P3

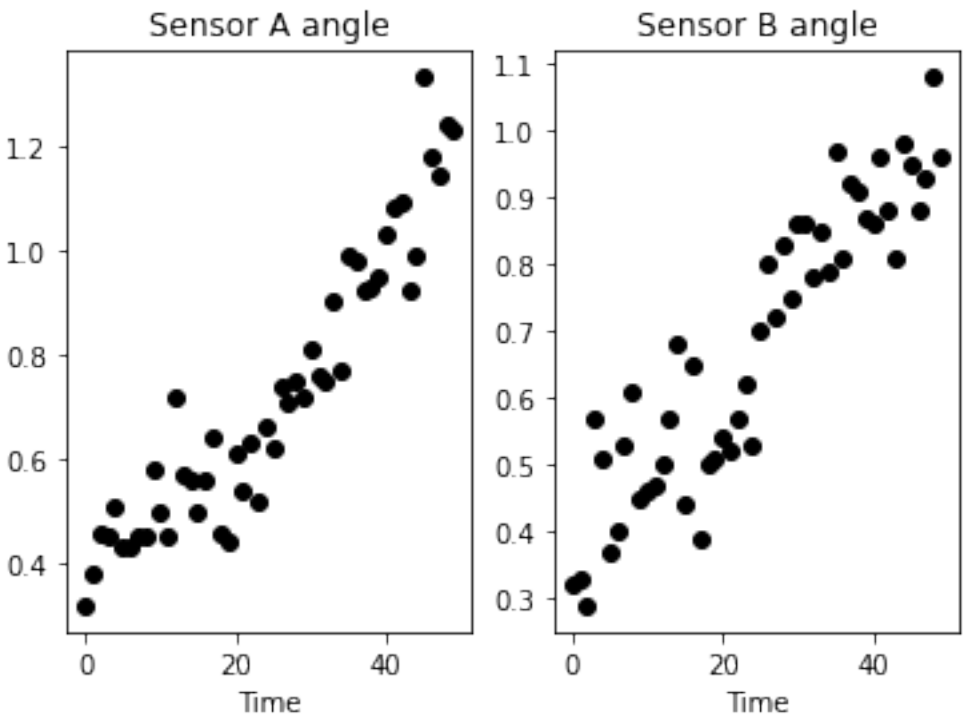
April 30, 2021

1 Particle filter implementation on ship tracking

1.0.1 In this project, the particle filter estimation on the ship's locations in a 2D-domain is implemented, where 10000 particles and 100 paritcles are studied to compare the performance over the size of the particle filter

[107]:
```
import numpy as np
import matplotlib.pyplot as plt
```

[108]:
```
N = 50 # number of time points

yA = np.loadtxt('sensorA.txt')
yB = np.loadtxt('sensorB.txt')
y = np.vstack((yB, yA))

plt.figure()
plt.subplot(121)
plt.plot(yA, 'ko')
plt.title("Sensor A angle")
plt.xlabel("Time")
plt.subplot(122)
plt.plot(yB, 'ko')
plt.title("Sensor B angle")
plt.xlabel("Time")
plt.show()
```



1.1 Set up the model

A surface vessel has state $\boldsymbol{x}_t = (\mathcal{E}_t, \mathcal{N}_t, \sqsubseteq_t, \sqcap_t)$, which represents the position along north and east, and the velocity along north and east respectively.

The initial state is $\boldsymbol{x}_t \sim \mathcal{N}_4(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1)$, where

$$\boldsymbol{\mu}_1 = \begin{bmatrix} 10 \\ 30 \\ 10 \\ -10 \end{bmatrix}$$

$$\boldsymbol{\Sigma}_1 = \begin{bmatrix} 10^2 & 0 & 0 & 0 \\ 0 & 10^2 & 0 & 0 \\ 0 & 0 & 5^2 & 0 \\ 0 & 0 & 0 & 5^2 \end{bmatrix}$$

The evolution model can be expressed as

$\boldsymbol{x}_{t+1} = \boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{\epsilon}_{t+1}$, where $\boldsymbol{A} = \begin{bmatrix} 1 & 0 & \delta & 0 \\ 0 & 1 & 0 & \delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, and $\delta = \frac{1}{60}$

$$\boldsymbol{\epsilon}_{t+1} \sim \mathcal{N}(\mathbf{0}, \begin{bmatrix} 0.1^2 & 0 & 0 & 0 \\ 0 & 0.1^2 & 0 & 0 \\ 0 & 0 & 0.5^2 & 0 \\ 0 & 0 & 0 & 0.5^2 \end{bmatrix})$$

While the observation model can be formulated as

$$\boldsymbol{y}_t = \begin{bmatrix} \arctan(E_t/E_t) \\ \arctan[(40 - N_t)/(40 - E_t)] \end{bmatrix} + \boldsymbol{\epsilon}_t, \text{ where}$$

$$\boldsymbol{\epsilon}_t \sim \mathcal{N}_2(\mathbf{0}, \begin{bmatrix} 0.1^2 & 0 \\ 0 & 0.1^2 \end{bmatrix})$$

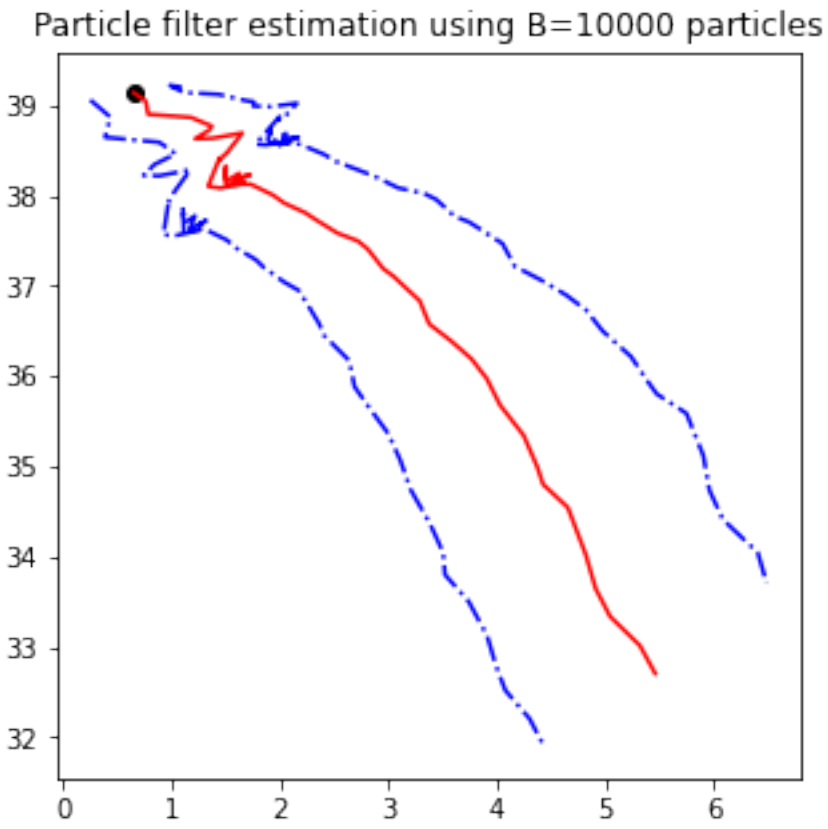
[109]:
```
A = np.eye(4)
delta = 1 / 60
A[0, 2] = delta
A[1, 3] = delta
```

[110]:
```
mu_prior = np.vstack((10, 30, 10, -10))
Sigma_prior = np.diagflat(np.vstack((10**2, 10**2, 5**2, 5**2)))
Sigma_PP = np.diagflat(np.vstack((.1**2, .1**2, .5**2, .5**2)))
Sigma_noise = np.diagflat(np.vstack((.1**2, .1**2)))
mu_prop = np.zeros([4, N])
mu_prop[:, 0] = mu_prior.squeeze()
mu_update = np.zeros([4, N])
Sigma_pred = np.zeros([4, 4, N])
Sigma_pred[:, :, 0] = Sigma_prior
Sigma_upd = np.zeros([4, 4, N])
Pred_err = np.zeros([2, N])
```

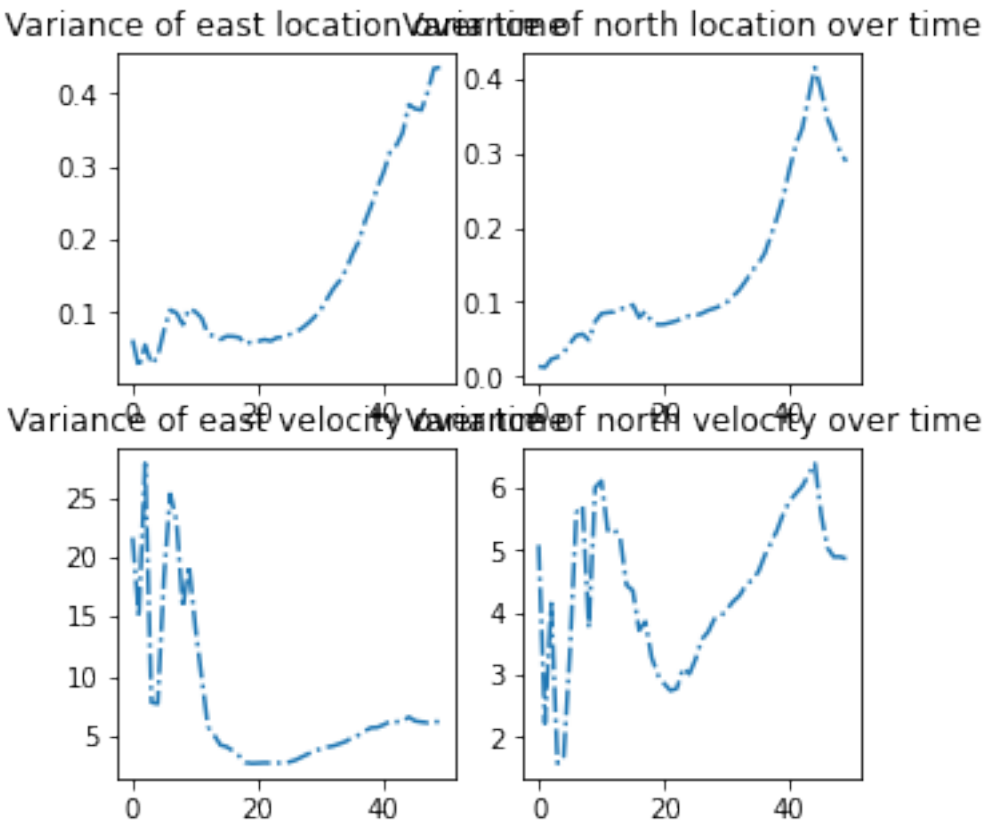
[111]:
```
B = 10000
L = np.linalg.cholesky(Sigma_prior)
xB = np.zeros([4, B])
xB = mu_prior * np.ones([1, B]) + np.dot(L, np.random.randn(4 * B).reshape(4,␣
↪B))
mm = np.zeros([4, N])
mvar = np.zeros([4, N])
mlow = np.zeros([4, N])
mhigh = np.zeros([4, N])
for i in range(N):
    yPB = np.vstack((np.divide(np.arctan(xB[0, :]), np.arctan(xB[1, :])),
                     np.divide(np.arctan(40 - xB[1, :]), np.arctan(40 - xB[0, :
↪]))))

    l = -.5 * np.linalg.inv(Sigma_noise[0, 0].reshape(-1, 1)) * np.multiply((
            np.dot(y[:, i].reshape(-1, 1), np.ones([1, B])) - yPB),
            (np.dot(y[:, i].reshape(-1, 1), np.ones([1, B])) - yPB))
    # why here only Sigma_noise used in the loglikelihood instead of the␣
↪sampling
```

```
    # reweighting
    w = l[0, :] + l[1, :]
    ew = np.exp(w)
    nw = ew / np.sum(ew) # updated weights pdf
    Fw = np.cumsum(nw) # cdf to be used for resampling
    # resampling using the weights
    xBupd = np.zeros([4, B])
    for kk in range(B):
        Ur = np.random.rand()
        ind = np.argwhere(Fw > Ur)[:, 0][0]
        xBupd[:, kk] = xB[:, ind]
    mm[:, i] = np.mean(xBupd, axis = 1)
    mvar[:, i] = np.var(xBupd, axis = 1)
    for ik in range(4):
        ms = np.sort(xBupd[ik, :])
        mlow[ik, i] = ms[int(np.ceil(.05 * B))]
        mhigh[ik, i] = ms[int(np.floor(.95 * B))]
    if i < N:
        xB = np.dot(A, xBupd) + np.dot(np.linalg.cholesky(Sigma_PP), np.random.
↪randn(4 * B).reshape(4, B))
```

[112]:
```
plt.figure(figsize = (5, 5))
plt.plot(mm[0,0], mm[1, 0], 'ko')
plt.title("Particle filter estimation using B={:d} particles".format(B))
plt.plot(mm[0, :], mm[1, :], 'r')
plt.plot(mlow[0, :], mlow[1, :], 'b-.')
plt.plot(mhigh[0, :], mhigh[1, :], 'b-.')
plt.show()
```



[113]:
```
plt.figure(figsize = (5, 5))
plt.subplot(221)
plt.plot(mvar[0, :], '-.')
plt.title("Variance of east location over time")
plt.subplot(222)
plt.plot(mvar[1, :], '-.')
plt.title("Variance of north location over time")
plt.subplot(223)
plt.plot(mvar[2, :], '-.')
plt.title("Variance of east velocity over time")
plt.subplot(224)
plt.plot(mvar[3, :], '-.')
plt.title("Variance of north velocity over time")
plt.show()
```

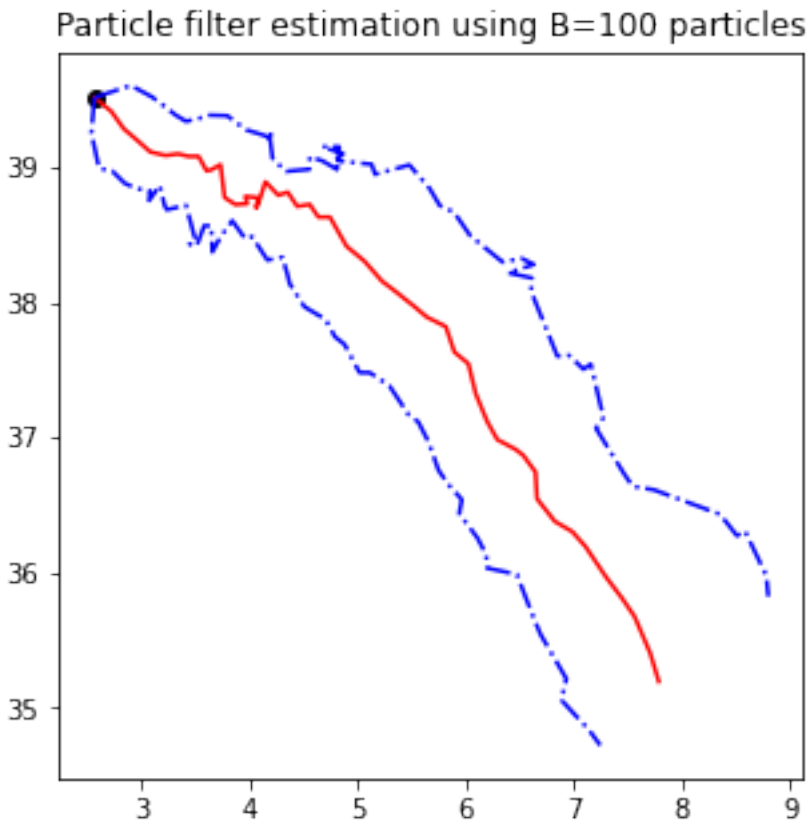


From the variances plot, one can tell the estimation variances for locations along east and north tend to increase over time, while the estimation variances for velocities tend to keep stable when the particle size is around 10000.

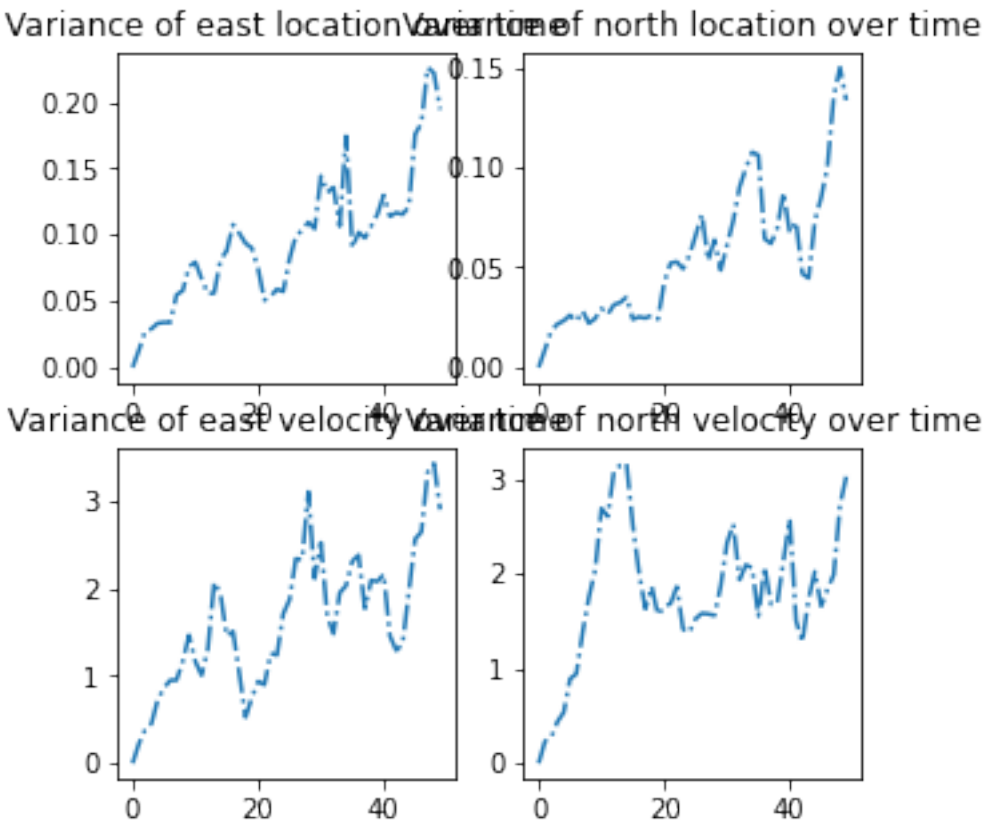
[114]:
```
B = 100
L = np.linalg.cholesky(Sigma_prior)
xB = np.zeros([4, B])
xB = mu_prior * np.ones([1, B]) + np.dot(L, np.random.randn(4 * B).reshape(4,␣
 ↪B))
mm = np.zeros([4, N])
mvar = np.zeros([4, N])
mlow = np.zeros([4, N])
mhigh = np.zeros([4, N])
for i in range(N):
    yPB = np.vstack((np.divide(np.arctan(xB[0, :]), np.arctan(xB[1, :])),
                     np.divide(np.arctan(40 - xB[1, :]), np.arctan(40 - xB[0, :
 ↪]))))

    l = -.5 * np.linalg.inv(Sigma_noise[0, 0].reshape(-1, 1)) * np.multiply((
            np.dot(y[:, i].reshape(-1, 1), np.ones([1, B])) - yPB),
            (np.dot(y[:, i].reshape(-1, 1), np.ones([1, B])) - yPB))
```

```
    # why here only Sigma_noise used in the loglikelihood instead of the␣
↪sampling
    # reweighting
    w = l[0, :] + l[1, :]
    ew = np.exp(w)
    nw = ew / np.sum(ew) # updated weights pdf
    Fw = np.cumsum(nw) # cdf to be used for resampling
    # resampling using the weights
    xBupd = np.zeros([4, B])
    for kk in range(B):
        Ur = np.random.rand()
        ind = np.argwhere(Fw > Ur)[:, 0][0]
        xBupd[:, kk] = xB[:, ind]
    mm[:, i] = np.mean(xBupd, axis = 1)
    mvar[:, i] = np.var(xBupd, axis = 1)
    for ik in range(4):
        ms = np.sort(xBupd[ik, :])
        mlow[ik, i] = ms[int(np.ceil(.05 * B))]
        mhigh[ik, i] = ms[int(np.floor(.95 * B))]
    if i < N:
        xB = np.dot(A, xBupd) + np.dot(np.linalg.cholesky(Sigma_PP), np.random.
↪randn(4 * B).reshape(4, B))
```

[115]:
```
plt.figure(figsize = (5, 5))
plt.plot(mm[0,0], mm[1, 0], 'ko')
plt.title("Particle filter estimation using B={:d} particles".format(B))
plt.plot(mm[0, :], mm[1, :], 'r')
plt.plot(mlow[0, :], mlow[1, :], 'b-.')
plt.plot(mhigh[0, :], mhigh[1, :], 'b-.')
plt.show()
```



[116]:
```
plt.figure(figsize = (5, 5))
plt.subplot(221)
plt.plot(mvar[0, :], '-.')
plt.title("Variance of east location over time")
plt.subplot(222)
plt.plot(mvar[1, :], '-.')
plt.title("Variance of north location over time")
plt.subplot(223)
plt.plot(mvar[2, :], '-.')
plt.title("Variance of east velocity over time")
plt.subplot(224)
plt.plot(mvar[3, :], '-.')
plt.title("Variance of north velocity over time")
plt.show()
```



When only having 100 particles, the estimation variances for velocities tend to become very large compared to the estimation variances for locations

1.2 To sum up, the particle filter provides an efficient way of estimating the location and velocities in this case, but it is important to choose the right size of the particles to both achieve the estimation accuracy and the computational efficency.