1. Is it possible that an event is independent of itself? If so, when?

Yes, example is the event “getting head in a coin toss of an unbiased coin”. Let’s say we are tossing the coin twice, the probability of the 2nd toss being head is not affected by the probability of the 1st toss being head.

1. Is it always true that if A and B are independent events, then Ac and Bc are independent events? Show that it is, or give a counterexample.

Yes.

𝑃(𝐴𝑐∩𝐵𝑐)

=1−𝑃(𝐴∪𝐵)

=1−𝑃(𝐴)−𝑃(𝐵)+𝑃(𝐴∩𝐵)

=1−𝑃(𝐴)−𝑃(𝐵)+𝑃(𝐴)𝑃(𝐵) -> since A and M are independent events, 𝑃(𝐴∩𝐵) is just (𝐴)𝑃(𝐵)

=(1−𝑃(𝐴))(1−𝑃(𝐵))

=𝑃(𝐴𝑐)𝑃(𝐵𝑐)