

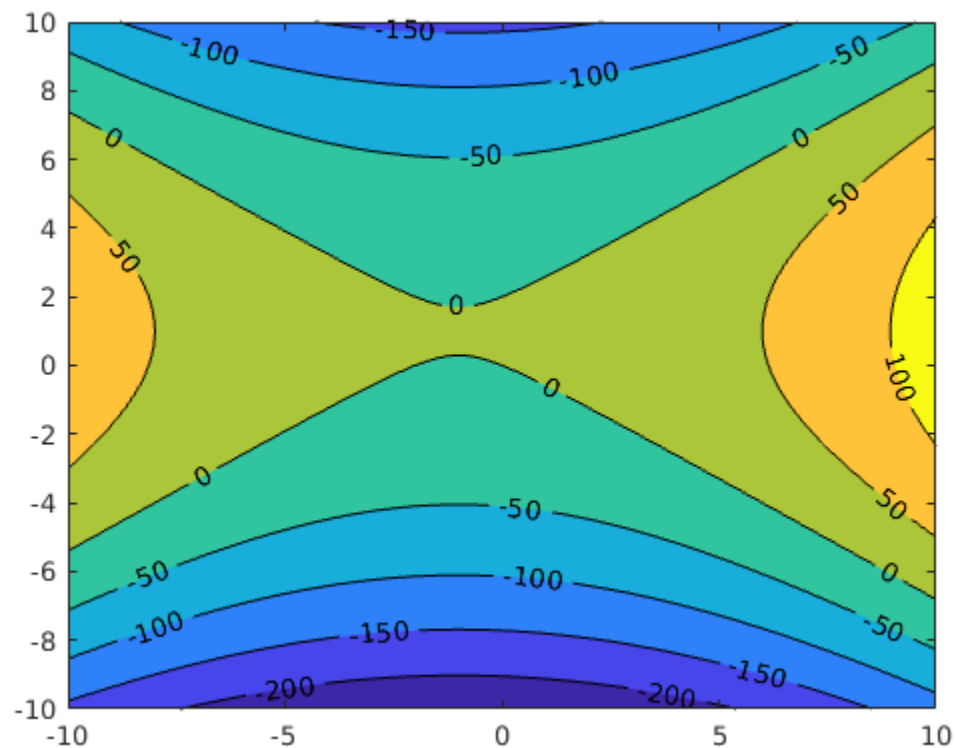
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# ASSIGNMENT 1: EXERCISE 1

## Part A

```
close all; clear all

f = @(x,y) 2.*x + 4.*y + x.^2 - 2.*y.^2;
n = 300;
x = linspace(-10,10,n+1);
y = x;
[X,Y] = meshgrid(x,y);
alpha = 1;
figure
[C,h] = contourf(alpha*X, alpha*Y, f(alpha*X,alpha*Y));
clabel(C,h)
```



## Part B

$$C = 2x + 4y + x^2 - 2y^2$$

$$C = (x+1)^2 - \frac{1}{2}(y-1)^2 - \frac{1}{2}$$

$$(y-1)^2 = 2(x+1)^2 - 1 + 2C$$

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$$y - 1 = \pm \sqrt{2(x + 1)^2 - 1 + 2C}$$

$$y = \pm \sqrt{2(x + 1)^2 - 1 + 2C} + 1$$

This function describes pretty well the figure shown above. Increasing the constant  $C$  (or  $z$ -axis) will cause the parabolas created by the square root to increase in distance in separation in the  $x$  direction.

Part C

$$f(x, y) = 2x + 4yx^2 - 2y^2$$

$$\frac{\partial f}{\partial x} = 2 + 2x$$

$$\frac{\partial f}{\partial y} = 4 - 4y$$

$$\frac{\partial f}{\partial x} = 0$$

$$x = -1$$

$$\frac{\partial f}{\partial y} = 0$$

$$y = 1$$

$$\frac{\partial^2 f}{\partial x^2} = 2$$

$$\frac{\partial^2 f}{\partial y^2} = -4$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

$$\frac{\partial^2 f}{\partial y \partial x} = 0$$

This point  $(-1, 1)$  is a stationary point but neither a maxima or minima. This point is an inflection point as shown by the hessian.

## ASSIGNMENT 1: EXERCISE 2

Part D

$$A = B^T B = x^T B^T B x = (Bx)^T (Bx) \geq 0$$

Therefore since this equality holds for all  $B$ ,  $A$  is positive semi-definite

Part E

$$f(y + \alpha(x - y)) - \alpha f(x) - (1 - \alpha)f(y) \leq 0$$

If this holds true, then the  $f(x)$  is convex.

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$$(y^T + (\alpha x^T) - (\alpha y)^T)A(y + \alpha x - \alpha y) - \alpha x^T A x - y^T A y + \alpha y^T A y \leq 0$$

$$0 \leq 0$$

All the terms cancel out. Therefore  $f(x)$  is convex.

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