

NUMERICAL OPTIMISATION

ASSIGNMENT 7

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EXERCISE 1

Implement the Gauss-Newton method for solution of nonlinear least square problems. As Gauss-Newton is a line search method, it can be easiest implemented inside the function `descentLineSearch.m`. More help is provided in `Cody Coursework`.

Submit your implementation via Cody Coursework.

[20pt]

EXERCISE 2 (optional)

You are given an implementation of the Levenberg-Marquardt method. This implementation plugs in the Levenberg-Marquardt solver `solverCMlevenberg.m` into the trust region function `trustRegion.m`. Briefly answer the following questions about the solver in `solverCMlevenberg.m`

- (i) What is the effect of the `for` loop in line 36 and what is the reason for sequential execution?
- (ii) Explain the formula for calculating the L-M direction p .
- (iii) Explain what is q and how it is calculated.
- (iv) Explain what is λ and the formula used for its update.

Submit your solution via Turnitin.

[0pt]

EXERCISE 3

Consider a model

$$\varphi(x_1, x_2, x_3; t) = (x_1 + x_2 t^2) \exp(-x_3 t)$$

with parameters (x_1, x_2, x_3) . Such models are relevant e.g. in optics (photon counting).

Simulate the measurements sampling this model for a fixed choice of parameters $(x_1, x_2, x_3) = (3, 150, 2)$ at 200 equi-spaced points in $t_i \in (0, 4]$ and adding Gaussian noise $n(t_i) \sim \mathcal{N}(0, \sigma^2)$ drawn from a normal distribution with 0 mean and standard deviation 5% of the maximal amplitude of the sampled model signal $\sigma = 0.05 \max_{t_i} |\varphi(t_i)|$,

$$\tilde{\varphi}(x_1, x_2, x_3; t_j) = \varphi(x_1, x_2, x_3; t_j) + n(t_j).$$

- (a) Formulate the least-squares problem for fitting the model φ and derive its Jacobian.

Submit your solution via Turnitin.

[30pt]

- (b) Estimate the parameters (x_1, x_2, x_3) from your simulated measurements using

- (i) Gauss-Newton (implemented in Ex 1)
- (ii) Levenberg-Marquardt (provided)

Specify all the relevant parameters and explain the results. Visualise the fit by plotting the estimated signal versus the measurements.

Submit your solution via Turnitin.

[50pt]

Remark. The submission to Turnitin should not exceed 4 pages. Avoid submitting code unless explicitly asked for and focus on explaining your results.