

Q1: In a country there are several airports. Airport A is directly connected to 23 other airports. Airport B has a direct connection to 3 other airports. Each airport, except A and B, is directly connected to 10 other airports. Prove that there is an airline route (maybe with flight changes) between A and B.

Ans: If not, they are in different connected components. That means A and B are in different graph. They are may or may not connected with other airports which degree equals to 10. Each of the components violates the handshaking lemma, having exactly one odd vertex. (Handshaking Lemma: The number of people who shook hands with and odd number of other people is even.)