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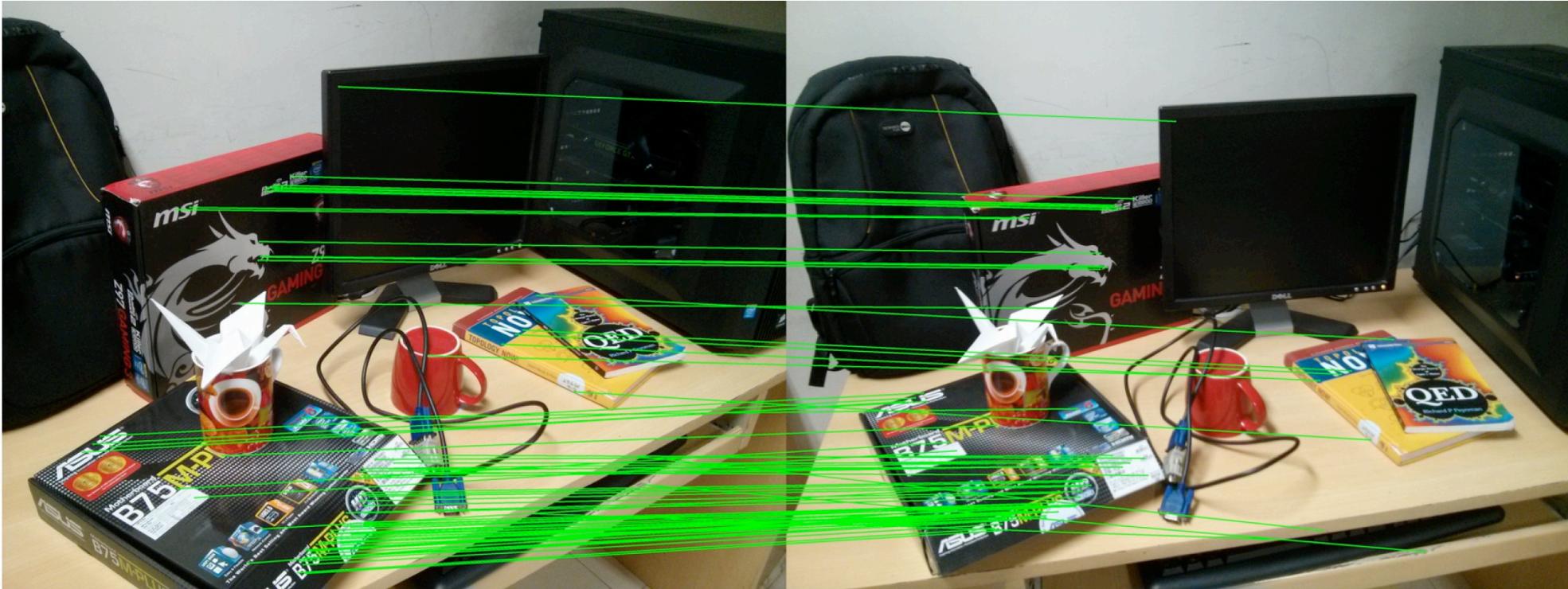
# Feature Detection and Matching: Detectors and Descriptors II

CS 6384 Computer Vision

Professor Yapeng Tian

Department of Computer Science

# Feature Detection and Matching

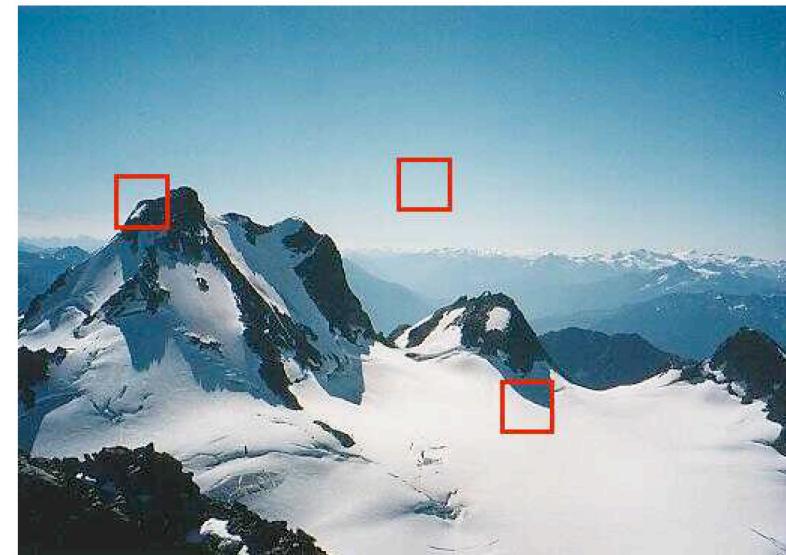
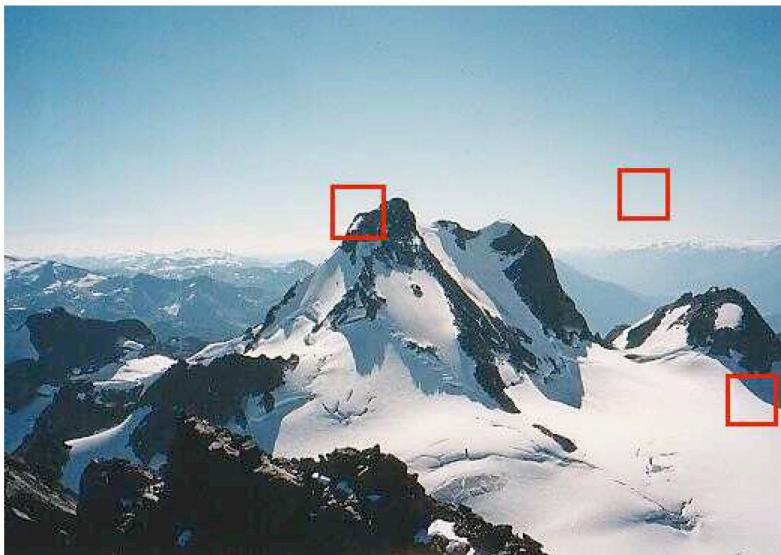


Geometry-aware Feature Matching for Structure from Motion Applications. Shah et al, WACV'15

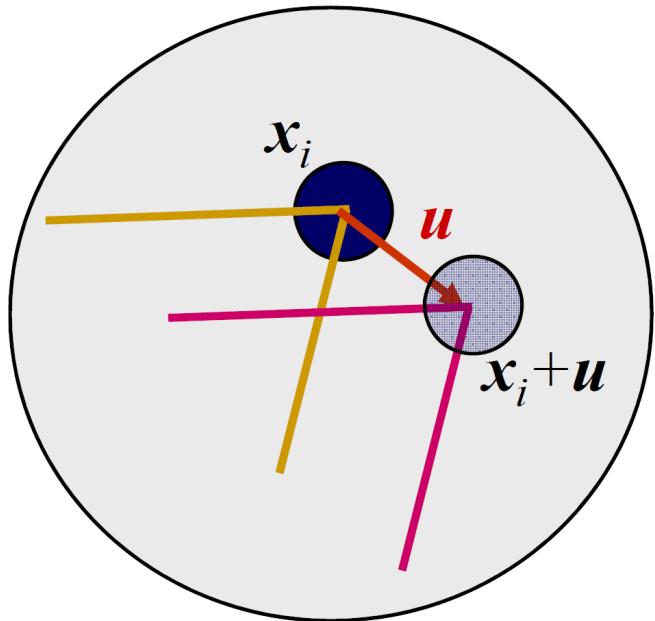
Applications: stereo matching, image stitching, 3D reconstruction, camera pose estimation, object recognition

# Feature Detectors

How to find image locations that can be reliably matched with images?

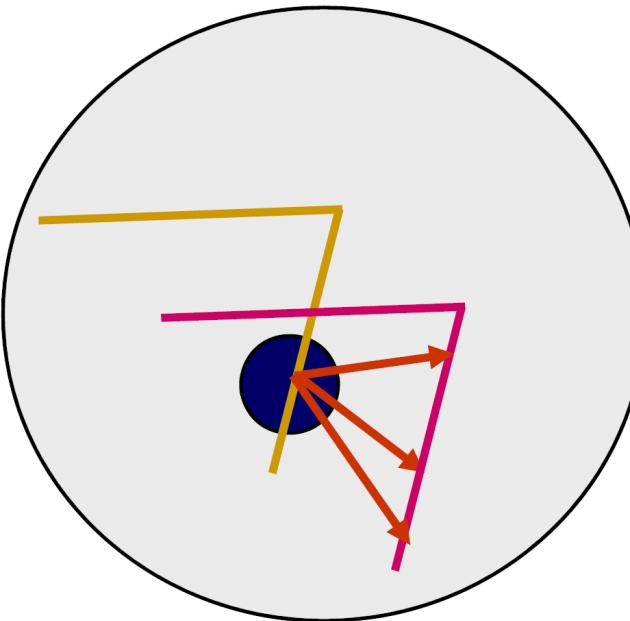


# Feature Detectors



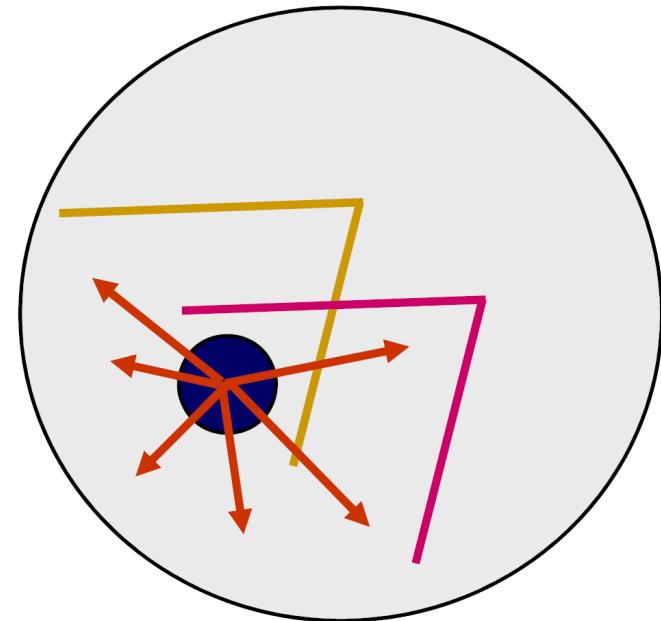
(a)

Corner



(b)

Edge



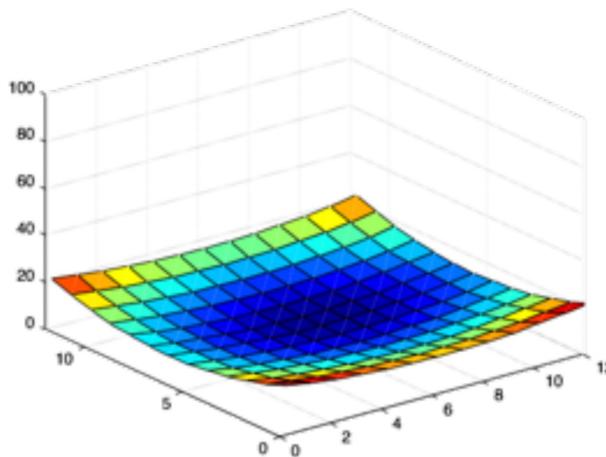
(c)

Textureless region

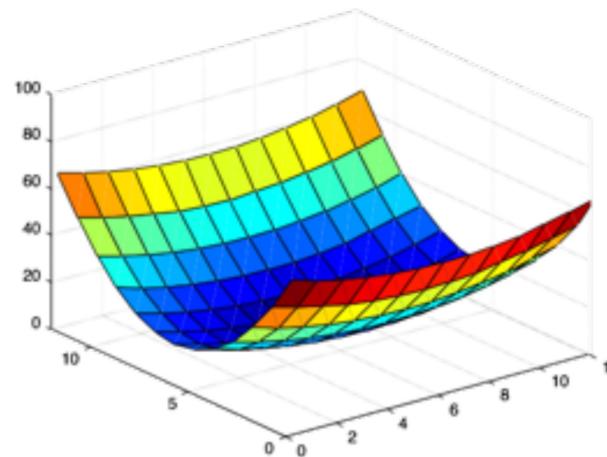
# Harris Corner Detector

$$f(\Delta x, \Delta y) \approx \sum_{x,y} w(x, y)(I_x(x, y)\Delta x + I_y(x, y)\Delta y)^2$$

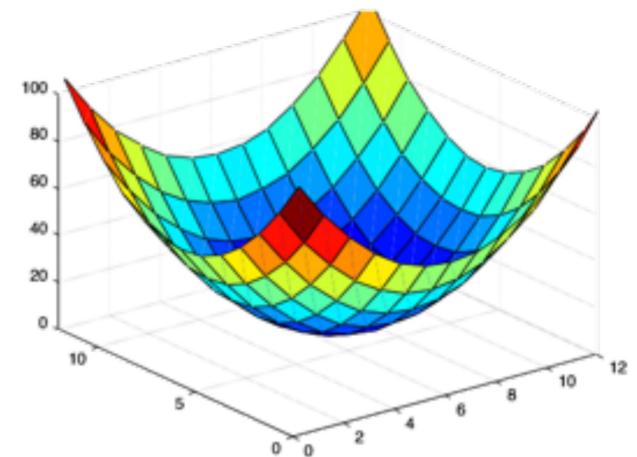
$$f(\Delta x, \Delta y) \approx (\Delta x \quad \Delta y) M \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \quad M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \sum_{x,y} w(x, y) I_x^2 & \sum_{x,y} w(x, y) I_x I_y \\ \sum_{x,y} w(x, y) I_x I_y & \sum_{x,y} w(x, y) I_y^2 \end{bmatrix}$$



Flat



Edge



Corner

# Invariance

Can the same feature point be detected after some transformation?

- Translation invariance

Are Harris corners translation invariance?

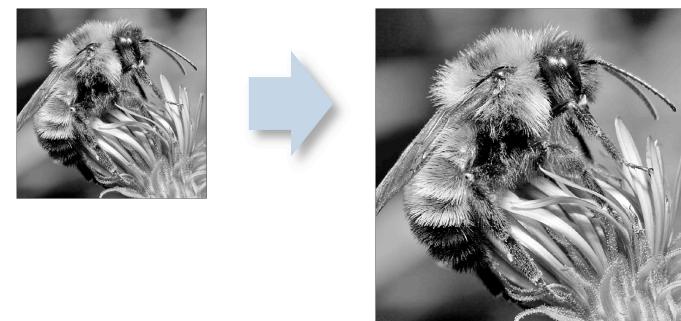
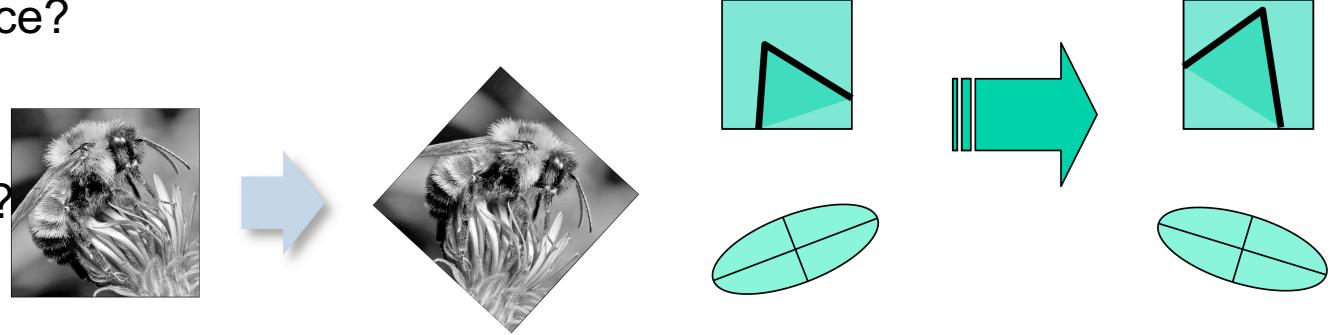
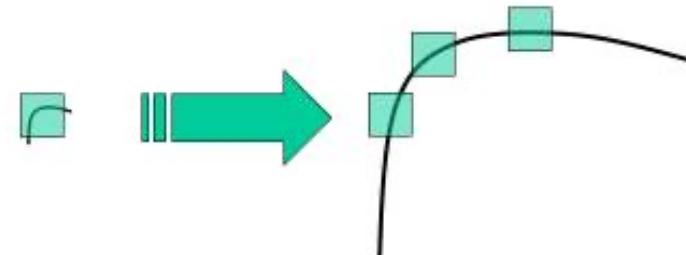
- 2D rotation invariance

Are Harris corners rotation invariance?

- Scale invariance

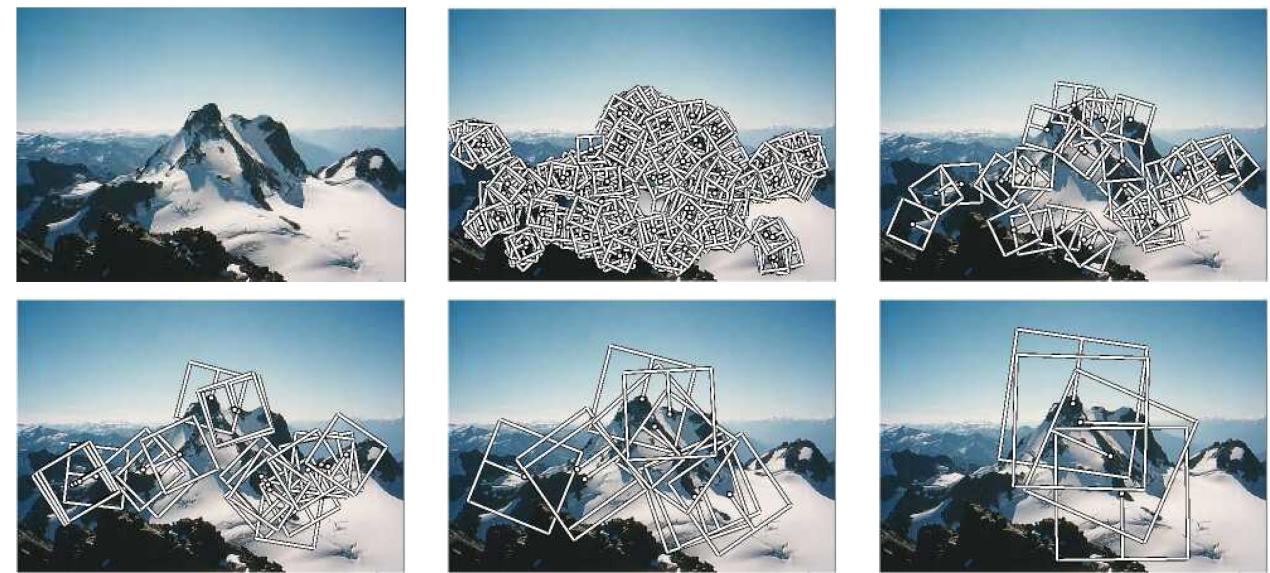
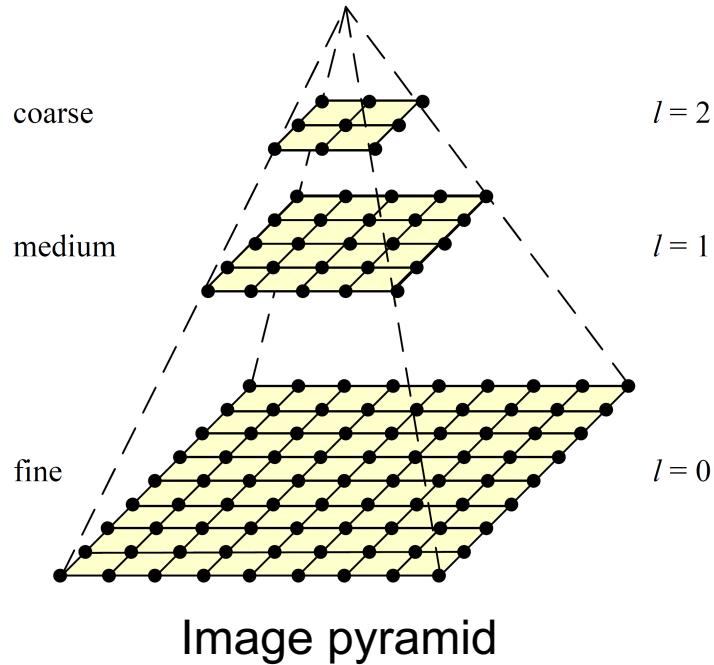
Are Harris corners scale invariance?

No



# Scale Invariance

Solution 1: detection features in all scales, matching features in corresponding scale (for small scale change)

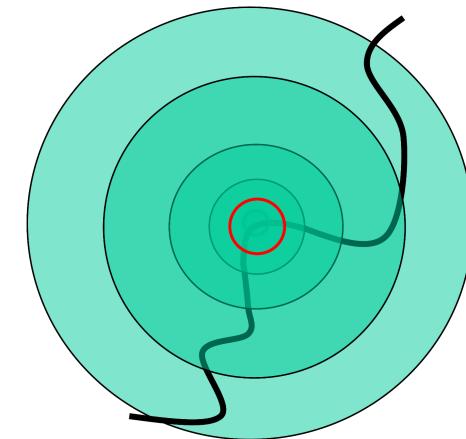
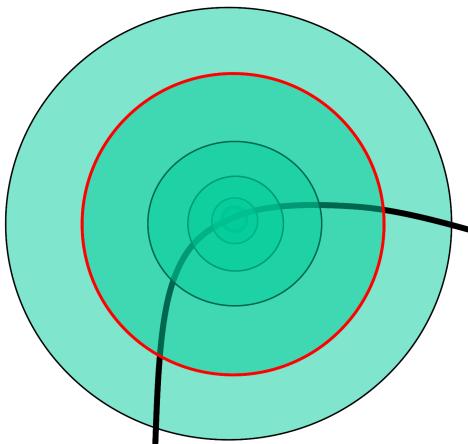


Multi-scale oriented patches (MOPS) extracted at five pyramid levels (Brown, Szeliski, and Winder 2005)

# Scale Invariance

Solution 2: detect features that are stable in both location and scale

Intuition: Find local maxima in both position and scale



What filter can we use  
for scale selection?

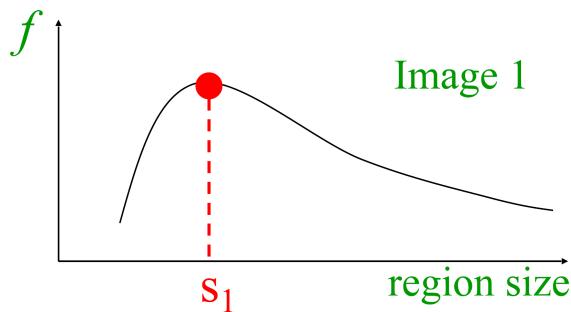


Image 1

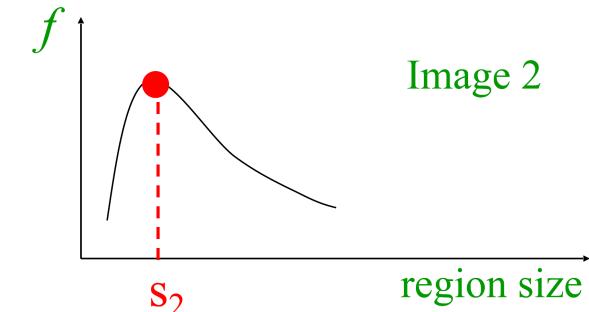


Image 2

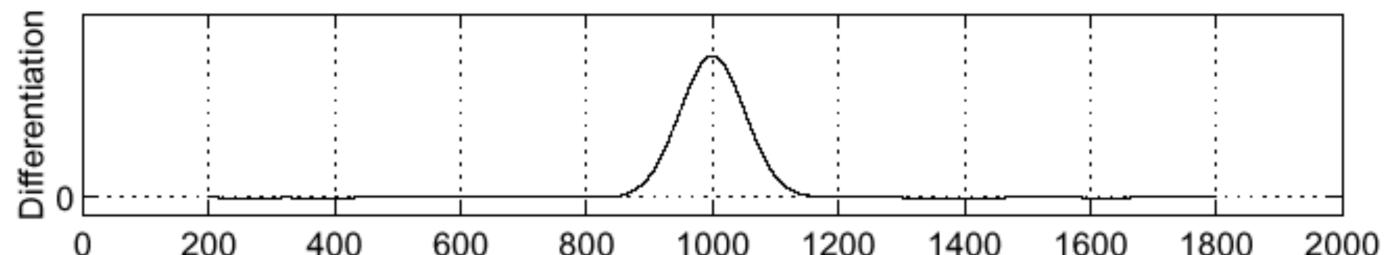
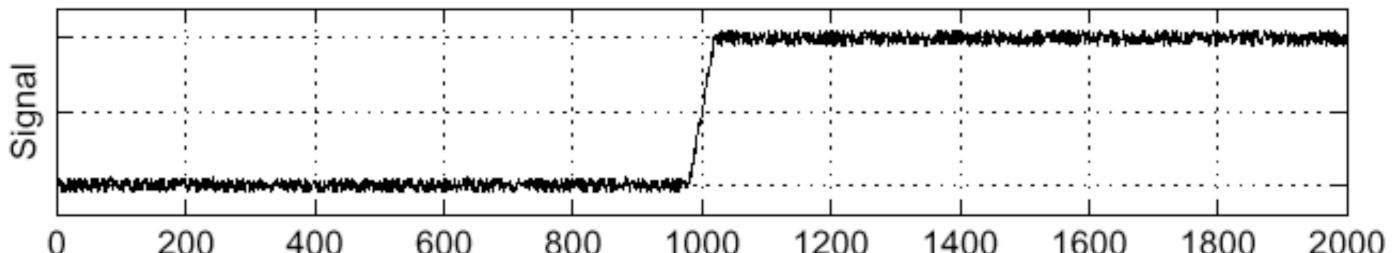
# Recall Derivative Filter

Central difference

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h}$$

-1	0	1
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X derivative



Find edge

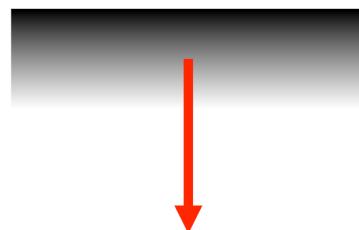
# Image Gradient

Gradient in x only



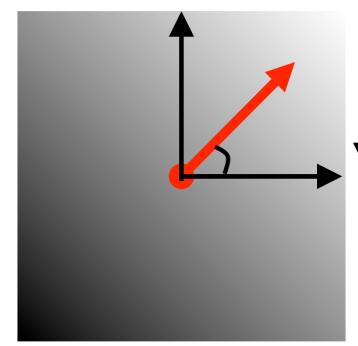
$$\nabla f = \left[ \frac{\partial f}{\partial x}, 0 \right]$$

Gradient in y only



$$\nabla f = \left[ 0, \frac{\partial f}{\partial y} \right]$$

Gradient in both x and y



$$\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

## Gradient direction

$$\theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

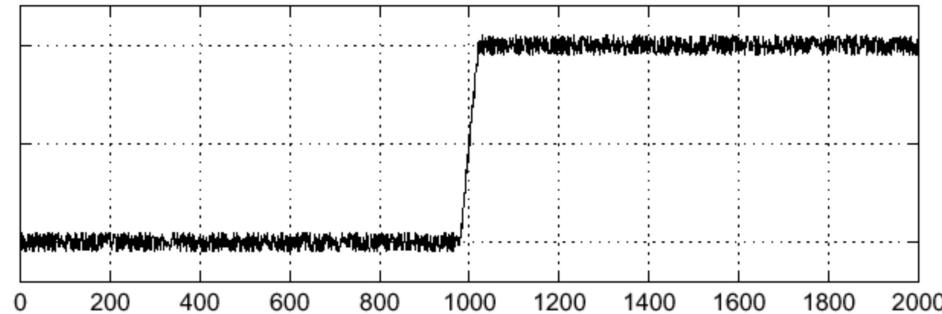
## Gradient magnitude

$$\|\nabla f\| = \sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2}$$

# Signal Noises

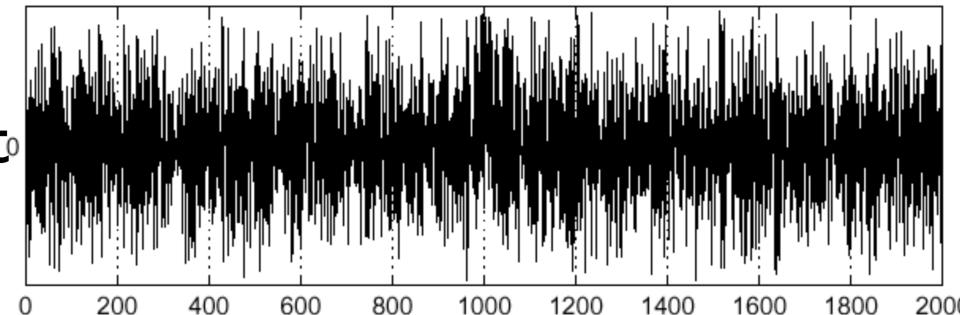
Derivative filters are sensitive to noises

Intensity plot



How to deal with noises?

Derivative plot



# Gaussian Filter

## Smoothing

1D 
$$g(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

2D 
$$g(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

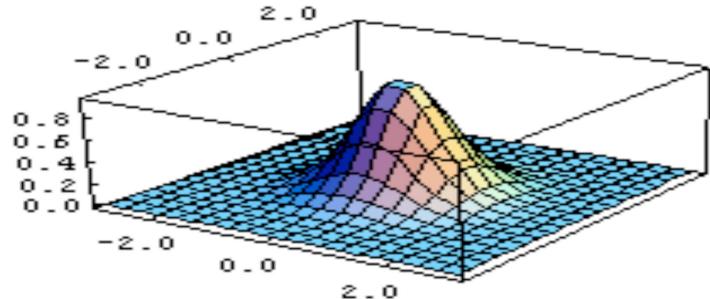
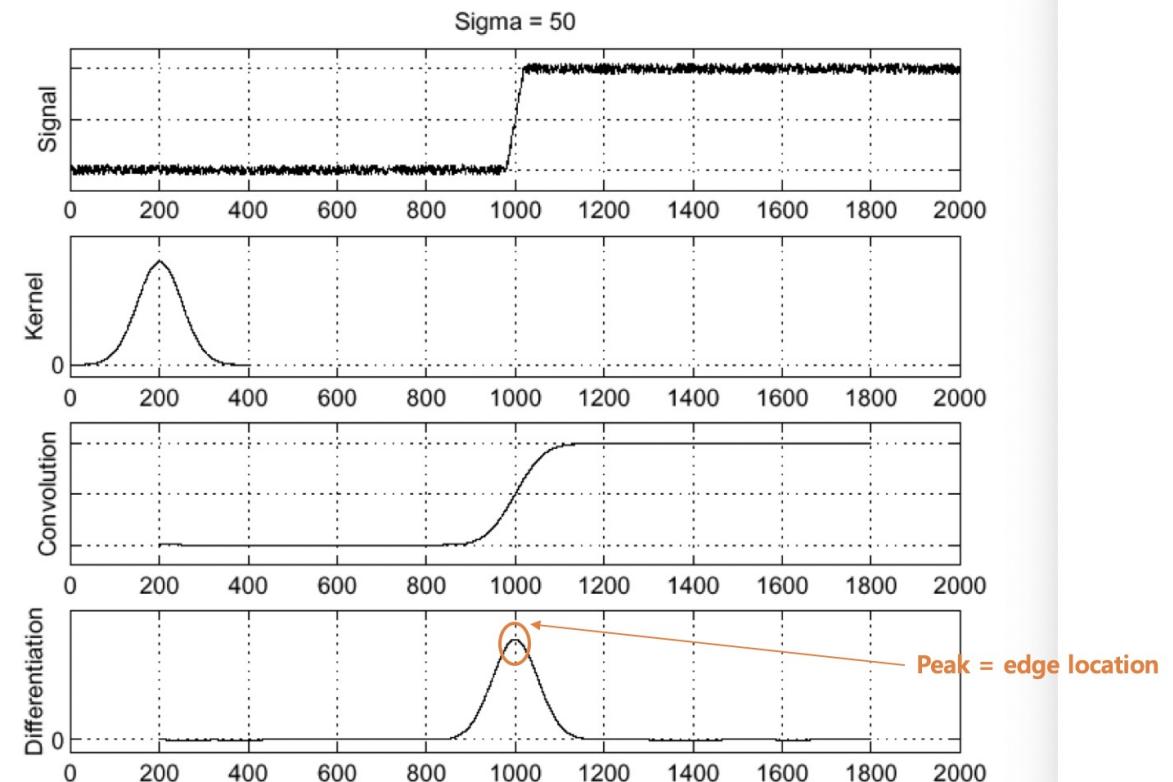


Image  $f$

Gaussian Filter  $h$

Convolution  $h \star f$

Derivative  $\frac{\partial}{\partial x}(h \star f)$



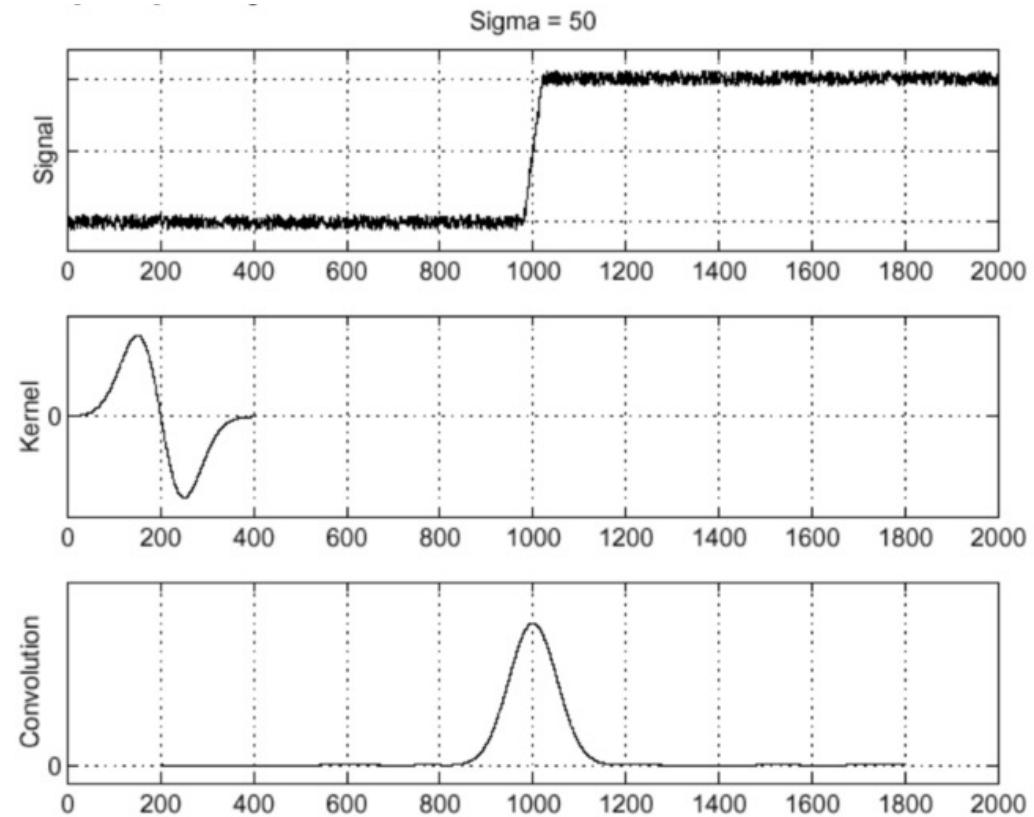
# Derivative of Gaussian Filter

- Convolution is associative  $\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$

$f$

Smoothing and derivative  $\frac{\partial}{\partial x}h$

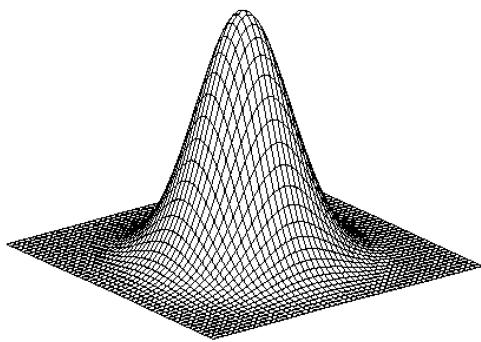
$(\frac{\partial}{\partial x}h) \star f$



# Derivative of Gaussian Filter

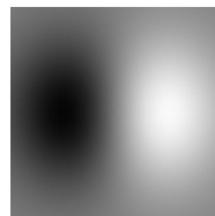
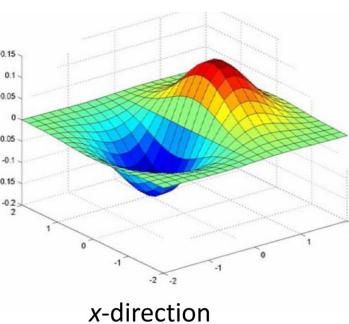
Convolution is associative

$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

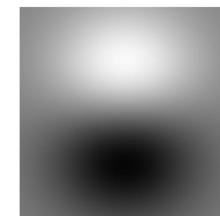
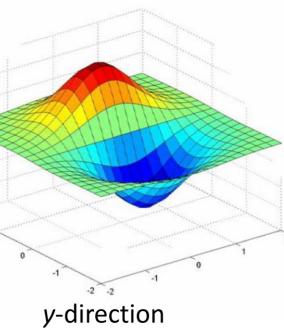


Gaussian

$$g(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



$$g_y(x, y) = \frac{\partial g(x, y)}{\partial y} = \frac{-y}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



# Laplace Filter

first-order  
finite difference

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h}$$

Derivative filter

-1	0	1
----	---	---

second-order  
finite difference

$$f''(x) \approx \frac{\delta_h^2[f](x)}{h^2} = \frac{\frac{f(x+h) - f(x)}{h} - \frac{f(x) - f(x-h)}{h}}{h} = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Laplace filter

1	-2	1
---	----	---

# Laplace Filter

2D

$$\nabla^2 \mathbf{I} = \frac{\partial^2 \mathbf{I}}{\partial x^2} + \frac{\partial^2 \mathbf{I}}{\partial y^2}$$

1	-2	1
---	----	---

1D Laplace filter

0	1	0
1	-4	1
0	1	0

2D Laplace filter

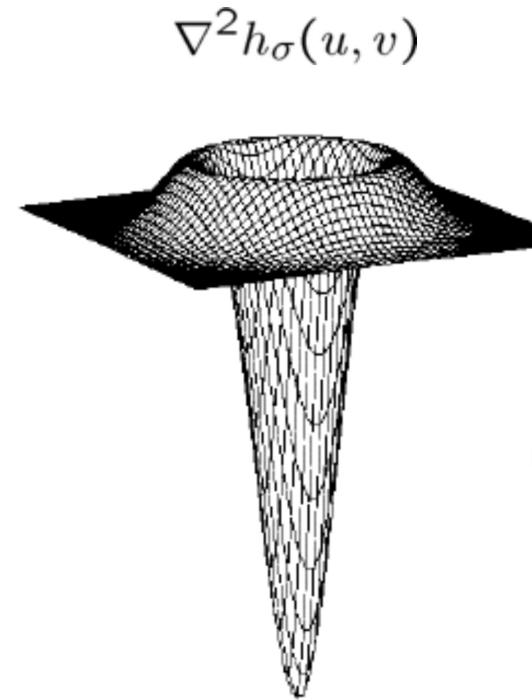
# Laplacian of Gaussian Filter

$$\nabla^2 \mathbf{I} = \frac{\partial^2 \mathbf{I}}{\partial x^2} + \frac{\partial^2 \mathbf{I}}{\partial y^2}$$

$$\nabla^2 \mathbf{I} \circ g = \nabla^2 g \circ \mathbf{I}$$

$$\nabla^2 g = \frac{x^2 + y^2 - 2\sigma^2}{\sigma^4} g(x, y)$$

Smoothing and second derivative

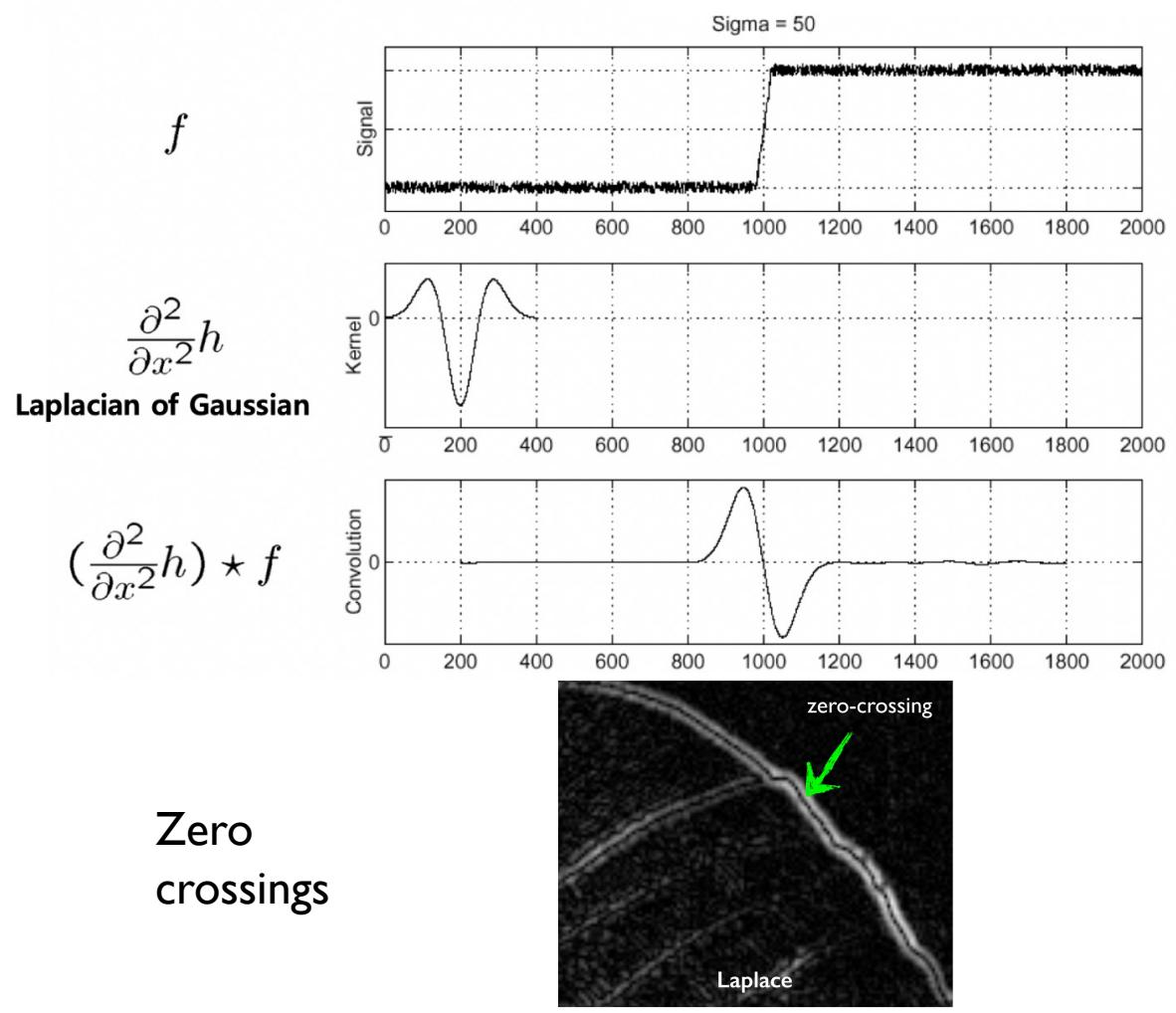
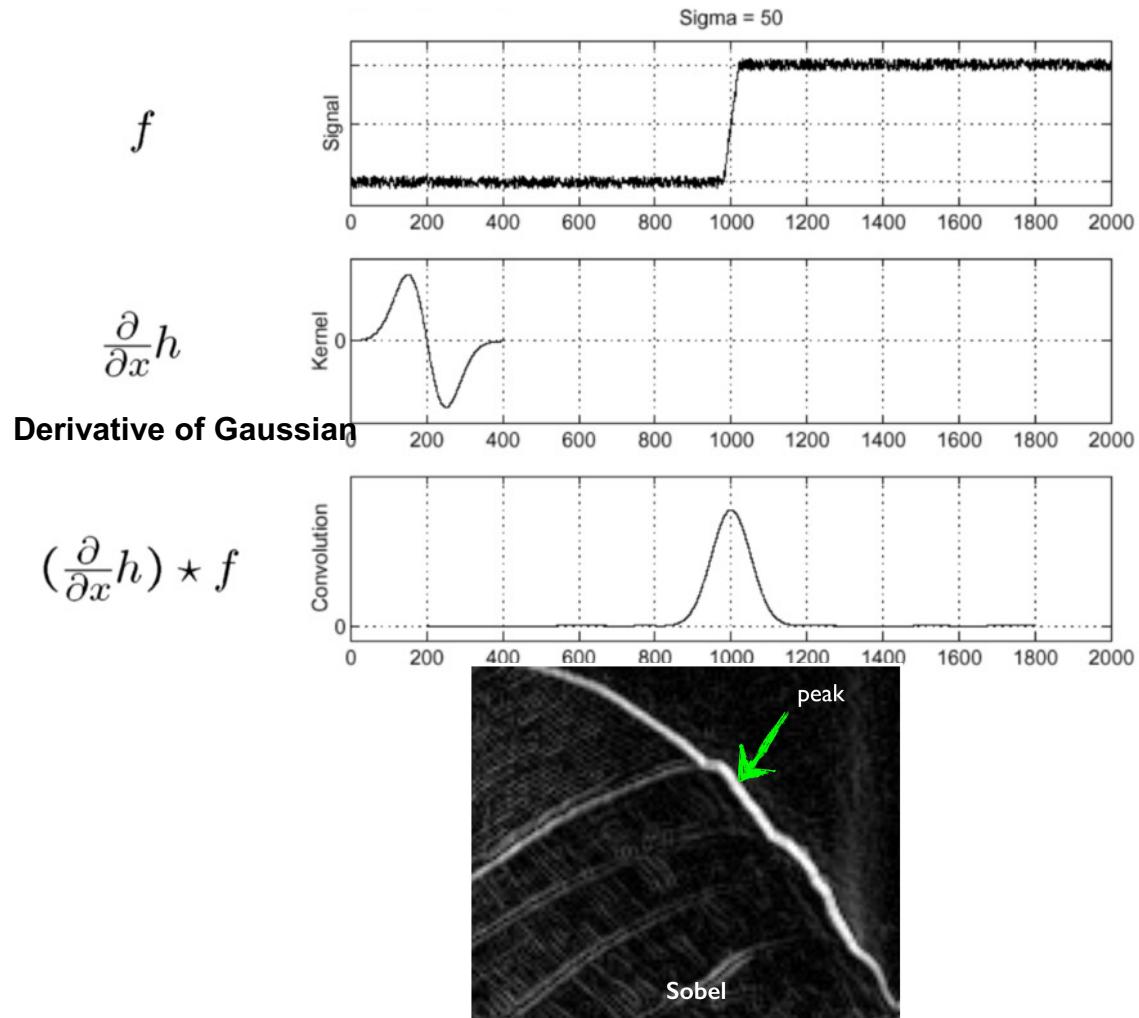


Laplacian of Gaussian



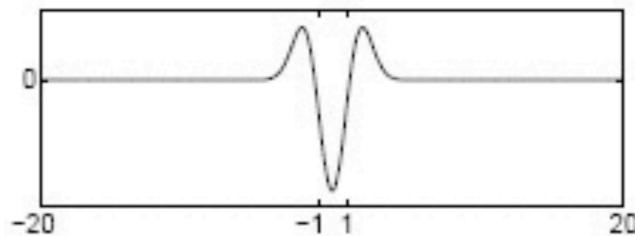
Mexican Hat Function

# Laplacian of Gaussian Filter

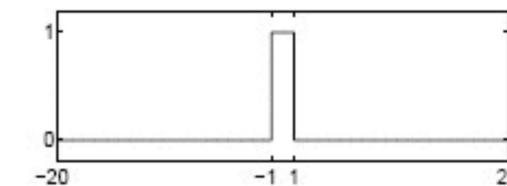
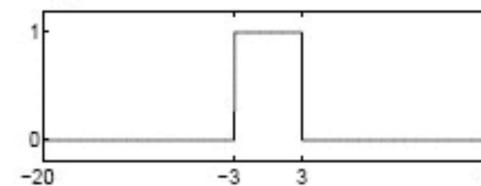
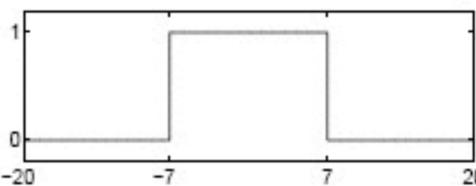
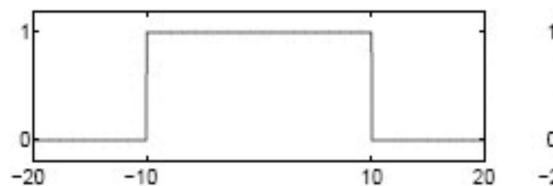


# Laplacian of Gaussian for Scale Selection

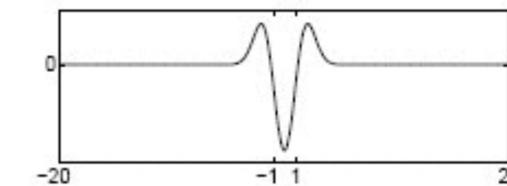
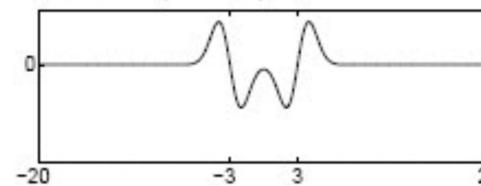
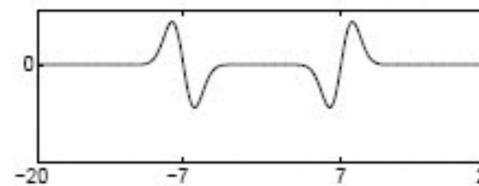
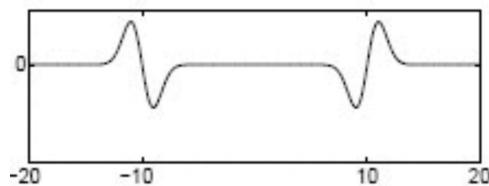
Laplacian filter



Original signal

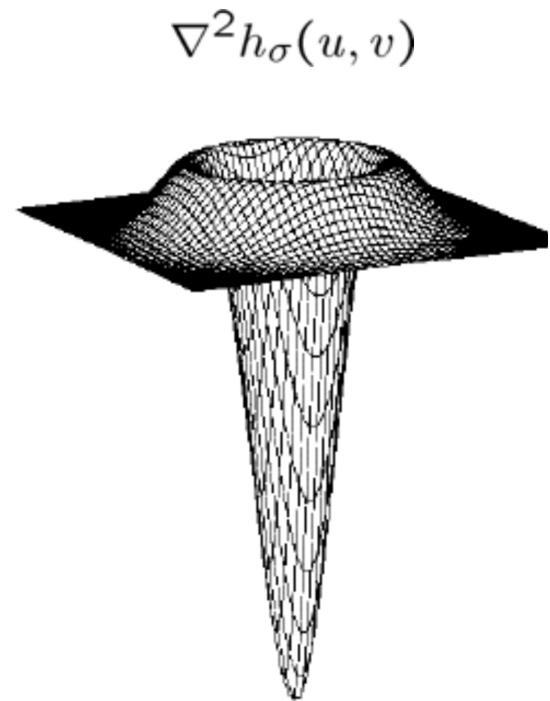
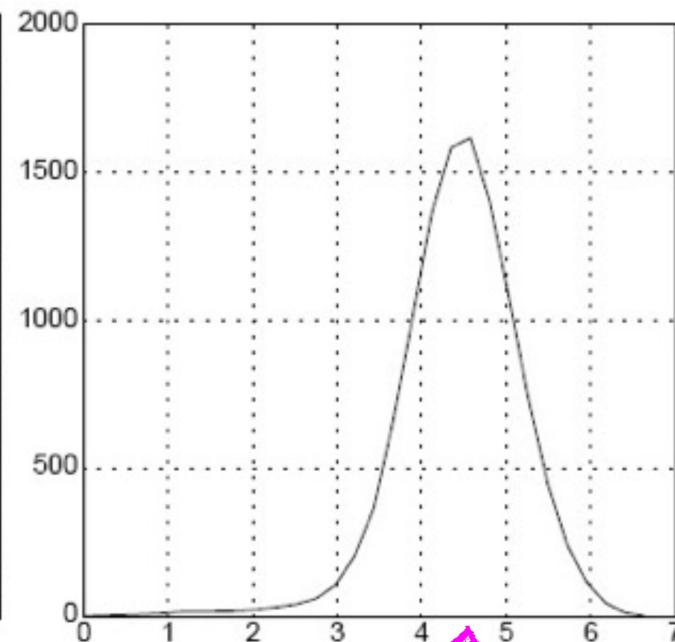
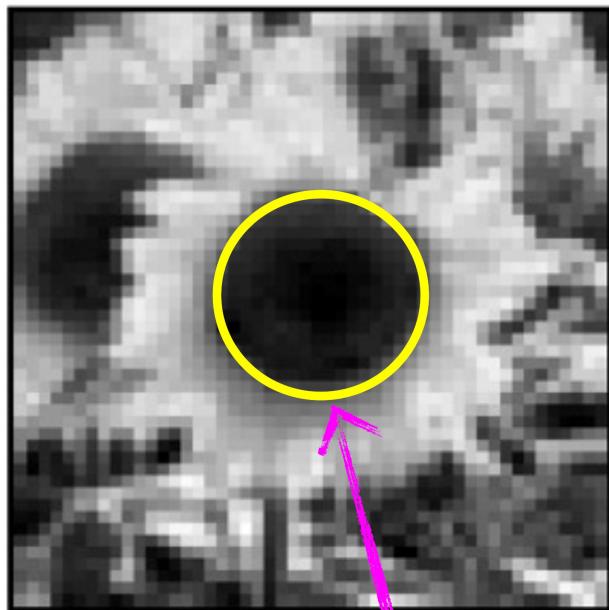


Convolved with Laplacian ( $\sigma = 1$ )



Highest response when the signal has the same **characteristic scale** as the filter

# Laplacian of Gaussian for Scale Selection

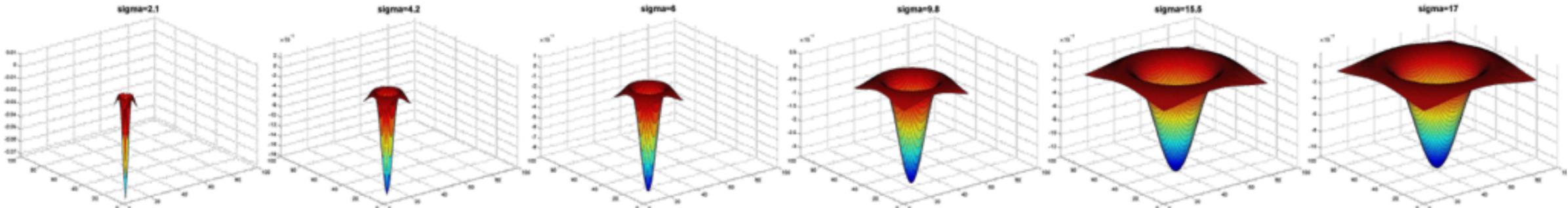


$\nabla^2 h_\sigma(u, v)$

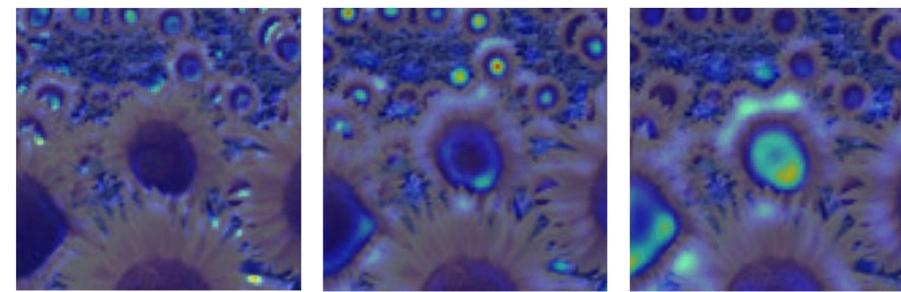
characteristic scale

Search over different scales  $\sigma$

# Laplacian of Gaussian for Scale Selection



2.1            4.2            6.0

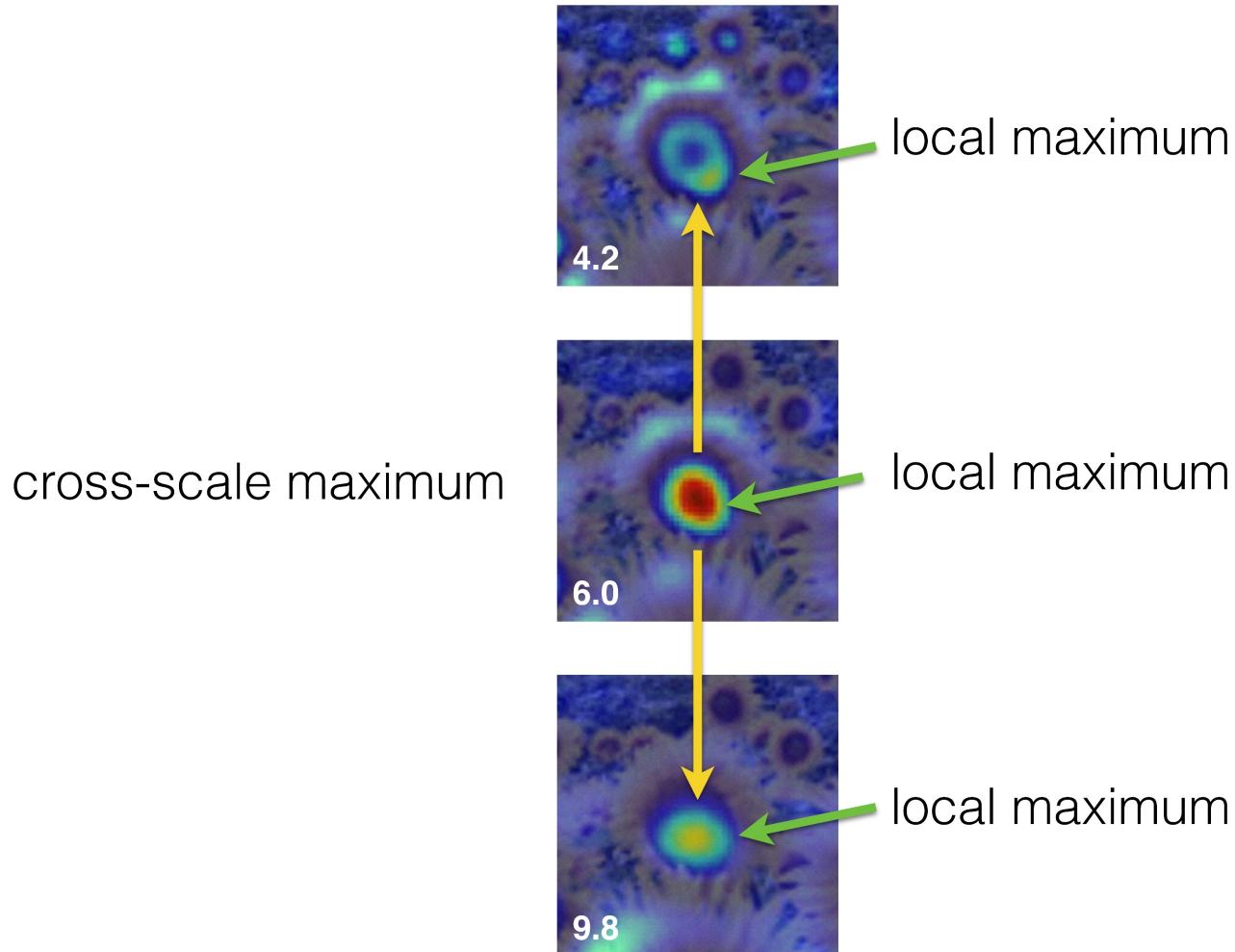


9.8            15.5            17.0



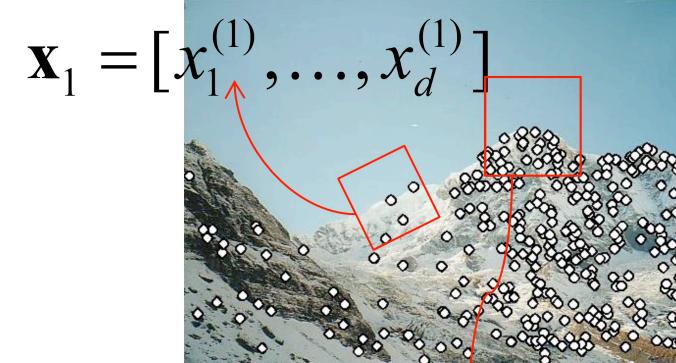
Multi-scale  
2D Blob  
detection

# Laplacian of Gaussian for Scale Selection



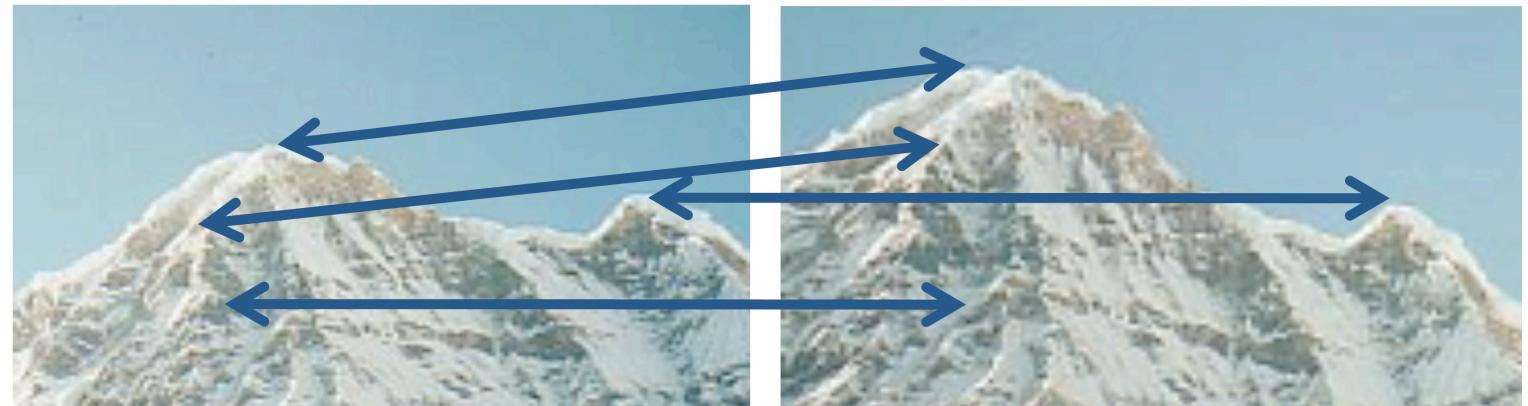
# Scale Invariance Feature Transform (SIFT)

Keypoint detection



Compute descriptors

Matching descriptors



David Lowe, Distinctive Image Features from Scale-Invariant Keypoints. IJCV, 2004

# SIFT: Scale-space Extrema Detection

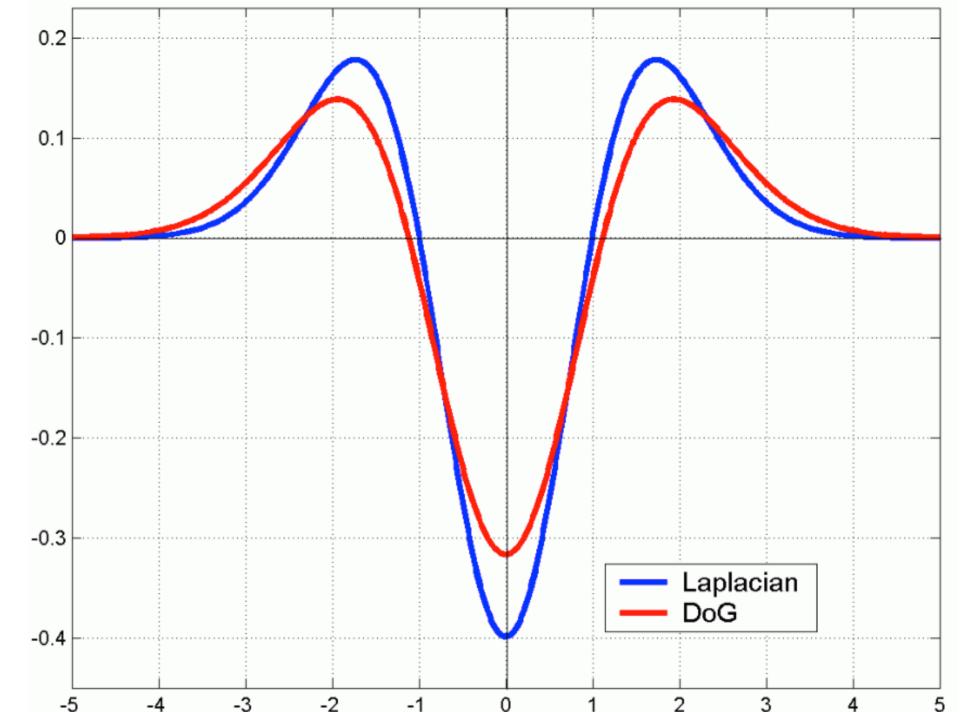
## Difference of Gaussian (DoG)

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

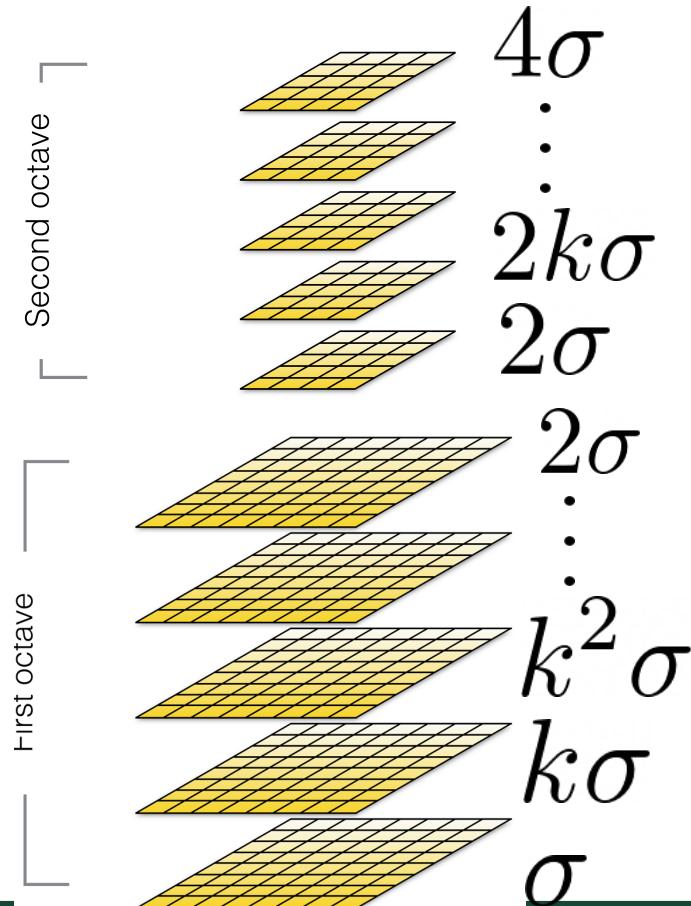
$$\begin{aligned} D(x, y, \sigma) &= (G(x, y, k\sigma) - G(x, y, \sigma)) * I(x, y) \\ &= L(x, y, k\sigma) - L(x, y, \sigma). \end{aligned}$$

Approximate of Laplacian of Gaussian  
(efficient to compute)



# SIFT: Scale-space Extrema Detection

## Gaussian pyramid



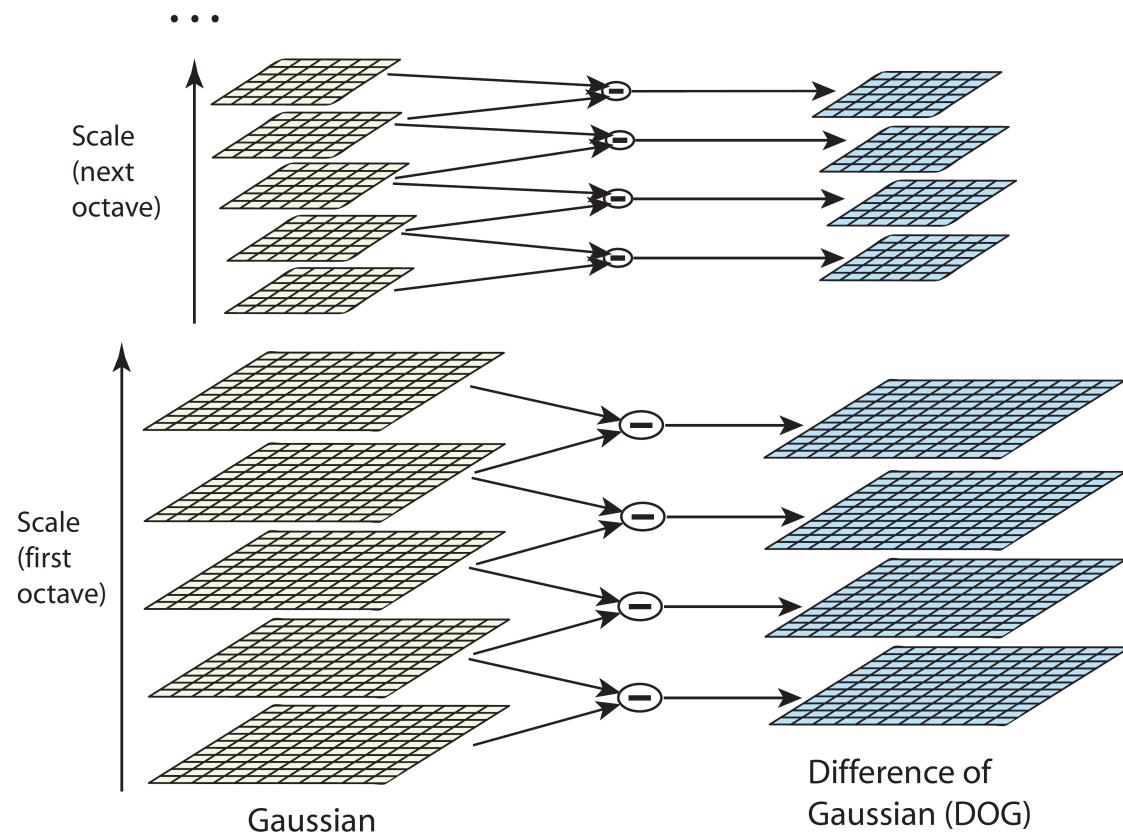
- Gaussian filters

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

$$G_{\sigma_1} * G_{\sigma_2} = G_{\sigma} \quad \sigma^2 = \sigma_1^2 + \sigma_2^2$$

- Sub-sampling by a factor of 2
  - Multiple the Gaussian kernel deviation by

# SIFT: Scale-space Extrema Detection

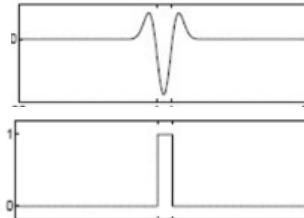
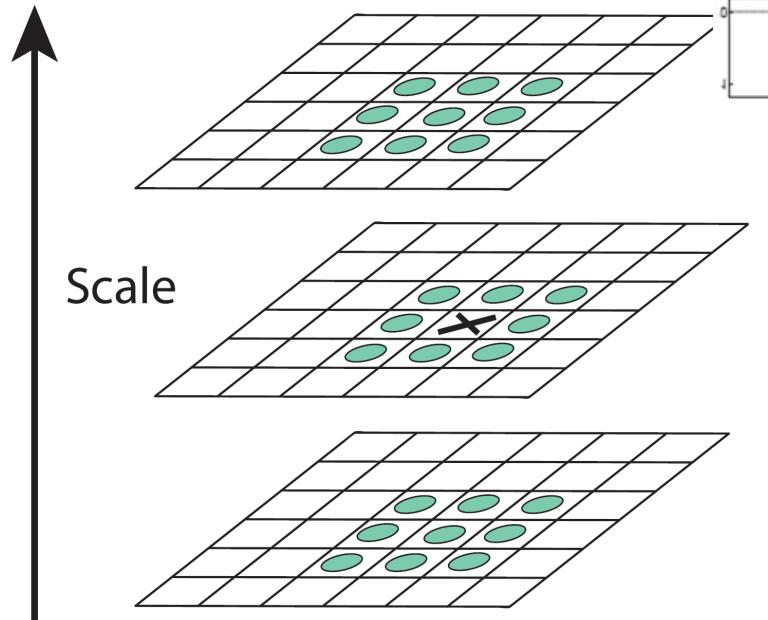


$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

$$D(x, y, \sigma) = (G(x, y, k\sigma) - G(x, y, \sigma)) * I(x, y)$$

$$= L(x, y, k\sigma) - L(x, y, \sigma).$$



# SIFT Descriptor

Image gradient magnitude and orientation

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

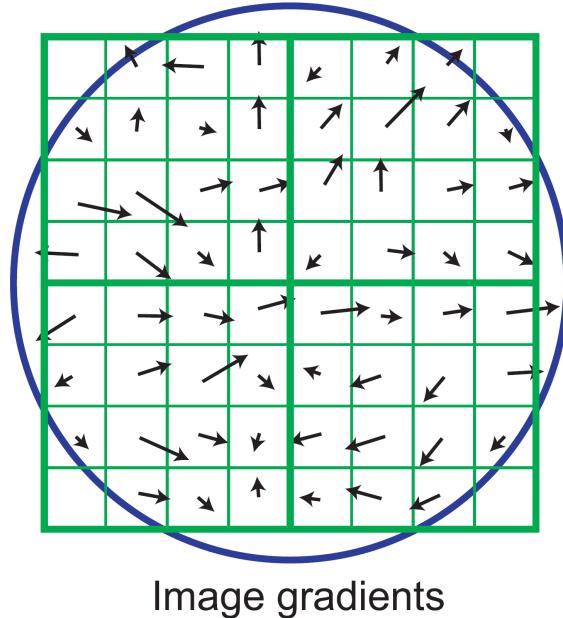


Image gradients

$$m(x, y) = \sqrt{(L(x + 1, y) - L(x - 1, y))^2 + (L(x, y + 1) - L(x, y - 1))^2}$$

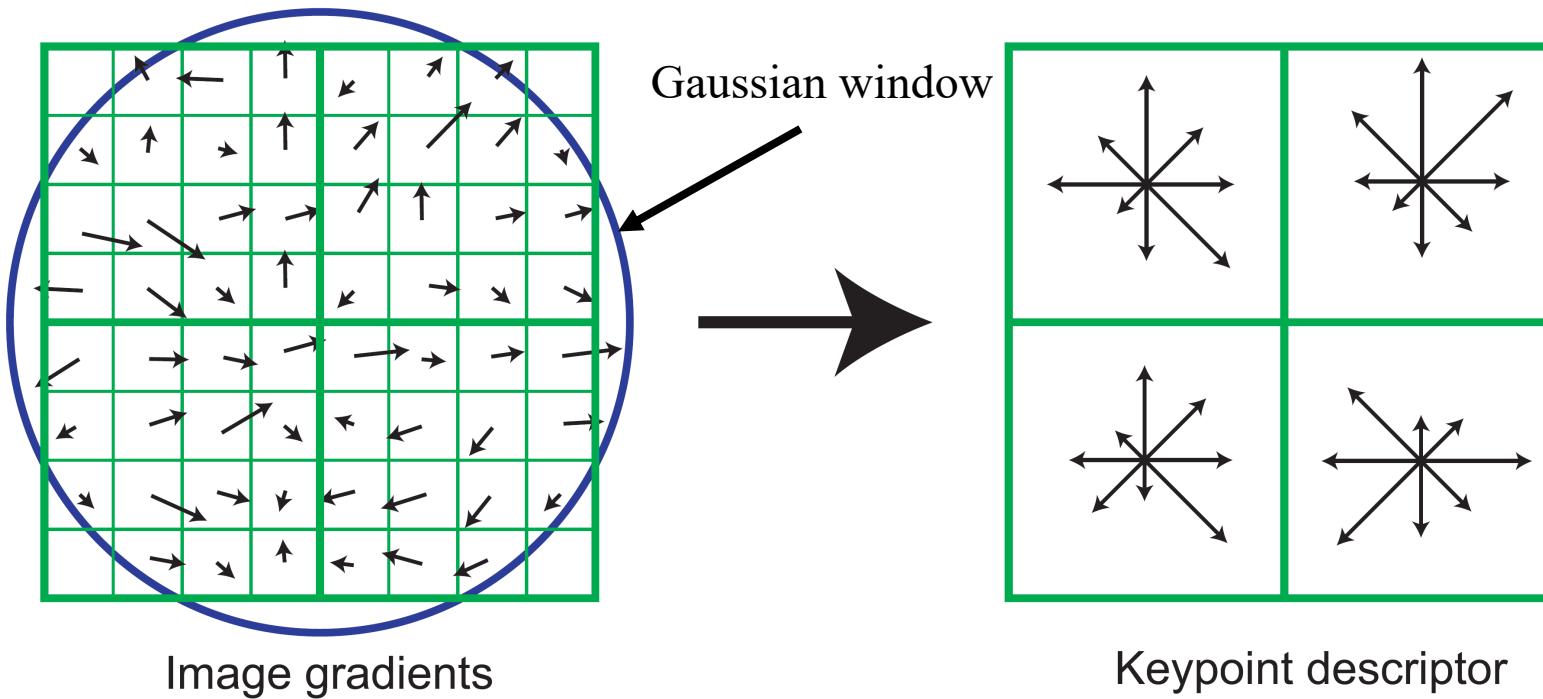
X-derivative    Y-derivative

$$\theta(x, y) = \tan^{-1}((L(x, y + 1) - L(x, y - 1)) / (L(x + 1, y) - L(x - 1, y)))$$

# SIFT Descriptor

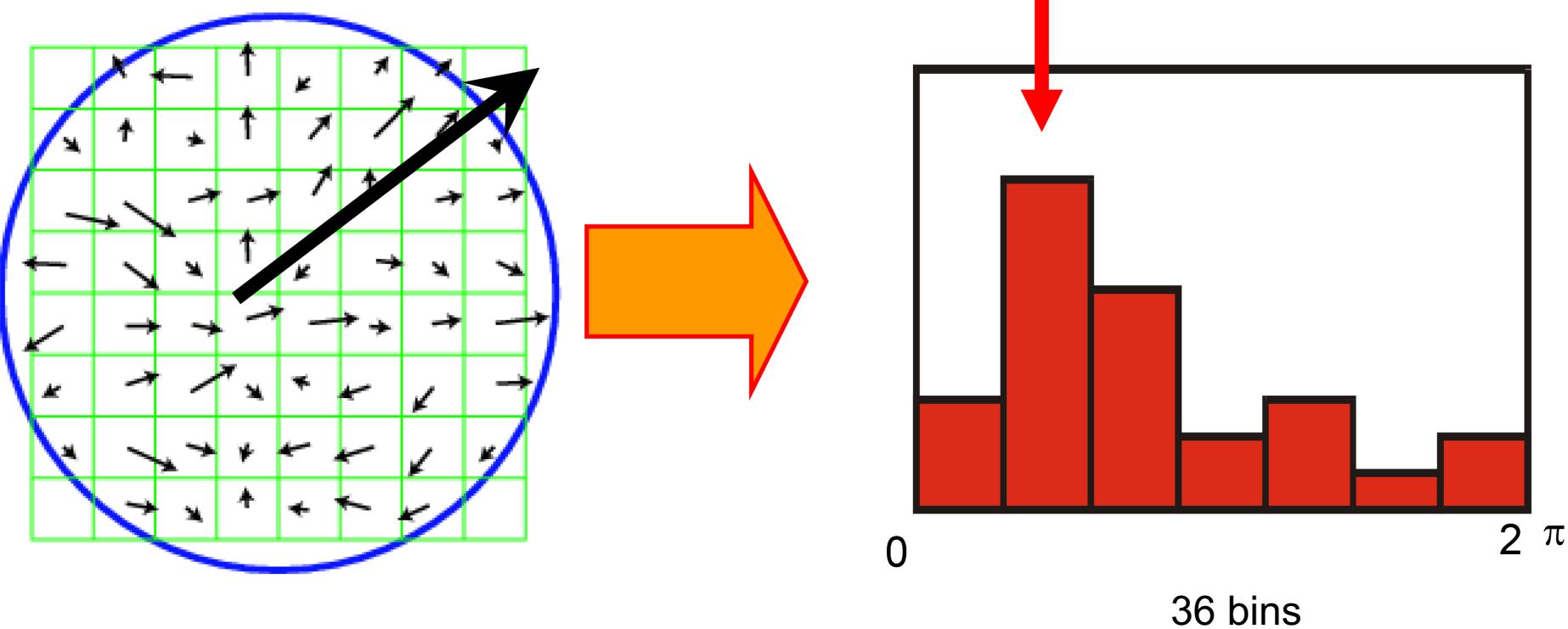
- Divide the 16x16 window into a 4x4 grid of cells (2x2 case shown below)
- Compute an orientation histogram for each cell
- 16 cells \* 8 orientations = 128 dimensional descriptor

Using the scale of  
the keypoint to  
select the level of  
Gaussian blur for  
the image



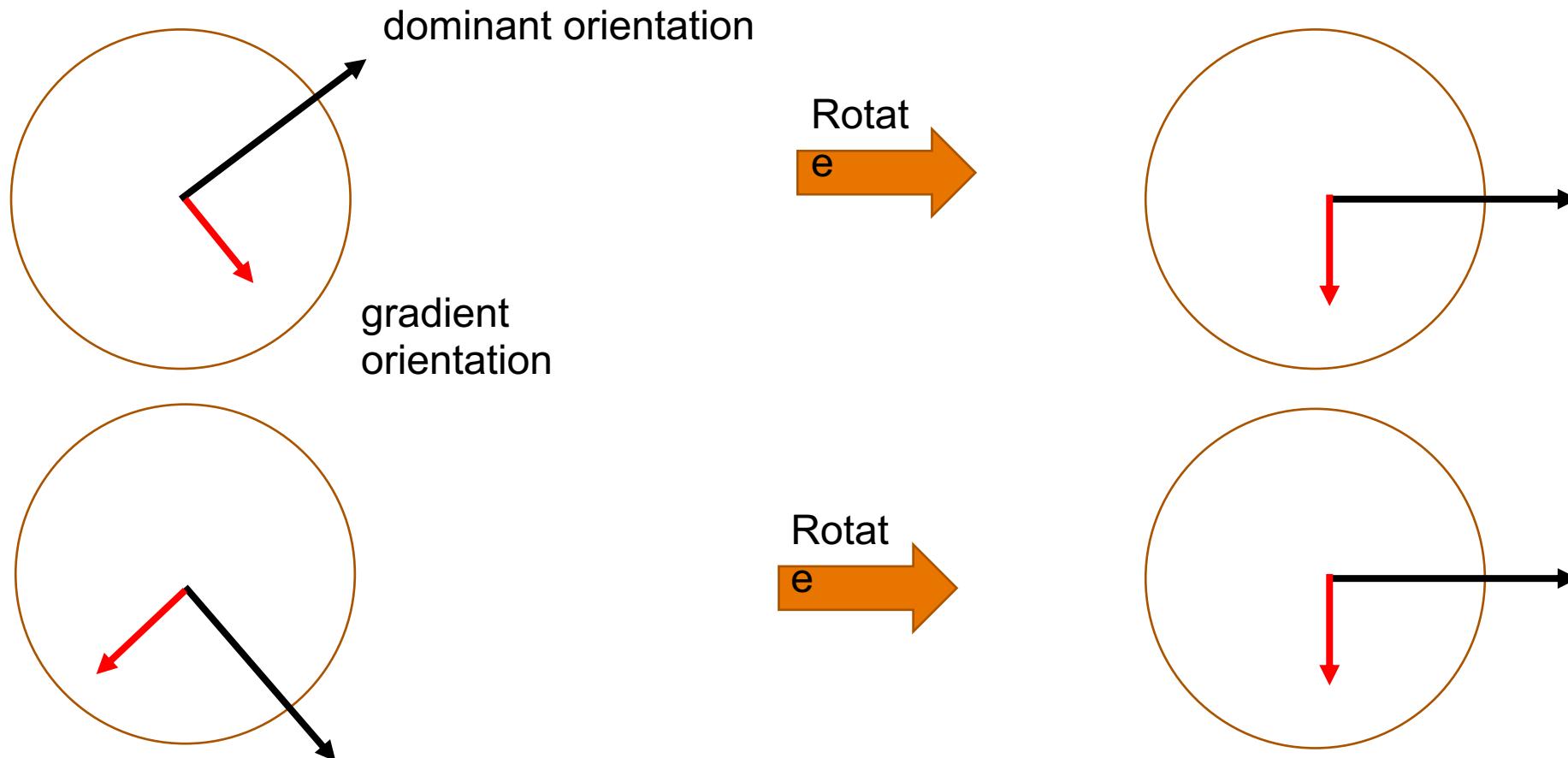
# SIFT: Rotation Invariance

Rotate all orientations by the dominant orientation



# SIFT: Rotation Invariance

Rotate all orientations by the dominant orientation



# SIFT Properties

Can handle change in viewpoint (up to about 60 degree out of plane rotation)

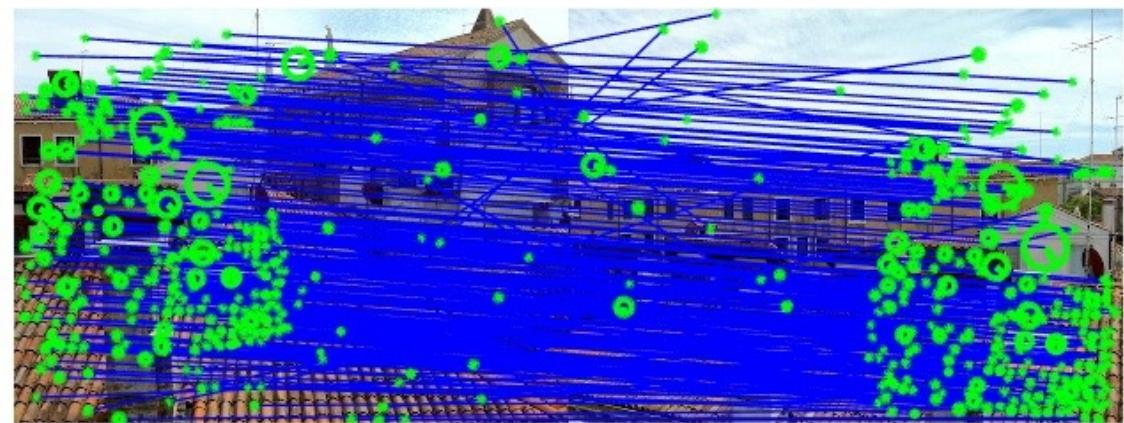
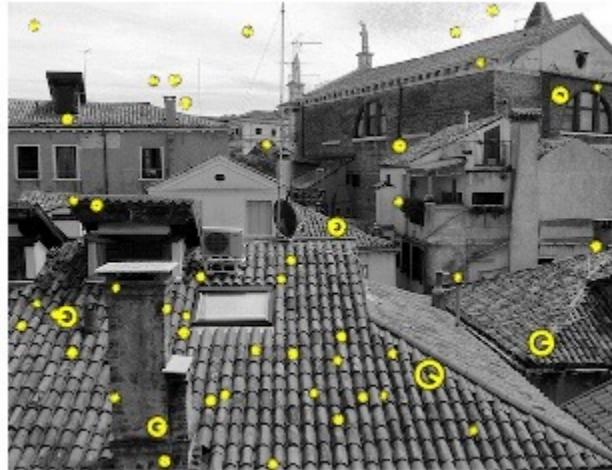
Can handle significant change in illumination

Relatively fast < 1s for moderate image sizes

Lots of code available

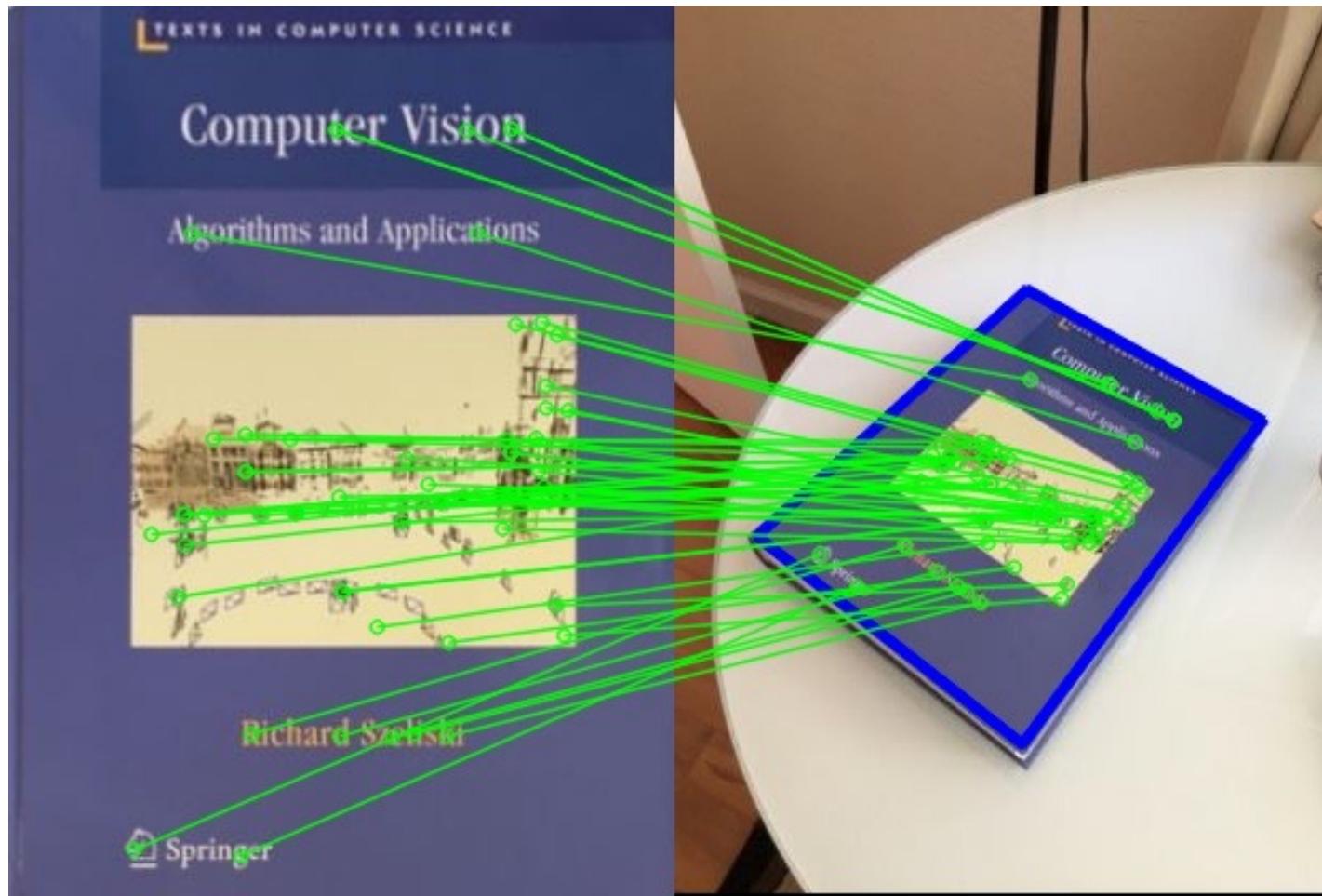
- E.g., <https://www.vlfeat.org/overview/sift.html>

# SIFT Matching Example



<https://www.vlfeat.org/overview/sift.html>

# SIFT Matching Example



# Further Reading

Section 7.1, Computer Vision, Richard Szeliski

David Lowe, Distinctive Image Features from Scale-Invariant Keypoints.  
IJCV, 2004

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