Project 3: Ship tracking from bearings

Please send your solutions to andrea.riebler@ntnu.no. **Deadline:** May 5th.

Consider three different filtering approaches (Extended Kalman filter, Particle filter, Ensemble Kalman filter) for the task of tracking a ship on the surface using two bearings-only observations.

There are two sensors measuring angles to a vessel. One sensor is located in east,north coordinate $(0, 0)$, the other at coordinate $(40, 40)$ in km units. See Figure 1 for a map view of this situation with the initial observation and an example of a realized ship trajectory.

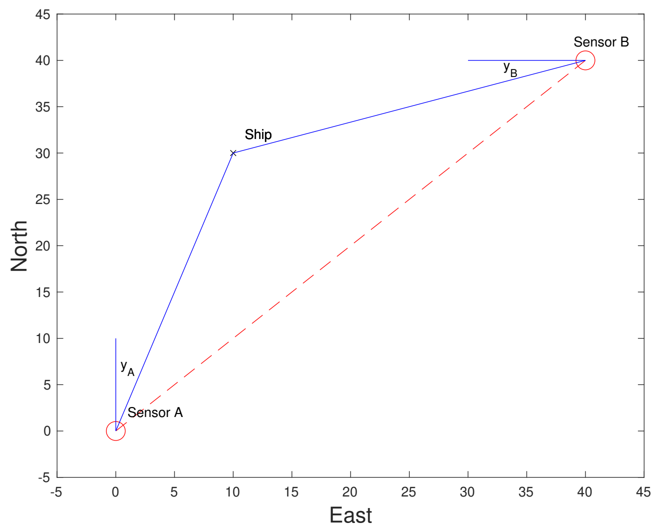

Figure 1: Sensor A and B in bearing-only tracking.

The state space model is defined as follows:

A surface vessel has state $\boldsymbol{x}_t = (E_t, N_t, v_t, u_t)^T$, where (E_t, N_t) is the east and north position at time t, while (v_t, u_t) is the associated velocity vector at time t.

The prior initial state is, $\boldsymbol{x}_1 \sim N_4(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1)$, where the expectation is $\boldsymbol{\mu}_1 = (10, 30, 10, -10)^T$ and the covariance matrix is $\boldsymbol{\Sigma}_1 = \text{Diag}(10^2, 10^2, 5^2, 5^2)^T$.

The prior dynamic process model for the vessel is defined by

$$
\begin{aligned}
\boldsymbol{x}_{t+1} &= \boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{\epsilon}_{t+1}, \\
\boldsymbol{A} &= \begin{bmatrix} 1 & 0 & \delta & 0 \\ 0 & 1 & 0 & \delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
\boldsymbol{\epsilon}_{t+1} &\sim N_4(\boldsymbol{0}, \text{Diag}[0.1^2, 0.1^2, 0.5^2, 0.5^2])
\end{aligned}
$$

for $t = 1, \ldots, T-1$, and with $\delta = 1/60$ which means sampling interval of every minute.

The observations made at sensors A and B, $\boldsymbol{y}_t = (y_{A,t}, y_{B,t})^T$, $t = 1, \ldots, T$, are modelled as conditionally independent and single-site response with additive Gaussian errors:

$$\boldsymbol{y}_t = \begin{bmatrix} \arctan(E_t/N_t) \\ \arctan[(40-N_t)/(40-E_t)] \end{bmatrix} + \boldsymbol{\epsilon}_t$$
$$\boldsymbol{\epsilon}_t \sim N_2(\mathbf{0}, \mathrm{Diag}[0.1^2, 0.1^2]),$$

where the inverse tangent function is defined as indicated in Figure 1, assuming that the ship is entering the field of interest for both sensors. Note that the noise level corresponds to a standard deviation of about 5 degrees in the angle.

The angle observations at sensor A and B can be downloaded from `https://folk.ntnu.no/joeid/MA8702/sensorA.txt` and `https://folk.ntnu.no/joeid/MA8702/sensorB.txt`. These observations are displayed over $T = 50$ time steps in Figure 2.

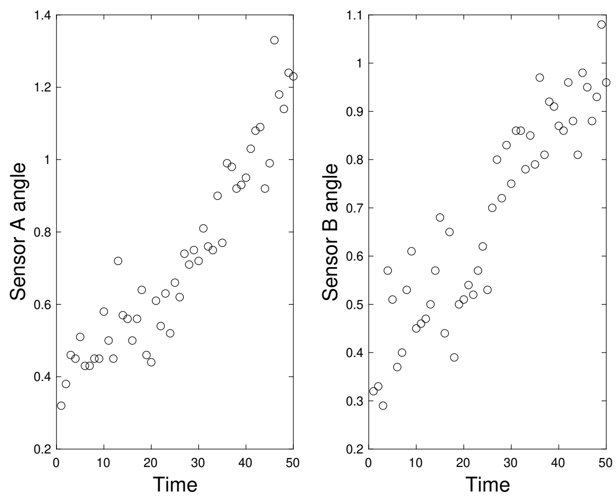

Figure 2: Observations of Sensor A and B.

TASKS

The objective of the study is to assess the filtering pdf $p(\boldsymbol{x}_t|\boldsymbol{y}_1, \ldots, \boldsymbol{y}_t)$, $t = 1, \ldots, T$.

- Implement an extended Kalman filter algorithm. This means linearizing the measurement equation around the predicted state. Plot the filtering solution in a map view, along with uncertainty bounds. Plot the filter variances of the position coordinates and the velocities over time. Discuss the results.

- Implement a standard particle filter algorithm with $B = 10000$ particles, using the state Markovian process model as proposal at each time step. Apply resampling in the particle filter. Plot the filtering solution in a map view, along with uncertainty bounds. Plot the filter variances of the position coordinates and the velocities over time. Activate also the particle filter with only $B = 100$ particles. Discuss the results.

- Implement a standard ensemble Kalman filter with $B = 1000$ ensemble members. Plot the filtering solution in a map view, along with uncertainty bounds. Plot the filter variances of the position coordinates and the velocities over time. Activate also the ensemble Kalman filter with only $B = 100$ ensembles. Discuss the results.

Finally, summarize your major experiences with the study.