Assignment 0: Exercise 1

```
clear all; close all
```

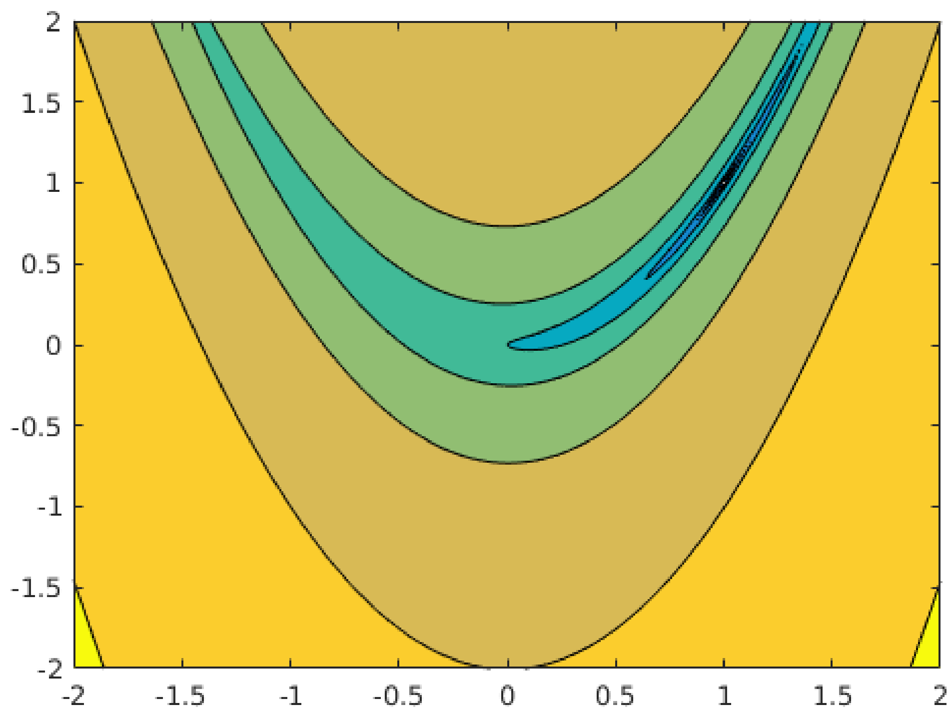
a) Rosenbrock function

```
f = @(x,y) 100.*(y - x.^2).^2 + (1 - x).^2;
```

b) Discretisation of the unit square. It can be scaled with factor α using simple multiplication $\alpha X, \alpha Y$
Plot the function of two variables

```
n = 300;
x = linspace(-1,1,n+1);
h = 2/n;
y = x;
[X,Y] = meshgrid(x,y);

alpha = 2;
%figure, surf(alpha*X, alpha*Y, f(alpha*X,alpha*Y), 'EdgeColor',
 'none')
%figure, surfc(alpha*X, alpha*Y, f(alpha*X,alpha*Y), 'EdgeColor',
 'none')
%figure, contour(alpha*X, alpha*Y, log(f(alpha*X,alpha*Y)))
figure, contourf(alpha*X, alpha*Y, log(f(alpha*X,alpha*Y)))
```



c) Gradient

$df/dx = -400(y - x^2)x - 2(1 - x)$

$df/dy = 200(y - x^2)$

c) Hessian

$d^2f/dx^2 = -400(y - 3x^2) + 2$

$d^2f/dxdy = -400x$

$d^2f/dydx = -400x$

$df^2/dy^2 = 200$

d) Find the minimiser x^* of the function f. Show that x^* is unique and that $\nabla^2 f(x^*)$ is positive definite.

We observe that $f(x, y) \geq 0$ because is the sum of two non-negative terms. Also,

$f(x, y) = 0 \Leftrightarrow y = x^2 \text{ and } x = 1$

This is only possible if $x = 1$ and $y = 1$. Hence, we have $x^* = (1, 1)^T$.

The Hessian at x^* is

$$\nabla^2 f(x^*) = \begin{pmatrix} 802 & -400 \\ -400 & 200 \end{pmatrix}$$

Hence, to verify that it is positive definite, for all $(x, y) \neq 0$ we must have:

$$(x \quad y) \begin{pmatrix} 802 & -400 \\ -400 & 200 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} > 0$$

This is equivalent to

$802x^2 - 800xy + 200y^2 > 0$

Finally, we have

$2x^2 + 200(2x - y)^2 > 0,$

that obviously holds for all $(x, y) \neq (0, 0)$.

e) Finite difference implementation of the gradient

```
% Construct central finite difference to be applied columnwise
D = 1/(2*h)*spdiags([-ones(n+1,1) ones(n+1,1)], [-1 1], n+1,n+1);
D(1,1:2) = 1/h*[-1 1];
D(n+1,n:n+1) = 1/h*[-1 1];

% Gradient via finate differences
% d/dy
Dyf = D*f(alpha*X, alpha*Y);
% d/dx, central finite difference applied rowwise
Dxf = (D*(f(alpha*X, alpha*Y).')).';

% Hessian via finite difference
% d^2/dxdy
Dxyf = D*Dxf;
% d^2/dx^2
Dxxf = (D*(Dxf.')).';
% d^2/dy^2
Dyyf = D*Dyf;
% d^2/dydx
Dyxf = (D*(Dyf.')).';
```

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