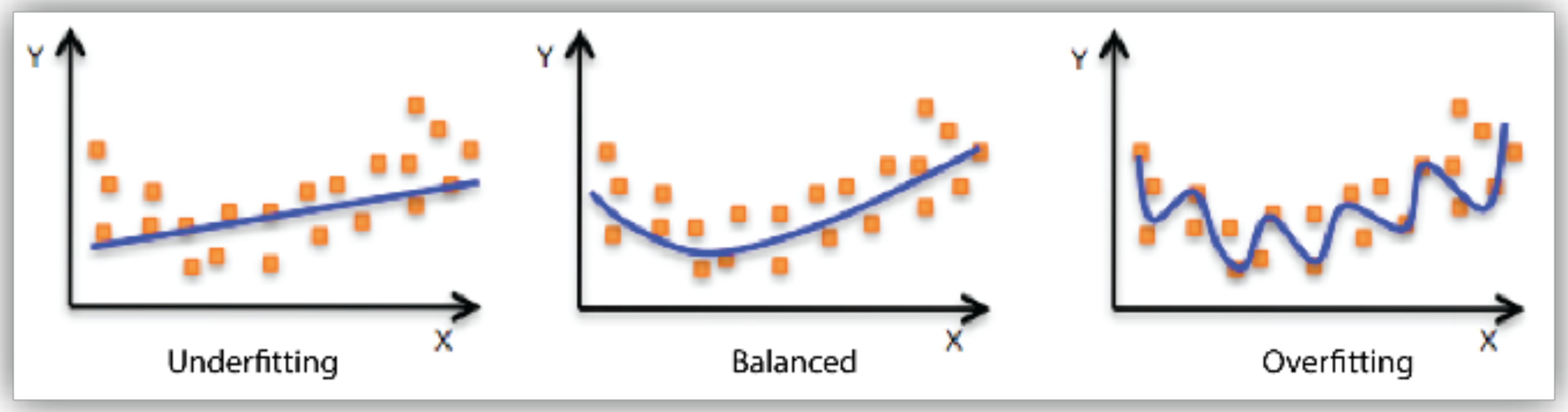
What is Underfitting?

When I don’t have a complex enough model to model my data.

Why is this bad?

Because we are losing information!

What is Underfitting/Overfitting?

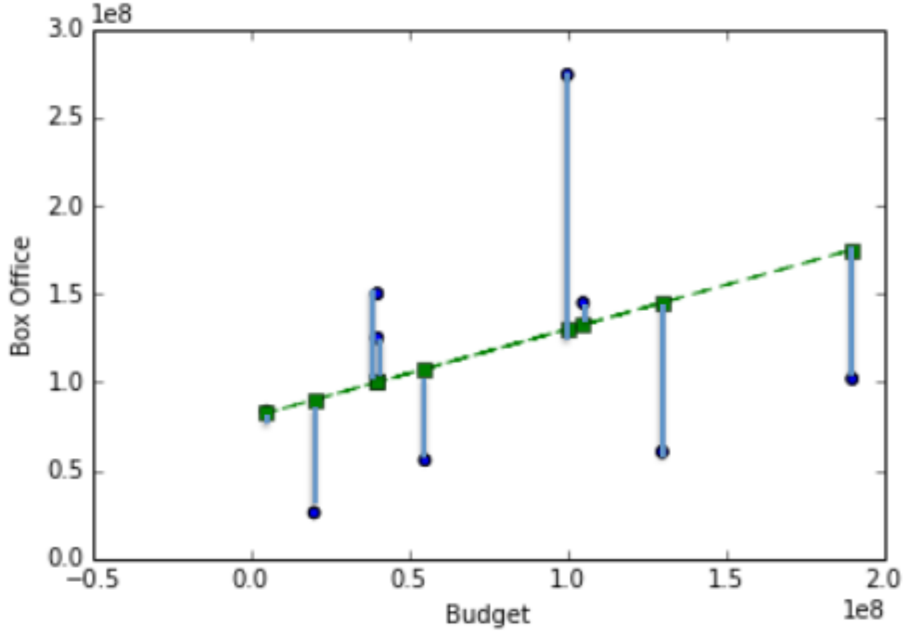


Regression

Regularization

While awarding goodness of fit, penalize model complexity

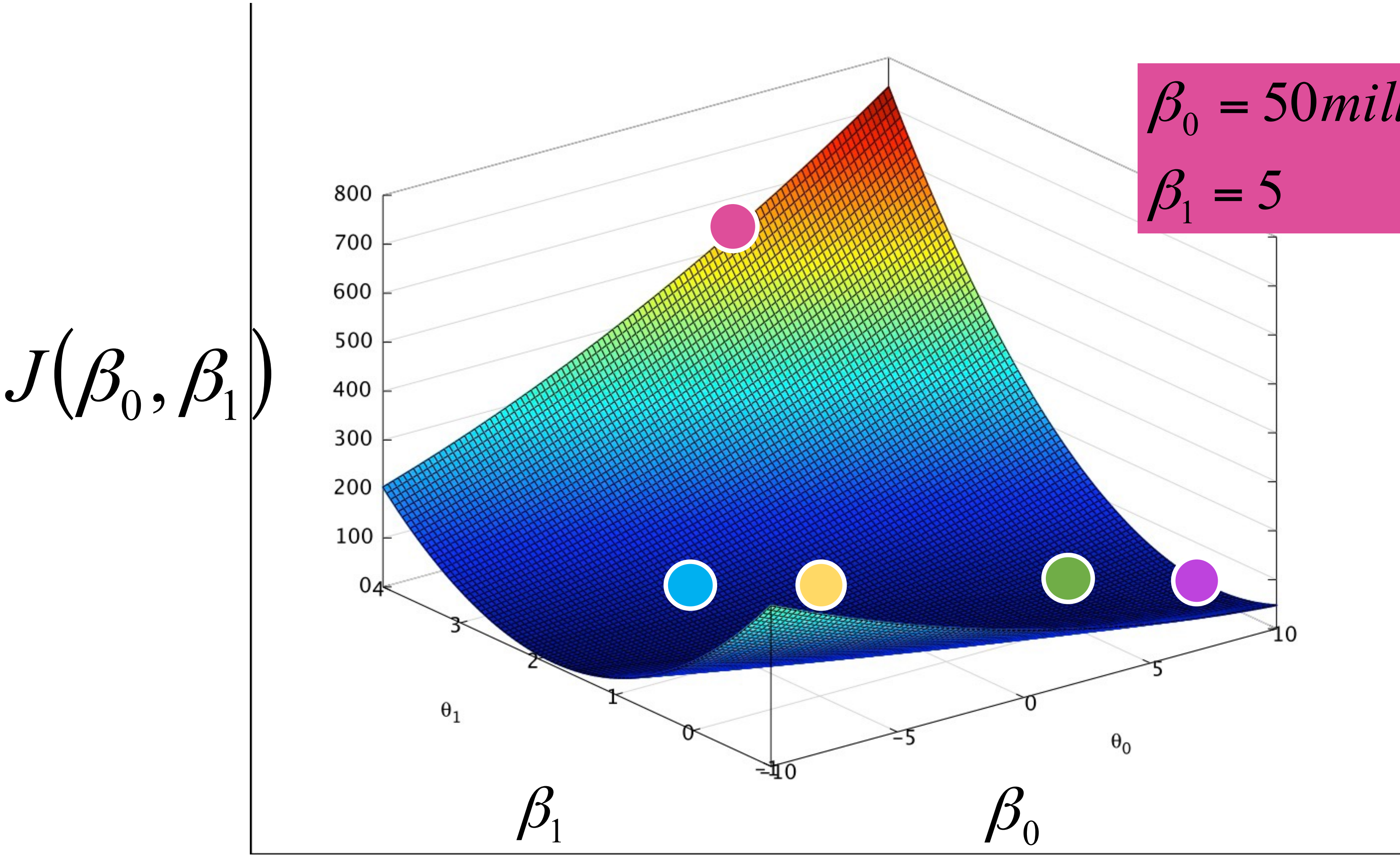
Why not do that while we are fitting?



Cost function

Takes a model (specific parameter values), returns a score

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$



$\beta_0 = 80 million$
$\beta_1 = 0.5$

$\beta_0 = 0$
$\beta_1 = 1.5$

$\beta_0 = 120 million$
$\beta_1 = 0.1$

$\beta_0 = 30 million$
$\beta_1 = 2$

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

Cost function

$$J\left(\beta_0, \beta_1\right) = \frac{1}{2m}\sum_{i=1}^{m}\left(\left(\beta_0 + \beta_1 x_{obs}^{(i)}\right) - y_{obs}^{(i)}\right)^2$$

Cost function

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

Cost function

Add a penalty for the size of each parameter!

$$J(\beta_0, \beta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)}\right)^2 + \lambda\sum_{j=1}^{k}\beta_j^2$$

L2 Regularization

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

Ridge Regression

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_{\beta}(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

Effect of λ?

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

zero

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

very small

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

“just right”

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

VERY LARGE

$$J(\beta_0, \beta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)}\right)^2 + \lambda\sum_{j=1}^{k}\beta_j^2$$

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

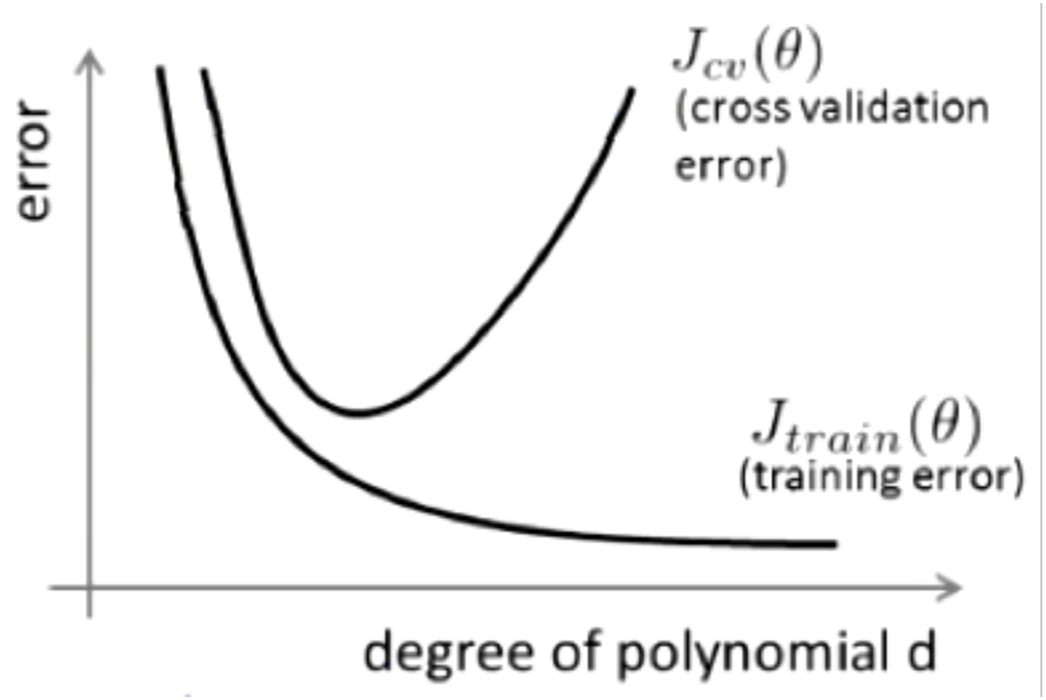
VERY LARGE

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

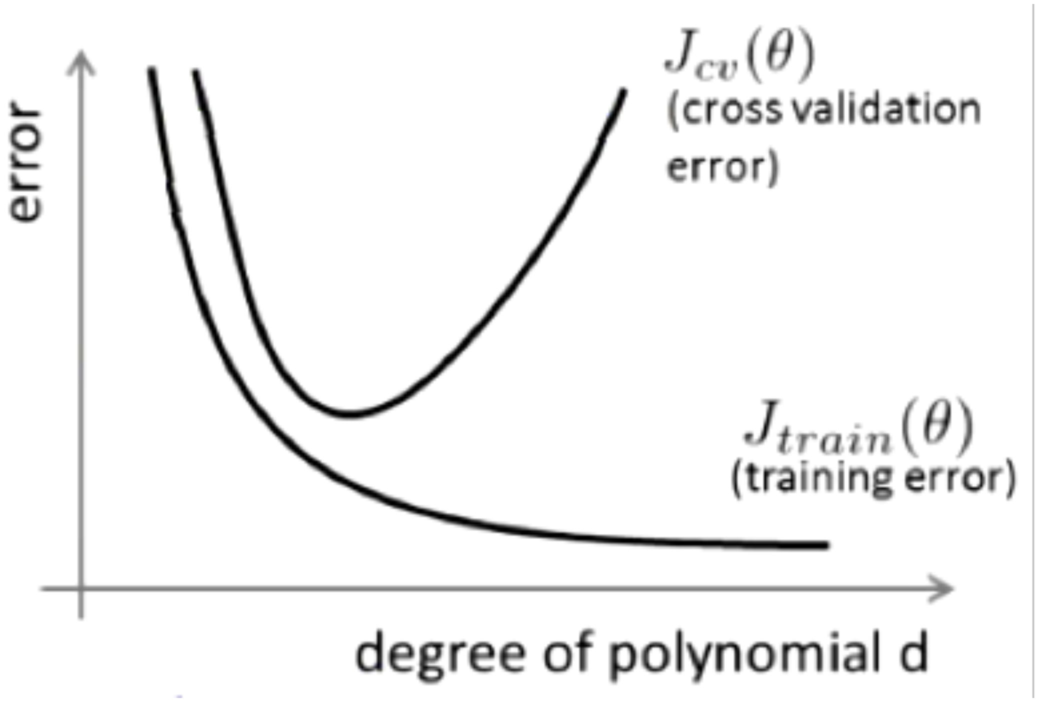
≈0 ≈0 ≈0 ≈0 ≈0

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

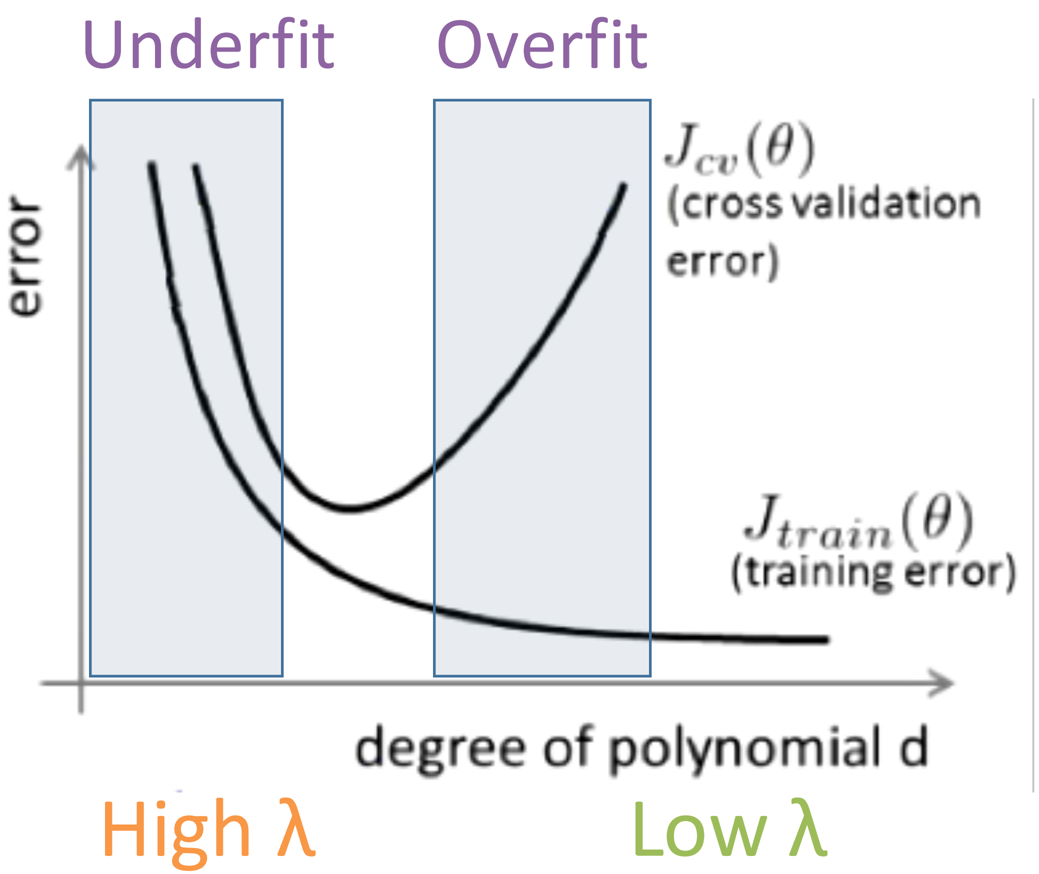
Error vs. regularization λ



Error vs. regularization λ



High λ Low λ


Underfit
Overfit
error
$J_{cv}(\theta)$
(cross validation error)
$J_{train}(\theta)$
(training error)
degree of polynomial d
High λ
Low λ

Ridge Regularization (L2)

$$J\left(\beta_0, \beta_1\right) = \frac{1}{2m}\sum_{i=1}^{m}\left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)}\right)^2 + \lambda\sum_{j=1}^{k}\beta_j^2$$

Ridge Regularization (L2)

$$J\left(\beta_0, \beta_1\right) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

Lasso Regularization (L1)

$$J\left(\beta_0, \beta_1\right) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \left| \beta_j \right|$$

Ridge Regularization (L2)

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

Lasso Regularization (L1)

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \left| \beta_j \right|$$

Elastic Net (L1 + L2)

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda_1 \sum_{j=1}^{k} \left| \beta_j \right| + \lambda_2 \sum_{j=1}^{k} \beta_j^2$$

How can I use this?

We were doing:

```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
model.fit(X,Y)
```

How can I use this?

We were doing:

```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
model.fit(X, y)
```

To use Ridge Regularization:

```
from sklearn.linear_model import Ridge
model = Ridge(1.0)
model.fit(X, y)
```

λ (sklearn Calls It alpha)

How can I use this?

We were doing:

```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
model.fit(X, y)
```

To use Lasso:

```
from sklearn.linear_model import Lasso
model = Lasso(1.0)
model.fit(X, y)
```

λ (sklearn Calls It alpha)

How can I use this?

We were doing:

```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
model.fit(X, y)
```

To use Elastic Net:

```
from sklearn.linear_model import ElasticNet
model = ElasticNet(1.0, l1_ratio = 0.5)
model.fit(X, y)
```

total weight for the full penalty term

ratio of l1/l2 penalty

My model is not
awesome
enough.

What do I do?

Try these and check test error
(and AIC,BIC,etc.) again:

Use a smaller set of features

Regularization: Increase/decrease λ

Try adding polynomials

Check functional forms for each feature

Try including other features

Use more data (bigger training set)