



THE UNIVERSITY OF TEXAS AT DALLAS

Image Processing: Filtering I

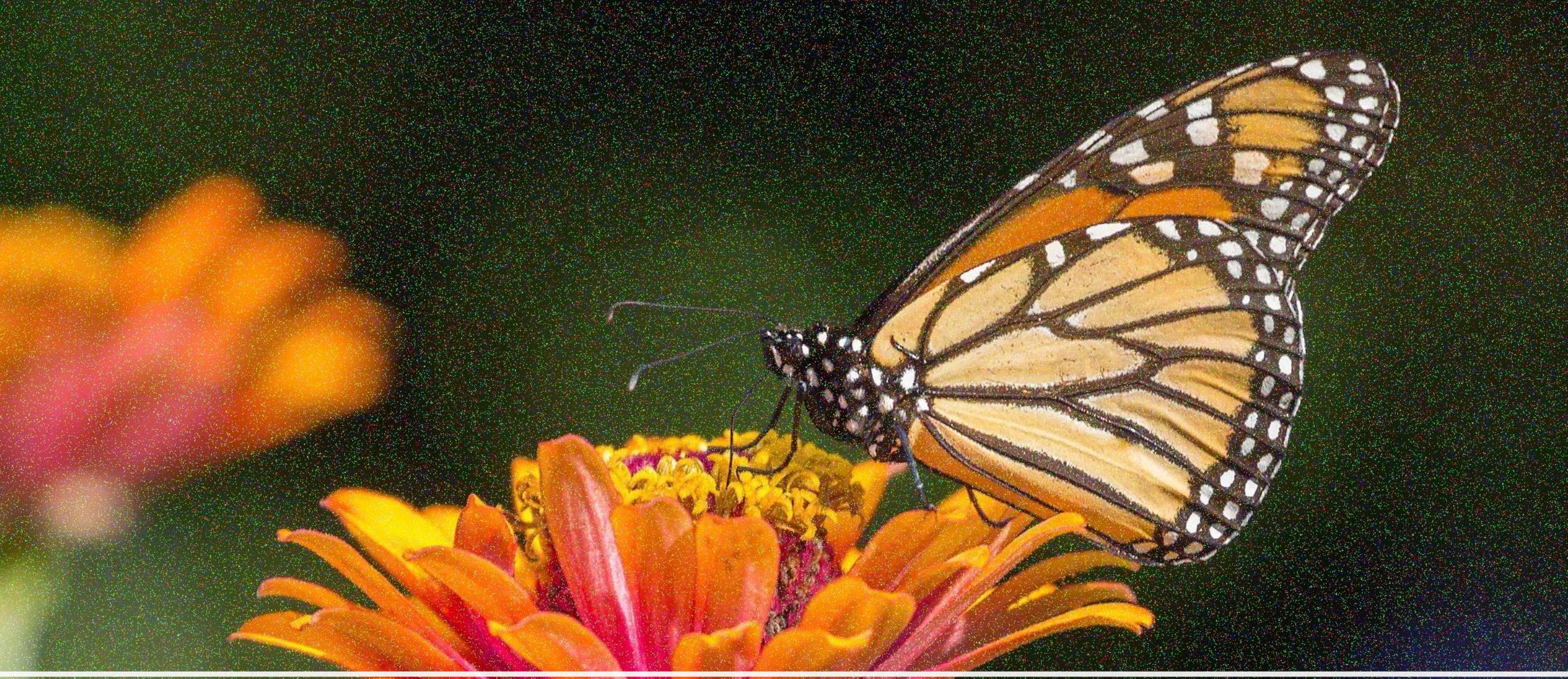
CS 6384 Introduction to Computer Vision

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Department of Computer Science



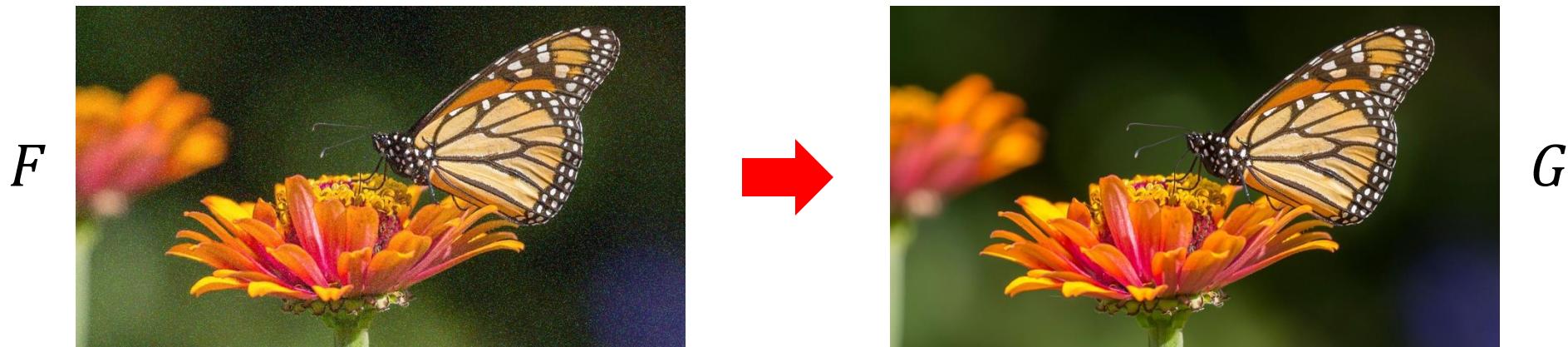




Question: How to reduce noises in an image?

Image Filtering

- Goal: generate a new image G whose pixel values are a combination of the original pixel values F
 - Enhance image quality (e.g., denoising, sharpening)
 - Extract visual features (e.g., edges, contours)
 - Basic computation unit in convolutional neural networks



Noise Reduction as An Example

How was the noisy image generated?



$$F[i, j, c] = I[i, j, c] + n[i, j, c]$$

i : row, *j*:column, *c*:color, *n*: additive noise

A close-up photograph of a monarch butterfly resting on a sunflower. The butterfly's wings are spread, showing its characteristic orange, black, and white patterns. It is positioned on a sunflower with large, yellow petals. The background is dark and appears to be a field of flowers, with some blurred colors visible.

How to remove the noise n from the noisy image ?

Characteristics of Noises and Natural Images

Image noises:

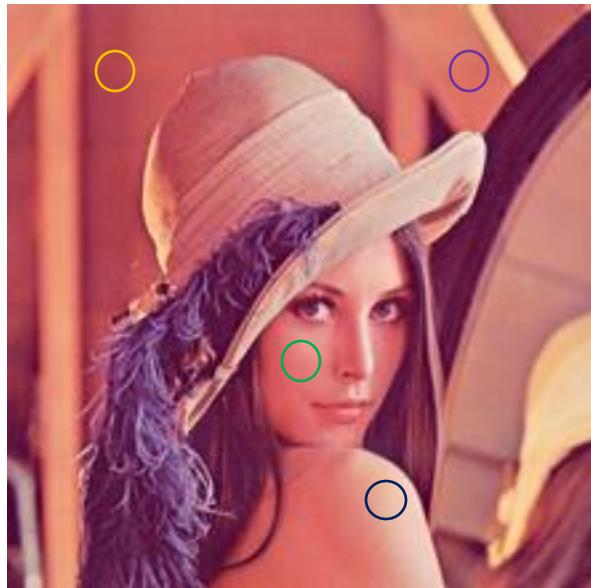
- Random and characterized by high frequency components
- Fewer details or finer textures

Natural images:

- Both low and high frequencies that are more evenly distributed
- More textures, patterns, and shapes with gradual changes in intensity or color

Image Prior: Local Smoothness

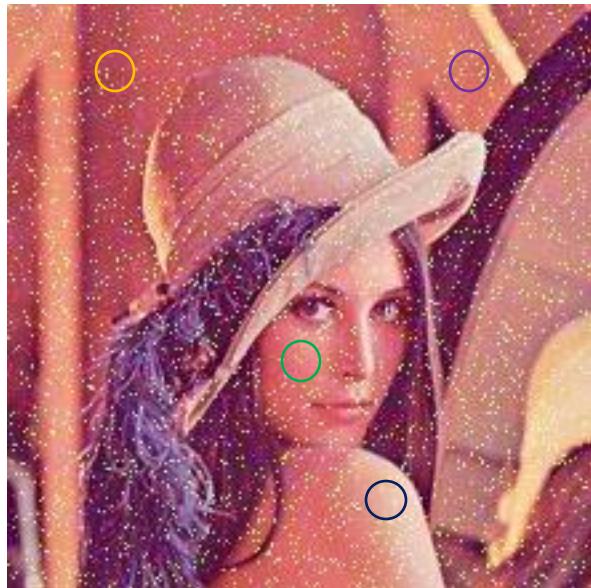
- Local natural image regions are typically smooth or uniform
- The overall structures or texture of a natural image often has a more subtle and gradual variation than image noise



- Image pixels in a small window (e.g., 5x5) usually are similar
- Noise values are dramatically changing at arbitrary directions

Image Prior: Local Smoothness

- Local natural image regions are typically smooth or uniform
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- Image pixels in a small window (e.g., 5x5) usually are similar
- Noise values are dramatically changing at arbitrary directions
- Due to noises, a noisy image have higher local variations than the clean image

Image Filtering for Noise Reduction

Reduce noises by enforcing local smoothness prior

- Make each pixel in a noisy image to be similar to its local neighborhoods
- How? There are many local neighborhoods (e.g., 9 in a 3x3 window)
 - A naïve method: replace each pixel value with the mean value of its local neighborhoods

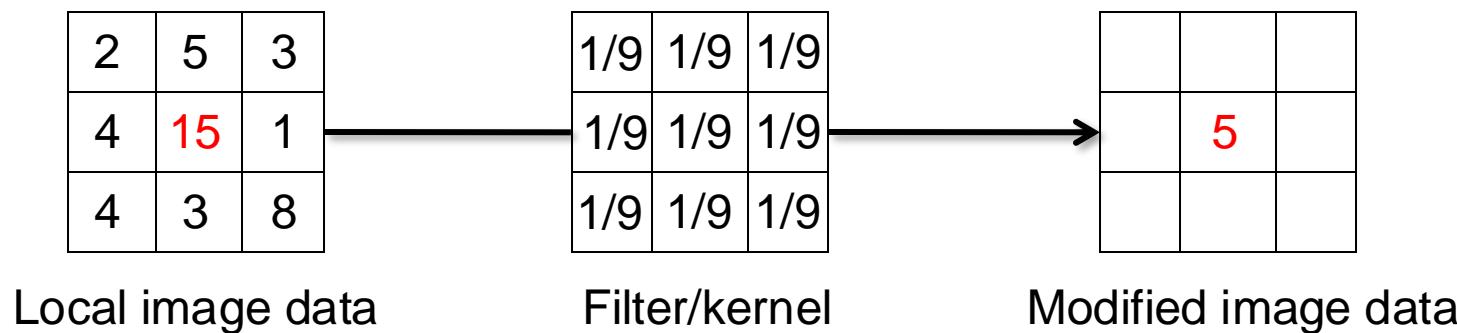


Image Filtering Process

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

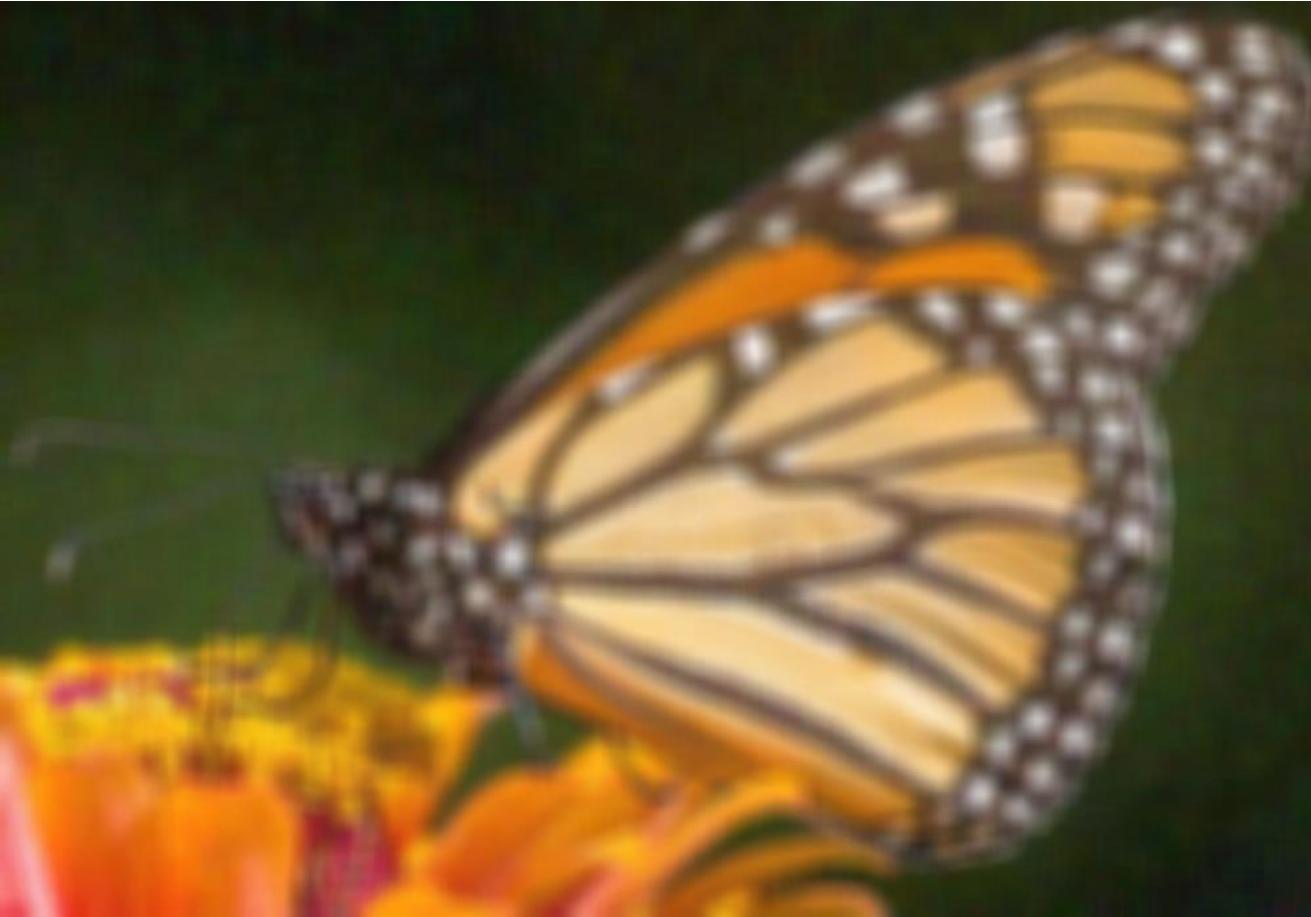


Noisy Image

Apply the filter to every pixel

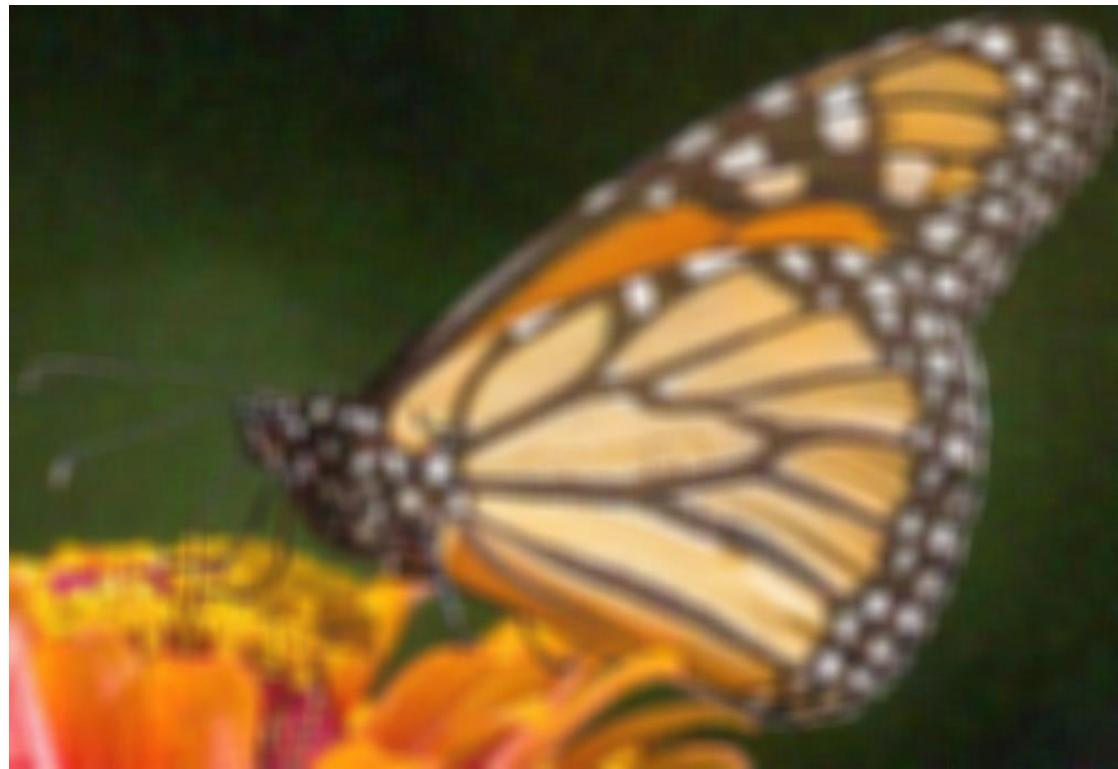
Image Filtering Process

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9



Filtered Image

Apply the filter to every pixel



Filtered Image



Noisy Image

Image Filtering

Modify the pixels in an image based on some function of a local neighborhood of each pixel

10	5	3
4	5	1
1	1	7

Local image data

Some function



	7	

Modified image data

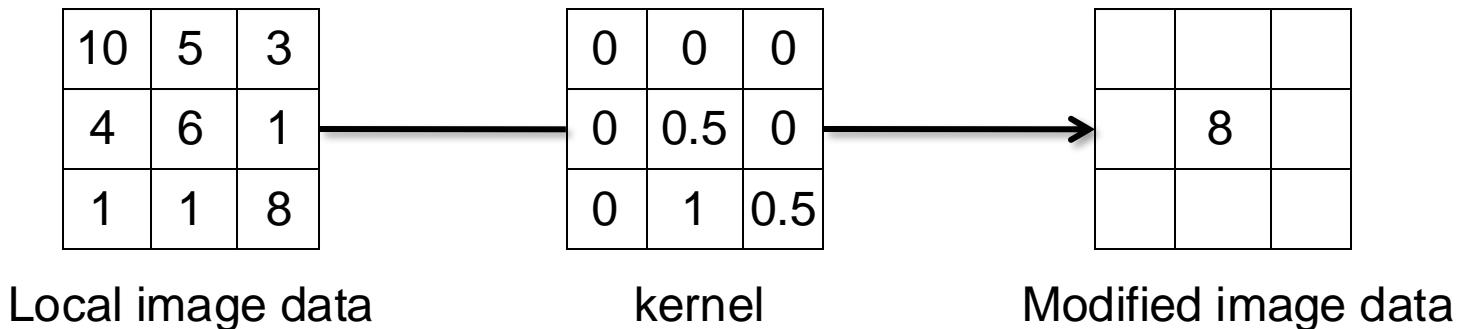
Slide credit: N. Snavely

Linear filtering

A simple filtering: linear filtering (cross-correlation/convolution)

- Replace each pixel by a linear combination (a weighted sum) of its neighbors

The prescription for the linear combination is called the “kernel” (or “mask”, “filter”)



Slide credit: N. Snavely

Cross-correlation

Let F be the image, H be the kernel (of size $2k+1 \times 2k+1$), and G be the output image

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v] F[i + u, j + v]$$

This is called a **cross-correlation**

operation:

$$G = H \otimes F$$

Can think of as a “dot product” between local neighborhood and kernel for each pixel

Slide credit: N. Snavely

Convolution

Same as cross-correlation, except that the kernel is “flipped”
(horizontally and vertically)

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k H[u, v]F[i - u, j - v]$$

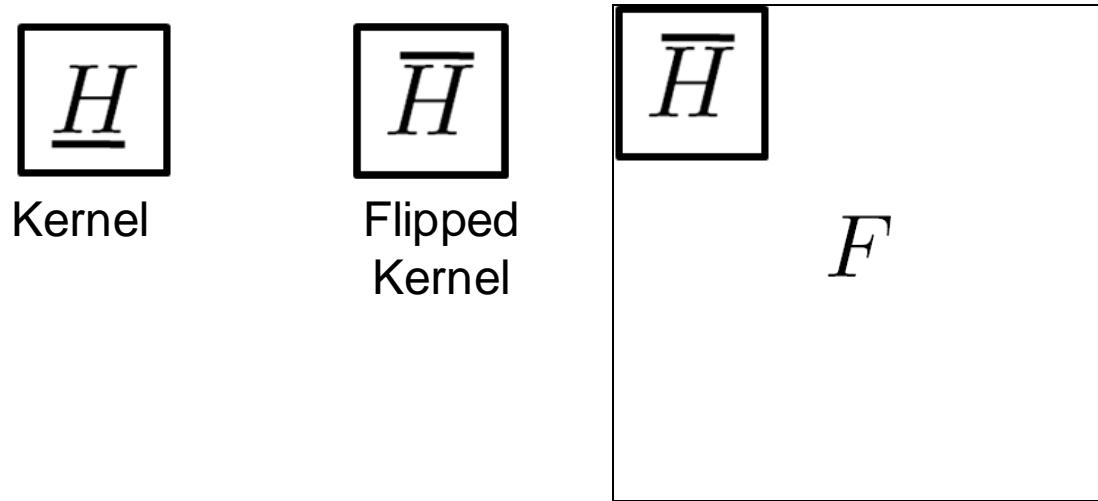
This is called a **convolution** operation:

Convolution is **commutative** and **associative**

$$G = H * F$$

Slide credit: N. Snavely

Convolution



Adapted from F. Durand

Mean filtering

$$\begin{matrix} \begin{matrix} & & \\ & & \\ & & \end{matrix} & * & \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\ 0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\ 0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\ 0 & 0 & 0 & 90 & 90 & 0 & 90 & 90 & 90 & 0 & 0 \\ 0 & 0 & 0 & 90 & 90 & 90 & 90 & 90 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 90 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & = & \begin{matrix} & & & & & & & \\ 0 & 10 & 20 & 30 & 30 & 30 & 20 & 10 \\ 0 & 20 & 40 & 60 & 60 & 60 & 40 & 20 \\ 0 & 30 & 60 & 90 & 90 & 90 & 60 & 30 \\ 0 & 30 & 50 & 80 & 80 & 90 & 60 & 30 \\ 0 & 30 & 50 & 80 & 80 & 90 & 60 & 30 \\ 0 & 20 & 30 & 50 & 50 & 60 & 40 & 20 \\ 10 & 20 & 30 & 30 & 30 & 30 & 20 & 10 \\ 10 & 10 & 10 & 0 & 0 & 0 & 0 & 0 \end{matrix} \end{matrix}$$

H

F

G

The diagram illustrates the mean filtering process. It shows three matrices: H , F , and G . Matrix H is a 3x3 kernel of zeros. Matrix F is a 10x10 input image with values ranging from 0 to 90. A 3x3 submatrix in F (rows 7-9, columns 4-6) is highlighted with a red box. Matrix G is the result of applying the mean filter from H to F . The value at position (7, 5) in G is 20, which is the average of the values in the red box of F .

Slide credit: N. Snavely

Mean filtering/Moving average

$F[x, y]$

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

$G[x, y]$

Slide credit: N. Snavely

Mean filtering/Moving average

$F[x, y]$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$G[x, y]$

0	10									

Slide credit: N. Snavely

Mean filtering/Moving average

$F[x, y]$

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

$G[x, y]$

0	10	20									

Slide credit: N. Snavely

Mean filtering/Moving average

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

			0	10	20	30			

Slide credit: N. Snavely

Mean filtering/Moving average

$$F[x, y]$$

$$G[x, y]$$

Slide credit: N. Snavely

Mean filtering/Moving average

$F[x, y]$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30	30	20	10		
	0	20	40	60	60	60	40	20		
	0	30	60	90	90	90	60	30		
	0	30	50	80	80	90	60	30		
	0	30	50	80	80	90	60	30		
	0	20	30	50	50	60	40	20		
	10	20	30	30	30	30	20	10		
	10	10	10	0	0	0	0	0		

Slide credit: N. Snavely

Linear filters: examples



*

0	0	0
0	1	0
0	0	0

Original

Source: D. Lowe

Linear filters: examples



*

0	0	0
0	1	0
0	0	0

=



Original

Identical image

Source: D. Lowe

Linear filters: examples

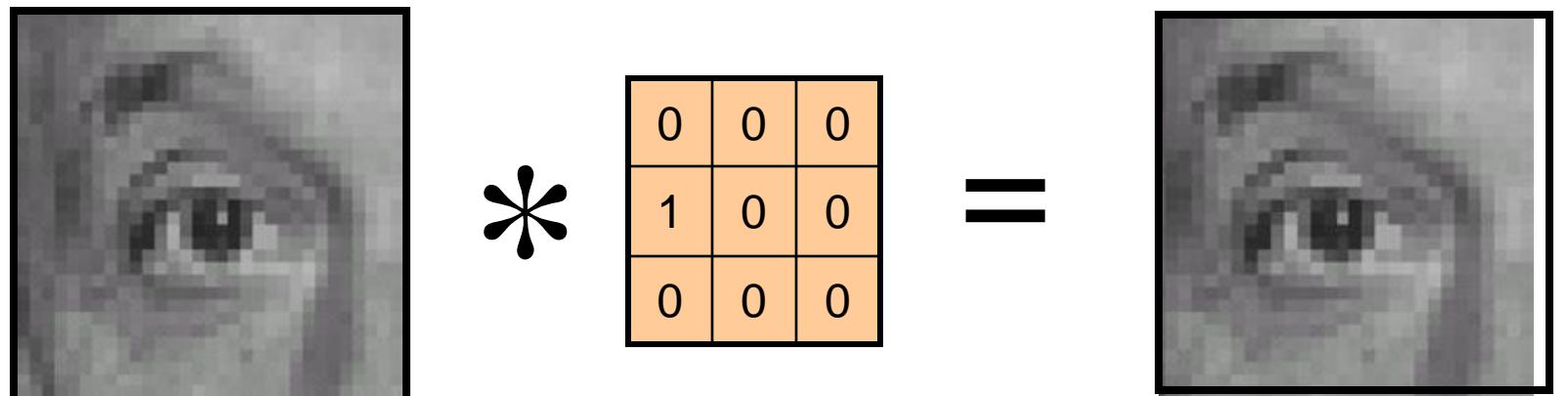


*

0	0	0
1	0	0
0	0	0

Source: D. Lowe

Linear filters: examples

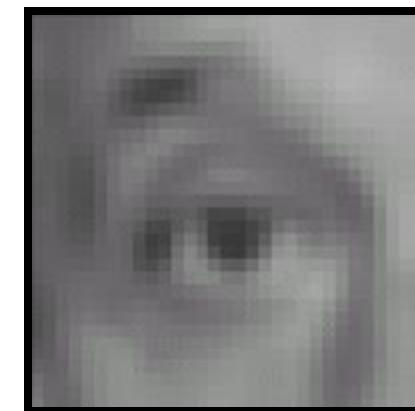

$$\text{Original} \quad * \quad \begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 1 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} = \text{Shifted left by 1 pixel}$$

Source: D. Lowe

Linear filters: examples



$$\ast \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} =$$



Original

Blur (with a mean/box filter)

Source: D. Lowe

Linear filters: examples

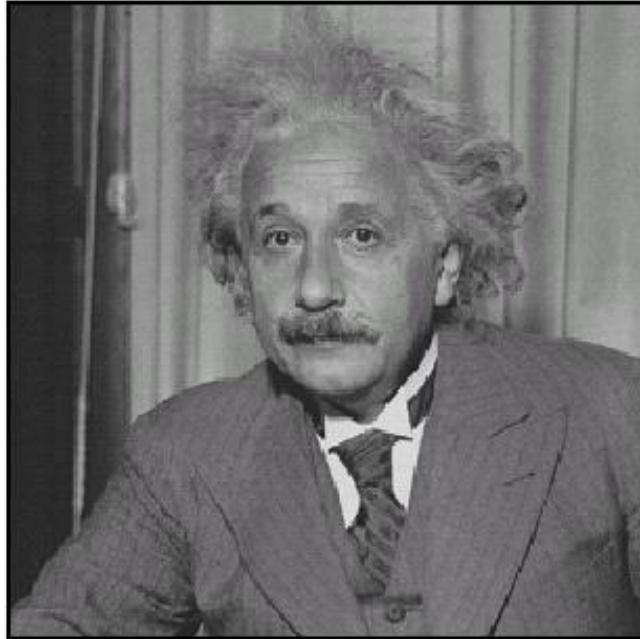
Original

$\ast \left(\begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 2 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} \right) - \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \right) =$

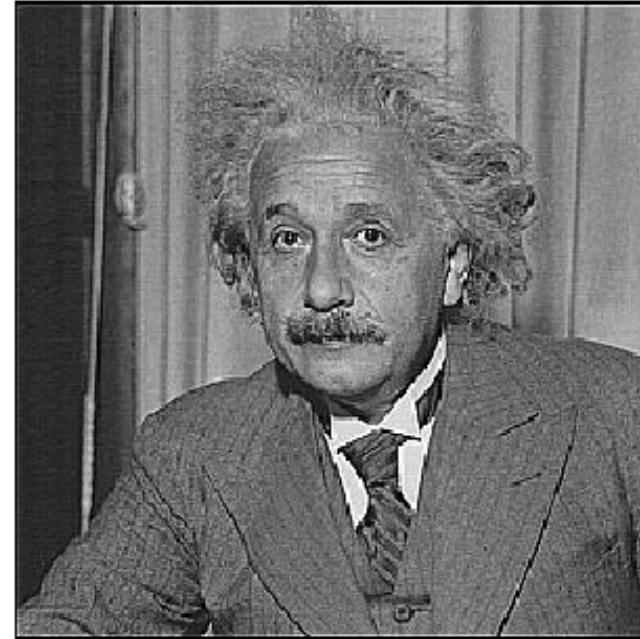
Sharpening filter

Source: D. Lowe

Sharpening



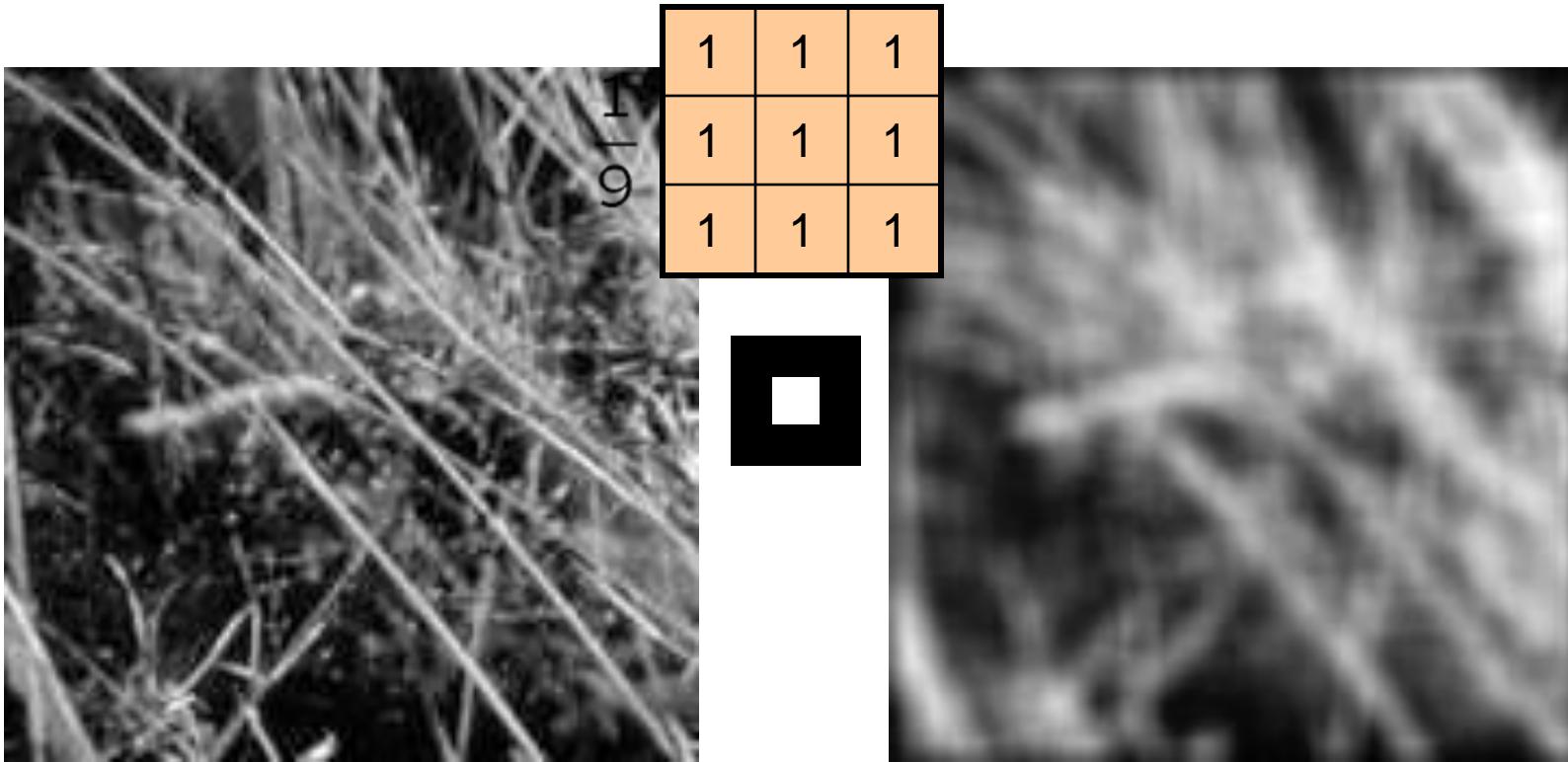
before



after

Source: D. Lowe

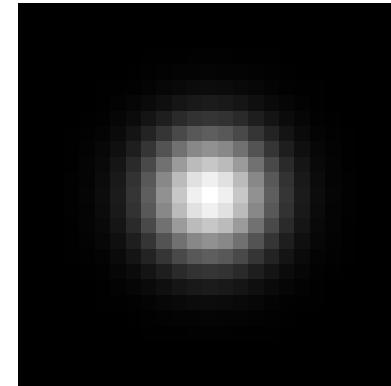
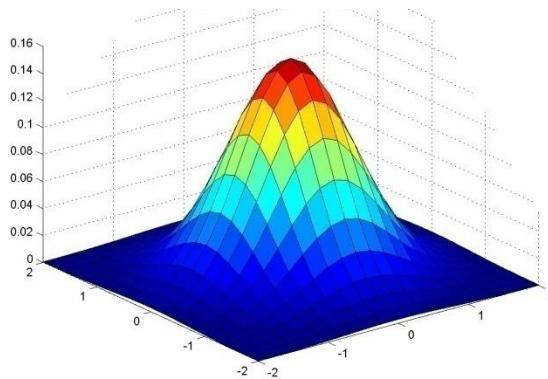
Smoothing with mean filter revisited



Block artifacts appear in the outputted image because non-relevant pixels are assigned the same weights during filtering

Source: D. Forsyth

Gaussian kernel

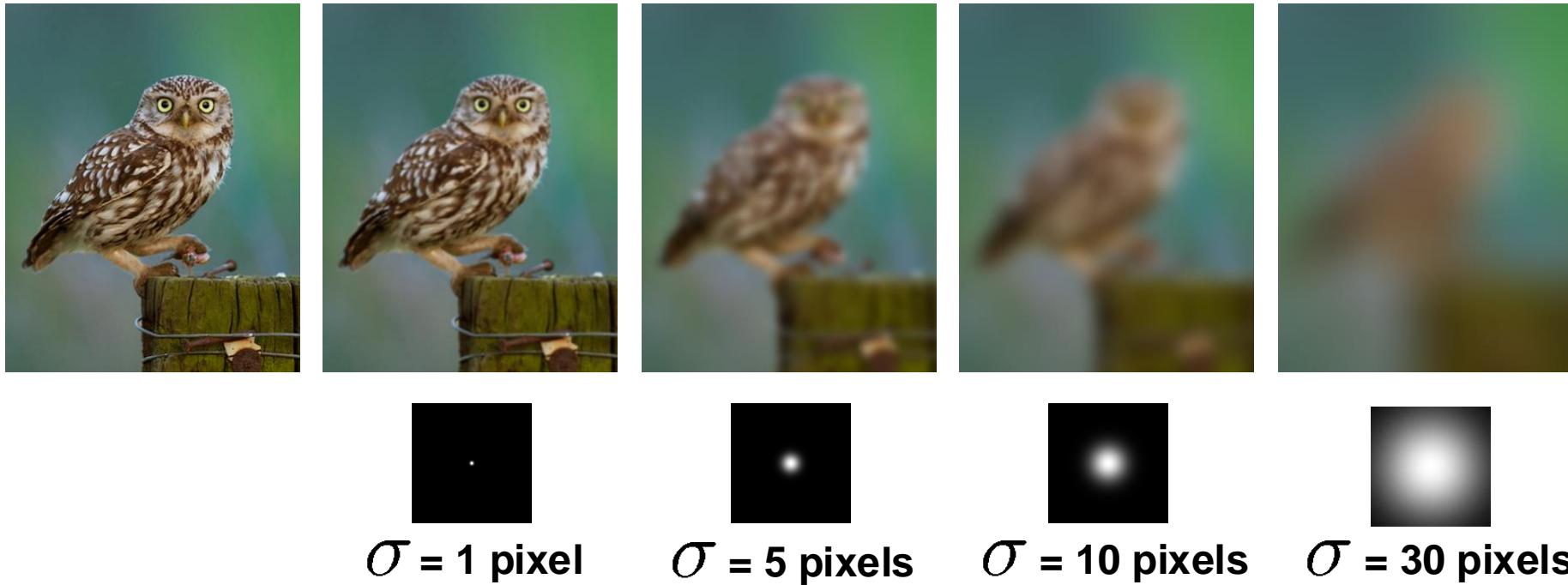


$$G_{\sigma} = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

- If a neighboring pixel is closer to the current pixel, it will be assigned a larger weight
- The σ controls the width of the kernel

Source: C. Rasmussen

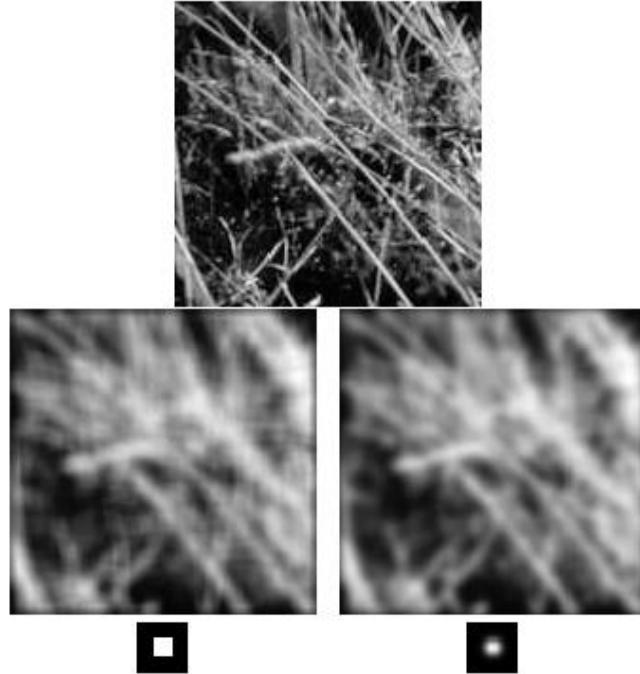
Gaussian filters



A Gaussian filter with a larger σ will produce a more blurred image

Slide credit: N. Snavely

Mean vs. Gaussian filtering



Both mean and Gaussian utilize local smoothness prior

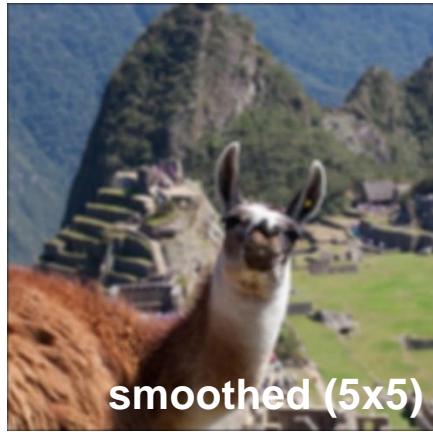
- Mean filter assumes all pixels in a local window are equally important
- Gaussian filter assumes pixels that are closer to the target pixel are more important

Source: N. Snavely

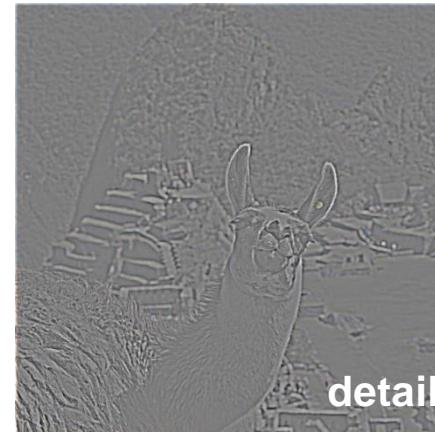
Sharpening revisited: What does blurring take away?



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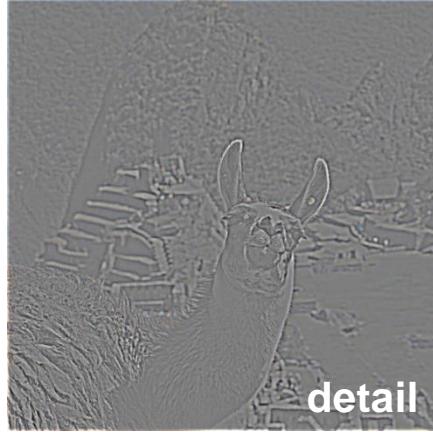
detail

(This “detail extraction” operation is also called a ***high-pass filter***)

Let's add it back:



+ α



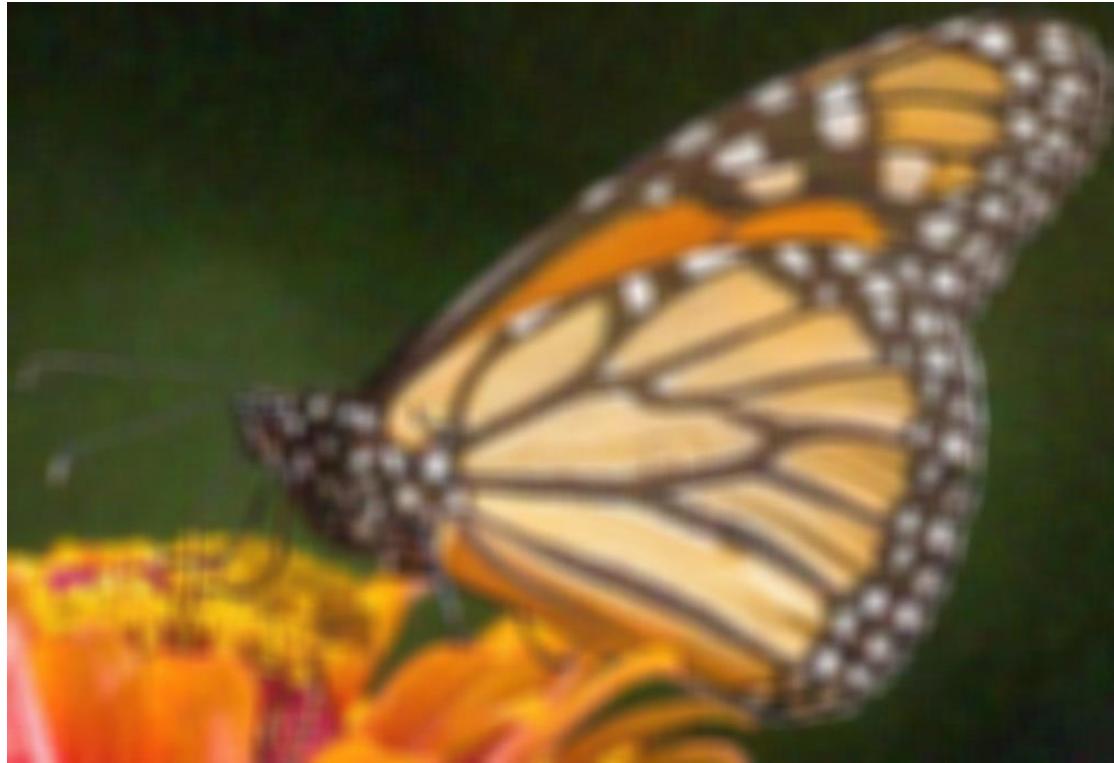
=



sharpened

Slide credit: N. Snavely

Filtered Image



Noisy Image



Question: How to handle blurry artifacts and preserve high-frequency details in the filtered image?

Nonlocal Means Filtering



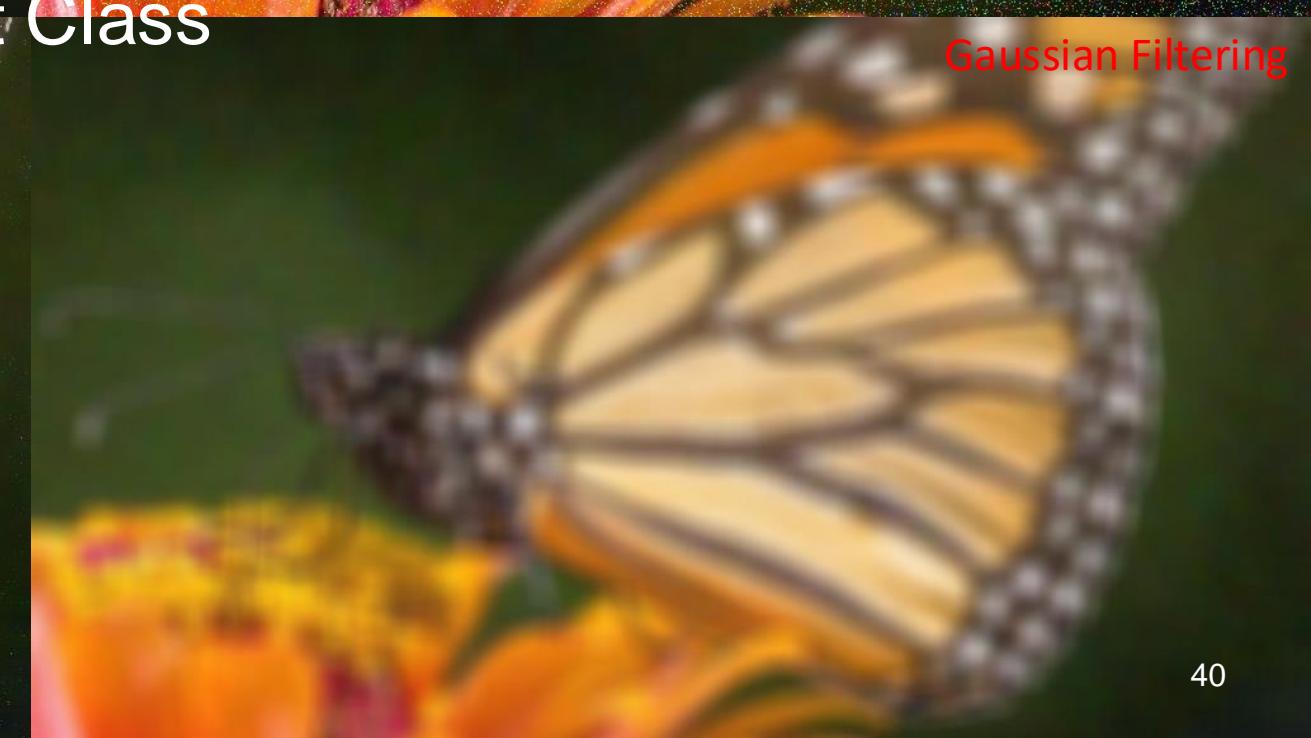
Bilateral Filtering



Next Class



Noisy Image



Gaussian Filtering

Further Reading

Chapters 3.1 and 3.2, Computer Vision: Algorithms and Applications,
Richard Szeliski