Mortgage Backed Valuation

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OAS calculations, Monte Carlo, OAS server and Linux cluster work by Alexander Belikoff, Kirill Levin, Harvey Stein, and Xusheng Tian, Quantitative Finance R&D group, Bloomberg.
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OAS application by Sean Dai, Mortgage group, Bloomberg.
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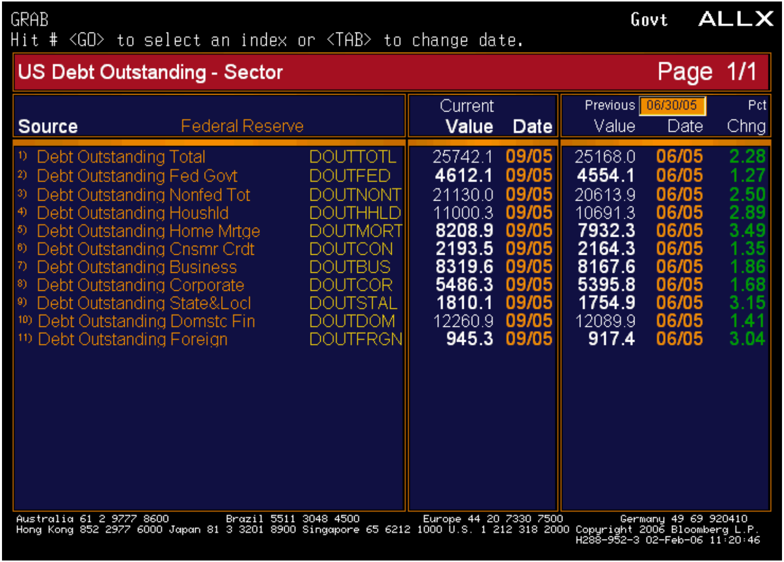
Outline

Size of mortgage market

- *The Case for U.S. Mortgage-Backed Securities for Global Investors*, Michael Wands, CFA, Head of U.S. Fixed Income, Global Fixed Income, State Street Global Advisors:
  - Lehman U.S. Aggregate Index - 2002
    - MBS — 35%
    - U.S. Credit — 27%
    - U.S. Treasury — 22%
    - U.S. Agency — 12%
    - ABS — 2%
    - CMBS — 2%
- *The U.S. Mortgage Market, Fannie Mae, and Freddie Mac — An IMF Study*:
  - March 2003 — $3.2 trillion in mortgage-backed issuance by Fannie and Freddie.

Outstanding debt



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Hit # <GO> to select an index or <TAB> to change date.

US Debt Outstanding - Sector Page 1/1

Source Federal Reserve

Source: Federal Reserve		Current Value	Current Date	Previous (06/30/05) Value	Previous Date	Pct Chng
1) Debt Outstanding Total	DOUTTOTL	25742.1	09/05	25168.0	06/05	2.28
2) Debt Outstanding Fed Govt	DOUTFED	4612.1	09/05	4554.1	06/05	1.27
3) Debt Outstanding Nonfed Tot	DOUTNONT	21130.0	09/05	20613.9	06/05	2.50
4) Debt Outstanding Houshld	DOUTHHLD	11000.3	09/05	10691.3	06/05	2.89
5) Debt Outstanding Home Mrtge	DOUTMORT	8208.9	09/05	7932.3	06/05	3.49
6) Debt Outstanding Cnsmr Crdt	DOUTCON	2193.5	09/05	2164.3	06/05	1.35
7) Debt Outstanding Business	DOUTBUS	8319.6	09/05	8167.6	06/05	1.86
8) Debt Outstanding Corporate	DOUTCOR	5486.3	09/05	5395.8	06/05	1.68
9) Debt Outstanding State&Locl	DOUTSTAL	1810.1	09/05	1754.9	06/05	3.15
10) Debt Outstanding Domstc Fin	DOUTDOM	12260.9	09/05	12089.9	06/05	1.41
11) Debt Outstanding Foreign	DOUTFRGN	945.3	09/05	917.4	06/05	3.04

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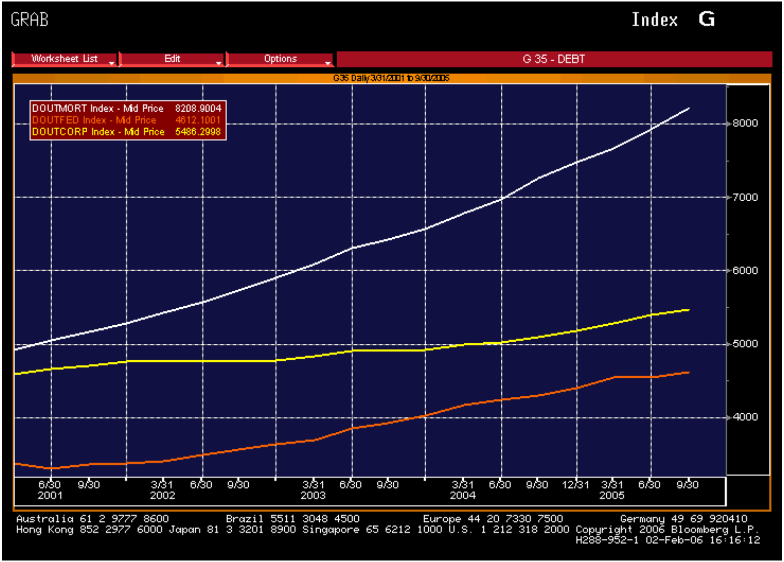
Size over time



MBS issuance

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Bloomberg MBS

AGENCY MBS POOL ISSUANCE Page 2 of 4

$ Billion Most Recent

	Jan06	Dec	Nov	Oct	Sep	Aug	Jul	Jun	May	Apr	Mar	YTD '06	'05	'04
TOTAL	87	82	87	89	116	94	86	78	75	81	66	87	987	1009
Issuer														
FHLMC	35.6	34.2	37.7	34.7	42.4	36.9	28.6	29.7	29.6	32.0	25.9	36	378	357
FNMA	45.5	42.1	42.7	47.4	65.2	50.3	49.2	41.8	37.7	41.9	33.5	46	523	527
GNMA1	2.7	2.9	3.3	3.7	4.4	3.8	4.1	3.4	3.7	3.6	3.2	3	42	58
GNMA2	3.6	2.5	3.1	3.7	3.8	3.4	4.3	3.6	3.9	3.8	3.6	4	44	68
Loan Type														
30yr	71.9	65.5	73.0	72.6	84.9	74.4	65.8	57.5	54.3	56.8	44.9	72	732	611
15yr	5.6	6.0	3.9	4.6	6.2	6.8	5.6	5.4	6.5	7.3	7.0	6	74	150
ARM	7.9	7.3	6.8	9.1	21.0	9.8	11.5	12.7	11.2	13.5	11.0	8	141	188
Other	1.9	3.0	3.1	3.1	3.7	3.2	3.3	2.9	2.9	3.7	3.2	2	38	61
Coupon (30yr Fixed)														
<4.5-	3.9	3.1	4.6	3.5	2.3	3.2	4.5	2.2	2.1	4.1	2.3	4	32	1
4.5-	1.6	1.8	3.0	6.5	13.0	8.6	4.2	2.9	3.1	3.2	3.5	2	54	14
5.0-	15.3	20.5	25.7	26.5	39.3	35.3	33.5	25.9	21.0	23.8	14.2	15	293	174
5.5-	26.0	25.0	29.0	29.2	24.3	21.4	18.0	19.3	21.4	20.8	20.7	26	272	273
6.0-	20.5	12.3	8.7	5.8	4.9	5.0	4.8	6.0	5.7	4.1	3.1	20	67	119
6.5-	3.7	2.1	1.4	.8	.7	.5	.5	.7	.7	.6	.8	4	10	22
7.0-	.9	.7	.5	.3	.3	.3	.2	.3	.2	.2	.2	1	4	5
>7.5	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1	.2	0	1	2

CMO issuance

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Bloomberg CMO

CMO ISSUANCE

$ Billion 2004-2006

Page 2 of 7

	Jan 06	Dec	Nov	Oct	Sep	Aug	Jul	Jun	May	Apr	Mar	Feb	YTD '06	'05	'04
TOTAL	38.9	100	78.4	93.4	119	95.3	89.5	91.8	88.2	64.3	85.8	79.1	39	1051	822
Collateral Source															
FHLMC	15.2	7.9	16.5	19.9	13.9	16.4	25.7	13.6	11.9	14.6	16.8	18.0	15	191	214
FNMA	10.7	7.5	6.6	9.2	19.3	8.4	9.4	10.6	11.9	4.1	12.0	9.7	11	116	81
GNMA	.3	2.5	1.3	2.4	6.1	3.3	2.1	2.8	1.5	1.4	3.0	3.0	0	32	48
WHOLE	12.8	82.4	54.0	61.9	79.7	67.2	52.2	64.8	62.9	44.1	53.9	48.3	13	713	479
Collateral Type															
30yr	23.0	34.6	41.7	48.3	55.3	42.6	45.3	38.9	39.5	29.1	47.5	42.3	23	500	485
15yr	-	.2	.2	.4	.2	.2	2.7	.1	-	3.1	.2	.5	-	8	7
ARM	15.2	63.9	30.3	44.3	62.4	48.1	39.9	51.3	47.8	27.7	36.8	31.5	15	514	299
Other	.7	1.6	6.2	.4	1.1	4.4	1.5	1.4	.9	4.3	1.3	4.8	1	30	30
Collateral WAC (Fixed Rate)															
<5.5	.3	2.3	6.4	1.4	2.4	4.3	4.5	1.3	1.6	4.7	2.2	9.7	0	54	130
5.5-	7.6	11.2	26.2	33.2	36.0	28.9	33.6	16.4	26.7	21.4	38.1	23.6	8	306	255
6.0-	14.4	16.6	13.2	13.2	13.5	12.1	10.4	20.5	10.3	10.0	6.6	10.1	14	147	97
6.5-	1.0	4.7	1.5	.8	4.1	.5	1.0	1.3	.7	-	1.0	3.6	1	20	26
7.0-	.3	1.0	-	-	-	1.0	-	.3	.7	.3	.1	.4	0	6	11
7.5-	-	-	-	-	.2	-	-	-	-	-	-	-	-	0	1
8.0-	-	-	-	-	-	-	-	.2	-	-	.9	-	-	1	-
>8.5	-	.6	.8	.5	.4	.4	-	.5	.3	.2	-	.2	-	4	3

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Mortgage market structure:

- Mortgages
- MBS pools
- CMOs

The subatomic particles - mortgages

People take out loans (mortgages) to buy homes.

- Fixed rate mortgages — fixed coupon, monthly payments, self amortizing, paying principal down to zero at maturity (15-30 years).
- Balloons – amortize on a 30 year basis, but expire in 5 or 7 years with payment of the remaining outstanding balance.
- Adjustable rate mortgages (ARMS) — floating coupon based on an index (LIBOR, treasury rates, . . .), typically with protection clauses against overly large coupon changes (lifetime and periodic caps, annual resets, . . .).

Low interest rates in recent years have sparked innovation — floating rate balloons with interest only payments, various sorts of built in protections, option ARMs, . . .

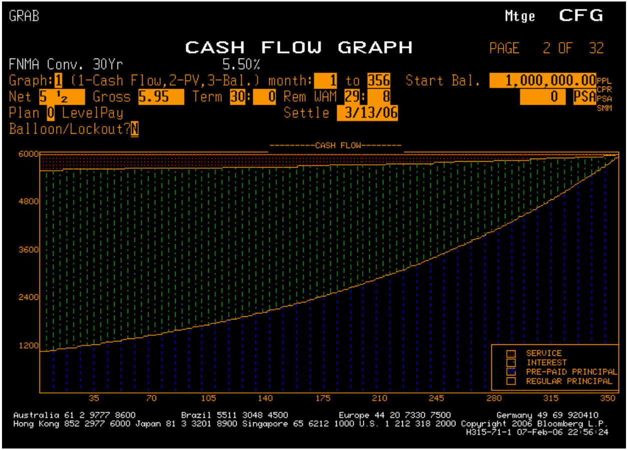
Mortgage behavior

While it looks like ARM valuation might require some work, in that they have embedded caps and ratchets and potentially float on a CMS rate, one might think that at least fixed rate mortgages would be easily valued.
With fixed monthly payments, monthly interest payments at a rate of C on the outstanding balance, N monthly payments and an initial balance of B, then monthly payments are:

$$\frac{C(1+C)^N B}{(1+C)^N - 1}.$$

Mortgage cash flows

Graphically, our cash flows look like:



so why not just discount and be done?

The painful intervention of reality

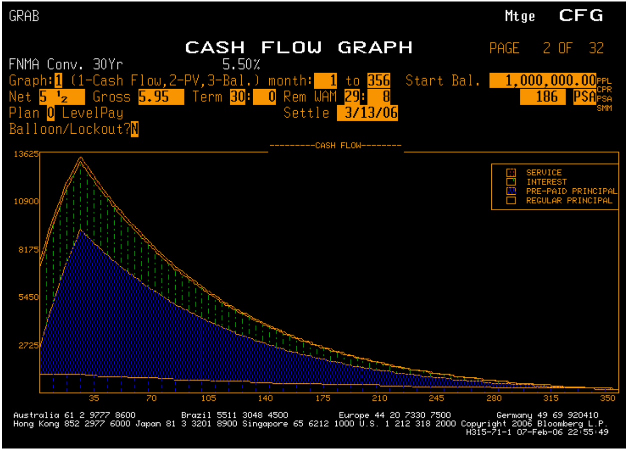
The problem with discounting the scheduled payments is that

- People move,
- refinance,
- default,
- make excess payments to pay off principal faster.

So, principal arrives randomly, or perhaps not at all (in default of uninsured loans).
Payments above the scheduled payments pay down the principal and are known as *prepayment*.
One of the major components of mortgage analysis is in modeling prepayment behavior.

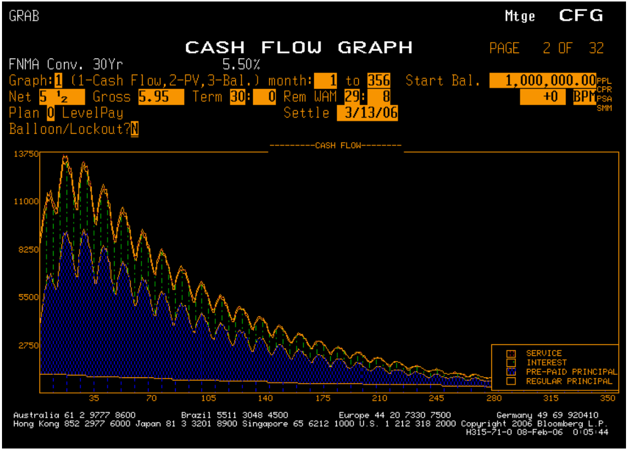
Impact of prepayment on cash flows

Thus, our simple mortgage, instead of having fixed cash flows, has cash flows that are dependent on prepayment rates. Under one prepayment assumption, we have:



Impact of prepayment on cash flows

Under a model that projects prepayments based on interest rates and loan characteristics, we see a different cash flow structure for each rate path. One example is:



The atoms — MBS Pools

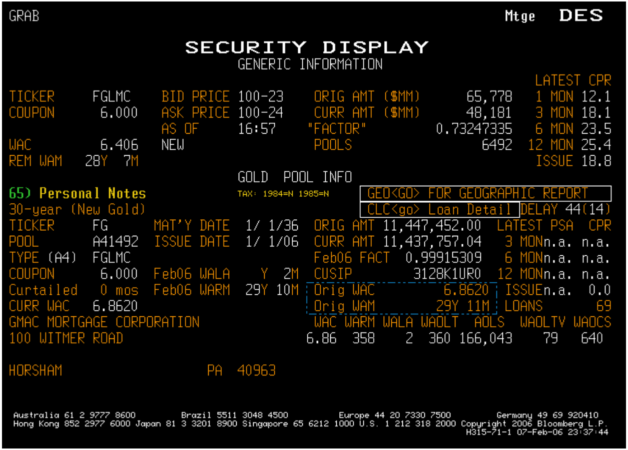
In 1938, the collapse of the national housing market led to the federal government's formation of the *Federal National Mortgage Association*, AKA "Fannie Mae". Now, we have Fannie Mae, Freddie Mac (*The Federal Home Loan Mortgage Corporation*), and Ginnie Mae *The Government National Mortgage Association*.

- Banks make mortgages.
- Government insures *conforming* mortgages.
- Agencies buy conforming mortgages.
- Banks have money to make more mortgages.
- Agencies sell shares of mortgages on secondary market.

Ginnie, Fannie and Freddie — three sets of rules for conforming loans.

MBS secondary market - pools

The agencies (Fannie, Freddie and Ginnie) buy mortgages, pool them together into MBS pools and sell shares. Pools are pass-through securities, in that the cash flows from the underlying collateral is passed through to the shareholder, minus a service fee.



MBS pool fine characteristics - Geographic data

These days, there's substantial information available about pool composition, such as geographic data:

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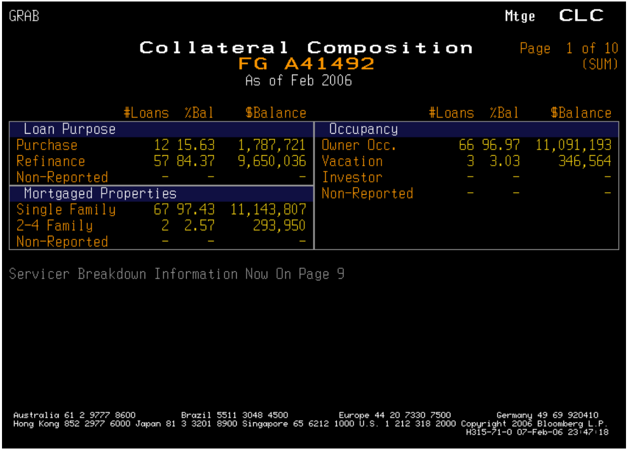
FHLMC GEOGRAPHIC DISTRIBUTION TABLE

POOL NUMBER: A41492 PAGE 1 OF 2

(S)tate or (Y)ear of origination? S

STATE	ORIG BALANCE	% OF TOTAL ORIG BAL	ORIG LOANS	% OF TOTAL ORIG LOANS	% OF TOTAL CURR BAL	CURR LOANS	% OF CURR LOANS
AL	192,638.02	1.68	2	2.90	1.68	2	2.90
AR	269,773.17	2.36	1	1.45	2.36	1	1.45
AZ	1,080,860.01	9.44	7	10.14	9.44	7	10.14
CA	800,824.58	7.00	4	5.80	7.00	4	5.80
CT	906,204.00	7.92	4	5.80	7.92	4	5.80
FL	1,427,623.42	12.47	8	11.59	12.47	8	11.59
IA	81,533.11	0.71	1	1.45	0.71	1	1.45
ID	119,894.12	1.05	1	1.45	1.05	1	1.45
IL	712,867.56	6.23	5	7.25	6.23	5	7.25
IN	76,935.31	0.67	1	1.45	0.67	1	1.45
KS	131,889.10	1.15	1	1.45	1.15	1	1.45
MI	471,994.57	4.12	4	5.80	4.12	4	5.80
MN	186,147.29	1.63	1	1.45	1.63	1	1.45
NC	175,452.47	1.53	1	1.45	1.53	1	1.45
ND	50,355.53	0.44	1	1.45	0.44	1	1.45
NE	71,080.03	0.62	1	1.45	0.62	1	1.45

MBS pool fine characteristics - Loan purpose



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Collateral Composition
FG A41492
As of Feb 2006

Page 1 of 10 (SUM)

	#Loans	%Bal	$Balance
Loan Purpose			
Purchase	12	15.63	1,787,721
Refinance	57	84.37	9,650,036
Non-Reported	-	-	-
Mortgaged Properties			
Single Family	67	97.43	11,143,807
2-4 Family	2	2.57	293,950
Non-Reported	-	-	-

	#Loans	%Bal	$Balance
Occupancy			
Owner Occ.	66	96.97	11,091,193
Vacation	3	3.03	346,564
Investor	-	-	-
Non-Reported	-	-	-

Servicer Breakdown Information Now On Page 9

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MBS pool fine characteristics - Loan to value ratio and credit ratings

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Collateral Composition
FG A41492

Page 2 of 10
(QRT)

Quartile Distributions as of Feb 2006

	Minimum	25%	Median	75%	Maximum	Wgtd. Avg
Note Rate (%)	6.250	6.875	6.875	7.000	7.000	6.862
Rem Mty (mths)	356	358	358	359	359	358
Loan Age (mths)	1	1	2	2	4	2
Orig Term (mths)	360	360	360	360	360	360
OLS ($bal)	39,000	141,000	190,000	240,000	345,000	194,614*
OLTV (%)	57	74	80	86	95	79
OCS	537	611	640	661	767	640

* Simple Average OLS = 166,043

MBS pool fine characteristics - loan rate distribution

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Collateral Composition Page 3 of 10

FG A41492 (WAC)

LRT WAVG: 6.86
LRT SDEV: 0.20

Collateral Loan Rate Distribution as of Issuance

LOAN RATE (PERCENT)	# LNS	BAL $MIL	% BAL
6.250- 6.374	5	.9	7.6
6.625- 6.749	6	.9	8.2
6.875- 6.999	28	4.6	40.1
7.000	30	5.0	44.1
TOTAL	69	11.4	100.0

MBS pool fine characteristics LTV distribution

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Collateral Composition Page 4 of 10

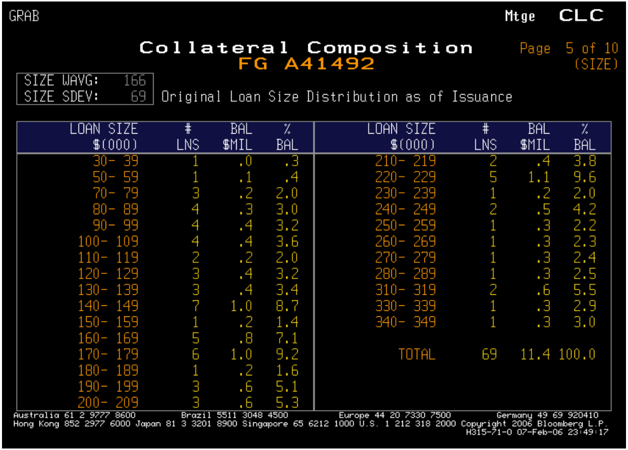
FG A41492 (LTV)

LTV WAVG: 79.0
LTV SDEV: 8.8

Amortized LTV Distribution as of Issuance

AMLTV (%)	# LNS	BAL $MIL	% BAL
50- 59	1	.2	1.9
60- 69	9	1.5	13.1
70- 74	8	1.8	15.9
75- 79	8	1.4	12.0
80- 84	25	3.5	30.7
85- 89	7	1.3	11.1
90- 94	5	.9	7.6
95- 99	6	.9	7.8
100	0	.0	.0
TOTAL	69	11.4	100.0

MBS pool fine characteristics
Loan size distribution



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Collateral Composition Page 5 of 10

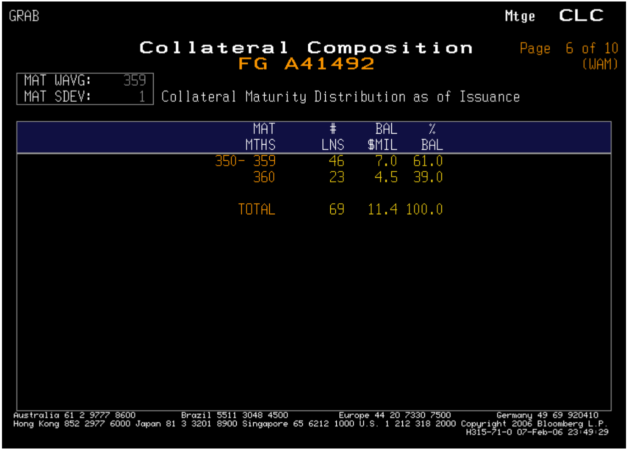
FG A41492 (SIZE)

SIZE WAVG: 166
SIZE SDEV: 69

Original Loan Size Distribution as of Issuance

LOAN SIZE $(000)	# LNS	BAL $MIL	% BAL	LOAN SIZE $(000)	# LNS	BAL $MIL	% BAL
30- 39	1	.0	.3	210- 219	2	.4	3.8
50- 59	1	.1	.4	220- 229	5	1.1	9.6
70- 79	3	.2	2.0	230- 239	1	.2	2.0
80- 89	4	.3	3.0	240- 249	2	.5	4.2
90- 99	4	.4	3.2	250- 259	1	.3	2.2
100- 109	4	.4	3.6	260- 269	1	.3	2.3
110- 119	2	.2	2.0	270- 279	1	.3	2.4
120- 129	3	.4	3.2	280- 289	1	.3	2.5
130- 139	3	.4	3.4	310- 319	2	.6	5.5
140- 149	7	1.0	8.7	330- 339	1	.3	2.9
150- 159	1	.2	1.4	340- 349	1	.3	3.0
160- 169	5	.8	7.1				
170- 179	6	1.0	9.2	TOTAL	69	11.4	100.0
180- 189	1	.2	1.6				
190- 199	3	.6	5.1				
200- 209	3	.6	5.3				

MBS pool fine characteristics
Maturity distribution



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Collateral Composition Page 6 of 10

FG A41492 (WAM)

MAT WAVG: 359
MAT SDEV: 1

Collateral Maturity Distribution as of Issuance

MAT MTHS	# LNS	BAL $MIL	% BAL
350- 359	46	7.0	61.0
360	23	4.5	39.0
TOTAL	69	11.4	100.0

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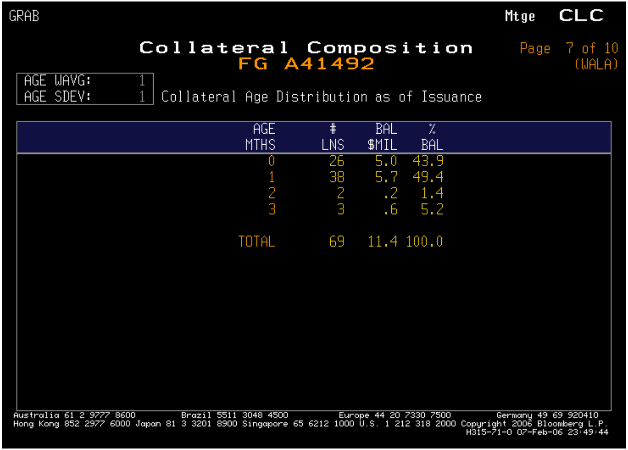
MBS pool fine characteristics Age distribution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Collateral Composition Page 7 of 10

FG A41492 (WALA)

AGE WAVG: 1
AGE SDEV: 1

Collateral Age Distribution as of Issuance

AGE MTHS	# LNS	BAL $MIL	% BAL
0	26	5.0	43.9
1	38	5.7	49.4
2	2	.2	1.4
3	3	.6	5.2
TOTAL	69	11.4	100.0

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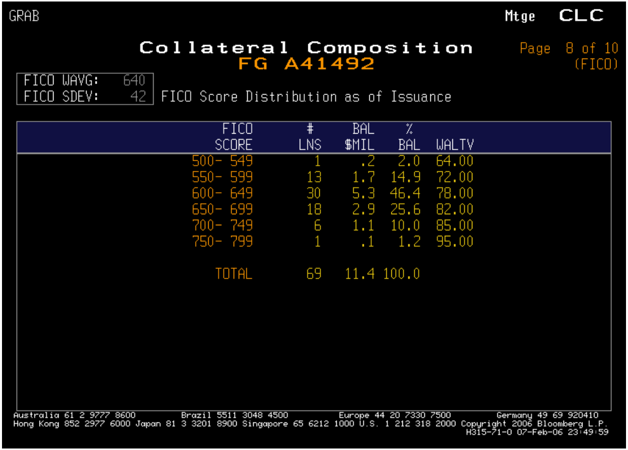
MBS pool fine characteristics
Credit rating distribution



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Collateral Composition
FG A41492

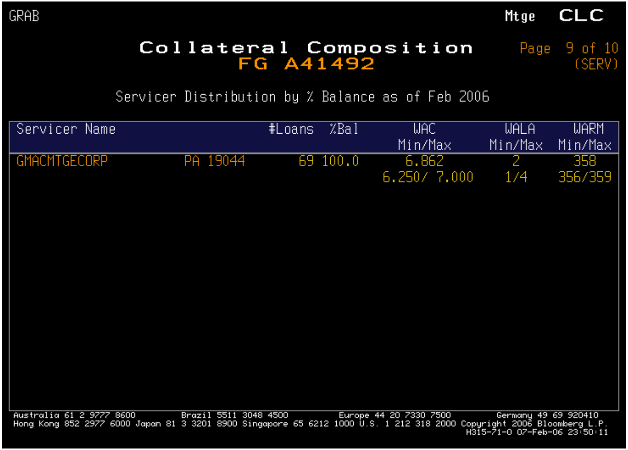
Page 8 of 10 (FICO)

FICO WAVG: 640
FICO SDEV: 42

FICO Score Distribution as of Issuance

FICO SCORE	# LNS	BAL $MIL	% BAL	WALTV
500- 549	1	.2	2.0	64.00
550- 599	13	1.7	14.9	72.00
600- 649	30	5.3	46.4	78.00
650- 699	18	2.9	25.6	82.00
700- 749	6	1.1	10.0	85.00
750- 799	1	.1	1.2	95.00
TOTAL	69	11.4	100.0	

MBS pool fine characteristics Servicers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Collateral Composition Page 9 of 10
FG A41492 (SERV)

Servicer Distribution by % Balance as of Feb 2006

Servicer Name		#Loans	%Bal	WAC Min/Max	WALA Min/Max	WARM Min/Max
GMACMTGECORP	PA 19044	69	100.0	6.862 6.250/ 7.000	2 1/4	358 356/359

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MBS pool fine characteristics
Sellers

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Collateral Composition
FG A41492

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(SELL)

Seller Distribution by % Balance as of Feb 2006

Seller Name		#Loans	%Bal	WAC Min/Max	WALA Min/Max	WARM Min/Max
GMACMTGECORP	PA 19044	69	100.0	6.862 6.250/ 7.000	2 1/4	358 356/359

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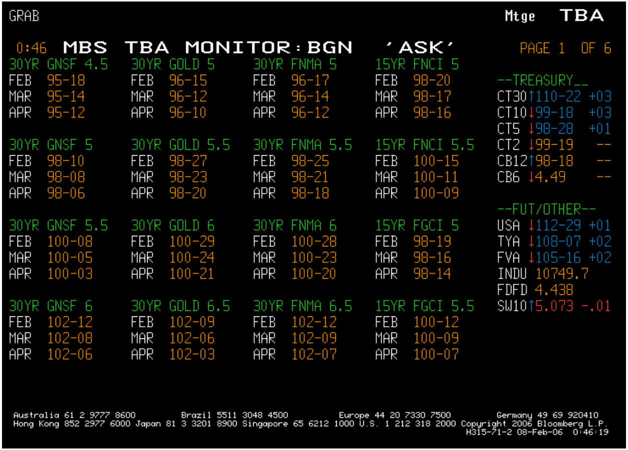
MBS secondary market - TBAs

Pools are also sold before being created, trading as TBAs:



Pool analysis

Pool analysis is like mortgage analysis, except that it benefits from safety in numbers.

- Backed by a substantial number of individual loans, so variance is reduced.

While it used to be the case that this was at the cost of only knowing gross aggregate data about the pool, these days the fine structure of the pool is often disclosed as well, telling us:

- Location of individual loans,
- Size of individual loans,
- LTV,
- and credit ratings.

The only thing missing is individual borrower details.

The molecules — CMOs

In 1983, Solomon Brothers and First Boston created the first *Collateralized Mortgage Obligation* (CMO).
They realized that more pools could be sold if the pool cash flows were carved up to stratify risk.
CMOs are:

- Backed by pools or directly by mortgages (whole loans), sometimes by as many as 20,000 of them.
- Split up cash flows of underlying collateral into a number of "bonds" or "tranches".
- By creating desirable risk structures, tranches can be sold to a wider audience, and at a profit.

CMOs are essentially arbitrary structured notes backed by mortgage collateral.

Tranche types

Tranches vary by how the principal and interest are carved up.

- Interest handling:
  - Fixed cpn — Can behave like a pool or very differently, depending on how principal is paid.
  - POs — Only principal payments from underlying collateral.
  - IOs — Only interest payments.
  - Floaters — Where there's a floater, there's an inverse floater (when you have fixed rate collateral).
  - Inverse floaters.
- Principal handling:
  - Sequential Pay — Sequence of tranches. First gets principal until paid, then 2nd gets principal, etc. Last one is most prepayment protected and behaves most like an ordinary bond.
  - PACs — Scheduled principal will be payed as long as prepayment remains in a specified band.
  - TACs — Scheduled principal will be payed when prepayment is at a specified level.
  - Etc.

Example CMO

GRAB Mtge

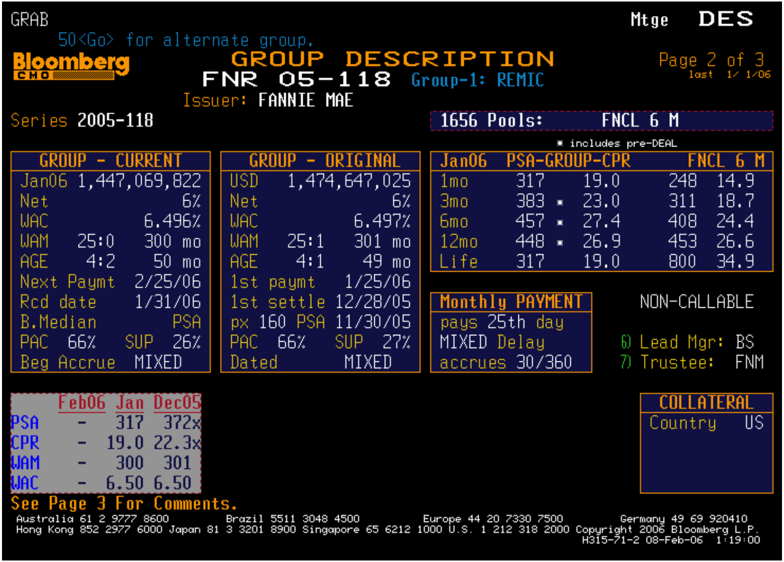
CMO/ABS SECURITIES Pg 1 of 4

All Classes for FNR 2005-118 FANNIE MAE

	Class	Orig Amt (000s)	Coupon	Orig WAL	Orig Maturity	CUSIP	GRADE	Description
* 1)	CI	127,338	6.000	2.50	1/25/32	31394VRE2	--	SC,IO,NTL
* 2)	DI	20,761	6.000	6.00	1/25/32	31394VRF9	--	SC,IO,NTL
* 3)	FA	362,762	5.370	12.10	1/25/32	31394VRG7	-	SC,FLT,DLY,SUP
* 4)	FI	96,555	4.930	4.00	1/25/32	31394VRH5	-	SC,IO,FLT,NTL
* 5)	FO	96,555	0.000	4.00	1/25/32	31394VRJ1	12AC	SC,PO,AD,+
* 6)	GS	11,277	13.560	17.50	1/25/32	31394VRK8	z46EE	SC,INV,DLY,SUP
* 7)	IJ	76,208	6.000	16.30	1/25/32	31394VRL6	--	SC,IO,NTL
* 8)	JO	76,208	0.000	16.30	1/25/32	31394VRM4	z32AB	SC,PO,PAC(11)
* 9)	PM	124,568	5.000	6.00	1/25/32	31394VRN2	5DC	SC,PAC(11)
* 10)	PN	153,898	6.000	8.00	1/25/32	31394VRP7	5ED	SC,PAC(11)
* 11)	PR	104,883	6.000	11.00	1/25/32	31394VRQ5	7ED	SC,PAC(11)
* 12)	PY	509,350	4.500	2.50	1/25/32	31394VRR3	2CC	SC,PAC(11)
* 13)	SA	8,750	13.560	12.10	1/25/32	31394VRS1	z36EE	SC,INV,DLY,SUP
* 14)	SC	10,203	13.560	6.10	1/25/32	31394VRT9	z19EE	SC,INV,DLY,SUP
* 15)	SI	16,093	12.420	4.00	1/25/32	31394VRU6	16EE	SC,IO,INV,NTL
* 16)	SO	16,093	0.000	4.00	1/25/32	31394VRV4	12AC	SC,PO,AD,+
* 17)	ZA	100	6.000	10.40	1/25/32	31394VRW2	6EE	SC,Z,PAC(22)
* 18)	CA	25,089	6.000	3.90	10/25/33	31394VRX0	2ED	SC,PAC(22)

Example CMO tranche

Example CMO collateral



CMO modeling

Computing the cash flows of a CMO tranche requires modeling the CMO, i.e. — converting the prospectus into a mathematical specification of the tranche payouts as a function of collateral cash flows.

CMO cash flow engine

The cash flow engine computes CMO cash flows for each scenario.
Inputs:

- CMO deal specification.
- Index projections.
- Current outstanding balance of each piece of collateral.

Given the above inputs, the cash flow engine parses and evaluates the deal specification, using the cash flows generated from running the prepayment model on each piece of collateral and amortizing it.

Prepayment modeling

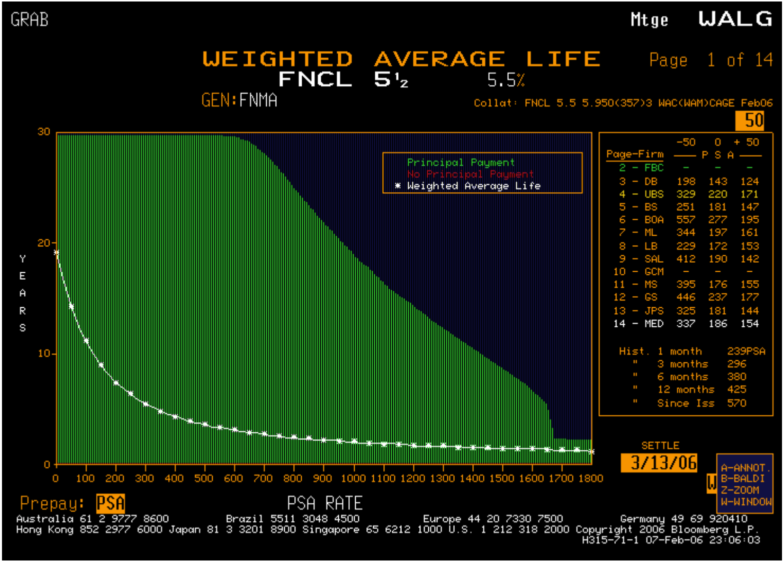
Prepayment speeds

Prepayment speeds are to prepayment modeling what yield calculations are to interest rate modeling.

- SMM — Percentage of remaining balance above scheduled paid: $100\frac{P'_i - P_i}{B_i}$.
- CPR — SMM annualized: $100(1-(1-\frac{\mathrm{SMM}}{100})^{12})$.
- PSA — "Prepayment Speed Assumption": 0.2% initially, increasing by 0.2% each month for the first 30 months, and 6.0% until the loan pays off. 200 PSA is double this rate, etc.
- MHP — PSA for manufactured housing (ABS, not MBS).

Looking at the value of a pool or CMO as a function of prepayment level is a useful analysis tool.

Prepayment speed graph



Prepayment modeling

Prepayment modeling is a major component of MBS and CMO valuation. In some sense, CMO and MBS valuation is Monte Carlo analysis of the prepayment model.

In prepayment modeling:

- Salient features of prepayment are proposed.
- Evidence is collected statistically.
- Models are developed for these relationships.

Major prepayment components

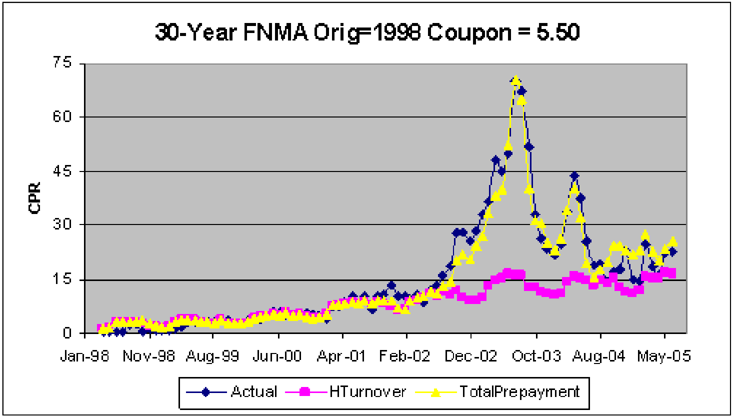
- Housing turnover.
- Refinancing.
- Curtailment and Default.

Housing turnover

- Appears as relatively constant baseline level of prepayment = total (existing) home sales divided by total housing stock.
- Seasonality — Less movement in the winter.
- Seasoning — Chances of moving increase with age of mortgage, but tend to level off. A function of WAM, loan type, and prepayment incentive.
- Lock-in effect — High rates relative to mortgage coupon are a disincentive to moving when LTV is high.
- Rarely over 10% CPR.

Housing turnover illustration

Housing turnover behavior typically dominates prepayment when rates are low.



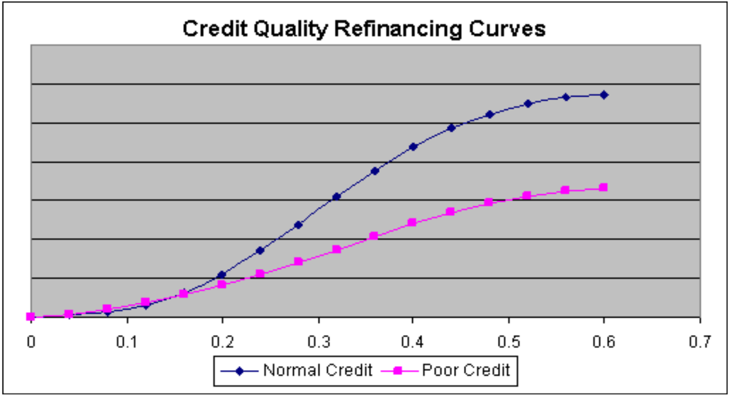
Refinancing

Refinancing is the major interest rate dependent component.

- "S-curve" — Zero when rates are above the mortgage coupon, picks up as rates drop and tops out at some maximum level (not everyone can refinance).
- Aging — New loans less likely to be refinanced due to refi costs, but mitigated by high refi incentive.
- Credit quality — Lower credit quality less likely to refi. Can use level of cpn above mortgage rates at issuance in lieu of FICO score.
- Burnout — As borrowers refinance out of a pool, the remaining borrowers are less likely to refinance (unaware or unable).
- Media effect — Prepayments tend to surge around multi-year lows, presumably induced by press coverage encouraging refinancing.
- Pipeline effect — Prepayment rate spikes are asymmetric. Prepayment drops slower than it grew. Due to mortgage broker capacity limits causing refi applications to back up.
- Lagged effects.

Refinancing S-curve

Sample refi S-curves for normal credit and poor credit:



Curtailment and Default

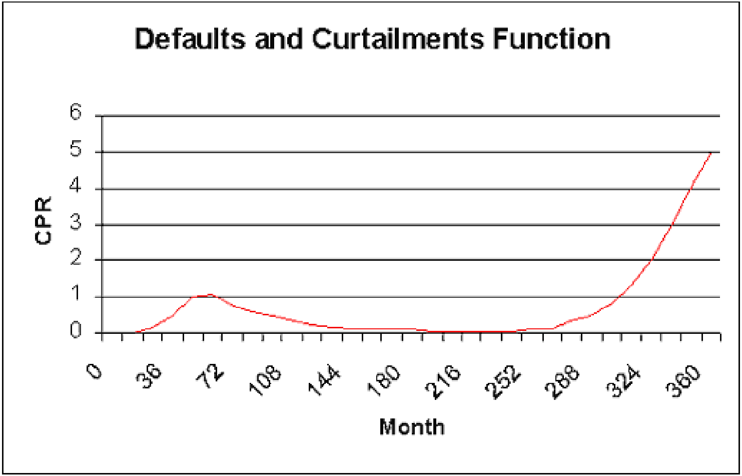
Default results in return of outstanding principal when property is sold.

Curtailment is the reduction in maturity due to additional partial payment of principal. Without reamortization, results in additional principal pay downs on a monthly basis. Can also be from paying off remainder of an old mortgage.

- Tend to be independent of interest rates.
- Default risk grows and then drops as LTV decreases.
- Curtailment picks up towards end of mortgage life.

Curtailment and Default graph

These considerations lead to the following typical curtailment and default graph.



Loan level characteristics

Increased disclosure allows improved prepayment modeling.
Primary loan attributes:

- LTV
- FICO score
- loan size

Secondary loan attributes:

- occupancy
- property type
- loan purpose

Yield calculations

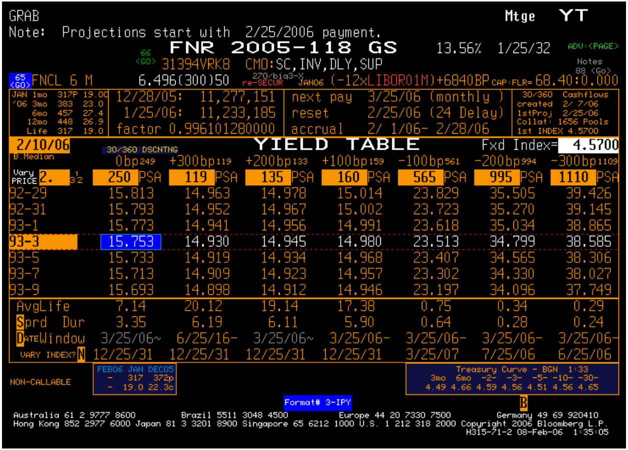
Yield calculations

Internal rate of return at a given price assuming a particular prepayment rate.

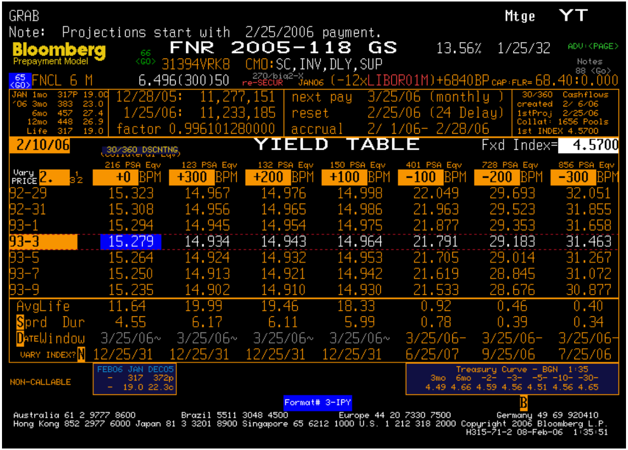
- Pick a prepayment speed.
- Generate cash flows.
- Solve for internal rate of return that gives quoted price.

Conceptually simple, but computationally intensive and involved when trying to value a CMO backed by 20,000 pools, with cash flows carved up across 100 tranches.

Yield example — PSA



Yield example — BPM



OAS Analysis

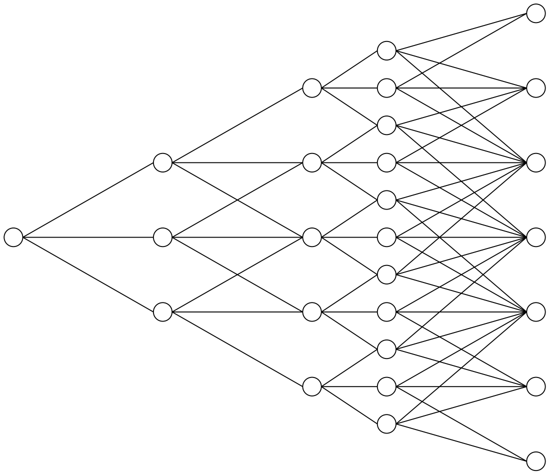
From Yield to OAS

Consider a semi-annual bond with cash flows C_i at times t_i (with principal payment included in C_N) and (dirty) price P. When the bond is not callable, OAS is basically Z-spread, which is basically a spread over a set of bonds which is basically a difference in yields.

$$
\begin{array}{lclr}
P & \approx & \displaystyle\sum_1^N \frac{C_i}{(1+\frac{Y}{2})^{2t_i}} & \text{Yield} \\
P & \approx & \displaystyle\sum_1^N \frac{C_i}{(1+\frac{R+S}{2})^{2t_i}} & \text{Spread} \\
P & \approx & \displaystyle\sum_1^N \frac{C_i}{(1+\frac{Y_i+S}{2})^{2t_i}} & \text{Z-Spread} \\
P & \approx & \displaystyle\sum_1^N \frac{C_i}{(1+\frac{Y_i+S}{2})^{2t_i}} & \text{OAS}
\end{array}
$$

Optionality in OAS

When the bond has embedded optionality, OAS attempts to value the optionality. Doing this requires some assumptions regarding the evolution of interest rates:



MBS and CMO OAS

Compute average value under a large set of realistic scenarios. The OAS is the shift in the discount rates needed to arrive at a specified price.

Because of the complexity and path dependency of the prepayment model and the CMO tranche calculations, OAS analysis is done via Monte Carlo.

- Calibrate interest rate model.
- Generate interest rate scenarios — discount rates, longer tenor rates, par rates.
- Generate indices — current coupons, district 11 cost of funds, and whatever else is needed.
- For each scenario:
  - Compute prepayments for each piece of collateral.
  - Amortize each piece of collateral.
  - Compute tranche cash flows.
  - Discount cash flows at specified OAS to generate scenario price.
- Average of scenario prices is the price at the specified OAS.

The Legality of Prepayment Modeling

Prepayment as determining a contingent claim

How can it be OK to introduce historical analysis (prepayment modeling) into a risk neutral valuation (OAS calculation)?

- Consider a population holding European options with a common (unknown) strike and varying maturity dates.
- Consider an historical study of the payoff of these options when exercised.
- The historical analysis would yield a discrete, random sampling of $\max(S-K,0)$, where K is the common strike.
- Historical analysis of the payoff would yield a good characterization of the actual payoff function.
- Using the historically estimated payoff function in option pricing would be perfectly correct. No risk neutral adjustment need be applied.

To the extent that prepayment is truly a contingent claim on interest rates (i.e. — that prepayment behavior is a function of interest rates), then there is no logical inconsistency in deducing it via historical analysis and using it directly in OAS analysis.

Problems with prepayment

Problems with the above analysis:

- Errors in deducing historical behavior.
- Aggregation of behavior.
- Behavior independent of rates — hedgeable?

Data and calibration

Data importance

Data:

- Most critical component of interest rate modeling.
- Simple model calibrated to good data is far better than a sophisticated model calibrated to the wrong data.
- Data must have good pricing and capture risks of securities being valued.
- Characterizing risk in terms of related liquid instruments will help to answer the question of which interest rate model to use.

Discounting component

Traditionally, the treasury curve. Thought to be closest in risk to agency default risk (i.e. negligible), but:

- Heterogeneous — coupon bonds, some liquid and some illiquid.
- Sparse data — large gaps between maturities leaves interim pricing unknown (subject to interpolation method).
- Market price of conditional cash flows (i.e. option prices = Volatility data) unavailable.

Newer market standard — calibrate to the swap curve.

- Denser data — monthly at short end, annually further out. Less subject to interpretation.
- Homogeneous — cash rates and par rates, not bond prices at varying coupons. Mostly on a clean, nominal basis.
- Rich volatility data — ATM and OTM caps and floors are well known. ATM swaptions of various tenors and maturities are well known. Only OTM swaption pricing is subject to discussion.
- Hedging — swap market often used to hedge MBSs. Calibrating to it makes calculating appropriate hedges easier.

Volatility calibration

What to calibrate to?

- Calibrate to full volatility cube? Too expensive to hedge.
- Calibrate to one volatility? Which one?

Compromise 1: Two year and ten year swaptions are commonly used for hedging, so calibrate to them.
Compromise 2: Rather than calibrating to the entire smile, calibrate to the ATMs and pick a model which does a reasonable job of capturing the overall smile. This is also good because OTM swaption data is hard to come by.

Interest rate models

Popular short rate models

Log normal short rate models have been popular for many years in the fixed income markets.

- Simple and intuitive.
- A priori, rates shouldn't go negative.

A log normal short rate model (LNMR)

One example is the first model we used for mortgage valuation. The short rate process is r_t, where:

$$\begin{aligned} r_t &= e^{R_t+\theta_t} \\ dR_t &= -aR_t dt + \sigma dW_t, \end{aligned}$$

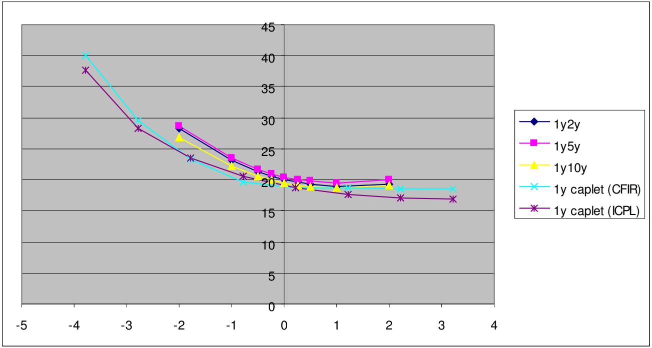
with a and σ constants, θ_t is a function of time and chosen to calibrate the model to the discount curve. Under this model R_t is a Gaussian process mean reverting to zero, and r_t is log normal and mean reverting as well.

Drawbacks of 1 factor LNMR

- Model skew is flat in log normal terms, which is not what's observed in the market today.
- Correlated rates.
- Calibration rigidity — can calibrate to the yield curve and two option prices. Mean reversion is known to be hard to calibrate. However, greater flexibility can be introduced with time dependent volatility (Black-Derman-Toy).

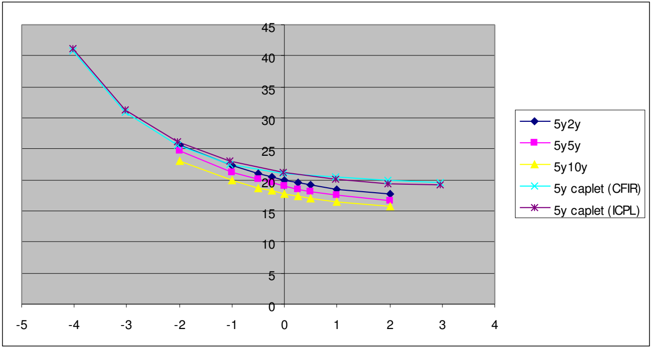
US skews 1 year out

It always pays to look at the data. Here are cap and swaption implied volatilities from the US market. The skew is substantial across maturities.



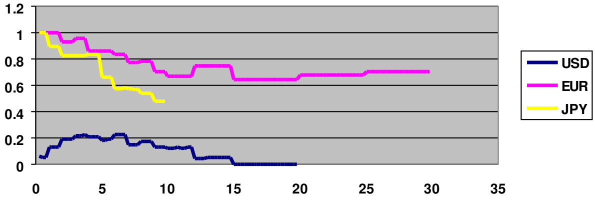
US skews 5 years out

The five year tenors are skewed as well.



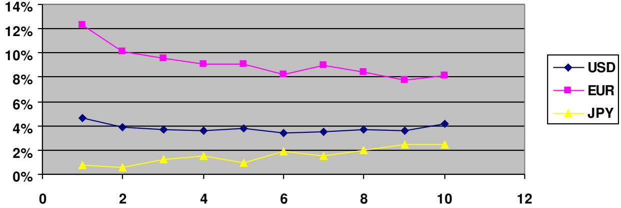
How normal is it?

Fitting the CEV model ($dr = \sigma r^{\beta} dW$) to caplet skews shows that the US market is fairly close to normal (β close to zero).



How normal is it?

Fitting errors for the CEV model to the US market aren't too large, so above betas are reasonable.



LGM

The Linear Gaussian Markovian models (AKA Hull-White models) are a family of extremely tractable models. The two factor form is:

$$
\begin{aligned}
r_t &= \theta_t + X_t + Y_t \\
dX_t &= -a_X X_t dt + \sigma_X(t) dW_1 \\
dY_t &= -a_Y Y_t dt + \sigma_Y(t) dW_2 \\
dW_1 dW_2 &= \rho dt
\end{aligned}
$$

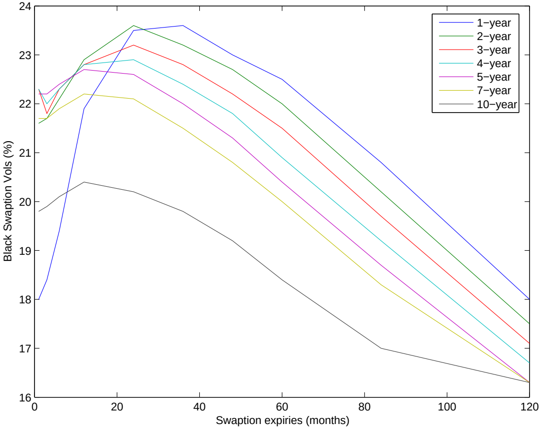
LGM advantages

- Fits overall historical behavior well with fixed mean reversion and correlation ($a_x = 0.03$, $a_y = 0.5$, $\rho = -0.7$).
- Can be calibrated to two term structures of volatility.
- Very tractable — Don't underestimate the value of closed form solutions (or decent approximations) for bond prices, caplet prices,and swaption prices and swap rates as a function of (X_t, Y_t).
- Much closer to market skew than log normal models.

LGM behavior

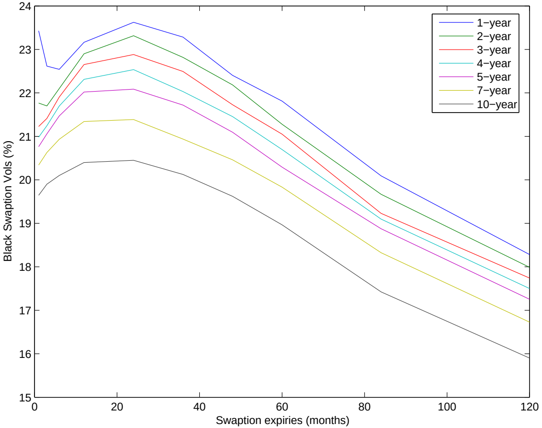
- High mean reversion factor dampens long term effect of that parameter, leaving long tenor volatility mostly determined by low mean reversion factor.
- Both factors impact short tenor volatility.
- Negative correlation reduces volatility of short maturities relative to long maturities and allows this behavior to persist in time.

Market volatility term structure



Swaption term structure — implied volatilities as a function of maturity for various tenors.

LGM volatility term structure



LGM model's swaption term structure — implied volatilities produced by the model, as a function of maturity for various tenors.

Index projection

Index projection problem

- Prepayment model needs appropriate inputs (such as monthly mortgage refi rates).
- Floater indices (such as D11COFI) need to modeled.
- Neither need be discount rates, and hence, can't be read directly from model.

Index projection solutions

- Not a problem for discount rates (LIBOR, swap rates, etc). Compute from model:
  - For one factor log normal — build lattice and compute future implied long tenor rates for each month as a function of the short rate.
  - For 2 factor LGM — rates are given by formulas.
- Avoid problem — calibrate prepayment model directly to discount rates (LIBOR, swap rates, etc).
- Simplify — to first order can ignore volatility of index relative to discount rates. How different can refi rates be on two different dates that have the same swap curve? Regress and model as a function of the rates. Or get fancy and take vol into account as well.

Chosen solution

We chose simple:

- Treat implied rates as a function of swap rates.
- Regress — proxy the current coupon for each collateral type as a weighted average of the 2 year and 10 year rates.
- Adjust for current actual value of current coupons.
- Adjust in prepayment model for fact that a proxy for the refi rate is being used.
- Potentially use lagged data (e.g. - D11COFI in *ARM Wrestling: Valuing Adjustable Rate Mortgages Indexed to the Eleventh District Cost of Funds*, by Stanton and Wallace: If D_i is the index in month i, and T_i is the 6 month treasury rate, then the model sets

$$D_i = 0.889 * D_{i-1} + 0.112 * T_i + 5.6\text{bp}.$$

Monte Carlo analysis

The need for speed

With 20,000 pieces of collateral, interpreted rules for paying out cash flows and time consuming prepayment projection calculations, running the 100,000 scenarios necessary for accurate valuation will take some time. If a scenario takes 0.0001 seconds on one piece of collateral, then we'll have to wait 2.3 days. 100 scenarios would take 3.3 minutes.

So, it pays to reduce the number of scenarios needed.

1 factor pseudo Monte Carlo

In the one factor case, the method we developed is the bifurcation tree approach.

- Begin with two paths starting at the current short rate.
- Evolve them to match the moments of the model relative to the last common point.
- Periodically allow the paths to split:

$$\begin{aligned} R^u &= \mathrm{E}[R_t | R_b = R] + \sqrt{\mathrm{Var}[R_t | R_b = R]} \\ R^d &= \mathrm{E}[R_t | R_b = R] - \sqrt{\mathrm{Var}[R_t | R_b = R]} \end{aligned}$$

1 factor LNMR

$\mathrm{E}[R_t|R_b = R]$ and $\mathrm{Var}[R_t|R_b = R]$ can be computed from the SDE for R_t by using e^{at} as an integrating factor, yielding:

$$R_t = e^{a(b-t)} R_b + \int_b^t \sigma e^{a(s-t)} dW_s,$$

so

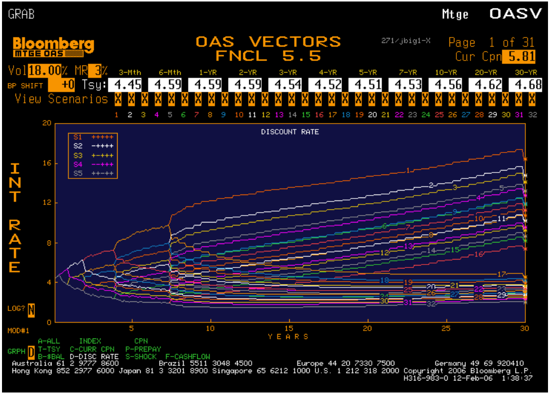
$$\mathrm{E}[R_t|R_b] = R_b e^{-a(t-b)},$$

and

$$\mathrm{Var}[R_t|R_b] = \frac{\sigma^2}{2a}(1 - e^{-2a(t-b)}).$$

1 factor pseudo Monte Carlo

The end result, with 5 selected bifurcation maturities, is the following tree:



Bifurcation tree behavior

Advantages:

- Reasonably straight forward.
- Conditional mean and variance of underlying R process is matched at bifurcation points.
- Results look reasonable with fairly small numbers of paths.

Disadvantages:

- Variability is coarse (semiannual steps).
- Unclear that overall variance is captured.
- Doesn't extend well to the multi-factor case.

LGM — VR1 — Numeraire considerations

We've investigated a variety of variance reduction techniques for the 2 factor LGM model.

The first is to recognize that the choice of numeraire has a major impact on Monte Carlo efficiency.

Consider valuation of a price process V under two different numeraires N and N', and corresponding equivalent martingale measures Q and Q'.

$$V_0 = N_0 E^Q[\frac{V_t}{N_t}] = N'_0 E^{Q'}[\frac{V_t}{N'_t}]$$

LGM — VR1 — Numeraire considerations

The relationship between N, N', Q and Q' is that

$$\frac{N_t/N_0}{N'_t/N'_0} = \frac{dQ}{dQ'},$$

where dQ/dQ' is the Radon-Nikodym derivative of Q with respect to Q'.

So, a numeraire that's higher on high rates will have an equivalent martingale measure that's correspondingly higher as well, and thus the Monte Carlo associated with the higher numeraire will sample more heavily from this region.

LGM — VR1 — Numeraire considerations

MBS gross behavior:

- High rates — Low prepayment — MBS behavior becomes more stable.
- Low rates — High prepayment — MBS behavior more interesting.

Numeraire selection — Choose numeraire that's low for high rates and high for low rates:

- Integrated form of LGM is very convenient — simplifies formulas. But, numeraire is low for low rate paths, causing poor Monte Carlo behavior.
- Standard money market numeraire is much better. Harder to work with, but does better importance sampling.

LGM — VR2 — path shifting

Adjust paths so that sample at least captures yield curve.

- Similar to doing control variate on the discount rates.
- $r_t = \theta_t + X_t + Y_t$, and X_t and Y_t are Gaussian and symmetric around zero, so just compute a new θ to get discounting correct for chosen path set.

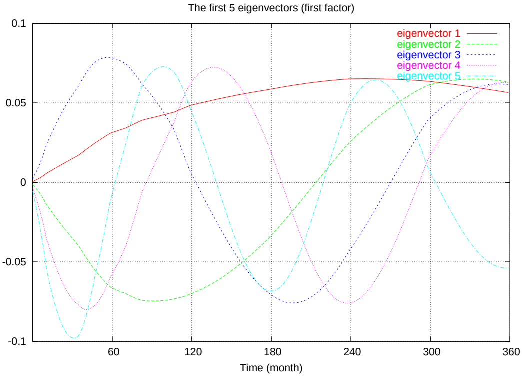
VR1 + VR2 yields pricing standard deviation of about 1bp for 2,000 paths, or ≤ 1 bp error $\approx 30\%$ of the time, and ≤ 2 bp error $\approx 66\%$ of the time.

LGM — VR3 — PCA

- Randomly sample paths $P = (X_{t_1}, \dots X_{t_n}, Y_{t_1}, \dots Y_{t_n})$
- X_{t_i} and Y_{t_i} are jointly distributed as a $2n$ dimensional Gaussian with zero mean.
- Let C be the covariance matrix of these $2n$ random variables.
- If $Z = (z_1, ..., z_{2n})^t$, and the z_i are IID $N(0,1)$, and $C = AA^t$, then AZ has the same distribution as P.
- Choose A to be the matrix whose columns are the eigenvectors of C, scaled by the square roots of their eigenvalues. Then $C = AA^t$.
- The best k factor approximation to P is given by using the first k columns of A.
- Small number of vectors capture most of variance.
- In 1 factor LGM — 7 vectors out of 360 for 95% of variance.
- In 2 factor LGM — 9 vectors out of 720 for 95% of variance.

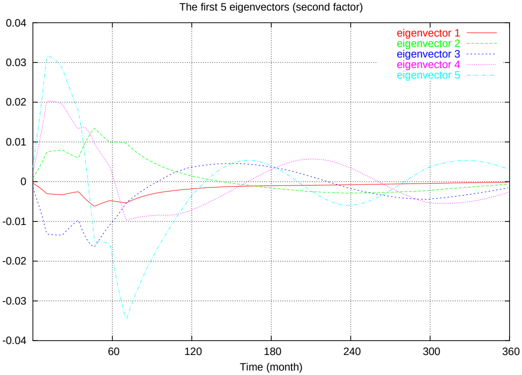
PCA impact - Low mr factor

The eigenvectors can be inspected by graphing the values for each factor separately. The first eigenvector for the low mean reversion factor captures the overall volatility. Subsequent eigenvalues capture higher and higher frequency changes.



PCA impact - High mr factor

The high mean reversion factor's eigenvectors are harder to interpret

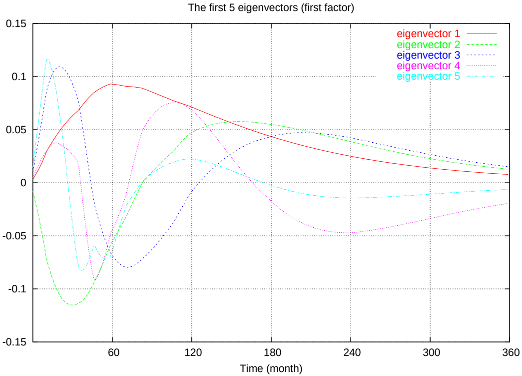


LGM — VR4 — Weighted PCA

- Most of action in pool is up front, both because of prepayment and because of discounting.
- Weight PCA with $e^{-\alpha t}$ to effect this.
- Weight two factors differently as well (High MR factor doesn't have as big an impact on MBS pricing as low MR factor).

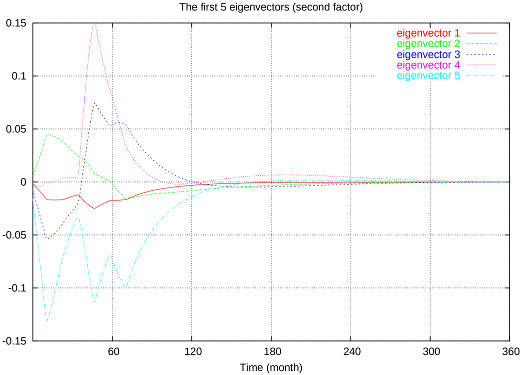
Weighted PCA impact - Low mr factor

As is expected, the impact of weighting is to capture more up front volatility at the cost of long term volatility:



Weighted PCA impact - High mr factor

Weighting dampens out much of the long term behavior of the high mean reversion factor:



LGM — VR5 — Local antithetic sampling

- Pure antithetic just makes sure sample mean is correct.
- Not so effective in interest rate MC.
- Do local antithetic instead (”uniform sampling with antithetic noise” (UWAN)) - Dupire and Savine:
  - Build a grid.
  - Pick antithetic pairs from each box.
  - Uses MC to eliminate convexity caused bias.
- Done on first few eigenvectors.

LGM — VR1+VR2+VR4+VR5 — Weighted PCA with local antithetic sampling

Combine methods:

- VR1 — Money market numeraire.
- VR2 — Path shifting.
- VR4 — Weighted PCA.
- VR5 — Local antithetic sampling.

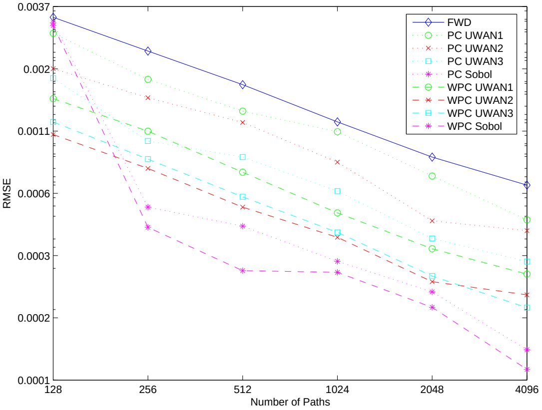
VR1+VR2+VR4+VR5 = standard deviation of 300 paths is lower than 5000 for VR1+VR2.

LGM — VR1+VR2+VR4 with Sobol sequences

Adding Sobol sequences to the mix:

- Use Sobol sequences to randomly scale eigenvectors.
- Further reduces variance.

Results


0.0037
0.002
0.0011
0.0006
0.0003
0.0002
0.0001
RMSE
128
256
512
1024
2048
4096
Number of Paths
FWD
PC UWAN1
PC UWAN2
PC UWAN3
PC Sobol
WPC UWAN1
WPC UWAN2
WPC UWAN3
WPC Sobol

Greeks

Greek errors

Greeks magnify pricing errors.

$$\frac{dP}{dS} \approx \frac{P(S+h) - P(S-h)}{2h}$$

If $P(S)$ is the true model price, and $\tilde{P}(S)$ is what we compute, then the error is $\epsilon(S) = \tilde{P}(S) - P(S)$.

$$P(S+h) = P(S) + P'(S)h + \frac{1}{2}P''(S)h^2 + \frac{1}{3!}P'''(S)h^3 + \ldots$$

so

$$\frac{P(S+h) - P(S-h)}{2h} = P'(S) + h^2(\frac{1}{3!}P'''(S) + \ldots)$$

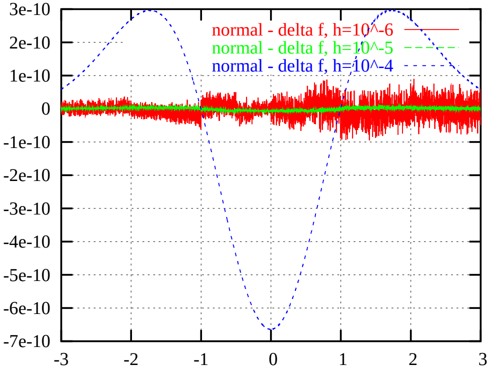
and

$$\begin{aligned}\frac{\tilde{P}(S+h) - \tilde{P}(S-h)}{2h} &= \frac{P(S+h) + \epsilon(S+h) - (P(S-h) + \epsilon(S-h}{2h} \\ &= dP/dS + h^2(\frac{1}{3!}d^3P/dS^3 + \ldots) + \Delta\epsilon/2h\end{aligned}$$

The error in the calculation is the error from dropping the higher order terms and from the change in error with respect to h.

Limits of computation

To see the magnitude of the problem, consider computing the derivative of the cumulative normal distribution by finite difference. Even with a computation as accurate as this, one should not exceed a step size of 10^{-5}.



Greek errors

Errors in derivative are caused by:

- High order terms corrupting finite difference ("convexity").
- Pricing error.

Error control:

- Small h reduces Greek error from convexity.
- Large h reduces Greek error from pricing error.
- Balance is needed — $h \approx \sqrt[3]{3\alpha f/f'''}$ if α is the relative error $\approx 10^{-5}$ for machine precision calculations with 64 bit doubles when $f/f''' \approx 1$.

Pricing error issues:

- Finite difference — ϵ flat, minimal problems.
- Monte Carlo — ϵ large and random — error goes to ∞ as $h \to 0$.

Solution for Monte Carlo:

- Make ϵ less random — Use the same paths, or the same random number seed.
- Accurate duration with 25bp shift, even when pricing variance is as large as 6bp.

Which Greeks?

To compute duration, rates are shifted while other inputs are held constant. How should the option data be held constant?

- Hold prices constant — Doesn't make sense. Option prices have to change as rates make them more or less in the money.
- Hold vol constant — Vol is a log normal vol, but LGM model is normal. If vol is held constant, then model volatility will change when the rates are shifted.
- Hold *normal* vol constant — Most consistent with LGM model.

The situation between normal vol and log normal vol is reversed for the LNMR model.

Validation

Validation

How to make sure entire process works?
General methods:

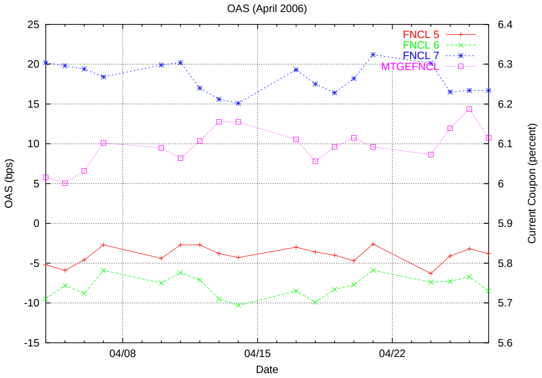
- Results are stable over time. Jumps can be explained by occurrence of a change in the market.
- Fitted parameters stable over time as well.
- Prices and OASs behave appropriately
  - Flat in neighborhood of current cpn.
  - Expected relationships between different securities hold.
  - Prices and OASs move as expected when input parameters are changed:
    - Yield curve shifts.
    - Volatility shifts.
    - Shifts of current index values.

Mortgage specific:

- Compare to empirical durations.

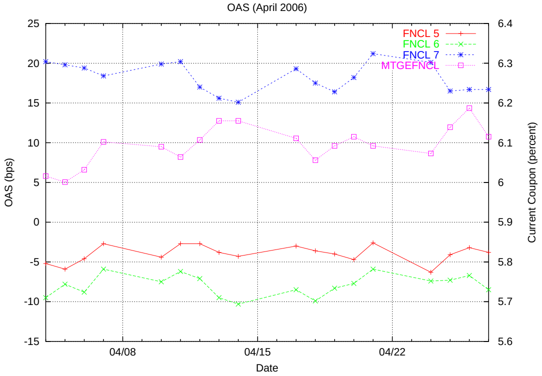
OAS stability

OASs are stable, but related to the current coupons, indicating a lack of linearity — the market demands a higher OAS when rates drop.



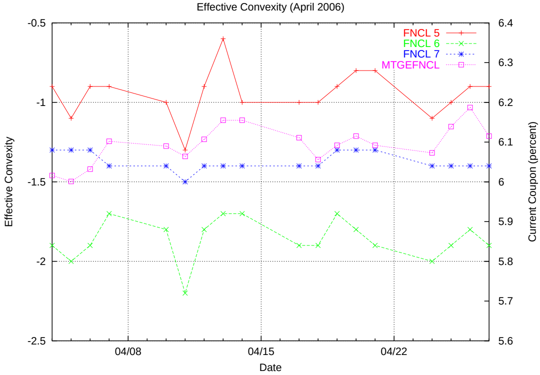
Duration stability

Durations are stable as well, and exhibit the appropriate relationship to the current coupon. Durations drop when rates drop, with the largest impact on coupons near but below the current coupon.



Convexity stability

Convexities also become more negative when the current coupon drops because heightened prepayment causes the pool to behave less bond-like.



Robust parallelization

Parallelization

Variance reduction alone is insufficient to compute OASs in real time. Parallelization is needed.

- Linux clusters.
- 6 clusters, 50 dual CPU PCs each = 100 CPUs per cluster
- Embarrassingly parallel sometimes isn't
  - Communication costs to farm out results and get back can render parallelization useless.
  - Old cluster - 750 MHZ, 100 MBPS ethernet — 60 seconds, 6 in parallel.
  - New cluster - 3.0 GHZ, 100 MBPS ethernet — 15 seconds, 4 seconds in parallel.
- Data dissemination problem — 2 GB of deal and collateral specifications.
- PCA parallelization — compute time vs communication speed.

Request flow

- User hits <go>
- Computation data is assembled:
  - Current values.
  - User selected values.
- Request sent to dispatcher.
- Dispatcher queues until an idle server is available. Retries failed requests.
- Server receives request.
- Explodes base calculation into individual path requests.
- Farms them out across the cluster.
  - Redundancy and robustness.
- Assembles the result.
- Farms out remaining requests.
- Assembles results.
- Replies to client.

Data dissemination

How to keep 2gb current on 300 machines?

- Layered approach.
- Source keeps head machines up to date.
- Head machines update remainder of cluster.
- Distribute within cluster in a tree fashion:
  - Copy from head to 4 children.
  - Each time a node is updated, it starts updating 4 new nodes, and its updater starts updating a new node.
  - Utilizes full bandwidth of ethernet switch — 100 MBPS from each machine to switch, but independent pairs of machines can sustain this up until the switch's backplane capacity.

Parallelization robustness

Preventing machine problems from causing calculation failures.
Errors encountered:

- Unstripable curves.
- Overheating machines.
- Flakey hard disks.
- Data unavailable.
- Bad data supplied.

Layered approach:

- Requests -> dispatcher -> OAS server -> slaves.
- If dispatcher gets an error (on a full request), it resends (up to the retry limit).
- If OAS server gets an error (on a path), it marks the slave as bad and tries again. If it gets confirmation of the error, the slave is marked good and the path is listed as bad. If not, the slave is no longer used.

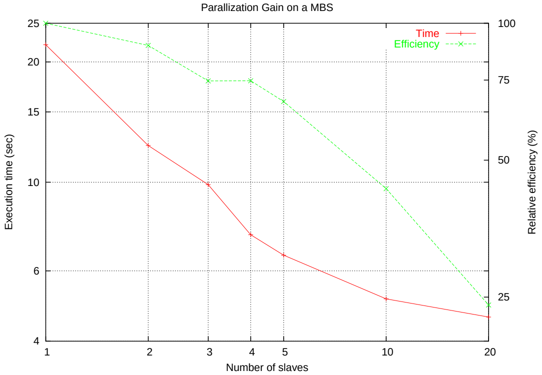
Parallelization issues

Even embarrassingly parallel problems might have trouble parallelizing.

- Even the simplest variance reduction adds up startup costs and communication overhead.
- If startup costs can't be distributed as well, then they yield a hard limit on parallelization speedup.
- PCA analysis is slow enough that it's hard to make it actually save time.
    - Compression of data being distributed.
    - Tree distribution of data.
    - Optimize PCA.
    - Parallelize PCA.
    - Partial PCA.

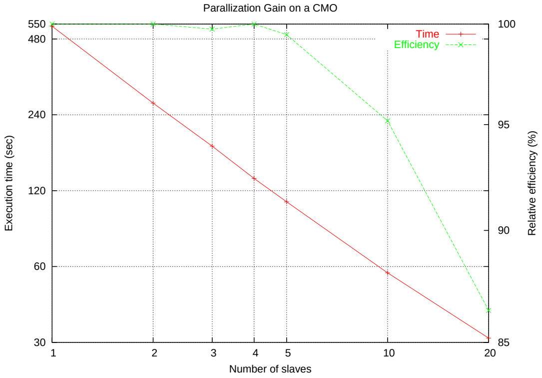
Effectiveness — pools

Parallelization is most effective when the ratio of scenario computation to communication is high. This is not the case for pools, where only one amortization and one prepayment model call are needed for each scenario, and there are no tranche cash flow calculations. But even with low efficiency, parallelization is effective in reducing run time.



Effectiveness – CMOs

CMOs are quite another story. With large collateral sets and complex rules, each scenario can be quite intensive, so parallelization at the path level is far more effective



Summary

CMO valuation is big science — lots of moving parts, with each one drawing on a different area:

- Prepayment modeling:
  - Statistical validation and modeling of economic and behavioral analysis.
- Data selection:
  - Risk analysis.
- Interest rate modeling:
  - Classic arbitrage pricing theory.
- Index projection:
  - Statistical analysis.
- Monte Carlo analysis:
  - Numerical methods.
  - Variance reduction techniques.
- Parallelization:
  - Building computation clusters.
  - Analysis and optimization of parallel algorithms.

As Emanuel Derman says, the best quants are interdisciplinarians. CMO valuation is one area that requires it.

Help x2

Press the <HELP> key twice for instant live assistance.

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