TOWARDS AN INDIRECT DETERMINATION OF THE MASS-BALANCE DISTRIBUTION OF GLACIERS USING THE KINEMATIC BOUNDARY CONDITION

BY
G.H. GUDMUNDSSON and A. BAUDER

VAW, ETH-Zentrum, Zürich, Switzerland

Gudmundsson, G.H. and Bauder, A., 1999: Towards an indirect determination of the mass-balance distribution of glaciers using the kinematic boundary condition. *Geogr. Ann.,* 81 A (4): 575–583.

ABSTRACT. The kinematic boundary condition at the surface is utilized to arrive at an estimate of the mass-balance distribution of the ablation region of Unteraargletscher, Bernese Alps, Switzerland. This is achieved without the use of any ground measurements. The terms of the kinematic boundary condition, involving surface-altitude changes with time, surface slopes, and horizontal surface velocities, are determined using high precision aerial photogrammetry. Estimating the vertical velocity distribution along the surface poses a major problem. Different approaches to solving this problem are discussed, and the potential of one particular approach is evaluated. This approach is, in essence, based on the assumption that the variation of vertical strain rates with depth is simple. The accuracy of the resulting indirect estimate of the mass balance distribution is assessed by a comparison with results from stake measurements made at about 40 different points at the surface. Although calculated values of mass balance are found to be within a reasonable range, they differ, as a function of altitude, in a systematic fashion from stake values. This suggests that the vertical strain-rate variation with depth is too complex to be parameterized in a simple manner.

Key words: glaciers, mass-balance distribution, Unteraargletscher.

Introduction

Measuring the mass balance of glaciers is important for a number of reasons. Temporal changes in the mass-balance distribution result primarily from changes in accumulation and melt along the surface. Monitoring changes in the mass-balance distribution may therefore be used to detect shifts in climate (Oerlemans and Fortuin 1992). Volume changes of small glaciers, i.e. changes in net balance, may also have a significant effect on global sea level (Meier 1984, 1990; Schwitter and Raymond 1993). In addition, glaciers regulate the runoff of water from glaciated areas by storing water during winter and releasing some (not necessarily the same amount) during summer.

Traditionally, the mass balance of glaciers has been determined by interpolating results from a number of *in situ* stake measurements (Østrem and Brugman 1991). Because ground-based methods are difficult, time consuming and expensive, estimates of mass-balance distributions are, in spite of their importance, only made for a limited number of glaciers. In the Swiss Alps, for example, financial constraints have led to the discontinuation of mass-balance measurements on some glaciers for which long mass-balance records exist. Furthermore, for the three glaciers for which measurements continue, the number of stakes is only about one-fifth of what is known to be necessary for an accurate mass-balance estimation (Hoinkes 1970; Funk *et al.* 1997).

A number of alternatives to the traditional stake method, all having the common goal of reducing the amount of field work as much as possible, have been suggested and tested on selected glaciers. These methods may be divided into two groups, depending on whether it is the spatially averaged annual mass balance (net balance of the whole glacier) or the mass-balance distribution that is determined.

Estimating the net mass balance requires information on the rate of change in glacier volume. This information can, for example, be obtained by comparing digital terrain models (DTMs) from different times. Examples of such estimates of volume changes of glaciers can be found in Chen and Funk (1990), Funk *et al.* (1997), Echelmeyer *et al.* (1996), and Sapiano *et al*. (1998).

For estimating the mass-balance distribution, some knowledge about the flow field of the glacier is needed. Most approaches used so far for arriving indirectly at the mass-balance distribution have been based on the use of the continuity equation:

$$\partial_t h + \nabla \cdot \mathrm{q} = \dot{b} \tag{1}$$

The mass-balance rate $\dot{b}$ is determined indirectly from the observed surface elevation changes with time ($\partial_t h$), and by estimating the horizontal divergence of the flux vector **q**. Examples for the use of this method, in the following referred to as the *continuity-equation* method, can be found in Reynaud *et al.* (1986), and Rasmussen (1988).

If mass-balance determinations are to be used to observe and to quantify climatic variability, estimates of temporal changes in the mass-balance distribution are to be preferred over estimates of the changes in net mass balance (rate-of-volume changes). This follows from the fact that – for a given instantaneous shift in climatic variables (average temperature, precipitation, cloudiness, etc.) – temporal changes in net mass balance are gradual and decrease towards zero with time. In this sense, the net mass balance does not constitute a true climatic signal. The mass-balance distribution, on the other hand, reacts immediately to climatic shifts and does not decrease towards zero as time proceeds.

Given the importance of estimating the mass-balance distribution of glaciers and considering the cost and time involved with the traditional stake method, we feel that serious efforts should be made towards developing and testing new approaches to mass-balance determination. Here we describe a method for estimating the mass-balance distribution of glaciers which does not depend on any direct surface measurements, except during an initial validation period. This approach differs significantly from the continuity-equation method as it is based on the kinematic boundary condition at the surface. It involves only quantities defined at the glacier surface, such as the surface velocity and the surface altitude. In an ongoing pilot study, the potential of this method, which we refer to as the *kinematic-equation* method, is currently being evaluated by applying it to the lower reaches of Unteraargletscher, Bernese Alps, Switzerland. Here we give a description of the method, present initial results and discuss some of the difficulties which we have encountered.

Method

The vertical component of the flow vector at the surface (v_z) is the only term of the kinematic boundary condition which cannot be determined directly using aerophotogrammetric methods. We describe two different approaches towards estimating v_z along the surface. The potential of both of these methods is currently being evaluated. However, in this paper our focus is on the simpler of the two approaches. In essence, our main assumption is that the variation of vertical strain rates with depth is simple and does not vary strongly across the surface of the glacier. From a comparison of the calculated mass-balance distribution with a number of *in situ* measurements, the accuracy of the results and the correctness of the underlying assumptions are assessed.

Description

The mass balance rate defined for a point at the surface, $\dot{b}(x,y)$, is related to the velocity ($\mathbf{v} = (v_x,v_y,v_z)$), the surface elevation changes with time ($\partial_t h(x,y)$), and the surface slopes ($\partial_x h(x,y)$, $\partial_y h(x,y)$) at that point. This relation is described by the kinematic boundary condition:

$$\partial_t h(x,y) + v_x(x,y)\, \partial_x h(x,y) + v_y(x,y)\, \partial_y h(x,y) - v_z(x,y) = \dot{b}\,(x,y) \qquad (2)$$

where h represents the surface elevation, (v_x,v_y,v_z) are the components of the surface velocity vector, and $\dot{b}$ is the mass-balance rate. Here the z axis of the coordinate system is vertical and points upwards, and the x and the y axes are horizontal. All terms in Equation 2 refer to the surface. The mass-balance function $\dot{b}(x,y)$ describes the flux (in units of velocity) through a surface element with a normal vector parallel to the z axis, in agreement with the common definition of the mass balance used in glaciology. Equation 2 does not depend on the rheological properties of the glacier, nor is it limited in its applicability to the surface of glaciers. Its derivation is found in common textbooks of continuum mechanics (e.g. Chandrasekharaiah and Debnath 1994) and in books on theoretical glaciology (Hutter 1983). Together with the incompressibility condition ($v_{i,i} = 0$), the kinematic boundary condition (Equation 2) can be used to derive the continuity equation (Equation 1).

The traditional glaciological method determines the single term on the right-hand side of Equation 2, i.e. the mass-balance rate $\dot{b}\,(x,y)$, from *in situ* measurements. An alternative approach, based on this equation, would be to determine each of the four terms on the left-hand side individually, and so to arrive at an indirect estimate of the mass-balance distribution. This is the basic idea behind the kinematic-equation method. The idea of exploiting the kinematic boundary condition for determining the mass-balance distribution is not new. A similar approach has been used previously on Griesgletscher, Swiss Alps, by Kääb (1996) and Kääb and Funk (1999).

Implementation

There are currently various remote-sensing methods available which can be used to determine some, or all, of the terms on the left-hand side of Equation 2. Our approach uses remote-sensing methods, such as aerophotogrammetrical monitoring and an airborne elevation-scanning system, for estimating the first three terms on the left-hand side of Equation 2.

The first term on the left-hand side ($\partial_t h$) of Equation 2 is obtained from aerophotogrammetric determination of DTMs using aerial photographs. Due to the lack of texture it may prove to be impossible to extract information on surface altitude from the higher-lying parts of the accumulation area. For this purpose, a number of laser-scanning experiments have been performed. Initial results using the laser-scanning system are presented and discussed by Favey *et al.* (1999). The horizontal velocity components (v_x and v_y) are determined from aerial photographs by simultaneous monoplotting of multitemporal stereo models. The method is described in Kääb *et al.* (1997).

One possibility to arrive at an estimate of v_z along the surface would be to use a fully three-dimensional numerical flow model that includes all the terms of the momentum equations. Such a model has been developed for most of Unteraargletscher (Gudmundsson 1999b). Another much simpler approach is to assume some simple variation of vertical strain rates with depth and to determine v_z from calculated values of horizontal strain along the surface using the condition of incompressibility. Here we evaluate the potential of this simpler approach.

To calculate the v_z distribution, the horizontal strain rates along the surface ($\dot{\epsilon}_{xx}$, $\dot{\epsilon}_{yy}$ and $\dot{\epsilon}_{xy}$) are first determined from photogrammetrically derived horizontal surface velocities. The vertical strain rates at the surface are then calculated using the incompressibility condition $\dot{\epsilon}_{ii} = 0$. Some spatial averaging or filtering may be needed to counteract amplification of measurement errors introduced in these initial steps. The vertical strain rates are then integrated over depth assuming some particular shape function. For example, one could assume that the vertical strain rates are constant throughout the depth, or, alternatively, that they vary linearly with depth. If sliding velocities are small in comparison to surface velocities, determining the vertical component of the surface velocity, using this approach, requires knowledge of (1) the bedrock geometry, and (2) the horizontal velocity distribution along the surface at a high spatial resolution.

Accuracy

In principle, the procedure described above can give estimates of the distribution of the mass balance with a high spatial and temporal resolution. The accuracy of the method is, however, an important question which must be addressed. The best method of assessing the accuracy of the estimate of the mass-balance distribution is through comparison with a number of *in situ* stake measurements. It should be realized that the principle behind the method is straightforward. The question is therefore not whether a mass-balance distribution can be calculated in this way, but how accurately it can be done. Once this question has been answered during an initial validation period, it is rather unlikely that errors will accumulate with time, because the errors introduced by the remote-sensing methods are not expected to vary in a systematic fashion from year to year.

The firn area poses some special problems. The photogrammetric method can, for example, only be used if the surface displays enough texture. Even if these problems were to be overcome, the kinematic-equation method does not eliminate the need for density-profile measurements in the firn area.

Results

Figure 1 shows an estimate of the mass-balance distribution of Unteraargletscher for the time period from 23 August 1996 to 15 August 1997. The mass-balance distribution was calculated by the kinematic-equation method, i.e. by individually estimating the four terms on the left-hand side of Equation 2. A few surface contours (black lines) are included for reference. All of the study area is below the firn line. The inset of Fig. 1 shows the distribution of the mass-balance estimates with elevation. The corresponding spatial distributions of the individual terms on the left-hand side of Equation 2 can be seen in Figs 2 (elevation changes, $\partial_t h$), 3 (horizontal velocities times surface slopes, $v_x \partial_x h + v_y \partial_y h$) and 4 (vertical velocities, v_z).

Surface elevation changes along a 50 m square grid were calculated from DTMs of both years. The DTMs were generated from aerial photographs. Their accuracy is estimated to be around 30 cm. Since surface roughnesses are of the same order, a

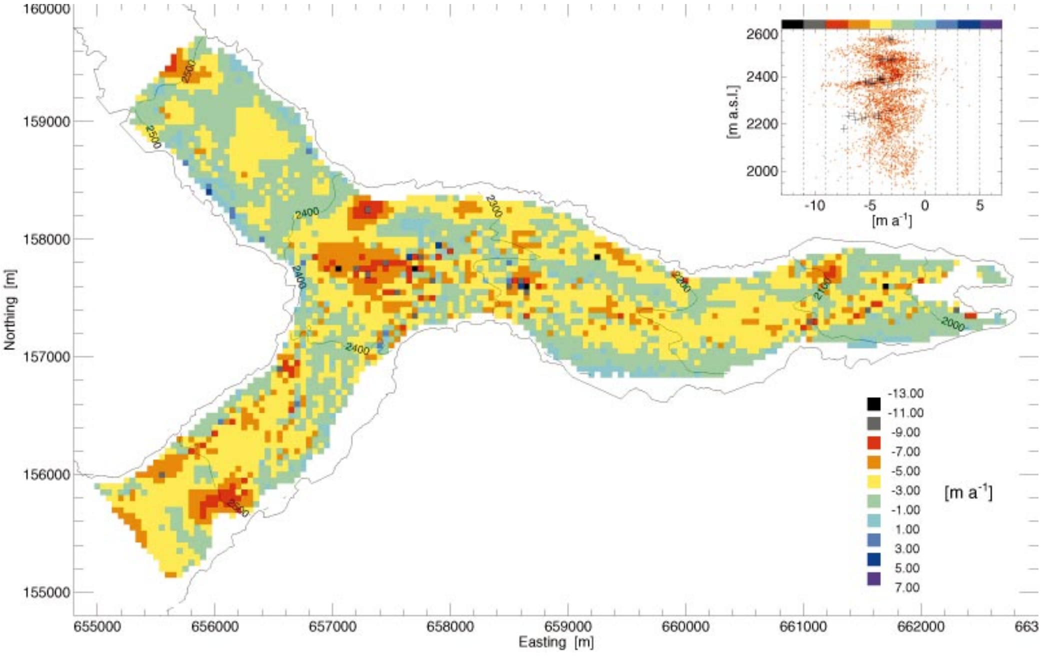

Fig. 1. Estimate of the mass-balance distribution of Unteraargletscher, Bernese Alps, Switzerland, for the time period 23 August 1996 to 15 August 1997. This estimate is based on the kinematic-equation method described in the text. Inset shows the distribution of both calculated (red points) and in situ measured (black crosses) values of mass balance as a function of altitude. Coordinates are in metres.

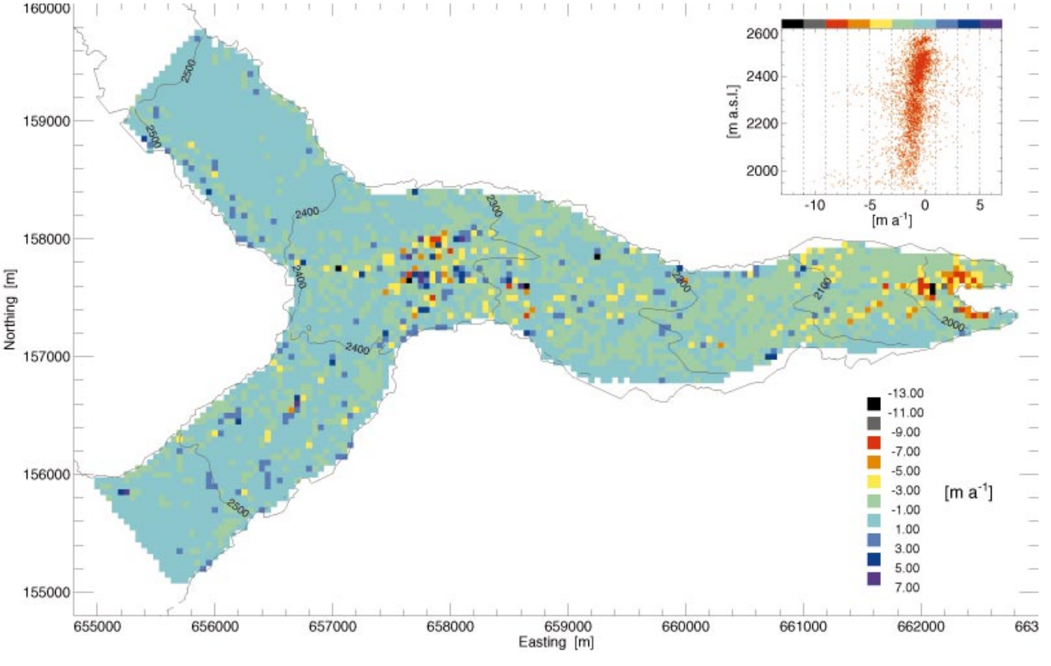

Fig. 2. Average changes in ice thickness for the time period 23 August 1996 to 15 August 1997. In the inset all grid-point estimates are depicted as a function of altitude.

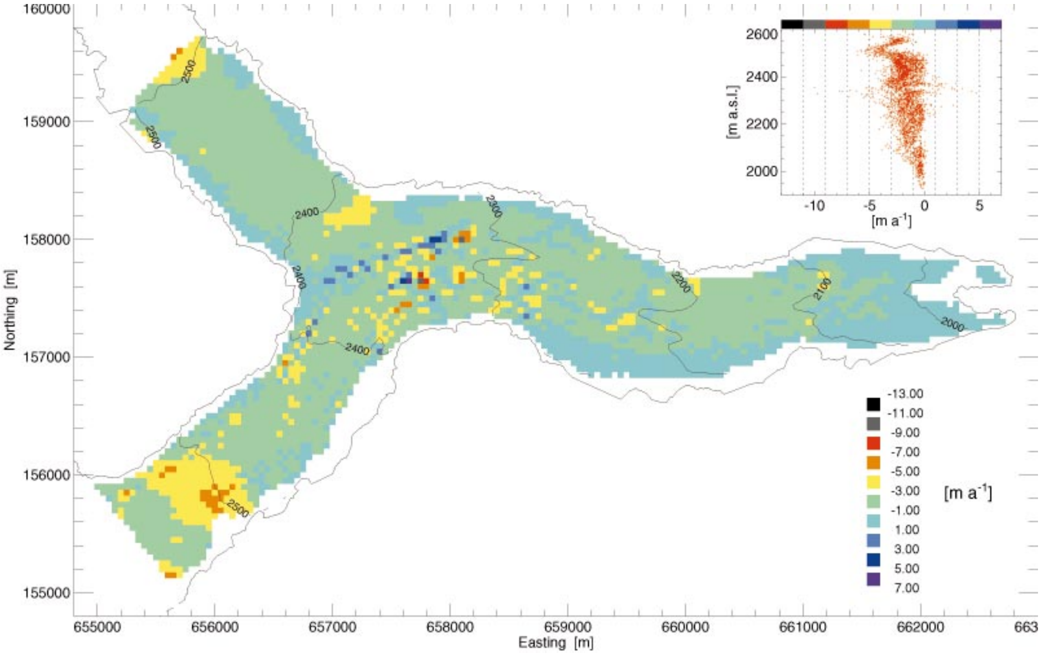

Fig. 3. Estimate of the sum $v_x \partial_x h + v_y \partial_y h$. The velocity distribution used for the estimate is shown in Fig. 5. The surface slopes were extracted from digital terrain models.

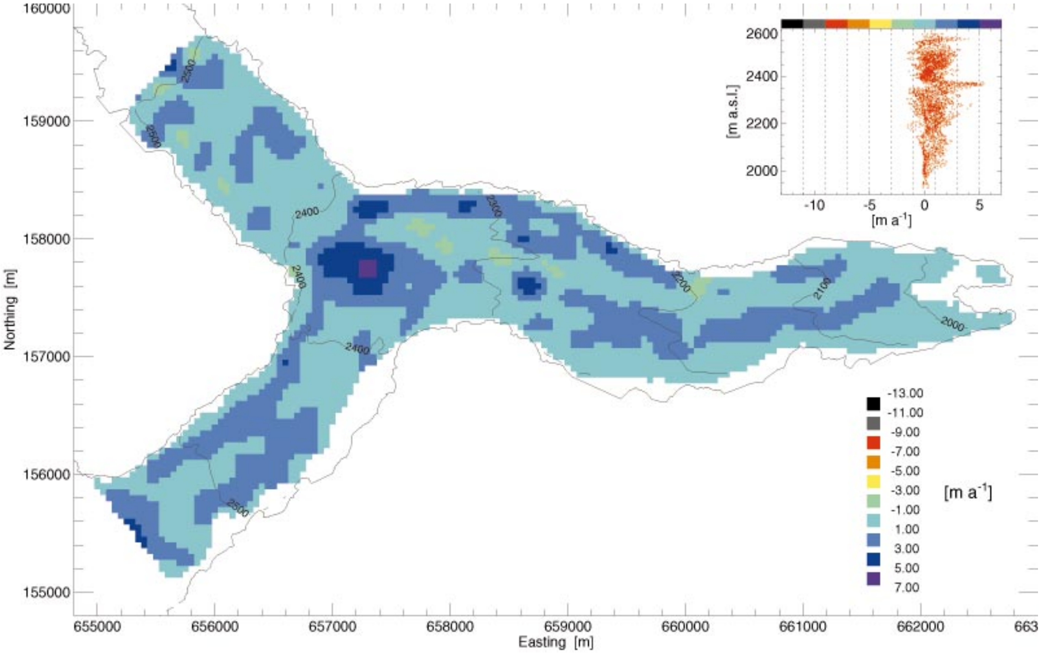

Fig. 4. Vertical surface velocities calculated from vertical surface strain rates and measured ice thicknesses by assuming a linear variation in vertical strain with depth. The altitude distribution of all surface strain-rate estimates is seen in the inset.

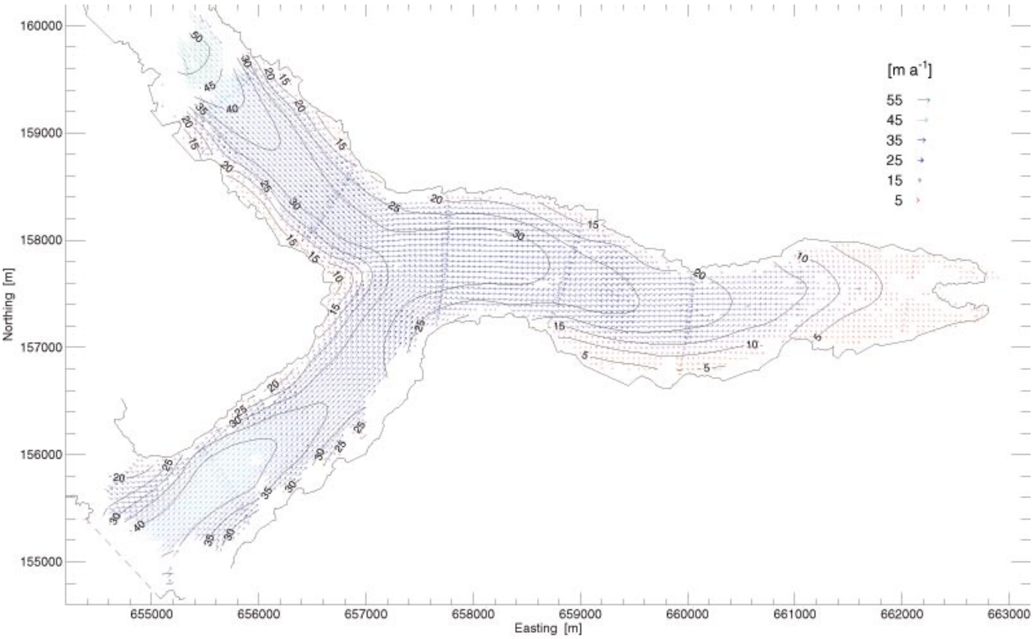

Fig. 5. Distribution of average horizontal surface velocities on Unteraargletscher, Bernese Alps, Switzerland, for the time period 23 August 1996 to 15 August 1997. A total of 6567 velocity estimates are shown.

better accuracy of the DTMs will hardly result in better estimates of changes in surface altitude.

The distribution of annual horizontal velocities (v_x and v_y) (Fig. 5) was extracted from aerial photographs through simultaneous monoplotting of multitemporal stereo models. More than 6000 velocity vectors were determined. Because surface velocities on Unteraargletscher are known to vary on a number of different time scales (Iken *et al.* 1983, Gudmundsson 1999a), the velocity vectors in Fig. 5 represent an annual average velocity distribution, but not necessarily actual velocities at some given point in time. Where measurements are available, winter velocities are about 75% of mean summer values (Gudmundsson *et al.* 1997).

Surface strain rates were calculated from the horizontal velocity distribution with a Fourier-transform method, and high-frequency errors suppressed with an optimal Wiener filter. This method of extracting surface strain rates from aerial photography is described in Gudmundsson *et al.* (1997). The incompressibility condition was used to determine the vertical strain rates at the surface. By assuming linear variation of vertical strain rates with depth, the vertical surface velocities were calculated through an integration over the glacier thickness.

For verifying purposes, the local mass balance was measured using the traditional stake-measurement method at about 40 different locations. Most of the stakes were located along five transverse profiles (Fig. 6). The stakes were installed between 29 September and 1 October 1996, and readings were made between 19 and 20 October 1997. Since the stake readings were made somewhat after the timing of the aerial photographs, direct estimates of local mass balance are expected to be a little larger than the indirect estimates. In the following discussion, the mass-balance distribution calculated indirectly using the kinematic-boundary condition is denoted by the symbol b^c. The distribution of direct stake measurements is denoted by the symbol b^m.

Discussion

A conspicuous feature of the calculated mass-balance distribution (b^c) is the large scatter in b^c (inset in Fig. 1). Generally, for any given altitude, a range of b^c values of $\pm$ 2 m a^{-1} or more is observed. The results from the stake measurements (b^m), depicted as black crosses in the inset of Fig. 1, show that a scatter within this range is a real feature of the actual mass-balance distribution. The annual mass balance along a transverse profile at an altitude of

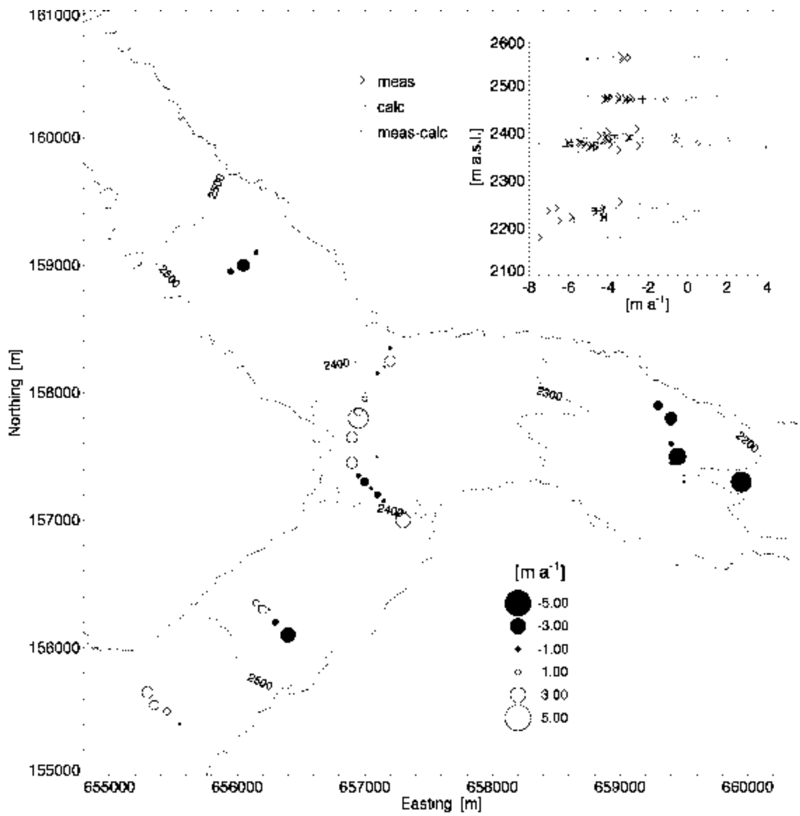

Fig. 6. Comparison of stake measurements with calculated values of mass balance. The positions of the circles give the locations of stake measurements. The radii of the circles are proportional to the difference between readings from stake measurements and the calculated values at each stake location. The inset depicts measured values (stake measurements), calculated values, and the corresponding numerical difference as a function of altitude.

2400 m a.s.l. varied, for example, between –5.9 and –1.8 m a^{-1}. This large spatial variability in melting conditions on Unteraargletscher is brought about by great amounts of very unevenly distributed supraglacial debris cover.

An unrealistic feature of Fig. 1 is the lack of an altitude gradient in the b^c distribution. In contrast, the results from the stake measurements (b^m) show a clear altitude gradient. Evidently, the altitude variation of one or more of the estimates of the individual terms on the left-hand side of Equation 2 must be incorrect.

The distribution of ice-thickness changes for the time period 23 August 1996 to 15 August 1997 (Fig. 2) displays the expected altitude gradient. Equally, there seems to be no reason to question the overall correctness of the estimates of the next two terms on the left-hand side of Equation 2 ($v_x \partial_x h$ and $v_y \partial_y h$, Fig. 3), although calculated values may, for a limited number of points, sometimes be in error. Thus, out of the four terms of the kinematic boundary condition, realistic estimates can be made for three of these terms using high-precision aerial photogrammetry. This is an important observation. Needless to say, this fact is, however, of limited use unless the fourth term (v_z) can be estimated with a similar accuracy. Not enough *in situ* measurements of vertical surface velocities are available to verify the v_z distribution directly. An assessment of the correctness of the v_z distribution must, therefore, focus on the main qualitative aspects of Fig. 4.

In general, the vertical components of the surface velocities (Fig. 4) are positive (upward oriented). For an ablation area, and for the typical surface slopes of Unteraargletscher, positive vertical velocities are to be expected along the surface. This is therefore a realistic feature of Fig. 4. The lack of an altitude gradient in the v_z distribution is, on the other hand, surprising. With increasing altitude and the usually concomitant reduction in ablation, the emerging velocities should decrease in magnitude. Changes in surface slope as a function of altitude are small and not sufficient to affect the emerging velocities significantly. The actual v_z distribution is, thus, expected to vary in a systematic fashion with altitude. Why does the calculated v_z distribution in Fig. 4 not display this expected altitude gradient? Since the data on ice thickness and the derived horizontal strain-rate distribution are unlikely to be seriously in error, the answer must be that the assumption of a linear vertical strain-rate variation with depth is wrong. Attempts to obtain a more realistic variation of vertical velocities with altitude by assuming various v_z depth profiles of different shapes did not lead to any considerable improvements in this regard. This suggests not only that the depth variation is not simple, but, more importantly, that it differs considerably in a qualitative sense between different areas of the glacier.

Analysing the short-scale (<h) features of the v_z distribution in Fig. 4 reveals both strengths and shortcomings of this distribution. The most striking short-scale features of Fig. 4 are (1) the strong positive v_z anomaly of the confluence area, and (2) a number of elongated strips aligned with the glacier flow direction of either higher- or lower-than-average vertical surface velocities. Pronounced short-scale surface undulations are expected to have a locally significant effect on the velocity distribution. On the basis of a numerical flow model of Unteraargletscher (Gudmundsson 1999b), negative vertical velocities have, for example, been predicted along the medial moraine.

The influence of the medial moraine on the vertical flow field is evident in Fig. 4, and appears, in a qualitative sense, quite realistic. Although it is important to have the means of detecting such short-scale variations in slope and velocity for arriving at correct indirect estimates of the mass-balance distribution, it is questionable to integrate surface strain rates, which are strongly affected by local surface perturbations, over the whole ice thickness to arrive at vertical surface velocities. Since the effects of local topographic disturbances on the strain-rate regime are expected to decay strongly with increasing depth, it is understandable that the strain-rate distribution across the glacier thickness may not be related in any simple way to such anomalous surface values.

The strongly positive v_z anomaly of the confluence area (Fig. 4) results from the transverse convergence over that area which far exceeds in magnitude the concomitant longitudinal extension. From theoretical consideration, and on the basis of numerical modelling of the flow at the confluence area, it is expected that the variation of vertical strain with depth is far from being linear at this location (Gudmundsson 1997, 1999b). This has also been demonstrated through direct measurements of vertical strain in boreholes (Gudmundsson *et al*. 1997).

Summary and outlook

A mass-balance distribution of Unteraargletscher has been calculated in an indirect way using remote-sensing methods and without resorting to any ground measurements (apart from those of the bedrock geometry). This was achieved by estimating various terms of the kinematic boundary condition. Of the four terms, which must be estimated indirectly for this purpose, three terms could be determined with a sufficient accuracy using high-precision aerial photogrammetry. The fourth term, the distribution of vertical velocities along the surface, was estimated by assuming that the variation of vertical strain rates with depth is both simple and does not vary significantly over the area of the glacier. The resulting v_z distribution shows a number of features, some of which appear quite realistic, while others are evidently incorrect. Where a direct comparison can be made, the calculated mass balance values are comparable in range and in magnitude to those obtained from stake measurements. The distribution of vertical velocities as a function of altitude does, however, not comply with the expected variation of emerging velocities.

By analysing the v_z distribution, two different types of errors have been identified. At some locations, the horizontal strain rates at the surface are strongly affected by the local surface topography. Although the vertical strain-rate variation, at these locations, might possibly be rather simple across most of the glacier thickness, errors are introduced as the anomalous strain rates are integrated over the whole depth of the glacier. For the confluence area, it has previously been suggested, both from an analysis of borehole measurements and from theoretical considerations, that the assumption of a simple vertical strain-rate variation is incorrect. The unrealistic anomaly in calculated vertical surface velocities found at the confluence area gives additional support to these earlier claims.

A considerably more accurate estimate of the v_z distribution along the surface may be obtained by the use of a fully three-dimensional flow model. An existing flow model of this type, covering a limited section of the Unteraargletscher, is currently being extended to include all of the ablation area. Another approach, also currently being considered, is to use filter methods to suppress the effects of short-scale surface undulations on the v_z estimation.

Acknowledgements

We thank H. Bösch for his help with extracting information on the surface velocities of Unteraargletscher from aerial photographs. We would also like to thank the engineering firm Flotron AG, Meiringen, for making DTMs of Unteraargletscher available to us, and the hydrological power company KWO for lending us a number of aerial photographs. The Swiss Army provided invaluable logistic support. U. H. Fischer, B. Hanson and W. Harrison gave comments on an earlier version of this paper which led to considerable improvements of the manuscript. We thank P. Jansson for his interest in this project. This work was supported by the Swiss National Science Foundation grants Nr. 2100-047095.96/1 and Nr. 2000-053561.98/1.

G.H. Gudmundsson and A. Bauder, VAW, Gloriastrasse 37/39, ETH-Zentrum, CH-8092 Zürich, Switzerland.

References

Chandrasekharaiah, D.S. and Debnath, L., 1994: Continuum Mechanics. Academic Press. Inc. San Diego.

Chen, J. and Funk, M., 1990: Mass balance of Rhonegletscher during 1882/83–1986/87. *Journal of Glaciology*, 36(123): 199–209.

Echelmeyer, K.A., Harrison, W.D., Larsen, C.F., Sapiano, J., Mitchell, J.E., DeMallie, J. and Rabus, B., 1996: Airbome surface elevation measurements of glaciers: a case study in Alaska. *Journal of Glaciology*, 42(142):53 547.

Favey, E., Geiger, A., Gudmundsson, G.H. and Wehr, A., 1999: Evaluating the potential of an airborne laser-scanning system for measuring the volume change of glaciers. *Geografiska Annaler*, 81A(4): 555–561.

Funk, M., Morelli, R. and Stahel, W., 1997: Mass balance of Gries gletscher 1961–1994: different methods of determination. *Zeitschrift für Gletscherkunde und Glazialgeologie*, 33(1): 41–55.

Gudmundsson, G.H., 1997: Ice deformation at the confluence of two glaciers investigated with conceptual map-plain and flow-line models. *Journal of Glaciology*, 43(145):537–547.

— 1999a: Measurements of temporal variations in vertical and longitudinal strain rates associated with a mini-surge event. *Journal of Glaciology* (submitted).

— 1999b: A three-dimensional numerical model of the confluence area of Unteraargletscher, Bernese Alps, Switzerland. *Journal of Glaciology*, 45(149) (150):219–230.

Gudmundsson, G.H., Iken, A. and Funk, M., 1997: Measurements of ice deformation at the confluence area of Unteraargletscher, Bernese Alps, Switzerland. *Journal of Glaciology*, 43(145):54 556.

Hoinkes, H., 1970: Methoden und Möglichkeiten von Massenhaushaltsstudien auf Gletschern. *Zeitschrift für Gletscherkunde und Glazialgeologie*, 6(2):37–90.

Hutter, K., 1983: Theoretical glaciology; material science of ice and the mechanics of glaciers and ice sheets. D. Reidel. Tokyo.

Iken, A., Röthlisberger, H., Flotron, A. and Haeberli, W., 1983: The uplift of the Unteraargletscher at the beginning of the melt season – a consequence of water storage at the bed? *Journal of Glaciology*, 29(101):28–47.

Kääb, A., 1996: Photogrammetrische Analyse zur Früherkennunggletscher- und permafrostbedingter Naturgefahren im Hochgebirge. Mitteilung 145, Versuchsanstalt für Wasserbau, Hydrologie und Glaziologie der ETH Zürich, Gloriastrasse 37–39, ETH-Zentrum, CH-8092 Zürich. pp. 182.

Kääb, A. and Funk, M., 1999: Modelling mass balance using photogram metric and geophysical data. A pilot study at Gries glacier, Swiss Alps. *Journal of Glaciology.* Accepted for publication.

Kääb, A., Haeberli, W. and Gudmundsson, G.H., 1997: Analyzing the creep of mountain permafrost using high precision aerial photogrammetry: 25 years of monitoring Gruben rock glacier, Swiss Alps. *Permafrost and Periglacial Processes*, 8(4):409–426.

Meier, M.F., 1984: Contribution of small glaciers to global sea level. *Science*, 226(4681):1418–1421.

— 1990: Reduced rise in sea level. *Nature*, 343(6254):115–116.

Oerlemans, J. and Fortuin, J.P.F., 1992: Sensitivity of glaciers and small ice caps to greenhous warming. *Science*, 258:11 117.

Østrem, G. and Brugman, M., 1991: Glacier mass-balance measurements–a manual for field and office work. Technical report. National Hydrology Research Institute. Science Report No. 4.

Rasmussen, L.A., 1988: Bed topography and mass-balance distribution of Colombia Glacier, Alaska, U.S.A, determined from sequential aerial photography. *Journal of Glaciology*, 34(117): 208–216.

Reynaud, L., Vallon, M. and Letreguilly, A., 1986: Mass-balance measurements: problems and two new methods of determining variations. *Journal of Glaciology*, 32(112):46–454.

Sapiano, J., Harrison, W. and Echelmeyer, K., 1998: Elevation, volume and terminus changes of nine glaciers in North America. *Journal of Glaciology*, 44(146):119–135.

Schwitter, M.P. and Raymond, C.F., 1993: Changes in the longitudinal profiles of glaciers during advance and retreat. *Journal of Glaciology*, 39(133):582–590.

Manuscript received August 1998, revised and accepted March 1999.