Supporting Information for "An evolutive linear kinematic source inversion"

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Text S1: Details on forward modeling.

As in the main paper, the convention used here is the following: non-underlined symbols represent scalars, single underlined are vectors and double underlined are matrices.

The source geometry is defined by a 2D surface, $\Sigma(\underline{\xi})$, embedded in a 3D known medium. The fault position vector $\underline{\xi}$ is defined over a 2D planar surface specified by strike ϕ and dip δ directions known prior to the slip inversion. More complex surfaces could be assumed if needed: the normal vector may vary along the fault surface. However, in this work we assumed that a planar surface assumption is a decent approximation with actual available data and it will simplified the notation. The forward problem is the computation of velocity recordings $\underline{v}(\underline{x}, t) = [v_x, v_y, v_z]^T$, at any receiver location $\underline{x} = [x_x, x_y, x_z]^T$ and time t during the interval $[t_1, t_2]$, given a certain spatio-temporal slip-rate distribution $\underline{\mathcal{V}}(\underline{\xi}, \tau)$, occurring at the position $\underline{\xi}$ and at a rupture time $\tau \in [\tau_1, \tau_2]$. Thanks to the representation theorem [*Aki and Richards*, 2002], the synthetic velocity field is computed as a double integral over the fault surface and the time window through the following expression

$$v_n(\underline{x}, t) = \int_{\tau_1}^{\tau_2} d\tau \iint_{\Sigma} \mathcal{V}_i(\underline{\xi}, \tau) \mathcal{T}_{ni}(\underline{x}, t - \tau; \underline{\xi}, 0) d\Sigma(\underline{\xi}),$$
$$n, i \in [x, y, z] \qquad \text{(S1)}$$

where the stress-state tensor $\mathcal{T}_{ni}$ is deduced from Green functions computed in the 3D known medium. However, we assume that there is no opening mode during an earthquake and, therefore, the slip-rate vector lies into a fault plane. Consequently, the general slip-rate vector $\underline{\mathcal{V}} = [\mathcal{V}_x, \mathcal{V}_y, \mathcal{V}_z]^T$ can be represented by only two components V_ϕ and V_δ along the local strike and dip directions respectively. Moreover, we assume that the fault surface is a plane with a unitary normal vector $\underline{\eta}$ everywhere. Using the linear transformation $\underline{\underline{\mathcal{P}}}_{3\times n}\underline{V}_{n\times 1}(\underline{\xi},t) = \underline{\mathcal{V}}_{3\times 1}(\underline{\xi},t)$, a reduction of the slip-rate vector from the cartesian coordinate system (3 unknowns) to a strike and dip coordinate system (2 unknowns) or, in case the rake angle (λ) is assumed as known, to a single scalar (1 unknown) controlling the amplitude of the assumed orientation of the vector, can be performed. In the first case, when the rake angle is unknown, such transformation is defined as

$$\underline{\mathcal{V}}(\underline{\xi},t) = \begin{bmatrix} \cos\phi & \eta_y \sin\phi \\ \sin\phi & -\eta_z \cos\phi \\ 0 & \eta_y \cos\phi - \eta_x \sin\phi \end{bmatrix} \begin{bmatrix} V_\phi(\underline{\xi},t) \\ V_\delta(\underline{\xi},t) \end{bmatrix}, \tag{S2}$$

while when the rake angle is assumed as known it turns into

$$\underline{\mathcal{V}}(\underline{\xi},t) = \begin{bmatrix} \cos\lambda\cos\phi + \sin\lambda\cos\delta\sin\phi \\ -\cos\lambda\sin\phi + \sin\lambda\cos\delta\cos\phi \\ \sin\lambda\sin\delta \end{bmatrix} V_s(\underline{\xi},t), \tag{S3}$$

with vector $\underline{\eta} = [-\sin\delta\sin\phi,\ -\sin\delta\cos\phi,\ \cos\delta]^T$ [*Stein and Wysession*, 2003]. If we have chosen parameters V_s and λ as unknowns, the inverse problem would have become non-linear, while the parameter set (V_ϕ, V_δ) we use preserves the linear property. In other words, the rake angle is never treated as a parameter to be inverted during this work and it will be only an attribute for which we may design hard or soft constraints.

For the complete description of the integral representation, the components of the stress-state tensor, $\underline{\underline{\mathcal{T}}}(\underline{x}, t-\tau; \underline{\xi}, 0)$, are detailed as

$$\underline{\underline{\mathcal{T}}}(\underline{x}, t-\tau; \underline{\xi}, 0) = \begin{bmatrix} \underline{\underline{\sigma}}^{(x)}(\underline{x}, t-\tau; \underline{\xi}, 0)\underline{\eta} \\ \underline{\underline{\sigma}}^{(y)}(\underline{x}, t-\tau; \underline{\xi}, 0)\underline{\eta} \\ \underline{\underline{\sigma}}^{(z)}(\underline{x}, t-\tau; \underline{\xi}, 0)\underline{\eta} \end{bmatrix}^T, \tag{S4}$$

where $\underline{\underline{\sigma}}^{(n)}(\underline{x}, t-\tau; \underline{\xi}, 0)\underline{\eta}$ represents the product of the stress tensor induced at the receiver position $\underline{x}$ and time $t-\tau$ by a unitary force applied at the source location $\underline{\xi}$ and time 0 along the direction described by the superscript n by the unitary normal vector to the fault. In other words, the columns of the stress-state tensor are formed by the corresponding unitary traction vectors.

Finally, thanks to the reciprocity property of Green functions and the specified linear transformation, the forward problem (S1) can be rewritten as the following expression

$$v_n(\underline{x}, t) = \int_{\tau_1}^{\tau_2} d\tau \iint_{\Sigma} \mathcal{P}_{ik}\, V_k(\underline{\xi}, \tau)\, \mathcal{T}_{in}(\underline{\xi}, t-\tau; \underline{x}, 0) d\Sigma(\underline{\xi}),$$
$$n, i \in [x, y, z] \text{ and } k \in [\phi, \delta] \text{ or } k \in [\lambda], \tag{S5}$$

which is the continuous integral form of our forward problem as specified in the main paper. This integral representation is more convenient because for real cases the number of receivers is always less than the number of nodes representing the fault surface.

References

Aki, K., and P. G. Richards (2002), *Quantitative seismology, theory and methods, second edition*, University Science Books, Sausalito,California.

Stein, S., and M. Wysession (2003), *An Introduction to Seismology, Earthquakes and Earth Structure*, Blackwell Publishing.