

Does there exist a graph with 5 vertices which have the following degrees: 2, 4, 4, 4, 4?

Ans: Yes.

1) Edges perspective: The upper bound edge number for a graph with n vertices is $n * (n-1) / 2$ and lower bound edge number is $n - 1$ (tree). In this case, the number of edges should be greater than 4 and less than 10. Sum of degree = 18. Number of edges = 9. The number of edges is greater than $4 * 3 / 2 = 6$, the graph is connected/ exists. No problem in this aspect.

2) No problem in handshaking Lemma.

3) Havel- Hakimi Theorem (which is applicable to simple graphs). Because there is no -1 appears in the whole process. No problem in this aspect.

Order: 4,4,4,4,2

Delete: 3,3,3,1

Delete: 2,2,0

Delete: 1,0

Delete: 0

```
bool Havel_Hakimi(){
    for(int i=0; i<n-1; ++i){
        sort(arr+i, arr+n, greater<int>());
        if(i+arr[i] >= n) return false;
        for(int j=i+1; j<=i+arr[i]; ++j){
            --arr[j];
            if(arr[j] < 0) return false;
        }
    }
    if(arr[n-1] != 0) return false;
    return true;
}
```

Hence, the graph exists.