COMPGV15: Assignment 8

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1 Exercise 1

a) KKT Conditions: Define Lagrangian as

$$L(x, \lambda) = \frac{1}{2}x^T Gx + c^T x + \lambda_1(Ax - b)$$

The constraint as

$$c(x) = Ax - b \leq 0$$

(i)

$$\frac{\partial L}{\partial x} \geq 0$$

$$\frac{\partial L}{\partial x} = x^T G + c^T + \lambda_1 A^T \geq 0$$

(ii)

$$\frac{\partial L}{\partial \lambda_1} \leq 0$$

$$Ax - b \leq 0$$

(iii)

$$x\frac{\partial L}{\partial x} = 0$$

$$\frac{\partial L}{\partial x} = x^T(Gx + c + \lambda_1 A) = 0$$

(iv)

$$\lambda_1 c(x) = 0$$

$$\lambda_1(Ax - b) = 0$$

(v)

$$x \geq 0$$

$$\lambda_1 \geq 0$$

b) Adding the vector of slack variables

$$y \in \Re^m, y \geq 0$$

$$g(x) = Ax - b + y = 0$$

$$L(x, \upsilon) = \frac{1}{2}x^T Gx + c^T x + \upsilon_1(Ax - b + y)$$

KKT Conditions can now be written as

(i)

$$\frac{\partial L}{\partial x} = x^T G + \upsilon_1 A = -c^T$$

(ii)

$$Ax + y = b$$

(iii)

$$x \geq 0$$

$$\upsilon_1 \geq 0$$

$$y \geq 0$$

c) To minimize L over x take gradient and set to zero.

$$\begin{aligned} \frac{\partial L}{\partial x} &= 0 \\ x^* &= -G^{-1}(\upsilon_1 A^T + c) \\ g(\upsilon) &= inf(L(x^*, \upsilon)) \end{aligned}$$

Subbing into the dual problem.

$$\begin{aligned} L(-G^{-1}(\upsilon_1 A^T + c), \upsilon) &= \frac{1}{2}(G^{-1}(\upsilon_1 A^T + c))^T G(G^{-1}(\upsilon_1 A^T + c)) + \upsilon_1(A(G^{-1}(\upsilon_1 A + c)) + b) \\ &= -\frac{\upsilon_1^2}{2}G^{-1} - \frac{1}{2}\upsilon_1 G^{-1}Ac \\ &- \frac{1}{2}\upsilon_1 G^{-1}c^T A - \frac{1}{2}G^{-1} - \upsilon_1^2 AG^{-1}A^T + G^{-1}c + \upsilon_1 b + \upsilon_1 y \end{aligned} \tag{1}$$

Minimum of augmented cost as function of Lagrange Multipliers λ. Can be $-\inf$ for some υ. g, the dual problem, is concave (even if f, the function to minimize, not concave).

2 Exercise 2

a) Setting up the Lagrangian:

$$L(x, y, \lambda) = x^2 - 4xy + 4y^2 + x^2 - 4x + 4 + \lambda(4 + y - x)$$

Solving for optimal (x^*, y^*, λ^*).

$$\frac{\partial L}{\partial x} = 2x^* - 4y * +2x * -\lambda^* = 0$$

$$\frac{\partial L}{\partial y} = -4x^* + 8y * +\lambda^* = 0$$

$$\frac{\partial L}{\partial \lambda} = 4 + y^* - x^* = 0$$

b) This system can be reformulated in the form $Ax = b$.

$$A = \begin{bmatrix} 4 & -4 & -1 \\ -4 & 8 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}$$

Using MATLAB to solve the system of linear equations

$$x = \begin{bmatrix} x^* \\ y^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 12 \end{bmatrix}$$