



# Lenses

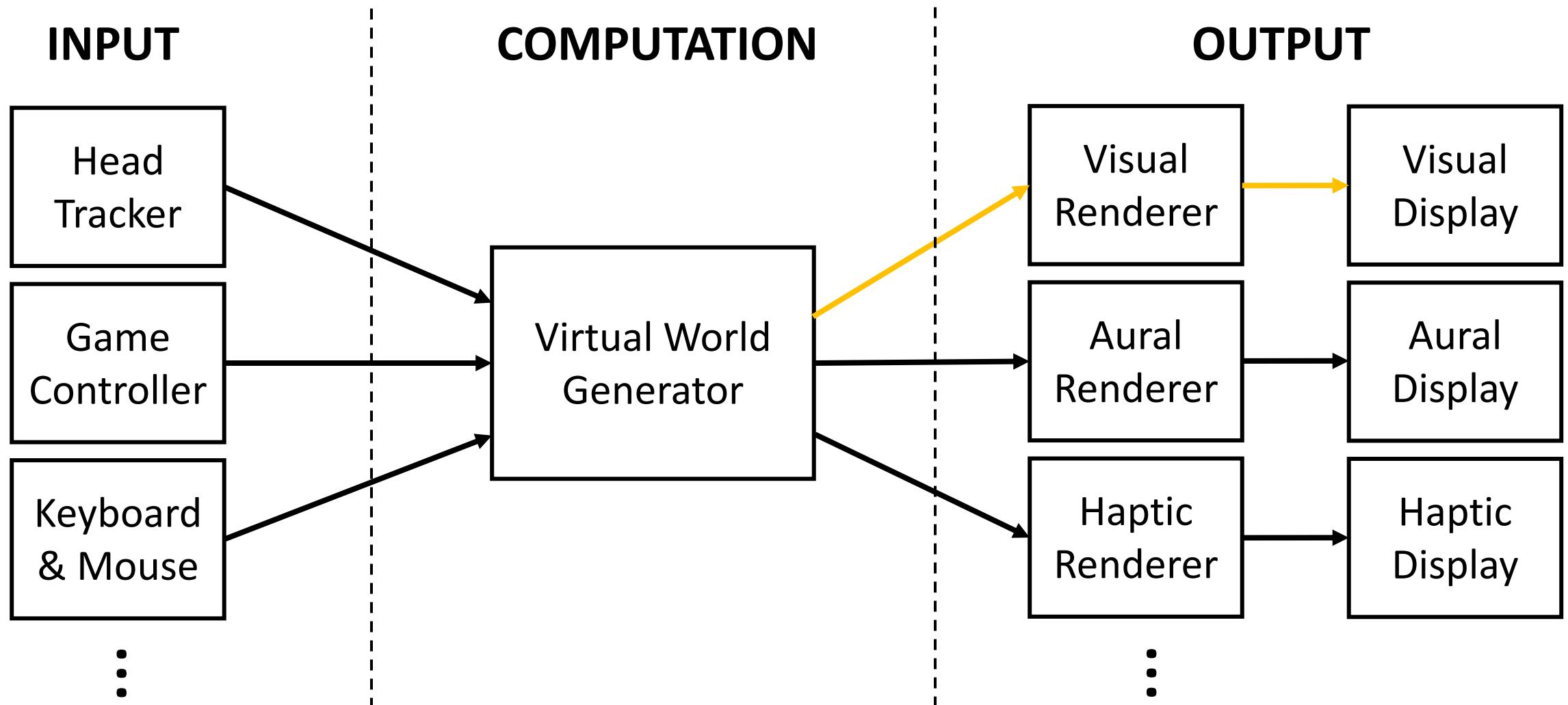
CS 6334 Virtual Reality

Professor Yapeng Tian

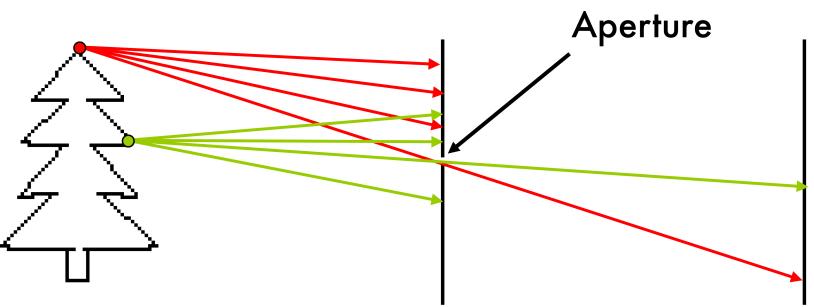
The University of Texas at Dallas

A lot of slides of course lectures borrowed from Professor Yu Xiang's VR class

# Review of VR Systems



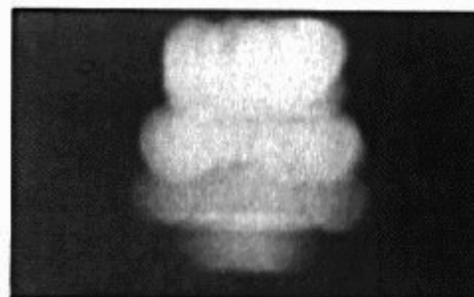
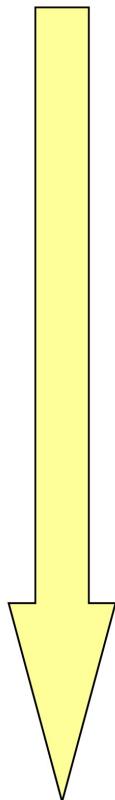
# Aperture Size of Pinhole Camera



Shrinking  
aperture  
size

What happen if the aperture is too small?

- Less light passes through
- Adding lenses



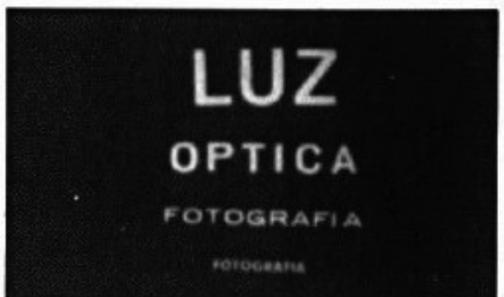
2 mm



1 mm

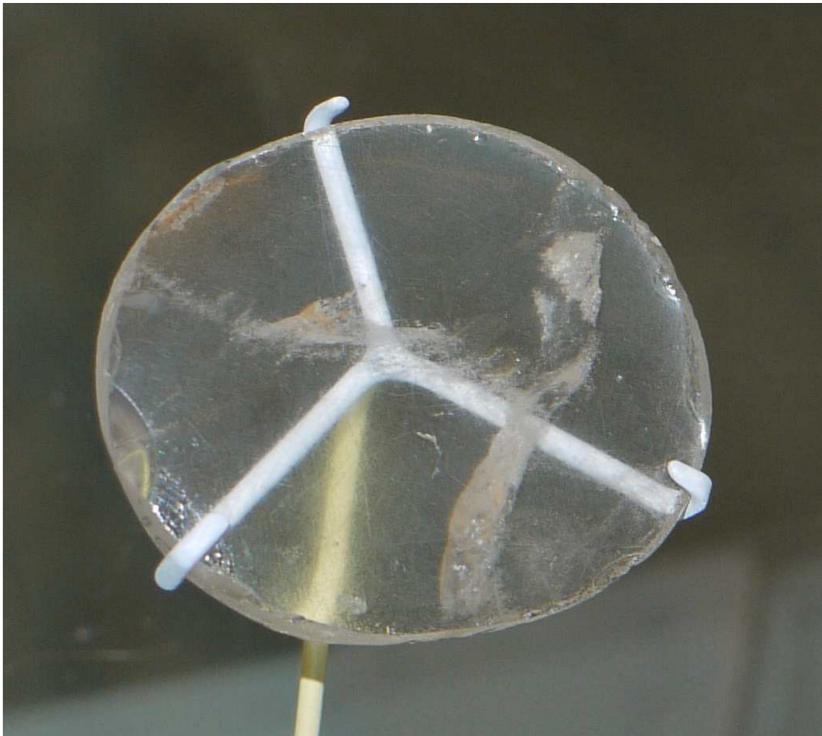


0.6mm

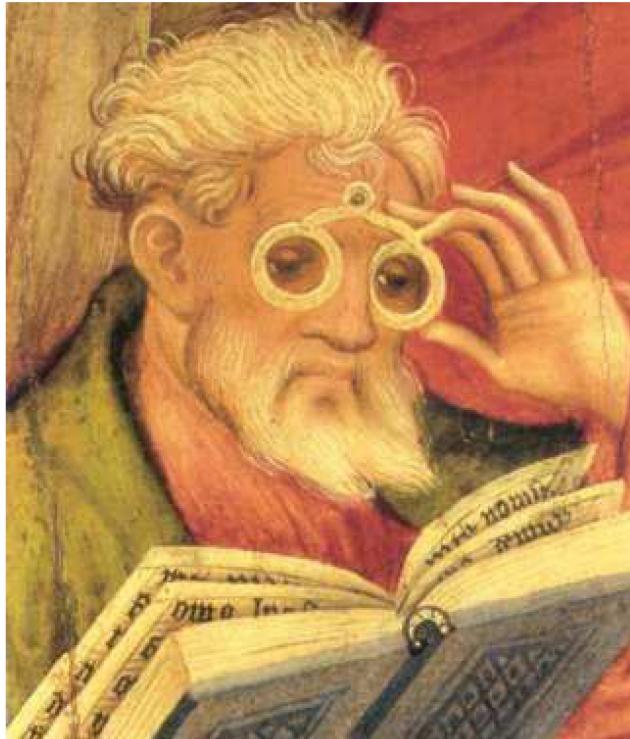


0.35 mm

# Lenses



(a)



(b)

Figure 4.8: (a) The earliest known artificially constructed lens, which was made between 750 and 710 BC in ancient Assyrian Nimrud. It is not known whether this artifact was purely ornamental or used to produce focused images. Picture from the British Museum. (b) A painting by Conrad von Soest from 1403, which shows the use of reading glasses for an elderly male.

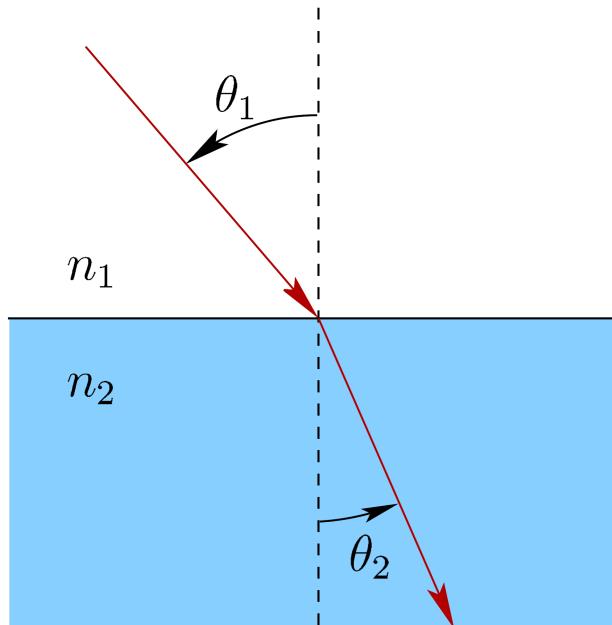
# Snell's Law

- How much rays of light bend when entering and exiting a transparent material

- Refractive index of a material  $n = \frac{c}{s}$
- Air  $n = 1.000293$ , water  $n = 1.33$
- Crown glass  $n = 1.523$

- Snell's Law

$$n_1 < n_2$$

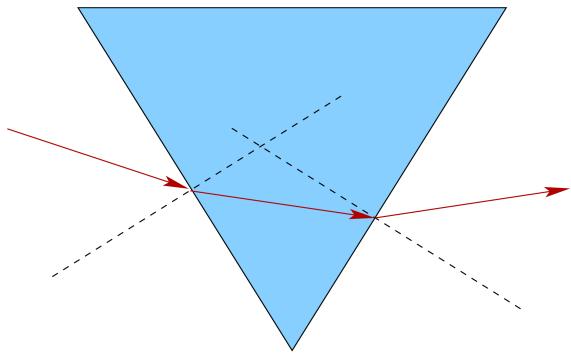
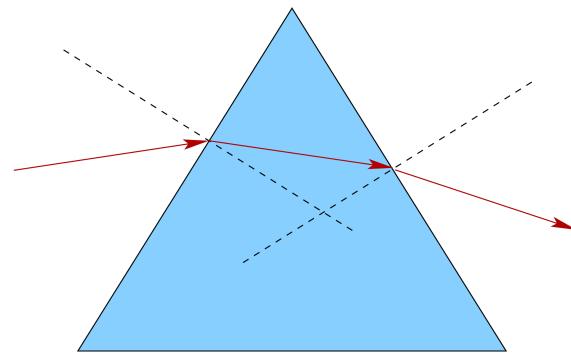


$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

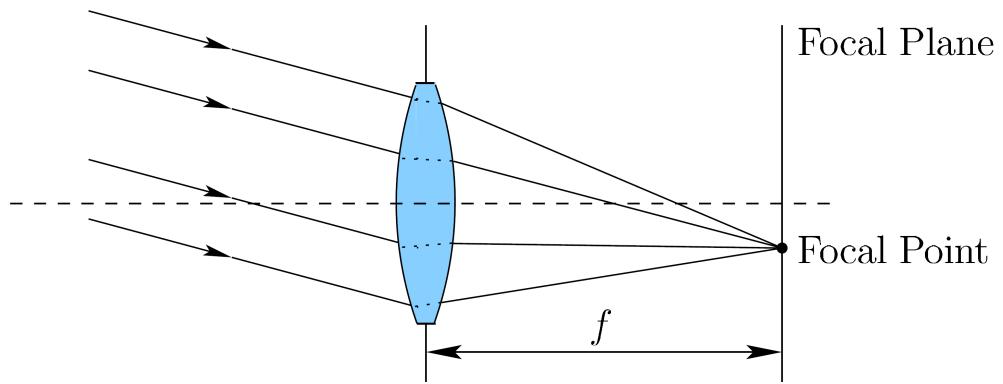
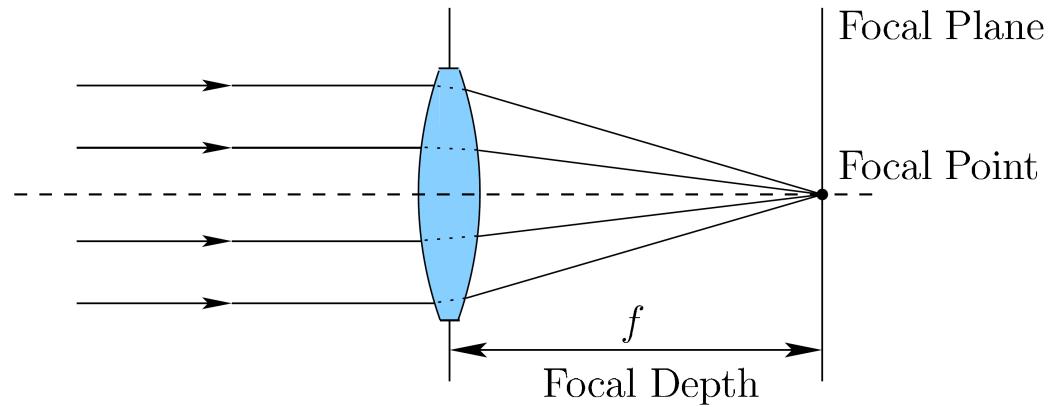
$$\theta_2 = \sin^{-1} \left( \frac{n_1 \sin \theta_1}{n_2} \right)$$

# Convex Lenses

- Prisms



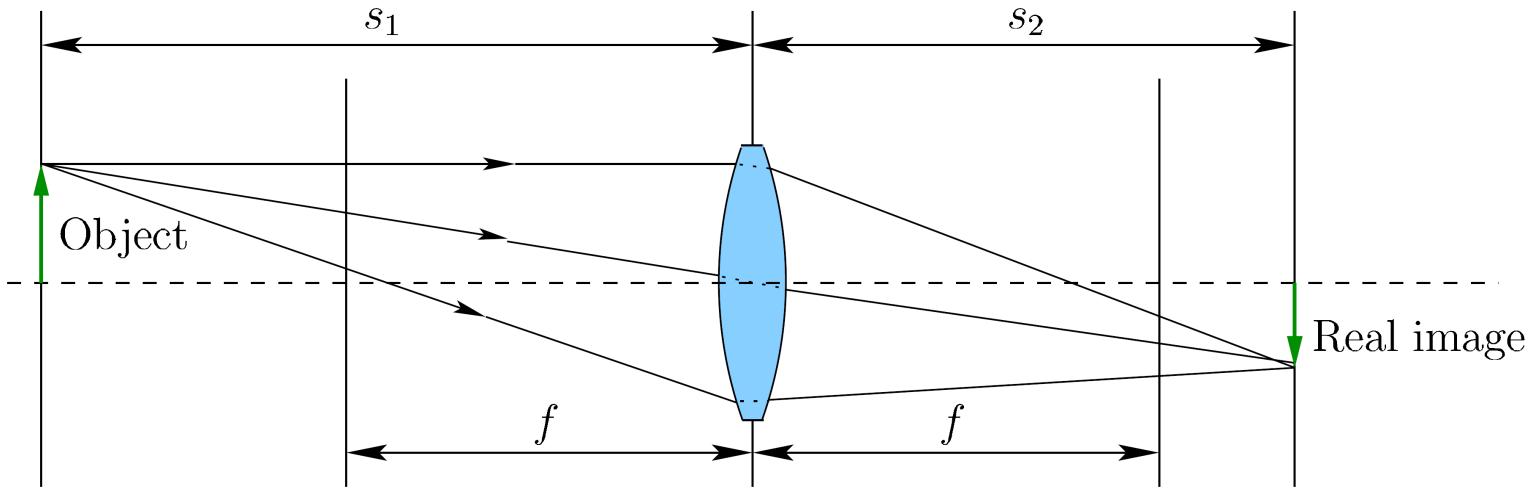
- A simple convex lens



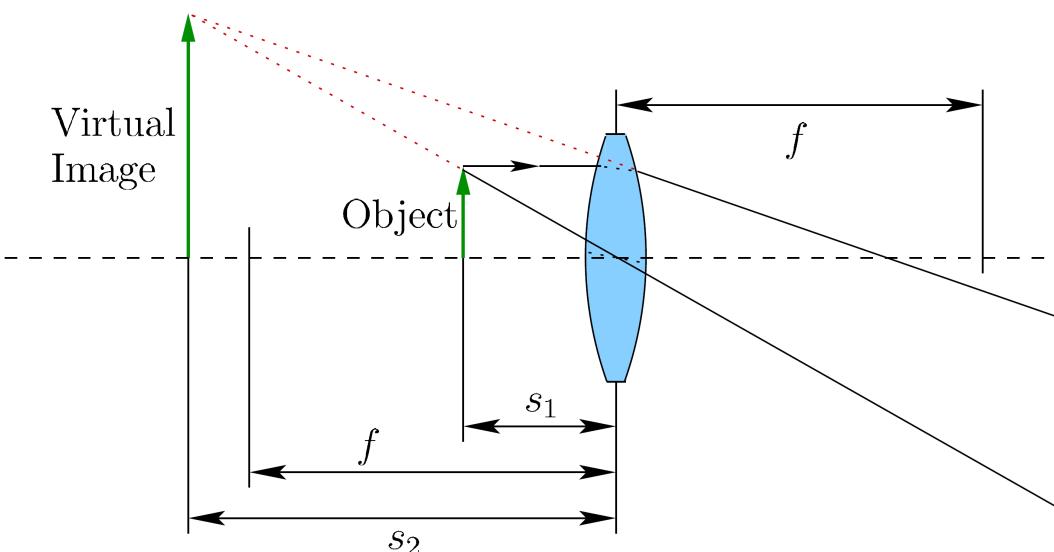
# Convex Lenses

- Objects in distance
  - Cameras

$$\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f}$$



- Objects very close
  - Magnification
  - VR headsets



# Controllable Aperture

- In the pinhole case, all depths are “in focus”, but there may not enough lights
- When using a convex lens, it focuses objects at a single depth

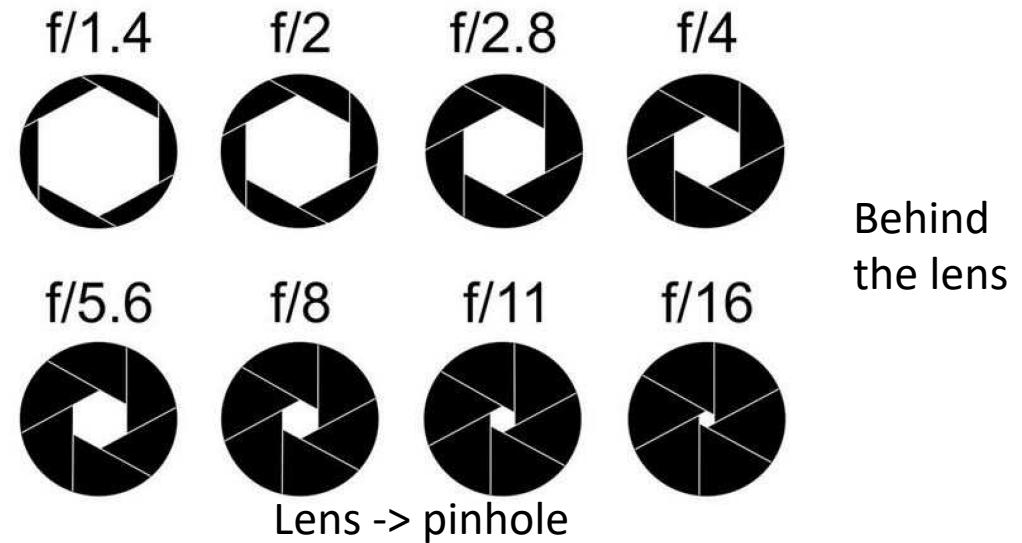
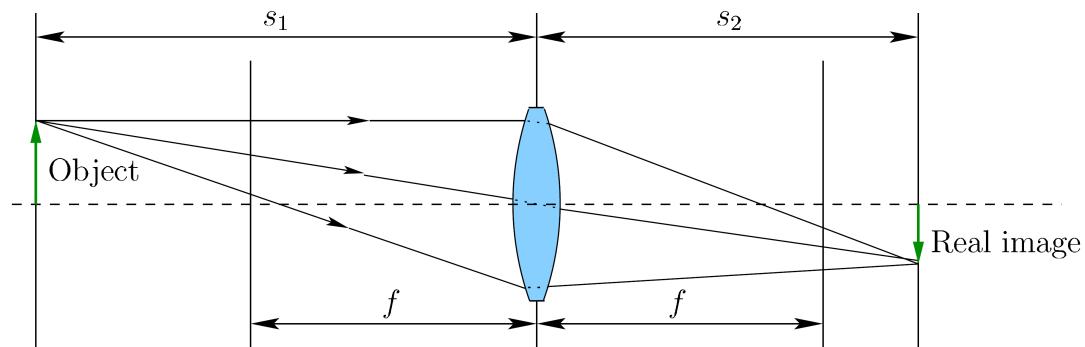


Figure 4.34: A spectrum of aperture settings, which control the amount of light that enters the lens. The values shown are called the *focal ratio* or *f-stop*.

# Controllable Aperture



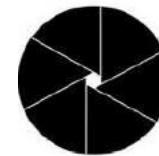
f/1.4



f/4



f/16



# Chromatic Aberration

- The speed of light through a medium depends on the wavelength
  - Solution: combining convex and concave lenses of different materials

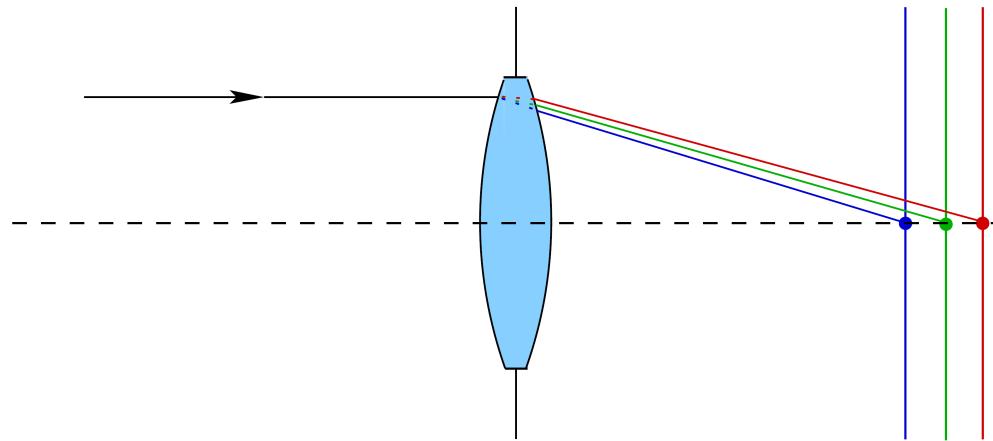


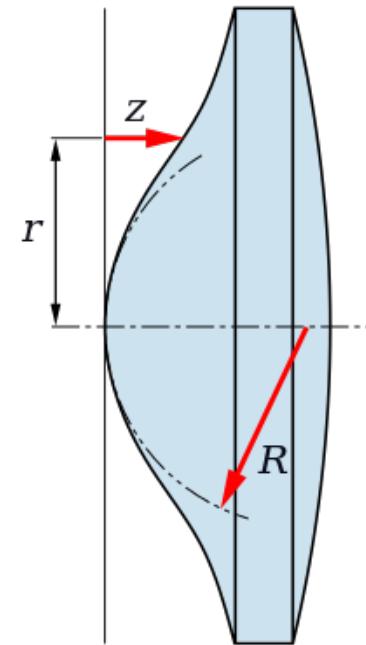
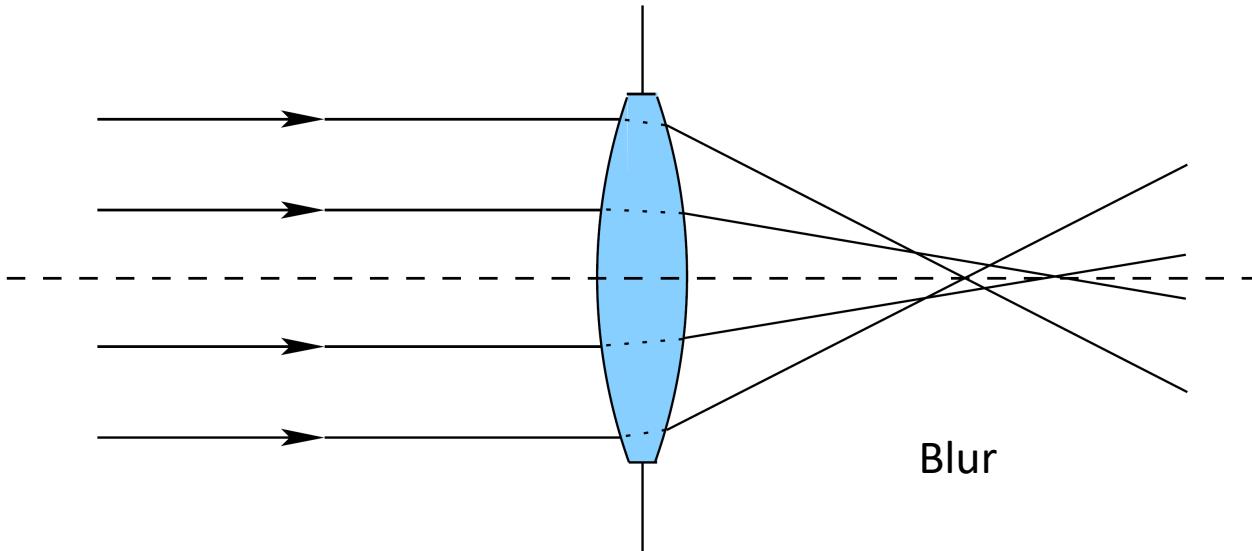
Figure 4.17: Chromatic aberration is caused by longer wavelengths traveling quickly through the lens. The unfortunate result is a different focal plane for different wavelengths or colors.



Figure 4.18: The upper image is properly focused whereas the lower image suffers from chromatic aberration. (Figure by Stan Zurek, license CC-BY-SA-2.5.)

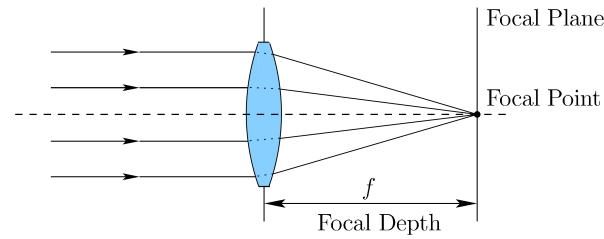
# Spherical Aberration

- Rays further away from the lens center being refracted more than rays near the center

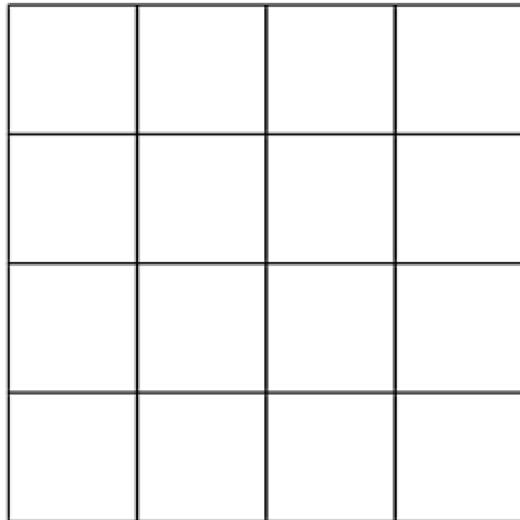


Aspheric lens

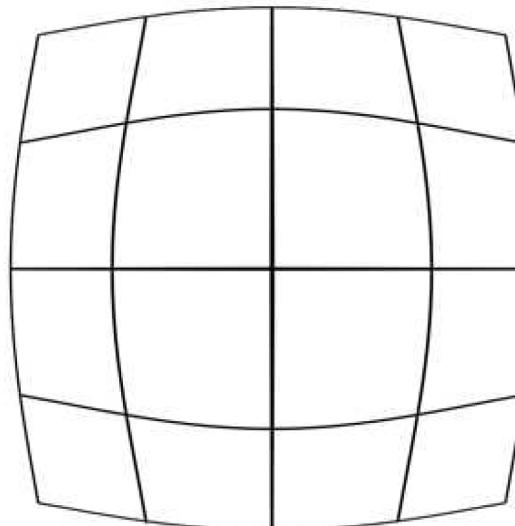
# Optical Distortion



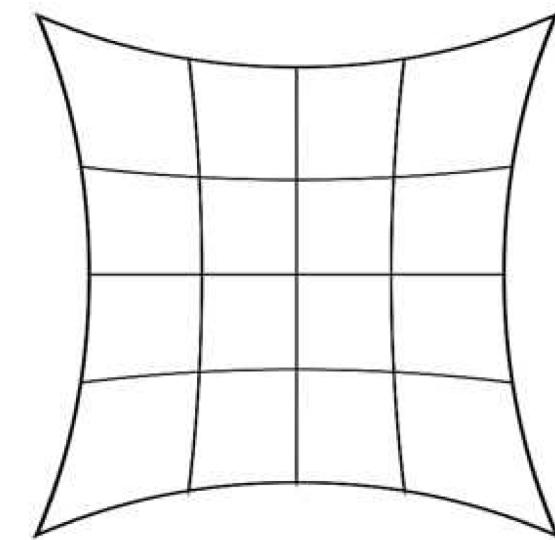
- The variation of refractive index towards the outer extremities of a rotational symmetric lens can cause magnification changes in the image space, depending on the distance from the principal axis.



No distortion



Barrel distortion  
(wide-angle lenses)



pincushion distortion  
(telephoto-lenses)

# Barrel Distortion of Fisheye Cameras



Figure 4.21: An image with barrel distortion, taken by a fish-eyed lens. (Image by Wikipedia user Ilveon.)

# Summary: Camera Models

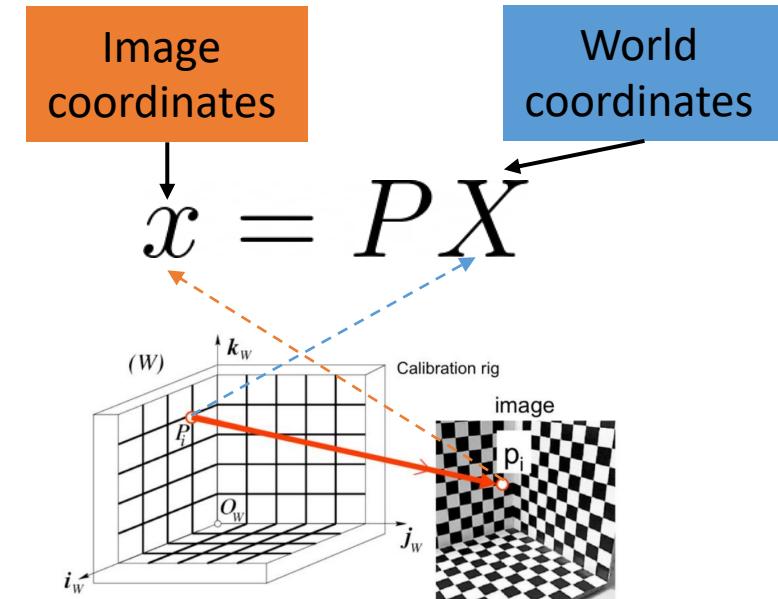
- Camera projection matrix: intrinsics and extrinsics

$$P = K[R|\mathbf{t}]$$

- Lens distortion
  - Radial distortion coefficients  $K_1, K_2, K_3, \dots$
  - Tangential distortion coefficients  $P_1, P_2, P_3, \dots$

# Camera Calibration

- Find the intrinsics, extrinsics and lens distortion coefficients of a camera
- Chess board camera calibration with OpenCV  
[https://docs.opencv.org/3.4/dc/dbb/tutorial\\_py\\_calibration.html](https://docs.opencv.org/3.4/dc/dbb/tutorial_py_calibration.html)



# Further Reading

- Section 4.2, 4.3, Virtual Reality, Steven LaValle
- Image formation by lenses  
<https://courses.lumenlearning.com/physics/chapter/25-6-image-formation-by-lenses/>
- Distortion (Wikipedia)  
[https://en.wikipedia.org/wiki/Distortion\\_\(optics\)](https://en.wikipedia.org/wiki/Distortion_(optics))