

## VOI formula in Gaussian case

For a Gaussian variable  $w$  with mean  $m$  and variance  $r^2$ :

$$\begin{aligned}
 PoV(x) &= E(\max\{0, w\}) \\
 &= \int \max\{0, w\} p(w) dw \\
 &= \int_0^\infty w p(w) dw \\
 &= \int_{-\frac{m}{r}}^\infty (m + rz) \phi(z) dz \\
 &= m \int_{-\frac{m}{r}}^\infty \phi(z) dz + r \int_{-\frac{m}{r}}^\infty z \phi(z) dz \\
 &= m(1 - \Phi(-\frac{m}{r})) + r \phi(-\frac{m}{r}) \\
 &= m\Phi mr + r\phi mr
 \end{aligned}$$

Since here it uses another standard Gaussian variable  $z$  to transform the given  $w$ . Therefore, according to standard Gaussian,

$$\begin{aligned}
 \Phi(z) &= \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} \exp^{-\frac{x^2}{2}} dx \\
 \phi(z) &= \frac{1}{\sqrt{2\pi}} \exp^{-\frac{z^2}{2}}
 \end{aligned}$$

Additional notes to understand the second part of the equation

$$\begin{aligned}
 r \int_{-\frac{m}{r}}^\infty z \phi(z) dz &= \int_{-\frac{m}{r}}^\infty z \exp(-\frac{z^2}{2}) dz \\
 &= \left[ -\frac{z}{z} \exp^{-\frac{z^2}{2}} \right]_{-\frac{m}{r}}^\infty - \int_{-\frac{m}{r}}^\infty \exp^{-\frac{z^2}{2}} (-z) dz \\
 &= \left[ -\frac{z}{z} \exp^{-\frac{z^2}{2}} \right]_{-\frac{m}{r}}^\infty - \int_{-\frac{m}{r}}^\infty \exp^{-\frac{z^2}{2}} d(-\frac{z^2}{2}) \\
 &= \left[ -\frac{z}{z} \exp^{-\frac{z^2}{2}} \right]_{-\frac{m}{r}}^\infty - \left[ \exp^{-\frac{z^2}{2}} \right]_{-\frac{m}{r}}^\infty
 \end{aligned}$$

$$\begin{aligned}
 r \int_{-\frac{m}{r}}^\infty z \phi(z) dz &= \int_{-\frac{m}{r}}^\infty z \exp(-\frac{z^2}{2}) dz \\
 &= - \int_{-\frac{m}{r}}^\infty \exp(-\frac{z^2}{2}) d(-\frac{z^2}{2}) \\
 &= \left[ \exp^{-\frac{z^2}{2}} \right]_{-\frac{m}{r}}^\infty
 \end{aligned}$$

## VOI for gaussian perfect information

Prior model for profits:

$$p(x) = N(0, \Sigma), \Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \quad (1)$$

Now, assume we know perfect information about 1 project, thus it becomes:

$$\begin{aligned} y &= x_1 \\ p(x_1) &= N(0, 1) \\ E(x_1) &= x_1 \\ E(x_2|x_1) &= E(x_1) + \Sigma_{x_2x_1} \Sigma_{x_2}^{-1} (x_1 - E(x_1)) = 0 + \rho \cdot 1 \cdot (1 - 0) = \rho x_1 \\ Var(x_1) &= 0 \\ Var(x_2|x_1) &= \Sigma_{x_2} - \Sigma_{x_2x_1} \Sigma_{x_2}^{-1} \Sigma_{x_1x_2} = 1 - \rho \cdot 1 \cdot \rho = 1 - \rho^2 \end{aligned} \quad (2)$$

### 0.1 Imperfect information

As for imperfect information, measurement error  $\tau^2$  feed into the system by adding additional variance, therefore, it becomes:

$$\begin{aligned} y &= x + N(0, \tau^2 \mathbf{I}) \\ p(y) &= N(\mathbf{0}, \tau^2 \mathbf{I} + \Sigma) = N(\mathbf{0}, \mathbf{C}) \\ E(x|y) &= \Sigma \mathbf{C}^{-1} \mathbf{y} \\ PoV &= E\{max\{E(x_i|y), 0\}\} = E(max(u_i, 0)) \\ u &= \Sigma \mathbf{C}^{-1} \mathbf{y} \sim N(0, \Sigma \mathbf{C}^{-1} \mathbf{C} \mathbf{C}^{-1} \Sigma) \sim N(0, \Sigma \mathbf{C}^{-1} \Sigma) \\ \mathbf{S} &= \begin{pmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} \end{pmatrix} \\ u_1 &\sim N(0, S_{11}) \\ u_2 &\sim N(0, S_{22}) \\ PoV(y) &= \sum_{i=1}^n (\mu_i) \Phi\left(\frac{\mu_i}{\sqrt{r_{ii}}}\right) + \sqrt{r_{ii}} \phi\left(\frac{\mu_i}{\sqrt{r_{ii}}}\right) = 0 + r \phi(0) = 0 + S_{ii} / \sqrt{2\pi} = \frac{\sqrt{S_{11}} + \sqrt{S_{22}}}{\sqrt{2\pi}} \end{aligned} \quad (3)$$