Fixed Indexed Annuity Fair Value Quantification and Valuation

PathWise Solutions Group

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Empower Results®

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Executive Summary

With the introduction of long-duration insurance targeted improvements (LDTI) for US GAAP, guaranteed minimum living and death benefit riders offered for Fixed Index Annuity (FIA) products are typically valued and reported using risk-neutral scenarios. But no clear instruction has been provided as to how exactly the market-consistent valuation should be done and a wide range of methodologies exist in practice.

Coincidentally, the draft principle-based reserving (PBR) framework for fixed and indexed annuities (VM-22[1]) by National Association of Insurance Commissioners (NAIC) introduces a stochastic reserve component similar to what VM-21 requires for variable annuity products. The transition from the current most deterministic framework to VM-22 is a challenge for many insurers.

In this paper, we present a comparison of two commonly used FIA valuation methodologies in order to help insurers decide on the best methodology to use under GAAP LDTI and VM-22. First, we compare two modeling approaches to project the future index credits with the annual point-to-point crediting formula: the stochastic equity crediting model and the deterministic option budget crediting model. Next, we extend the analysis to the monthly average crediting formula. We then highlight the advantages and disadvantages of the two equity return projecting approaches. Finally, we discuss how different levels of available option budgets and policy durations would impact the analysis.

We conclude that the stochastic equity crediting model is more capable of capturing the fair value of the riders across two crediting methods and different assumption settings. The valuation of the embedded guarantees involves complex mathematics and requires sophisticated stochastic-on-stochastic simulations. The option budget crediting model is a shortcut approach which could produce material discrepancies in some situations compared to the more robust stochastic equity crediting model. Furthermore, the option budget crediting model produces a single best estimate instead of a distribution of possible values required for PBR reserving purposes.

[1] National Association of Insurance Commissioners NAIC Valuation Manual VM-22

1. Introduction

The Accounting Standards Update (ASU) 2018-12 (titled *Targeted Improvements to the Accounting for Long-Duration Contracts*) issued by the US Financial Accounting Standards Board (FASB) aims to improve the disclosure requirements under US GAAP for certain long duration contracts including deferred annuity contracts.

One of the areas of focus within ASU 2018-12 is the reporting of a new Market Risk Benefit (MRB) classification of liabilities, which must be valued in a market-consistent way following LDTI requirements. MRBs are reported at par (valued at $0) at inception and defined as the difference in present values between the future benefits and the ascribed fees. The scope of MRBs include various types of guaranteed living and death benefit riders (also known as GMxBs) offered on deferred annuity contracts including variable annuities (VAs) and fixed index annuities (FIA). Under the pre-LDTI GAAP requirements, there are inconsistencies in reporting these benefit guarantees: the insurance accrual model (formally known as Statement of Position SOP 03-1) or as an embedded derivative under the fair value model. The new standards require these guarantees to be accounted for only based on fair value.

Additionally, the current US statutory framework for FIA follows Actuarial Guidelines XXXIII (AG-33) and XXXV (AG-35). These two Actuarial Guidelines are formula-based approaches that follow the Commissioners' Annuity Reserving Valuation Method (CARVM). To align with the recent product development and risk exposure to the financial market, NAIC has proposed a new principle-based reserving (PBR) framework for non-variable annuities, including FIA with guaranteed lifetime withdrawal benefit (GLWB) riders. Under the new PBR framework (VM-22), the reserve for FIA that have embedded guarantees with market risk will likely be measured by conditional tail expectation (CTE) at the 70th percentile.

In this paper, we will attempt to provide insights into how insurers can properly determine the fair value for FIA riders and comply with the new reporting frameworks.

The rest of this paper is structured as follows:

- Section 2 introduces the two models to project the indexed crediting rates: the stochastic equity crediting model and the deterministic option budget crediting model. It provides an overview on four interest rate and equity model combinations. It then discusses modelling the available option budget level and the role it plays in the analysis and presents a summary of the data used for the analysis.
- The third section recaps the annual point-to-point crediting method and compares FIA rider valuation results across these model combinations.
- The fourth section recaps the monthly average crediting method and extends the analysis to this crediting method.
- The last section focuses on the importance of policy duration to the analysis and compares the results across different contract durations.

2. Methodology, Data and Assumptions

Two Modelling Approaches to Project Index Credits

Two models are commonly used to project index credits: the stochastic equity crediting model and the deterministic option budget crediting model.

Stochastic Equity Crediting Model

The stochastic equity crediting model requires a market-calibrated stochastic equity simulation model to generate a large number of equity scenarios under risk-neutral settings. The algorithm also calculates option prices (with different strike prices) for each simulated scenario. Based on these prices and the available option budget level, we can then calculate the resulting cap rate, participation rate and index spread. These parameters are reset at the beginning of each year and held constant throughout the year.

After applying the parameters to the index change, we will arrive at the indexed return under the stochastic equity crediting model.

This model is sophisticated and requires a state-of-the-art simulation model and computational resources.

Deterministic Option Budget Crediting Model

The deterministic option budget crediting model is a shortcut model which does not require simulation of equity returns. Crediting the account value is simply based on the available option budget level. At the beginning of each year, the model records the available option budget level and then accumulates the available option budget level at the risk-free rates during the year.

To allow it to be comparable to the stochastic equity model, the crediting rate does not fall below a floor rate which corresponds to the minimum cap rate, and the floor rate is backed out by the Black-Scholes formula.

This approach is fast and simple. One immediate disadvantage of this approach is that it does not capture the full distribution of the results for tail risk management and reserving purposes.

Models

To see which model is more capable of capturing the fair value of FIA riders, we compare results under four different models, ranging from simple (deterministic option budget crediting model with deterministic interest rate model) to relatively complex (stochastic equity crediting model with stochastic interest rate model).

Table 1. Four Models for Comparison

Model	Equity Returns	Interest Rates	Crediting Rate Model
1 - Deterministic	Not modelled	Deterministic	Deterministic option budget crediting model
2 - Stochastic Rates	Not modelled	HW2F	Deterministic option budget crediting model
3 - Stochastic Equity	Bates	Deterministic	Stochastic equity crediting model
4 - Stochastic Rates and Equity	Bates	HW2F	Stochastic equity crediting model

For risk-neutral equity simulations, the Bates model is used. Compared to the traditional Black-Scholes framework, the Bates model is capable of simulating both stochastic volatilities and stochastic jumps. The Bates model is one of the more sophisticated market risk models used by life insurers.

For interest rates simulations, the Hull-White 2-Factor (HW2F) short rate model is used. Compared to the its 1-factor counterpart (HW1F), the HW2F model is extended to capture non-parallel movements of the yield curve and can produce a more realistic interest rate volatility term structure. The HW2F model is considered a generalized version of the HW1F model.

In our analysis, both models were calibrated to the prevailing capital market environment to generate market-consistent results.

The two deterministic option budget crediting models do not involve equity return simulations. The annual indexed return for account values is assumed to be equal to the available option budget level. The account value is accumulated at the risk-free rates within each year.

The two stochastic equity crediting models utilize the Bates model to generate 10,000 stochastic risk-neutral equity scenarios, which are then combined with the available option budget levels to solve for crediting parameters that are reset periodically. The indexed account crediting depends on the index change and is subject to these parameters mentioned above.

To set the available option budget level, we follow the practice of taking the difference between the general account yield and the required pricing spread. The general account yields are simulated. However, the required pricing spread is a more subjective assumption, as it depends on the insurer's profitability expectation. Therefore, in order to remain unbiased, we perform result comparisons on different levels of required pricing spreads.

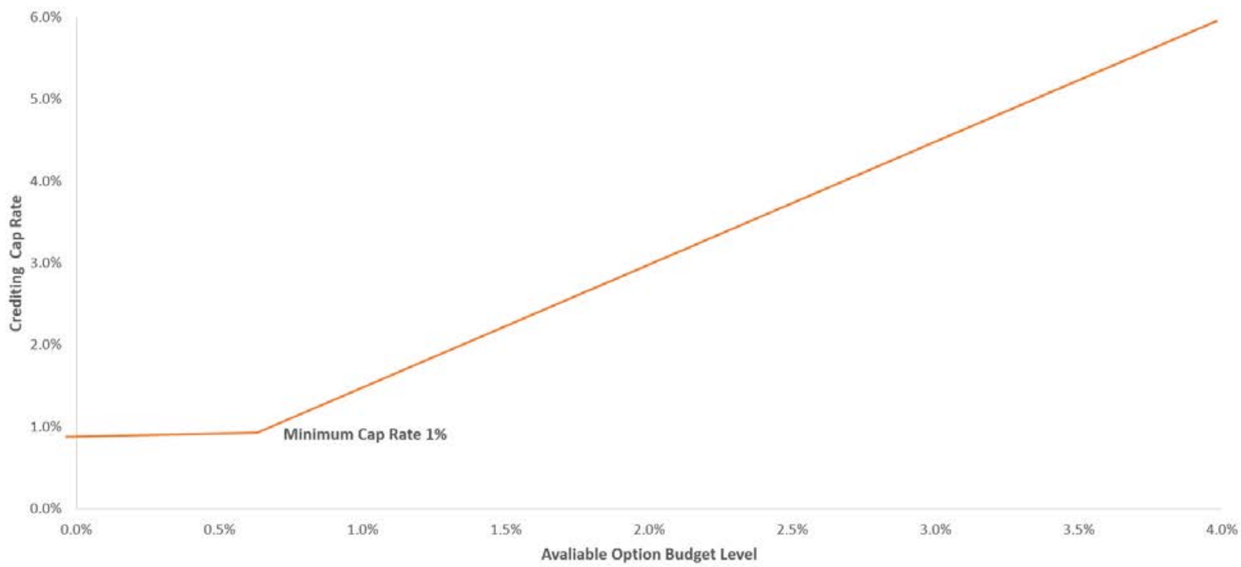
We also model dynamic lapse in our analysis. A policyholder's surrender incentive depends on the moneyness of a policy. It is assumed that a policyholder rationally compares the present value of expected benefits to the surrender value when making surrender decisions. When the surrender value is sufficiently higher than the present value of expected benefits, the policyholder will have a higher incentive to lapse the policy. The lapsing behavior varies across different simulated interest rate and equity return scenarios.

Available Option Budget Level

One way to model the available option budget level is to take the difference between the general account yield and the required pricing spread. The general account yield depends on the investment performance of a designated investment portfolio which is mostly comprised of fixed income assets. The required pricing spread is set by the insurers to meet profitability requirements. Within the option budget level available, insurers typically execute 1-year term equity strategies to hedge the crediting rate guarantees. Since the option prices and the general account yield fluctuate, the parameters are reset annually.

With a higher level of available option budget, the indexed return can potentially be higher. For example, a higher level of available option budget will provide a higher cap rate if all other parameters are held constant.

Figure 1. Cap Rate vs. the Available Option Budget Level[2]



When the available option budget level is below a certain threshold, the cap rate stays at its minimum level.

Product Features

We compare a group of 1,000 FIA contracts with an annual reset cap strategy, GLWB and Guaranteed Minimum Death Benefit (GMDB) riders. The resetting cap rate never falls below 1.00%, in other words, the minimum cap rate is 1.00%. The benefit base for the illustrated GLWB rider has an annual roll-up between 6.50% and 7.00% (depending on the rider chosen by the policyholder) compounding annually until activation of the benefit. The annual rider fee ranges between 0.95% and 1.10% of the benefit base. The annual maximum withdrawal percentage depends on the policyholder’s attained age at activation of withdrawals and whether the FIA contract is a joint policy.

Table 2. In-force Policy Book Profile

Number of Policies:	1,000
Total Initial Premium:	32,561,633
Average Age at Issue:	61 years old
Total Initial Income Base:	31,216,625

[2] Option prices are simulated by the Bates equity model.

Assumptions

We compared key model assumptions against industry observations to ensure realistic parameters were used in our analysis. One of the key inputs is the rider fee assumption:

Table 3. Rider Fee Assumptions Applied

Fee %	Income Base Roll-Up %[3]	Income Base Credit Keep Difference[4]	Income Base Roll-Up Period (in Yrs)	Earliest Age for Penalty-Free Withdrawals
0.00%	0.00%	No	0	0
0.95%	7.00%	No	10	60
0.95%	7.00%	Yes	10	60
1.10%	6.50%	Yes	10	55
1.00%	6.50%	Yes	10	55
1.10%	X%[5]	Yes	-	50

Each of the 1,000 policyholders chooses one of the rider products listed in Table 3 and the corresponding fees are deducted from their account value on an annual basis.

The above fee structure is comparable to the that of products offered by American Equity Investment Life Insurance Company® and Nationwide® Mutual Insurance Company:

Table 4. Typical Rider Fees

Fee %	Income Base Roll-Up %	Compounding Interest	Income Base Roll-Up Period (in Years)	Disability Benefits	Earliest Age for Penalty-Free Withdrawals
1.00%	7.00%	No	10	No	45
0.00%	4.00%	Yes	15	No	50
1.10%	7.25%	No	7	No	50
1.20%	7.25%	No	7	Yes	50
0.90%	6.00% for 10 Years, Then Reset (3.00% Minimum)	Yes	10 (Locked-In) + 10 (Reset)	No	50
1.00%	6.00% for 10 Years, Then Reset (3.00% Minimum)	Yes	10 (Locked-In) + 10 (Reset)	Yes	50
1.10%	6.00% for 10 Years, Then Reset (3.00% Minimum)	Yes	10 (Locked-In) + 10 (Reset)	No	50
1.20%	6.00% for 10 Years, Then Reset (3.00% Minimum)	Yes	10 (Locked-In) + 5 (Reset)	Yes	50

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[3] Income base roll-up is compounded annually.

[4] This feature provides flexibility to the policyholders. During the income base crediting phase, when there is a small amount of withdrawal, the income base will still be credited by the difference between the income base roll-up percentage and the withdrawal percentage.

[5] If there are already withdrawals, roll-up rate = 1.5 × account value growth. If there is not yet withdrawal, roll-up rate = 2.5 × account value growth.

3. Baseline Analysis: Annual Point-to-Point Crediting Method

Product Overview

The indexed crediting on the policy account value depends on both the performance of an external index (such as S&P 500®) and the account value crediting method chosen. The performance of the policy account value has a direct impact on the rider value.

The annual point-to-point method is the most common type of crediting in FIA markets. The index change depends only on the difference between the equity index end point and its starting point and ignores the points in-between. The index change using this method is calculated as follows:

$$\text{Index Change} = \frac{End\ Value}{Starting\ Value} - 1 \qquad (1)$$

To arrive at the indexed return, an index spread is deducted from the index change, and the difference is multiplied by a participation rate. The indexed interest rate is subject to a cap rate and never falls below 0%.

Table 5. Indexed Return Parameters

Indexed Return Parameter	Description
Index Spread	The return percentage to be deducted from the stock index change
Participation Rate	The portion of the gain in the stock index to be credited
Cap Rate	The maximum indexed return possible

$$\text{Indexed Return} = Max\left(Min\left((\text{Index Change} - \text{Index Spread}) \times Participation\ Rate, Cap\ Rate\right), 0\%\right)$$
(2)

For example, given the following parameters:

Table 6. Sample Indexed Return Parameters

Indexed Return Parameter	Value
Index Spread	2%
Participation Rate	80%
Cap Rate	5%

Assuming the beginning stock index value is 100, and it is 120 at the end of the year, the index change will be 20%. The resulting indexed return will be capped at 5% $= Max\left(Min\left((20\% - 2\%) \times 80\%, 5\%\right), 0\%\right)$.

The insurance company has the right to periodically reset the indexed return parameters, which are subject to the fluctuating available option budget level.

Results

1% Pricing Spread

Table 7 below presents the results across four model combinations in table 1. The “PV Benefits” column in the table quantifies the actuarial present values of the insurance company’s future market risk liabilities using the models specified in each row. The "Net PV" column presents the fair value result which is the difference in present values between the future benefits and ascribed fees. The last column "Option Budget Approach Discrepancy" compares the result of the deterministic option budget crediting model with the stochastic equity option budget crediting model given the same interest rate model.

Table 7. Results (Pricing Spread = 1%)

Index Crediting Model	Interest Rates	Equity Returns	PV Benefits	PV Fees	Net PV	Net PV / Initial Premium [6]	Option Budget Approach Discrepancy[7]
1 - Deterministic option budget crediting	Deterministic	Not modelled	$6,526,364	$3,569,483	$2,956,882	9.08%	0.58%
2 -Deterministic option budget crediting	HW2F	Not modelled	$6,539,299	$3,565,169	$2,974,129	9.13%	0.65%
3 - Stochastic equity crediting	Deterministic	Bates	$6,482,429	$3,542,585	$2,939,844	9.03%	
4 - Stochastic equity crediting	HW2F	Bates	$6,494,483	$3,539,500	$2,954,983	9.08%	

As we can see, with a 1% pricing spread, there is no material difference among the models. All four models produce nearly identical results. In the rest of this section, we will compare the results across different pricing spread assumptions.

1% Pricing Spread and Parallel Shift Up General Account Yield by 1%

Table 8. Results (Pricing Spread = 1% and Shift Up General Account Yield by 1%)

Index Crediting Model	Interest Rates	Equity Returns	PV Benefits	PV Fees	Net PV	Net PV / Initial Premium	Option Budget Approach Discrepancy
1 -Deterministic option budget crediting	Deterministic	Not modelled	$4,922,844	$3,577,970	$1,344,874	4.13%	-12.08%
2 -Deterministic option budget crediting	HW2F	Not modelled	$5,004,251	$3,592,078	$1,412,173	4.34%	-13.01%
3- Stochastic equity crediting	Deterministic	Bates	$5,105,162	$3,575,557	$1,529,606	4.70%	
4 - Stochastic equity crediting	HW2F	Bates	$5,204,015	$3,580,732	$1,623,283	4.99%	

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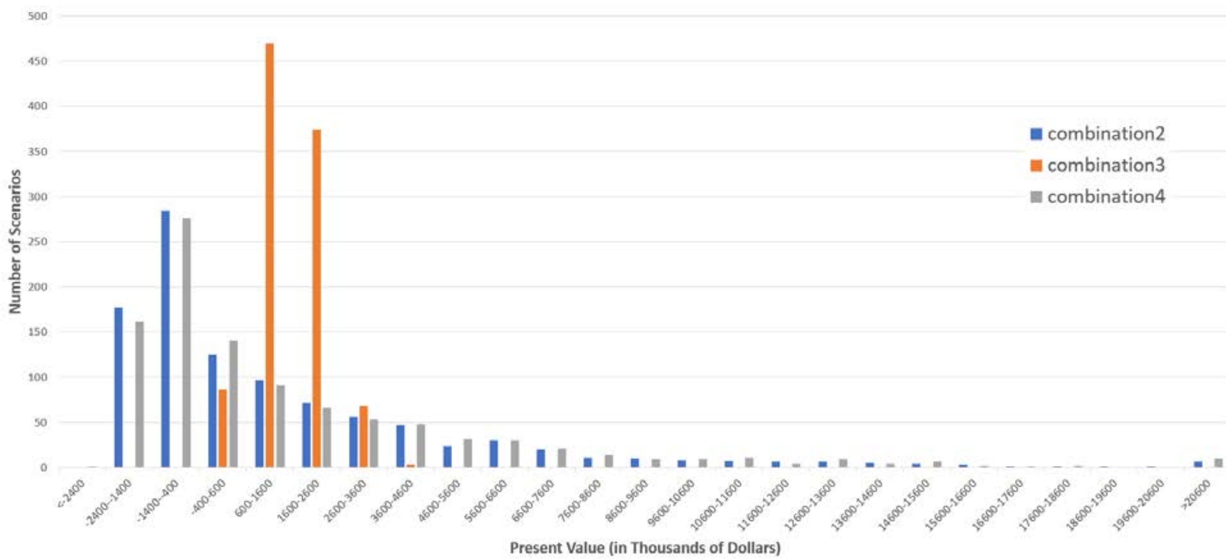
6 Total initial premium for 1,000 FIA contracts is $32,561,633
7 $2,956,882 / $2,939,844 – 1 = 0.58% and $2,974,129 / $2,954,983 – 1 = 0.65%.

This setting maintains the pricing spread at 1% while parallel shifts the general account yield curve up by 1%. It provides a higher level of option budget than the prior case and is plausible during a rising interest rate environment.

The deterministic option budget crediting model underestimates the liabilities by more than 12% in comparison to the stochastic equity crediting model and fails to capture the full dynamics of the equity scenarios.

Figure 2. Net PV Frequency Distribution Across Combinations in Table 8[8].



All model combinations but the first one (a deterministic interest rate model with equity return not modelled) are able to produce present value distributions. Figure 2 shows the different distributions. We see a dramatic impact of the interest rate model on the distribution of the present values. Therefore, model selection is important when it comes to managing the product risk across a wide range of scenarios and the tail scenarios in particular. The deterministic modelling approach is insufficient in this case.

2% Pricing Spread

The interest rate model was calibrated with the market prices observed in December 2019 when the 10-year US treasury yield was below 2%. If the insurer requires a relatively aggressive pricing spread of 2%

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Combination in Figure 2	Index Crediting	Interest Rates	Equity Returns
Not Included	1 -Deterministic option budget crediting	Deterministic	Not modelled
Combination2	2 -Deterministic option budget crediting	HW2F	Not modelled
Combination3	3- Stochastic equity crediting	Deterministic	Bates
Combination4	4 - Stochastic equity crediting	HW2F	Bates

under such an interest rate environment, the available option budget level will be low and even reach 0% for some simulated scenarios.

When the available option budget level is 0%, the minimum cap rate guarantee will be activated. In other words, the cap rate will stay at the minimum 1.00% level.

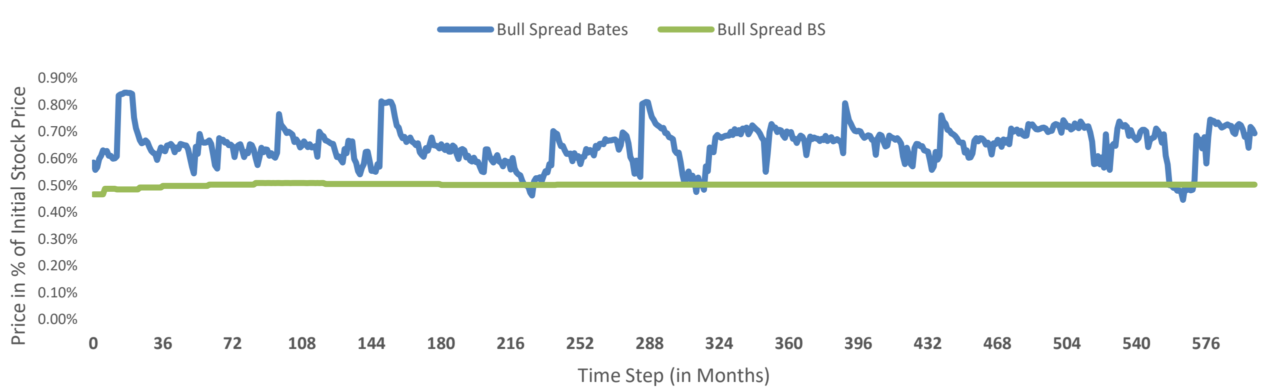
- Under the stochastic equity crediting model, the account value will be credited by a bull spread option strategy with a floor of 0% and a cap of 1.00%.
- Under the deterministic option budget crediting model, the account value will be credited directly by the price of this bull spread option strategy. Because there is no equity simulation involved in this model, by default, the pricing is produced by the Black-Scholes formula.

Table 9. Results (Pricing Spread = 2%)

Index Crediting Model	Interest Rates	Equity Returns	PV Benefits	PV Fees	Net PV	Net PV / Initial Premium	Option Budget Approach Discrepancy
1 - Deterministic option budget crediting	Deterministic	Not modelled	$7,574,722	$3,530,928	$4,043,795	12.42%	5.99%
2 -Deterministic option budget crediting	HW2F	Not modelled	$7,540,553	$3,528,047	$4,012,505	12.32%	5.01%
3 - Stochastic equity crediting	Deterministic	Bates	$7,328,249	$3,512,955	$3,815,294	11.72%	
4 - Stochastic equity crediting	HW2F	Bates	$7,323,767	$3,502,669	$3,821,098	11.73%	

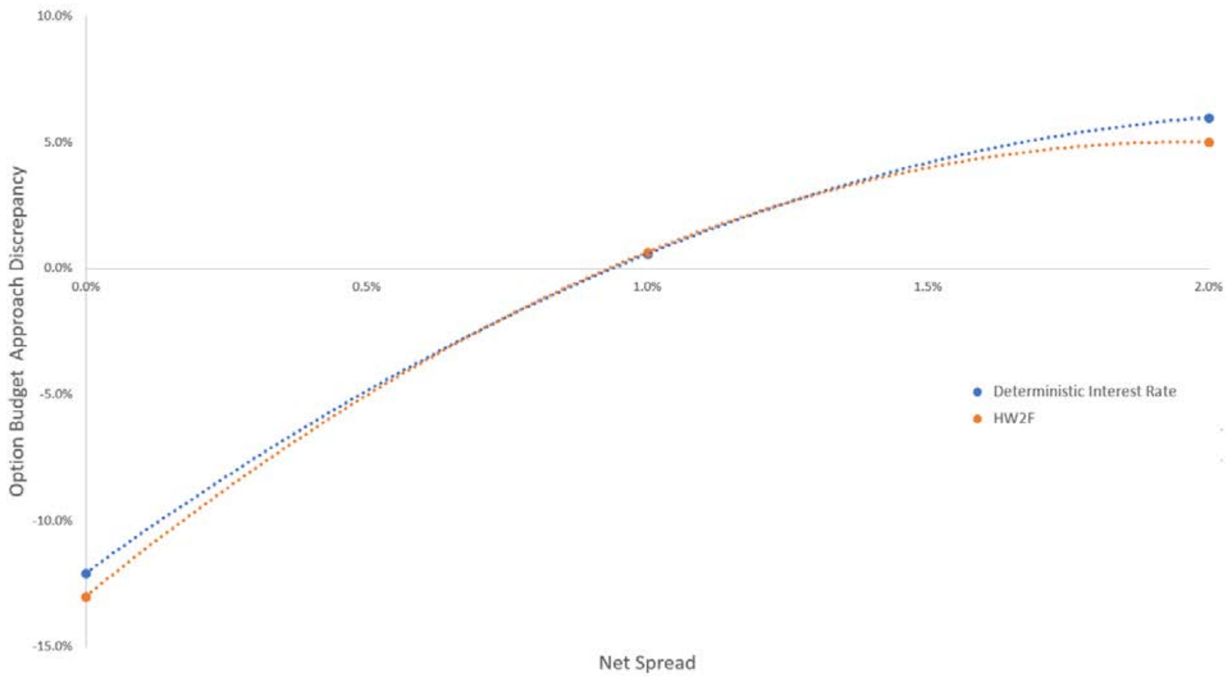
In this case, the deterministic option budget crediting model overestimates the liabilities by more than 5%. The Black-Scholes formula does not capture the stochastic nature of volatilities and jumps. Therefore, it undervalues the bull spread option strategy under most scenarios. The figure below compares the prices simulated by the Bates model to the Black-Scholes prices under a representative interest rate scenario:

Figure 3. Bull Spread Option Strategy Price Comparison between Bates and Black-Scholes (BS)



Comparison

Figure 4. Net PV Ratio between Two Equity Modelling Approaches Across Different Net Spreads[9]



For the annual point-to-point FIA product, there are non-linear differences in the results between the deterministic option budget crediting model and their stochastic counterparts across different available option budget levels and interest rate models. A higher level of available option budget generally generates a larger gap between the two models.

4. Extended Analysis: Monthly Average Crediting Method

To generalize the analysis, in this section, we extend the comparisons of the four model combinations to the monthly average crediting method.

Product Overview

The monthly average crediting method records the values of a stock index at the end of each month, then averages twelve consecutive month-end index values to arrive at a 12-month rolling average. This crediting method compares this 12-month rolling average to the index value at the beginning of the year to determine the index change.

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[9] This chart summarizes the last columns in table 7,8 and 9.

Net Spread on the x-axis is defined as the pricing spread less by the upward shift in the general account yield.

$$Index\ \text{Change} = \frac{(\sum_{1}^{12} Monthly\ End\ Point_i)/12}{Starting\ Value} - 1 \quad (3)$$

Compared to the annual point-to-point crediting method, this crediting method reduces the index volatility by averaging the index values and has a limited potential up-side. On the modelling front, the path dependent nature of this crediting method is more challenging to model.

Same as the annual point-to-point crediting method, the indexed return also follows formula (2) for this product. And the insurance company has the right to reset the parameters including the cap rates, participation rates and index spreads as they are subject to the available option budget level.

Modelling Approaches to Projecting Index Credits

To conduct comparison analysis, we will apply the two previously discussed modelling approaches to project index credits: the stochastic equity crediting model and the deterministic option budget crediting model. Because the index change is path-dependent, the stochastic equity crediting model requires a model to be able to generate monthly values along each path.

Results

We apply the four models in Table 1 for comparisons. The comparison is on the same group of 1,000 FIA contracts illustrated earlier, instead of annual point-to-point, the account value crediting follows the monthly averaging crediting method.

1% Pricing Spread

Table 10. Results (Pricing Spread = 1%)

Index Crediting Model	Interest Rates	Equity Returns	PV Benefits	PV Fees	Net PV	Net PV / Initial Premium	Option Budget Approach Discrepancy
1 - Deterministic option budget crediting	Deterministic	Not modelled	$6,526,378	$3,528,780	$2,997,598	9.21%	-0.80%
2 -Deterministic option budget crediting	HW2F	Not modelled	$6,539,246	$3,526,813	$3,012,433	9.25%	-1.12%
3 - Stochastic equity crediting	Deterministic	Bates	$6,540,239	$3,518,529	$3,021,711	9.28%	
4 - Stochastic equity crediting	HW2F	Bates	$6,562,193	$3,515,708	$3,046,485	9.36%	

Similar to the annual point-to-point crediting method, no material difference is observed when the pricing spread is 1%.

1% Pricing Spread and Upward Parallel Shift General Account Yield by 1%

Table 11. Results (Pricing Spread = 1% And Upward Shift General Account Yield by 1%)

Index Crediting Model	Interest Rates	Equity Returns	PV Benefits	PV Fees	Net PV	Net PV / Initial Premium	Option Budget Approach Discrepancy
1 - Deterministic option budget crediting	Deterministic	Not modelled	$4,922,810	$3,544,533	$1,378,277	4.23%	-20.65%
2 -Deterministic option budget crediting	HW2F	Not modelled	$5,004,667	$3,556,373	$1,448,294	4.45%	-23.65%
3 - Stochastic equity crediting	Deterministic	Bates	$5,290,471	$3,553,552	$1,736,918	5.33%	
4 - Stochastic equity crediting	HW2F	Bates	$5,449,203	$3,552,406	$1,896,797	5.83%	

Under this setting, the deterministic option budget crediting model under-estimates the liabilities by more than 20% in comparison to the stochastic counterparts.

2% Pricing Spread

Table 12. Results (Pricing Spread = 2%)

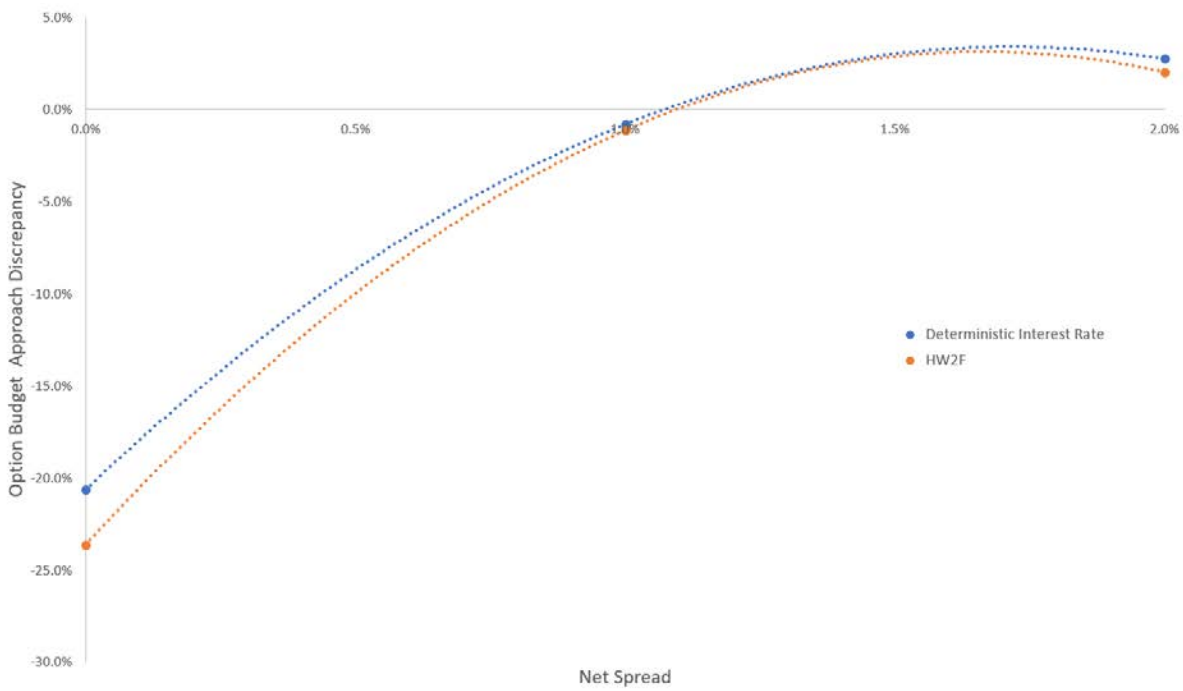
Index Crediting Model	Interest Rates	Equity Returns	PV Benefits	PV Fees	Net PV	Net PV / Initial Premium	Option Budget Approach Discrepancy
1 - Deterministic option budget crediting	Deterministic	Not modelled	$7,574,705	$3,493,849	$4,080,856	12.53%	2.75%
2 -Deterministic option budget crediting	HW2F	Not modelled	$7,540,449	$3,484,645	$4,055,804	12.46%	2.03%
3 - Stochastic equity crediting	Deterministic	Bates	$7,459,811	$3,488,107	$3,971,703	12.20%	
4 - Stochastic equity crediting	HW2F	Bates	$7,454,608	$3,479,383	$3,975,224	12.21%	

When the pricing spread is 2%, the minimum cap of 1% is triggered for most of the interest rate scenarios because insufficient option budget is available to purchase options. Therefore, the deterministic option budget crediting model credits accounts at the minimum level which corresponds to the minimum cap. This minimum crediting level is backed out by the Black-Scholes formula.

Under this setting, a moderate difference is observed because the account crediting rates are narrowly distributed for all four models.

Comparison

Figure 5. Net PV Comparison between Two Equity Modelling Approaches Across Different Net Spreads[10]



The pattern in figure 5 is similar to that in figure 4. When the option budget level is limited, the valuation differences are small because the account crediting is of a narrower distribution. In other words, there is limited range of possible outcomes.

When the available option budget level is higher, there are more possible scenarios for the crediting rates. In that case, the deterministic option budget crediting model fails to properly capture the full distribution.

5. Contract Duration Sensitivities

To investigate the impact of contract duration on the model selection, we will re-examine the results by looking the contract duration sensitivities.

The average age of 1,000 policies in the prior comparison is 61 years old. Younger policyholders are subject to a higher interest rate risk because their contract durations are longer. In order to investigate the impact of policy duration, we uniformly decrease all 1,000 policyholders' ages while maintaining the pricing spread at 1% and examine the results.

[10] This chart summarizes the last columns in table 10, 11 and 12.
Net Spread on the x-axis is defined as the pricing spread less by the upward shift in the general account yield.

Results

Average Age: 45 Years Old

We first uniformly decrease all policyholders' ages by 16 years to arrive at an average age of 45 years old.

Table 13. Ultra-Long Duration Contract Results (Average Age: 45 Years Old)

Index Crediting Model	Interest Rates	Equity Returns	PV Benefits	PV Fees	Net PV	Net PV / Initial Premium	Option Budget Approach Discrepancy
1 - Deterministic option budget crediting	Deterministic	Not modelled	$11,456,190	$5,944,153	$5,512,038	16.93%	1.62%
2 -Deterministic option budget crediting	HW2F	Not modelled	$11,302,883	$5,670,083	$5,632,801	17.30%	0.74%
3 - Stochastic equity crediting	Deterministic	Bates	$11,343,390	$5,919,444	$5,423,946	16.66%	
4 - Stochastic equity crediting	HW2F	Bates	$11,237,032	$5,645,876	$5,591,155	17.17%	

The assumptions of the above results are the same as those in Table 7 except for the policyholder age. In comparison to Table 7, the policy duration extension increases both the PV benefits, PV fees and the resulting liabilities to the insurer.

Due to the longer duration, PV fees and PV benefits are impacted by any small movements in the yield curve.

Given a 16 years uniform decrease in all ages, a different impact is observed under the two different interest rate approaches. Under the deterministic interest rate model, the deterministic option budget crediting model over-estimates the liabilities by 1.62%, and it is 0.74% under the stochastic interest rate model.

Average Age: 40 Years Old

Next, we uniformly decrease all policyholders' ages by 21 years to arrive at an average age of 40 years old.

Table 14. Ultra-Long Duration Contract Results (Average Age: 40 Years Old)

Index Crediting Model	Interest Rates	Equity Returns	PV Benefits	PV Fees	Net PV	Net PV / Initial Premium	Option Budget Approach Discrepancy
1 -Deterministic option budget crediting	Deterministic	Not modelled	$10,240,575	$6,747,378	$3,493,198	10.73%	4.13%
2 -Deterministic option budget crediting	HW2F	Not modelled	$10,033,682	$6,272,517	$3,761,165	11.55%	1.88%

3 - Stochastic equity crediting	Deterministic	Bates	$10,087,528	$6,732,772	$3,354,757	10.30%	
4 - Stochastic equity crediting	HW2F	Bates	$9,952,422	$6,260,776	$3,691,645	11.34%	

PV benefits are lower than those in Table 13 because policyholders need to wait longer for the benefits to be effective. We observe larger differences between the deterministic option budget crediting model and the stochastic counterparts when the policy durations are extended further.

Average Age: 35 Years Old

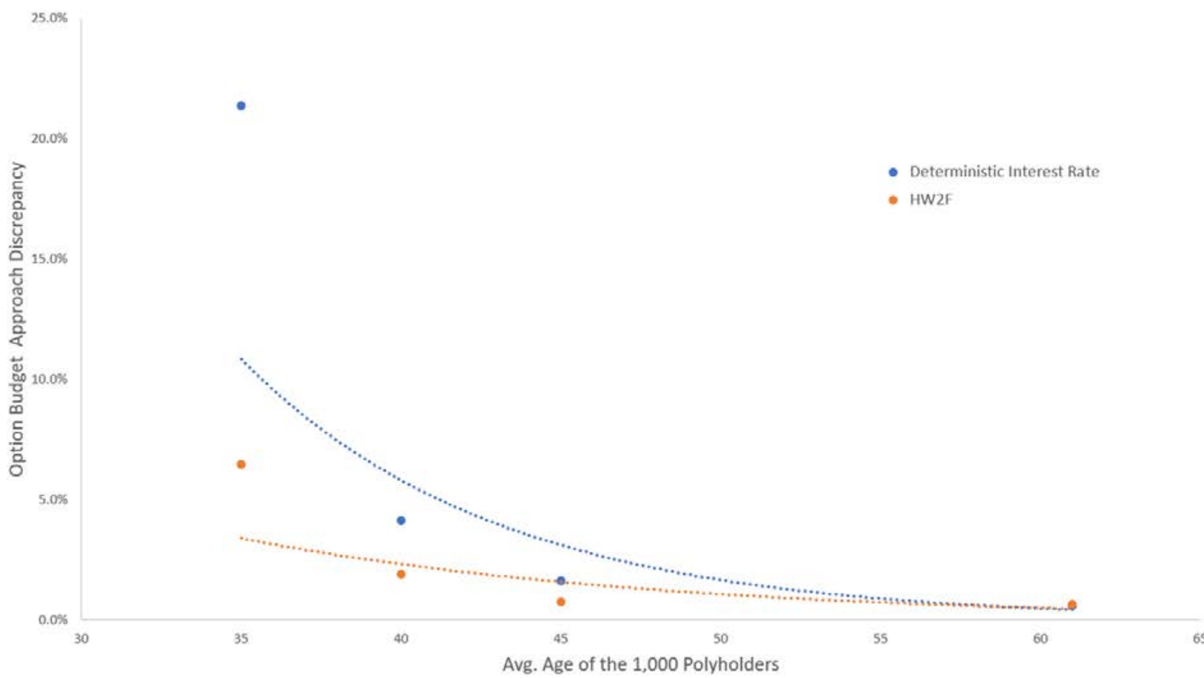
Table 15. Ultra-Long Duration Contract Results (Average Age: 35 Years Old)

Index Crediting Model	Interest Rates	Equity Returns	PV Benefits	PV Fees	Net PV	Net PV / Initial Premium	Option Budget Approach Discrepancy
1 - Deterministic option budget crediting	Deterministic	Not modelled	$8,655,091	$7,579,931	$1,075,160	3.30%	21.35%
2 -Deterministic option budget crediting	HW2F	Not modelled	$8,450,052	$6,881,153	$1,568,900	4.82%	6.44%
3 - Stochastic equity crediting	Deterministic	Bates	$8,462,781	$7,576,782	$886,001	2.72%	
4 - Stochastic equity crediting	HW2F	Bates	$8,352,609	$6,878,696	$1,473,913	4.53%	

As can be seen, the longer the policy durations, the bigger the discrepancy becomes as the deterministic option budget crediting model is unable to properly capture the risks.

Comparison

Figure 6. Net PV Comparison between Two Equity Modelling Approaches Across Different Durations[11]



When comparing the deterministic and stochastic models, the impact from interest rate modelling approach needs to be considered in addition to the pricing spread levels. This is important especially for ultra-long duration policies which carry a substantial amount of interest rate risk. As we can see from figure 6, the longer the policy duration, the larger the gap between the two approaches becomes.

6. Conclusion

In this paper, we have compared the fair value quantification of FIA riders with four combinations of equity models and interest rate models under various deterministic and stochastic settings and different assumptions including available option budget, policy duration and product crediting method.

The deterministic option budget crediting model produces a simple best-estimate valuation and is faster in computation. But we observed this shortcut model does not always produce results consistent with those from stochastic models which use a large number of market-consistent equity return scenarios. It also fails to capture the entire distribution of valuations due to the simplistic nature of the approach.

The MRB reporting introduced by ASU No. 2018-12 generally requires FIA riders to be reported at fair value post LDTI. Additionally, under the newly introduced PBR framework for fixed annuities (VM-22), reserving for FIAs riders will likely require the use a CTE 70 measure. Therefore, in order to meet the

[11] This chart summarizes the last columns in table 7, 13, 14 and 15.

requirements of latest reporting frameworks, companies need to consider the various modelling approaches and pros and cons associated with each approach.

Appendix

Hull-White Two-Factor (HW2F)

The formulaic representation of the Hull-White Two-Factor (HW2F) interest rate model is as follows:

$$r_t = x_t + y_t + \varphi_t$$

$$dx_t = -ax_t dt + \sigma dW_t^x$$

$$dy_t = -by_t dt + \eta dW_t^y$$

$$Corr(W^x, W^y) = \rho_{xy}$$

where r_t represents the instantaneous short rate.

r_t is governed by two correlated stochastic processes: x_t and y_t. Compared to the HW1F model, the extra source of randomness allows us to generate a more realistic yield curve.

Bates equity model

$$dS_t = S_t\left((r_t - \lambda\bar{k})dt + \sqrt{v_t}dB_t^s + dJ_t\right)$$

$$J_t = \int_0^t \int_{-\infty}^{\infty} z\, M(dz, dt) = \sum_{i=1}^{N_t} k_i$$

$$\log(1 + k_i) \sim N\left(\log(1 + \bar{k}) - \frac{1}{2}\delta^2;\ \delta^2\right)$$

$$dv_t = \kappa(\theta - v_t)dt + \nu\sqrt{v_t}dB_t^v$$

$$Corr(B^s, B^v) = \rho_{sv}$$

where

dJ_t is a Poisson process generating stochastic jumps and

dv_t is the variance dynamics that follows a Cox-Ingersoll-Ross process.