



THE UNIVERSITY OF TEXAS AT DALLAS

Optical Flow

CS 4391 Introduction to Computer Vision

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Motion Perception

Separate moving figure from a stationary background

Motion for 3D perception

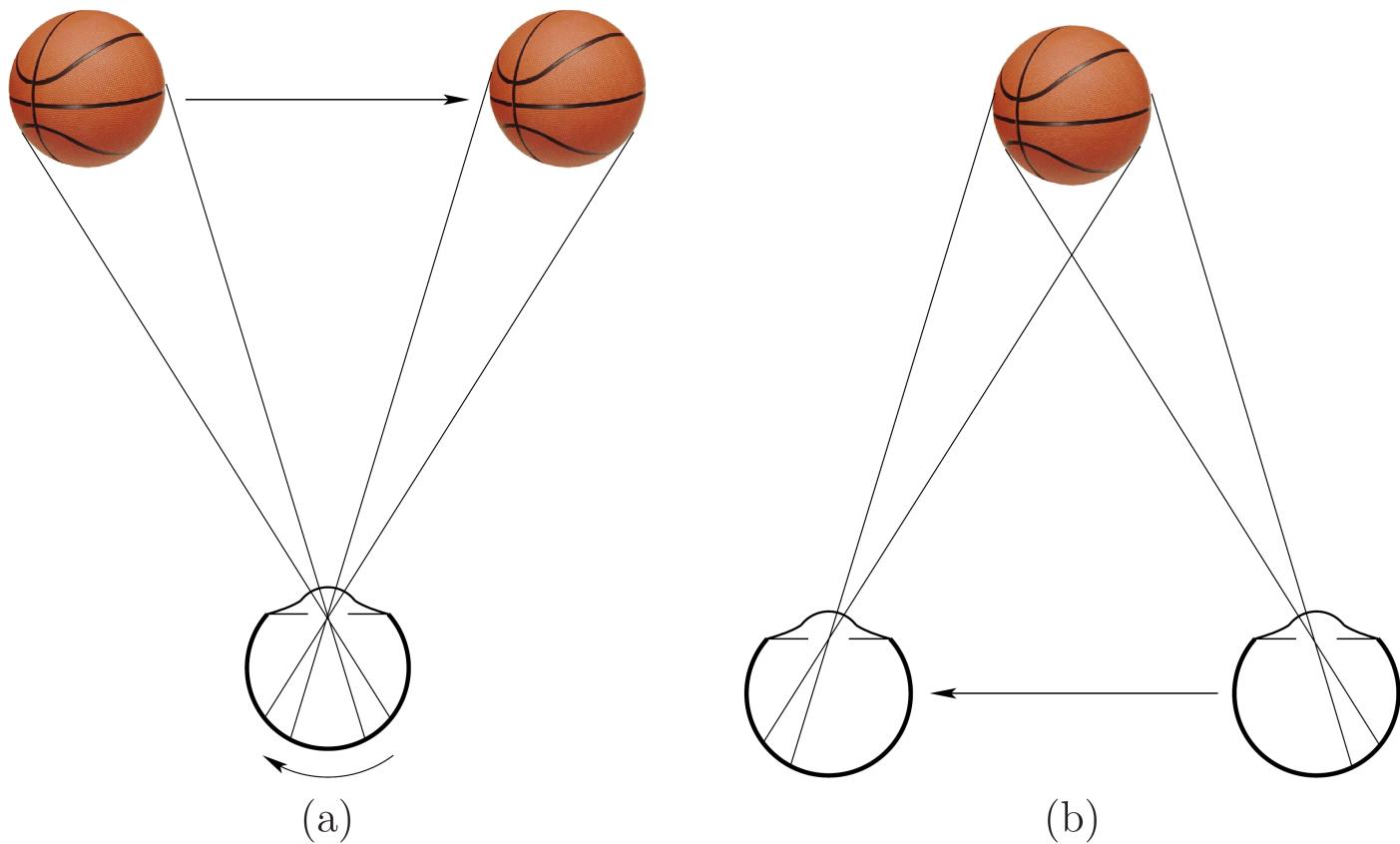
- Look at a fruit by rotating it around

Guide actions

- Walking down the street or hammering a nail



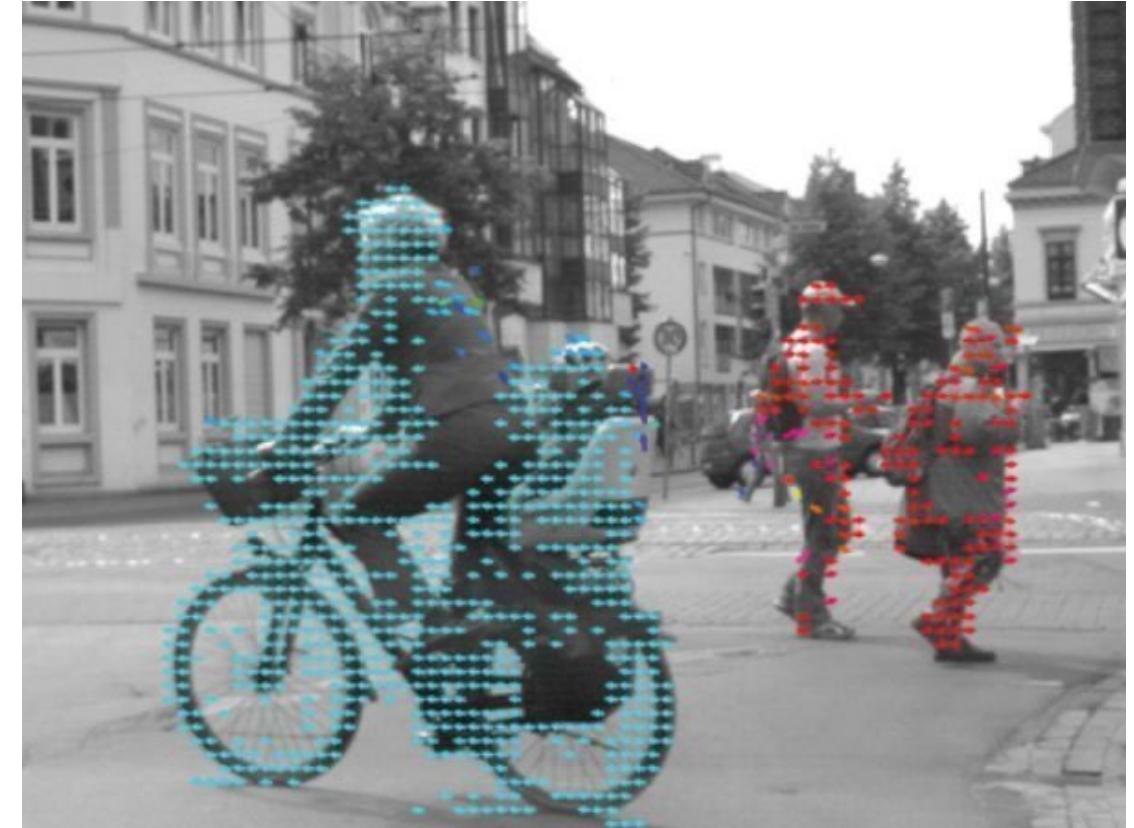
Object Motion vs. Eye Movement



Two motions that cause equivalent movement of the image on the retina

- **Saccadic suppression:** the brain selectively block visual processing during eye movements, suppress motion detectors in the second case
- **Proprioception:** the body's ability to estimation its own motions due to motor commands (i.e., use of eye muscles)
- **Information is provided by large-scale motion:** if the entire scene is moving, the brain interprets the user must be moving

Motion from Object/Camera Movement in Videos

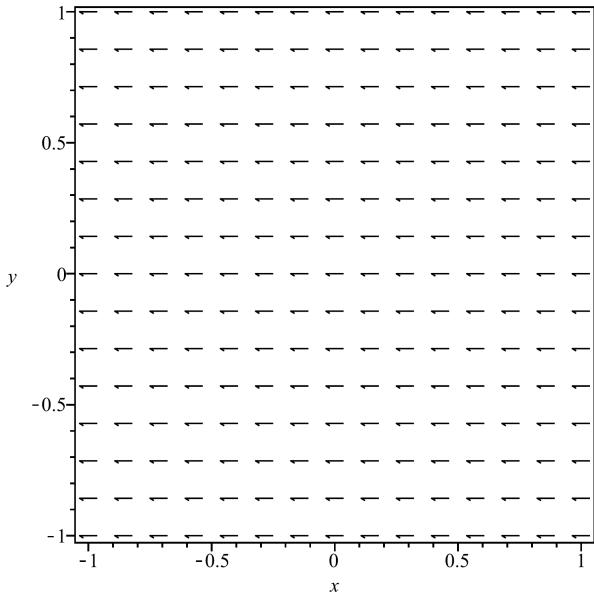


Optical Flow

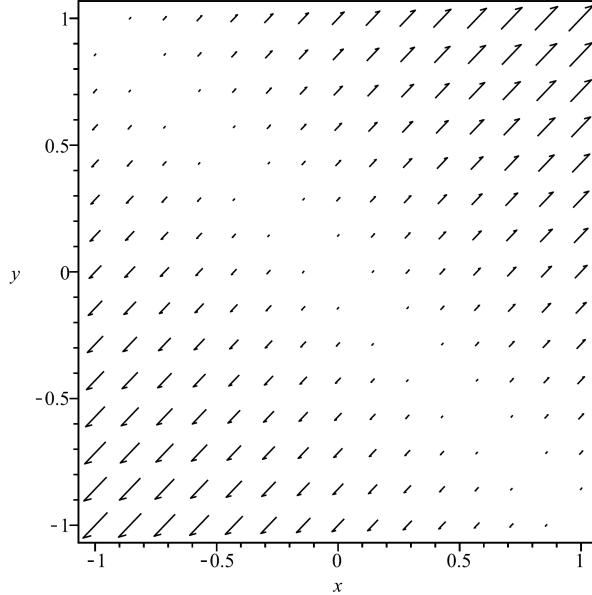
The pattern of apparent motion of objects, surfaces and edges in a visual scene caused by the relative motion between an observer and a scene

Velocity field

$$(v_x, v_y) = \left(\frac{dx}{dt}, \frac{dy}{dt} \right)$$

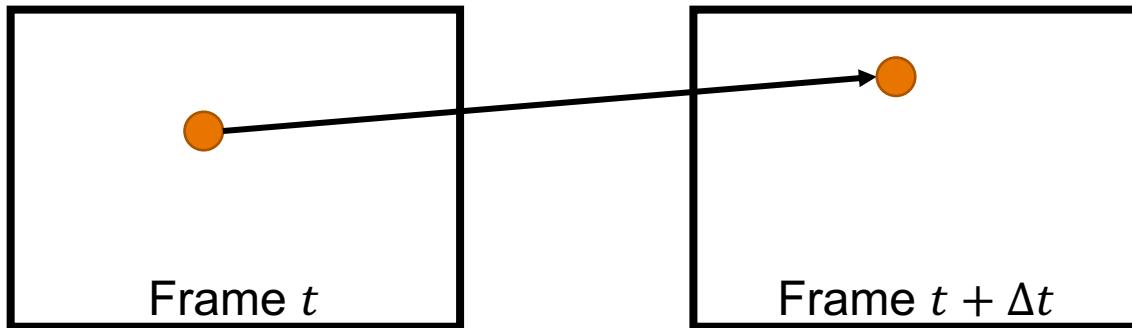


$$(x, y) \mapsto (-1, 0)$$



$$(x, y) \mapsto (x + y, x + y)$$

Brightness Constancy Constraint

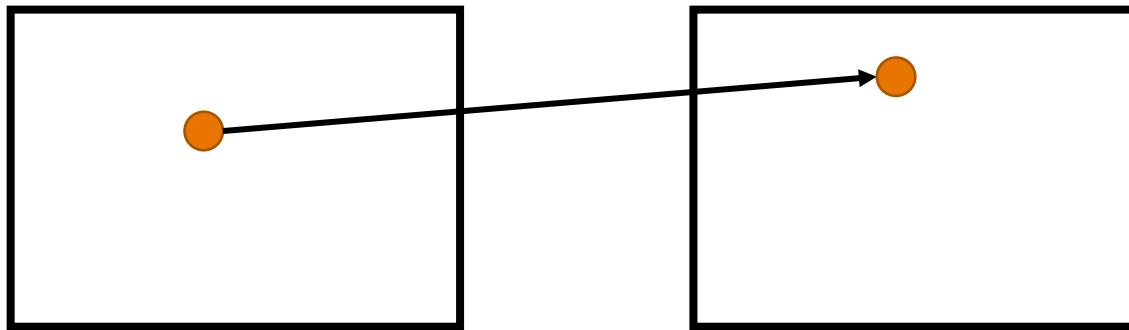


$$I(x, y, t) = I(x + \Delta x, y + \Delta y, t + \Delta t)$$

Taylor series

$$I(x + \Delta x, y + \Delta y, t + \Delta t) = I(x, y, t) + \frac{\partial I}{\partial x} \Delta x + \frac{\partial I}{\partial y} \Delta y + \frac{\partial I}{\partial t} \Delta t + \text{higher-order terms}$$

Brightness Constancy Constraint



$$\frac{\partial I}{\partial x} \Delta x + \frac{\partial I}{\partial y} \Delta y + \frac{\partial I}{\partial t} \Delta t = 0$$

$$\frac{\partial I}{\partial x} \frac{\Delta x}{\Delta t} + \frac{\partial I}{\partial y} \frac{\Delta y}{\Delta t} + \frac{\partial I}{\partial t} \frac{\Delta t}{\Delta t} = 0$$

$$\frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

Brightness Constancy Constraint

$$\frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

$\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}$ (spatial gradient; we can compute this!)

$\frac{dx}{dt}, \frac{dy}{dt} = (u, v)$ (optical flow, what we want to find)

$\frac{\partial I}{\partial t}$ (derivative across frames. Also known,
e.g. frame difference)

Image Gradient

Sobel Filter

$$\begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array}$$

Sobel

=

$$\begin{array}{|c|} \hline 1 \\ \hline 2 \\ \hline 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline \end{array}$$

x-derivative

weighted average
and scaling

$$S_x = \begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array}$$

$$\frac{\partial f}{\partial x} = S_x \otimes f$$

$$S_y = \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

$$\frac{\partial f}{\partial y} = S_y \otimes f$$

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

Frame Difference

t

1	1	1	1	1
1	1	1	1	1
1	10	10	10	10
1	10	10	10	10
1	10	10	10	10
1	10	10	10	10

$t + 1$

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	10	10	10
1	1	10	10	10
1	1	10	10	10

-

=

$$I_t = \frac{\partial I}{\partial t}$$

0	0	0	0	0
0	0	0	0	0
0	9	9	9	9
0	9	0	0	0
0	9	0	0	0
0	9	0	0	0

(Example of a forward difference)

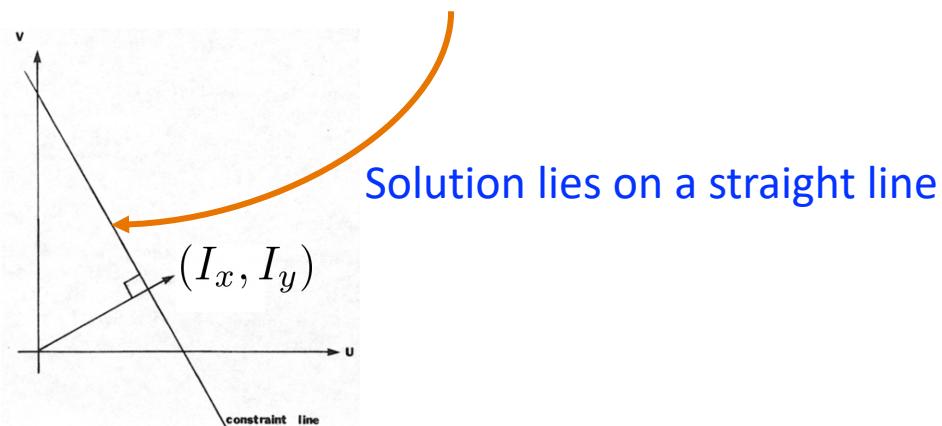
Brightness Constancy Constraint

$$I_x u + I_y v + I_t = 0$$

Known (spatial and temporal gradients)

Unknown (optical flow)

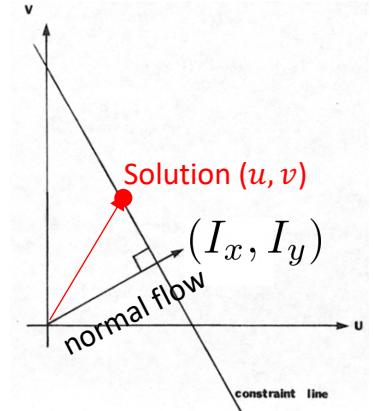
- For each pixel, there are two unknowns



The solution cannot be determined uniquely with a single constraint (a single pixel)

Brightness Constancy Constraint

$$I_x u + I_y v + I_t = 0$$



- The component of the flow vector in the gradient direction is determined (called normal flow) (Recall vector projection geometry)

$$\frac{1}{\sqrt{I_x^2 + I_y^2}} (I_x, I_y) \cdot (u, v) = \frac{-I_t}{\sqrt{I_x^2 + I_y^2}}$$

- The component of the flow vector orthogonal to this direction cannot be determined.

https://en.wikipedia.org/wiki/Dot_product

Lucas-Kanade Method

$$I_x u + I_y v + I_t = 0$$

Assumption: the flow is constant in a local neighborhood of a pixel under consideration

Use two or more pixels to compute optical flow 5x5 window

$$\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ I_x(p_2) & I_y(p_2) \\ \vdots & \vdots \\ I_x(p_{25}) & I_y(p_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(p_1) \\ I_t(p_2) \\ \vdots \\ I_t(p_{25}) \end{bmatrix}$$

A
 25×2

d
 2×1

b
 25×1

Lucas-Kanade Method

Solve the least squares problem

$$\begin{matrix} A & d = b \\ 25 \times 2 & 2 \times 1 & 25 \times 1 \end{matrix} \longrightarrow \text{minimize } \|Ad - b\|^2$$

$$(A^T A) \begin{matrix} d \\ 2 \times 2 \end{matrix} = A^T b \begin{matrix} 2 \times 1 \\ 2 \times 1 \end{matrix}$$

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

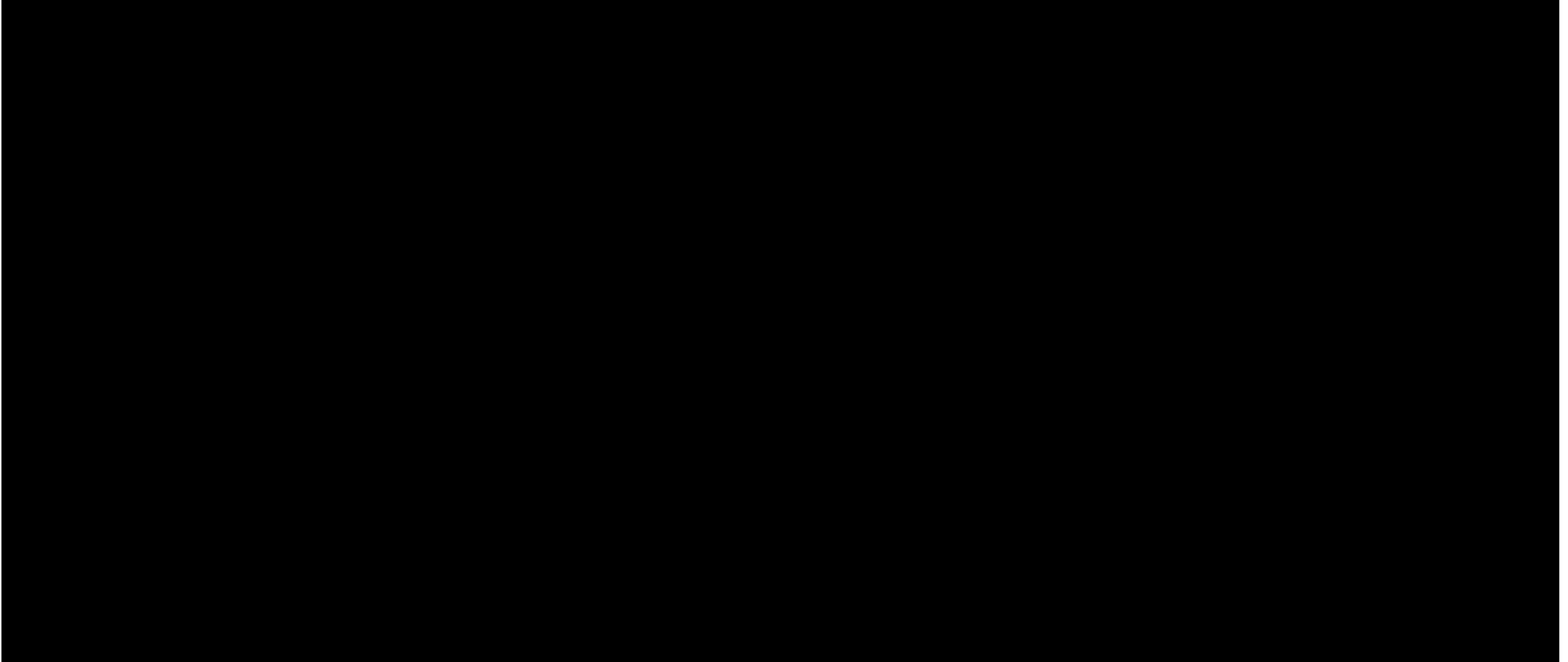
$$A^T A \qquad \qquad \qquad A^T b$$

https://en.wikipedia.org/wiki/Proofs_involving_ordinary_least_squares#Least_squares_estimator_for_.CE.B2

Optical Flow Example



Video Demo



Source: <http://clim.inria.fr/Datasets/SyntheticVideoLF/>

Video Demo



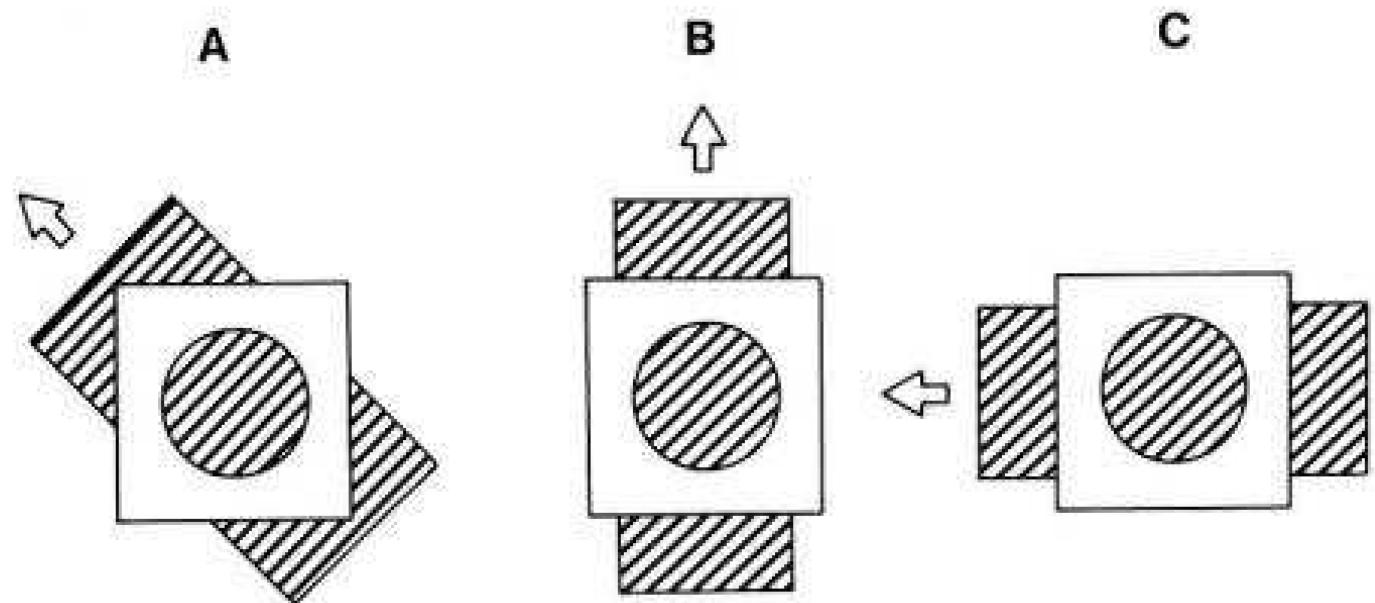
Source: <http://clim.inria.fr/Datasets/SyntheticVideoLF/>

Aperture Problem in Optical Flow Estimation

Motion detectors are local

Our visual system infers the global motion

The aperture problem



Horn–Schunck method introduces a global constraint of smoothness to solve the problem [\[wiki\]](#)

Next Lecture

- Deep neural networks for optical flow estimation
- Applications of optical flow

Further Reading

Lucas–Kanade method

https://en.wikipedia.org/wiki/Lucas%E2%80%93Kanade_method

Determine Constant Optical Flow, Berthold K.P. Horn

https://people.csail.mit.edu/bkph/articles/Fixed_Flow.pdf