POWER SPECTRUM ANALYSIS OF THREE-DIMENSIONAL REDSHIFT SURVEYS

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Abstract

We develop a general method for power spectrum analysis of three dimensional redshift surveys. We present rigorous analytical estimates for the statistical uncertainty in the power and we are able to derive a rigorous optimal weighting scheme under the reasonable (and largely empirically verified) assumption that the long wavelength Fourier components are Gaussian distributed. We apply the formalism to the updated 1-in-6 QDOT IRAS redshift survey, and compare our results to data from other probes: APM angular correlations; the CfA and the Berkeley 1.2Jy IRAS redshift surveys. Our results bear out and further quantify the impression from e.g. counts-in-cells analysis that there is extra power on large scales as compared to the standard CDM model with $\Omega h \simeq 0.5$. We apply likelihood analysis using the CDM spectrum with Ωh as a free parameter as a phenomenological family of models; we find the best fitting parameters in redshift space and transform the results to real space. Finally, we calculate the distribution of the estimated long wavelength power. This agrees remarkably well with the exponential distribution expected for Gaussian fluctuations, even out to powers of ten times the mean. Our results thus reveal no trace of periodicity or other non-Gaussian behavior.

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1. INTRODUCTION

The IRAS QDOT survey (see e.g. Efstathiou et al. 1990) was designed for the study of large—scale structure, and has been analyzed statistically in a number of ways: counts in cells (Efstathiou et al. 1990) topology (Moore et al. 1992) etc. It has also been used to predict the peculiar motions of the local group (Rowan–Robinson et al. 1990; Strauss et al. 1990; Strauss et al. 1992) and of other galaxies (Saunders et al. 1991) and thereby give an estimate of the density of the universe. Here we shall present the results of power–spectrum analysis of this survey, preliminary results of which have appeared elsewhere (Kaiser 1992; Feldman 1993).

The present study follows the program laid down by Peebles in his pioneering series of papers (Yu & Peebles 1969; Peebles 1973; Peebles & Hauser 1974) for statistical analysis of galaxy catalogues via low order correlation functions. The power spectrum P(k) is the Fourier transform of the spatial two–point function $\xi(r)$ and, as such, can only provide a partial description of the nature of the large–scale structure. While limited in information, the two–point function does hold a rather special place: if the initial fluctuations were Gaussian then the power spectrum provides a complete description of the fluctuations. It also provides a good starting point for higher–order analyses and, in any case, our knowledge of the two–point function is as yet far from complete, particularly on large scales.

While Peebles emphasized the role of the power spectrum as a tool for two-point analysis, most work on real data has tended to use the auto-correlation function as the primary estimator. Recently, the power spectrum has made something of a comeback and has been applied to the CfA survey(s) (Baumgart & Fry 1991; Vogeley et al. 1992), to pencil beam surveys (Broadhurst et al. 1990; see also Kaiser & Peacock 1991) and to both 1-in-6 0.6 Jy (Kaiser et al. 1991) and complete 1.2-Jy surveys (Strauss et al. 1990; Fisher et al. 1993) derived from Version 2 of the IRAS point source catalogue (Chester, Beichmann & Conrow 1987). Power spectrum analysis has also been applied to radio galaxies (Webster 1977; Peacock & Nicholson 1991), to clusters of galaxies (Peacock & West 1992; Einasto, Gramann & Tago 1993) and to the Southern Sky Redshift Survey (Park, Gott & da Costa 1992).

Since P(k) and $\xi(r)$ are Fourier transform pairs, one might question what is gained by all this analysis. While complete knowledge of the two–point function $\xi(r)$ is equivalent to complete knowledge of the power spectrum P(k) since they are a Fourier transform pair, the same is not true of estimates $\hat{\xi}$ and \hat{P} derived from finite and noisy observational samples. There may be benefits to be derived from having both estimates and, depending on the details of the survey at hand, there will be relative advantages of one method over the other.

Whereas the auto–correlation function measures the excess probability of finding a pair of galaxies in two volumes separated by some distance, the power spectrum directly measures the fractional density contributions on different scales. This, in general, is a more natural quantity, and one that is being supplied by theories that describe the state of the Universe at early times e.g. inflationary theories. Furthermore the correlation function, especially on large scales, is very sensitive to assumptions we make about the mean density \bar{n} (the correlation function $\xi_{12} + 1 \equiv n_1 n_2 / \bar{n}^2$ where 1, 2 refer to two volumes); in fact, the scales where $\xi \ll 1$ are the ones we are most interested in when studying large–scale clustering. In contrast, the power spectrum, which gives us a direct measurement of $\delta \rho / \rho$, scales as \bar{n}^{-1} and so determines the density field on large scales more robustly. Uncertainty in our knowledge of the mean density may amplify the error in the correlation function; however, the shape of the power spectrum should be unaffected by incomplete knowledge of \bar{n} which is measured by the k=0 mode.

Another general advantage of the power spectrum derives from the fact that the true P(k) is a positive quantity. Thus, in interpreting an estimate $\hat{P}(k)$ — which will not necessarily be positive — we have important extra information at our disposal. It is therefore possible to recognize directly unphysical results which may indicate some problem with data or analysis program. For the auto-correlation function this information translates into an infinite number of integral constraints: the integral of $\xi(\mathbf{r})$ times $\exp(i\mathbf{k} \cdot \mathbf{r})$ must be positive for every value of \mathbf{k} , but this seems relatively obscure.

A related advantage of the power–spectrum analysis involves the error analysis. In general, the statistical uncertainty in any two–point function involves the 4–point function which is at best poorly known. An interesting error estimate — and the relevant one if one is interested in testing models like CDM — is obtained if we assume that the large–scale structure resembles a Gaussian random field. When we take the Fourier transform of the galaxies in the survey we are, roughly speaking, transforming the product of the infinite random sea of density fluctuations $f(\mathbf{r})$ with a window determined by the survey geometry. The transform will therefore be the convolution of the Fourier coefficients $f(\mathbf{k})$ with a 'point–spread function' (psf) which, for the IRAS survey, is a compact ball with

width equal to the inverse of the depth of the survey. For a Gaussian field, the coefficients $f(\mathbf{k})$ are totally uncorrelated with one another, so, aside from the a short range coherence imposed by the convolution, estimates of the power will also be statistically independent. In the correlation function representation the fluctuations in $\xi(\mathbf{r})$ at different \mathbf{r} will be correlated in some rather complicated way.

These advantages are obtained at a price, and that price is particularly high when the survey geometry is highly irregular. With a pencil beam survey, for instance, the psf is a pancake with transverse dimension 1/w, where w is the width of the needle, and the direct estimate of the power at low spatial frequencies will be strongly contaminated by 'aliasing' from high spatial frequencies (Broadhurst *et al.* 1990; Kaiser & Peacock 1991). The interpretation of the results are then quite difficult and requires careful modeling of the convolution. In a direct estimate of $\xi(r)$ no such problem arises, although estimating the statistical error in ξ is still problematic for highly irregular geometries.

In this paper we concentrate on the development of a formalism that is designed to construct a descriptive statistic that measures the power spectrum of the underlying density fluctuation field assuming that the fluctuation field is some homogeneous and statistically isotropic random process. We apply the formalism to surveys with a large baseline in all three dimensions, in particular, to the flux limited 1–in–6 IRAS QDOT survey that covers 73.9% of the sky and has 1824 galaxies with redshift that corresponds to radii of $20 h^{-1} \text{ Mpc} < R < 500 h^{-1} \text{ Mpc}$ (where $h \equiv H_0/100 \,\mathrm{kms}^{-1} \mathrm{Mpc}^{-1}$, as usual) and galactic latitude $|b| > 10^{\circ}$. We have used a revised version of the QDOT database in which a redshift error that afflicted approximately 200 southern galaxies has been corrected (courtesy of A. Lawrence). Since the QDOT survey is deep and covers nearly the whole sky we feel that in this case, and in general for surveys of that type, power–spectrum analysis would seem to be the method of choice.

The layout of the paper is as follows: In §2 we describe the method; this is essentially the use the power spectrum of the weighted galaxy counts, with the weight optimized for Gaussian fluctuations. A new feature of this work as compared with previous studies is the rigorous error analysis and optimized weighting scheme. In §3 we apply the method to the QDOT 1-in-6 survey. The results which confirm the impression from e.g. the counts-in-cells analysis that there is an excess of large-scale power relative to the CDM predictions (if we normalize to fluctuations at the $\sim 8h^{-1}$ Mpc scales) and that the excess appears as a 'bump' in P(k) around wavelengths $\lambda \sim 100-150h^{-1}$ Mpc. In §4 we compare

the results with other probes of the large–scale power spectrum, theoretical models and analysis from other surveys. In §5 we examine the statistical fluctuations in the power and compare these with the Gaussian expectation. We conclude in §6.

2. METHOD

In §2.1 we construct our estimator of the power $\hat{P}(k)$. In §2.2 we calculate the variance in $\hat{P}(k)$ under the assumption that the Fourier transform of the galaxies is approximately Gaussian distributed. In §2.3 we find the optimum weighting scheme, and in §2.4 we will convert the results to practical formulae to implement on the computer. In §2.5 we present the covariance matrix.

2.1 The Estimator

We make the usual assumption that the galaxies form a Poisson sample (Peebles 1980) of the density field $1 + f(\mathbf{r})$:

$$P(\text{vol element } \delta V \text{ contains a galaxy}) = \delta V \overline{n}(\mathbf{r}) [1 + f(\mathbf{r})]$$
 (2.1.1)

where $\overline{n}(\mathbf{r})$ is the expected mean space density of galaxies given the angular and luminosity selection criteria, and we wish to estimate the power spectrum

$$P(k) = P(\mathbf{k}) \equiv \int d^3 r \, \xi(\mathbf{r}) \, e^{i\mathbf{k} \cdot \mathbf{r}}$$
 (2.1.2)

where $\xi(\mathbf{r}) = \xi(r) = \langle f(\mathbf{r}')f(\mathbf{r}' + \mathbf{r}) \rangle$. Our Fourier transform convention is $f(\mathbf{k}) = \int d^3r \, f(\mathbf{r}) \exp(i\mathbf{k} \cdot \mathbf{r})$ so $f(\mathbf{r}) = (2\pi)^{-3} \int d^3k \, f(\mathbf{k}) \exp(-i\mathbf{k} \cdot \mathbf{r})$. Since we are dealing with a random field of infinite extent, the numerical value of $f(\mathbf{k})$ is not well defined, but if we set $f(\mathbf{r}) = 0$ outside of some enormous volume V then $\langle f(\mathbf{k})f^*(\mathbf{k})\rangle/V = \int d^3r \, \langle f(\mathbf{r}')f(\mathbf{r}' + \mathbf{r})\rangle \exp(i\mathbf{k} \cdot \mathbf{r})$ tends to a well defined limit and it is this quantity that we call P(k).

Our approach is to take the Fourier transform of the real galaxies minus the transform of a synthetic catalogue with the same angular and radial selection function as the real galaxies but otherwise without structure. We also incorporate a weight function $w(\mathbf{r})$ which will be adjusted to optimise the performance of the power–spectrum estimator. We define the weighted galaxy fluctuation field, with a convenient normalization, to be

$$F(\mathbf{r}) \equiv \frac{w(\mathbf{r}) \left[n_g(\mathbf{r}) - \alpha n_s(\mathbf{r}) \right]}{\left[\int d^3 r \, \overline{n}^2(\mathbf{r}) \, w^2(\mathbf{r}) \right]^{1/2}}$$
(2.1.3)

where $n_g(\mathbf{r}) = \sum_i \delta(\mathbf{r} - \mathbf{r}_i)$ with \mathbf{r}_i being the location of the i^{th} galaxy and similarly for a synthetic catalogue which has number density $1/\alpha$ times that of the real catalogue.

Taking the Fourier transform of $F(\mathbf{r})$, squaring it and taking the expectation value we find:

$$\langle |F(\mathbf{k})|^2 \rangle = \frac{\int d^3r \int d^3r' \, w(\mathbf{r}) \, w(\mathbf{r}') \langle [n_g(\mathbf{r}) - \alpha n_s(\mathbf{r})][n_g(\mathbf{r}') - \alpha n_s(\mathbf{r}')] \rangle e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}}{\int d^3r \, \overline{n}^2(\mathbf{r}) \, w^2(\mathbf{r})} \quad (2.1.4)$$

With the model of equation 2.1.1, the two point functions of n_g , n_s are

$$\langle n_g(\mathbf{r}) n_g(\mathbf{r}') \rangle = \overline{n}(\mathbf{r}) \overline{n}(\mathbf{r}') [1 + \xi(\mathbf{r} - \mathbf{r}')] + \overline{n}(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}')$$

$$\langle n_s(\mathbf{r}) n_s(\mathbf{r}') \rangle = \alpha^{-2} \overline{n}(\mathbf{r}) \overline{n}(\mathbf{r}') + \alpha^{-1} \overline{n}(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}')$$

$$\langle n_g(\mathbf{r}) n_s(\mathbf{r}') \rangle = \alpha^{-1} \overline{n}(\mathbf{r}) \overline{n}(\mathbf{r}')$$
(2.1.5)

(see appendix A), so

$$\langle |F(\mathbf{k})|^2 \rangle = \int \frac{d^3k'}{(2\pi)^3} P(\mathbf{k'}) |G(\mathbf{k} - \mathbf{k'})|^2 + (1+\alpha) \frac{\int d^3r \,\overline{n}(\mathbf{r}) \, w^2(\mathbf{r})}{\int d^3r \,\overline{n}^2(\mathbf{r}) \, w^2(\mathbf{r})}$$
(2.1.6)

where

$$G(\mathbf{k}) \equiv \frac{\int d^3 r \, \overline{n}(\mathbf{r}) \, w(\mathbf{r}) \, e^{i\mathbf{k} \cdot \mathbf{r}}}{\left[\int d^3 r \, \overline{n}^2(\mathbf{r}) \, w^2(\mathbf{r})\right]^{1/2}}.$$
(2.1.7)

The content of this result is readily understood. The density field in our data is the true infinite density field times some mask, so in Fourier space we obtain a convolution between the transforms of the true density field and of the mask. The fair—sample hypothesis assumes that there are no phase correlations between density and mask, so that the power spectrum of the data is the true power spectrum convolved with that of the mask. For Poisson—sampled density fields, we obtain in addition a constant shot—noise contribution to the power. This arises because the discrete density field has a delta function in its correlation function. Since $\xi(0)$ is assumed to be better behaved than this for the underlying density field, this discreteness contribution is usually subtracted.

For the IRAS survey $G(\mathbf{k})$ is a rather compact function with width $\sim 1/D$, where D characterizes the depth of the survey, Provided we restrict attention to $|\mathbf{k}| \gg 1/D$, which is really just the requirement that we have a 'fair sample', and provided $P(\mathbf{k})$ is locally smooth on the same scale, then

$$\langle |F(\mathbf{k})|^2 \rangle \simeq P(\mathbf{k}) + P_{\text{shot}},$$
 (2.1.8)

so the raw power spectrum $|F(\mathbf{k})|^2$ is the true power spectrum plus the constant shot noise component

$$P_{\text{shot}} \equiv \frac{(1+\alpha) \int d^3 r \, \overline{n}(\mathbf{r}) \, w^2(\mathbf{r})}{\int d^3 r \, \overline{n}^2(\mathbf{r}) \, w^2(\mathbf{r})}.$$
 (2.1.9)

Our estimator of $P(\mathbf{k})$ is just

$$\hat{P}(\mathbf{k}) = |F(\mathbf{k})|^2 - P_{\text{shot}},\tag{2.1.10}$$

and to obtain our final estimator of P(k) we average $\hat{P}(\mathbf{k})$ over a shell in k-space:

$$\hat{P}(k) \equiv \frac{1}{V_k} \int_{V_k} d^3 k' \hat{P}(\mathbf{k}'), \qquad (2.1.11)$$

where V_k is the volume of the shell.

Equations 2.1.3, 2.1.9–11 provide our operational definition of $\hat{P}(k)$. To use these we must specify the weight function $w(\mathbf{r})$ which so far has been arbitrary, and we must choose some sampling grid in k-space. In order to set these wisely — and also to put error bars on our estimate of the power — we need to understand the statistical fluctuations in $\hat{P}(\mathbf{k})$.

2.2 Statistical Fluctuations in the Power

From equation (2.1.11) the mean square fluctuation in $\hat{P}(k)$ is

$$\sigma_P^2 \equiv \left\langle \left(\hat{P}(k) - P(k) \right)^2 \right\rangle = \frac{1}{V_k^2} \int_{V_k} d^3k \int_{V_k} d^3k' \langle \delta \hat{P}(\mathbf{k}) \delta \hat{P}(\mathbf{k}') \rangle. \tag{2.2.1}$$

which depends on the two point function of $\delta \hat{P}(\mathbf{k}) \equiv \hat{P}(\mathbf{k}) - P(k)$. Since $\hat{P}(\mathbf{k})$ is itself a two-point function of $F(\mathbf{r})$ this depends on the four-point function which is poorly known. An interesting model for the two point function of $\delta \hat{P}(\mathbf{k})$ is to assume that the Fourier coefficients $F(\mathbf{k})$ are Gaussian distributed, in which case $\langle \delta \hat{P}(\mathbf{k}) \delta \hat{P}(\mathbf{k}') \rangle = |\langle F(\mathbf{k}) F^*(\mathbf{k}') \rangle|^2$ (see appendix B). There are several factors which motivate the Gaussian assumption. The Fourier transform of $F(\mathbf{r})$ at some low spatial frequency \mathbf{k} will receive contributions both from real low frequency density fluctuations and from small scale clustering and discreteness of galaxies. The latter will tend to produce Gaussian fluctuations by virtue of the central limit theorem, and, in the simplest inflationary scenarios at least, the long-wavelength fluctuations will also be Gaussian distributed. Perhaps a stronger motivation is that the Gaussian hypothesis seems to agree well with the observations; see §5.

We can calculate $\langle F(\mathbf{k})F^*(\mathbf{k}')\rangle$ by a simple generalisation of the steps leading to 2.1.6, and we find

$$\langle F(\mathbf{k})F^*(\mathbf{k}')\rangle = \int \frac{d^3k''}{(2\pi)^3} P(\mathbf{k}'')G(\mathbf{k} - \mathbf{k}'')G^*(\mathbf{k}' - \mathbf{k}'') + S(\mathbf{k}' - \mathbf{k}), \qquad (2.2.2)$$

where we have defined

$$S(\mathbf{k}) \equiv \frac{(1+\alpha) \int d^3 r \, \overline{n}(\mathbf{r}) \, w^2(\mathbf{r}) \, e^{i\mathbf{k}\cdot\mathbf{r}}}{\int d^3 r \, \overline{n}^2(\mathbf{r}) \, w^2(\mathbf{r}),}$$
(2.2.3)

and, in the same approximation that led to equation (2.1.8) we obtain

$$\langle F(\mathbf{k})F^*(\mathbf{k} + \delta \mathbf{k})\rangle \simeq P(\mathbf{k})Q(\delta \mathbf{k}) + S(\delta \mathbf{k})$$
 (2.2.4)

where

$$Q(\mathbf{k}) \equiv \frac{\int d^3 r \, \overline{n}^2(\mathbf{r}) \, w^2(\mathbf{r}) \, e^{i\mathbf{k} \cdot \mathbf{r}}}{\int d^3 r \, \overline{n}^2(\mathbf{r}) \, w^2(\mathbf{r})}, \tag{2.2.5}$$

and therefore

$$\langle \delta \hat{P}(\mathbf{k}) \delta \hat{P}(\mathbf{k}') \rangle = |P(\mathbf{k}) Q(\delta \mathbf{k}) + S(\delta \mathbf{k})|^2.$$
 (2.2.6)

Under the Gaussian assumption, $F(\mathbf{k})$ and $\delta \hat{P}(\mathbf{k})$ take the form of locally homogeneous random processes with 2-point functions whose shapes are determined by the survey geometry. The variance in the power $\langle \delta \hat{P}(\mathbf{k})^2 \rangle$ is just the square of the total power (i.e. signal power plus shot noise power), and the two point function of the power is again a compact function with width $\sim 1/D$. Thus, the estimator of the power behaves like an incoherent random field smoothed on scale $\delta k \sim 1/D$. Note the relation between $Q(\delta \mathbf{k})$ and $G(\delta \mathbf{k})$ defined in 2.1.7; these are both measures of the coherence in k space. The fact that G depends on \bar{n} and Q on \bar{n}^2 is just the usual difference between the transform of a field and the transform of its two-point function.

In this regard the power spectrum estimator is very different from the correlation function estimator. If we have some continuous field f(r) that we view through a survey 'window' of scale D, then as D becomes large the estimator of the two-point function tends asymptotically to $\xi(r)$. The microscopic fluctuations in $\hat{P}(\mathbf{k})$, in contrast, remain of order unity even as $D \to \infty$, but the coherence length for the fluctuations shrinks so when we average over some finite volume of frequency space the averaged power tends to P(k), and the fractional fluctuations in $\hat{P}(k)$ will be on the order of the square root of the number of coherence volumes averaged over in equation 2.1.11.

Equation 2.2.1, 2.2.6 can be used in a number of ways: With $P(k) = \hat{P}(k)$ we obtain self-consistent error bars; with $P(k) = P_{\text{CDM}}(k)$, for instance, we obtain the expected fluctuations for this specific theory, and with P(k) = 0 we obtain the 'Poisson' error bars widely used in the past, though these of course tend to underestimate the real uncertainty.

It should be stressed that the Gaussian model is only an assumption. An alternative would be to estimate $\langle \delta \hat{P}(\mathbf{k}) \delta \hat{P}(\mathbf{k} + \delta \mathbf{k}) \rangle$ directly from $\hat{P}(\mathbf{k})$ — as well as providing an empirical error estimate, by comparing this directly with equation 2.2.6 we could obtain an interesting test of the Gaussian nature of the fluctuations. This issue will be discussed in greater depth in §5.

2.3 Optimum Weighting

If the shell we average over in equation 2.1.11 has a width which is large compared to the coherence length then the double integral in 2.2.1 reduces to

$$\sigma_P^2(k) \simeq \frac{1}{V_k} \int d^3k' |P(k)Q(\mathbf{k'}) + S(\mathbf{k'})|^2,$$
 (2.3.1)

so, with the definition of $Q(\mathbf{k})$ and $S(\mathbf{k})$ and using Parseval's theorem, the fractional variance in the power is

$$\frac{\sigma_P^2(k)}{P^2(k)} = \frac{(2\pi)^3 \int d^3r \,\overline{n}^4 \, w^4 [1 + 1/\overline{n}P(k)]^2}{V_k \left[\int d^3r \,\overline{n}^2 \, w^2 \right]^2}.$$
 (2.3.2)

We seek $w(\mathbf{r})$ which minimises this. Writing $w(\mathbf{r}) = w_0(\mathbf{r}) + \delta w(\mathbf{r})$ and requiring that $\sigma_P^2(k)$ be stationary with respect to arbitrary variations $\delta w(\mathbf{r})$ we obtain

$$\frac{\int d^3 r \,\overline{n}^4 \, w_0^3 \left(\frac{1+\overline{n}P}{\overline{n}P}\right)^2 \delta w(\mathbf{r})}{\int d^3 r \,\overline{n}^4 \, w_0^4 \left(\frac{1+\overline{n}P}{\overline{n}P}\right)^2} = \frac{\int d^3 r \,\overline{n}^2 \, w_0 \delta w(\mathbf{r})}{\int d^3 r \,\overline{n}^2 \, w_0^2}$$
(2.3.4)

and it is easy to see by direct substitution that this is satisfied if we take

$$w_0(\mathbf{r}) = \frac{1}{1 + \overline{n}(\mathbf{r})P(k)}. (2.3.5)$$

This is the optimal weighting (under the assumption that the fluctuations are Gaussian). It is the analogue of the " $1 + 4\pi \overline{n} J_3$ " weighting scheme often used in correlation analysis, and, just as in that case, the procedure is somewhat circular since one needs a preliminary estimate of P(k) in order to set the weighting.

The asymptotic behavior of this weighting scheme is very reasonable: Say we want to measure density fluctuations on a particular scale λ . Since $\overline{n}(r)$ is a rapidly decreasing function we will have two regimes: For small r, we will have many galaxies per λ^3 volume, so the error will be dominated by the finite number of independent 'fluctuation volumes' and consequently one would like to give equal weight per volume, or equivalently to weight galaxies in proportion to $1/\overline{n}$. At large radius we are dominated by shot—noise and consequently one would like to weight galaxies equally. Equation 2.3.5 is in accord with these notions. Note that the optimal weight depends on the spatial frequency. Insofar as the power tends to decrease with frequency this means that when we measure long wavelengths we will be inclined to give greater weight to the more distant galaxies.

An important exception to the above analysis arises when the sampling of the density field is not Poissonian. This arises in practice when the sky is divided into a number of zones, and a fixed number of redshifts per zone are measured, independent of the actual number of galaxies present (e.g. the Campanas redshift survey: Shectman *et al.* 1992). Clearly, a gross underestimate of the power spectrum would result if we were simply to treat the measured redshift data as a Poisson sample. The correct procedure is to weight the galaxies according to the sampling factor: if a given galaxy is part of the fraction f sampled from a given field, then we form the density field $n_g(\mathbf{r}) = \sum_i f_i^{-1} \delta(\mathbf{r} - \mathbf{r_i})$. This gives an estimate of the density field which is corrected for the variable sampling, and all that alters is the shot noise term. The first of equations 2.1.5 now becomes

$$\langle n_g(\mathbf{r}) n_g(\mathbf{r}') \rangle = \overline{n}(\mathbf{r}) \overline{n}(\mathbf{r}') [1 + \xi(\mathbf{r} - \mathbf{r}')] + \overline{n}(\mathbf{r}) f^{-1}(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}')$$
(2.2.7)

so that the shot term in 2.1.6 is greater then it would have been for the same number of galaxies with constant sampling. Otherwise, the analysis goes through as before. This may seem a little surprising, given that the sampling factors are not imposed in advance, but 'know' about the large-scale properties of the density field. However, study of the derivation of equation 2.1.5 given in appendix A shows that there is no problem. The critical term is the evaluation of $\langle n_i n_j \rangle$ for two different cells. Even if the mean density of objects has been adjusted to reflect the average density perturbation over some region, galaxies are still distributed at random within that region, which is all that is required in order to write $\langle n_i n_j \rangle$ in terms of $\xi(\mathbf{r_i} - \mathbf{r_j})$.

2.4 A Practical Algorithm.

We now summarize the results, and write all the integrals in terms of discrete sums as will be evaluated on the computer. Let us assume that we are provided with the coordinates for the real and synthetic catalogues plus a function which returns the value of $\overline{n}(\mathbf{r})$ (the details of how the synthetic catalogue was actually constructed for the IRAS survey are given below). Alternatively, the local value of \overline{n} may be provided along with the coordinates of the synthetic galaxies.

We need to evaluate $F(\mathbf{k})$ and also the auxiliary functions $Q(\mathbf{k})$ and $S(\mathbf{k})$. We know that these will be smooth on scale $\delta k \sim 1/D$, so we evaluate these on a cartesian grid. We would like to emphasize that the geometry of the survey requires a cartesian grid, and so we sample k-space linearly and present the results as linear-log plots rather than the traditional log-log plots. A reasonable value for the grid spacing for the IRAS survey is $\delta k = 0.02h$ Mpc⁻¹. Since the functions $Q(\mathbf{k})$ and $S(\mathbf{k})$ will be rather compact we need only evaluate these on a fairly small grid. A convenient way to perform the spatial integrals is to use $\int d^3r \, \overline{n}(\mathbf{r}) \dots \to \alpha \sum_s \dots$, where the sum is over the synthetic galaxies which we assume are sufficiently numerous to define \overline{n} . In order to set the weight scheme (equation 2.3.5) we need to assume some value for P(k). The approach we have taken is to use a range of values for P(k)— each of which provides a legitimate though not necessarily optimal estimate — and then one can select, for any range of wavenumber, the appropriate optimal estimator. Having chosen a value for P(k) it is convenient to adjust the normalization of the weight function so that

$$\int d^3 r \, \overline{n}^2(\mathbf{r}) \, w^2(\mathbf{r}) \to \alpha \sum_s \overline{n}(\mathbf{r}_s) \, w^2(\mathbf{r}_s) = 1$$
 (2.4.1)

we then evaluate

$$F(\mathbf{k}) = \int d^3r \ w(\mathbf{r})[n_g(\mathbf{r}) - \alpha n_s(\mathbf{r})]e^{i\mathbf{k}\cdot\mathbf{r}} \to \sum_g w(\mathbf{r}_g)e^{i\mathbf{k}\cdot\mathbf{r}_g} - \alpha \sum_s w(\mathbf{r}_s)e^{i\mathbf{k}\cdot\mathbf{r}_s}, \quad (2.4.2)$$

$$Q(\mathbf{k}) = \int d^3 r \, \overline{n}^2(\mathbf{r}) \, w^2(\mathbf{r}) \, e^{i\mathbf{k}\cdot\mathbf{r}} \to \alpha \sum_s \overline{n}(\mathbf{r}_s) \, w^2(\mathbf{r}_s) e^{i\mathbf{k}\cdot\mathbf{r}_s}$$
(2.4.3)

and

$$S(\mathbf{k}) = (1+\alpha) \int d^3 r \, \overline{n}(\mathbf{r}) \, w^2(\mathbf{r}) \, e^{i\mathbf{k}\cdot\mathbf{r}} \to \alpha (1+\alpha) \sum_s w^2(\mathbf{r}_s) e^{i\mathbf{k}\cdot\mathbf{r}_s}. \tag{2.4.4}$$

Our radially averaged power spectrum estimator is

$$\hat{P}(k) = \frac{1}{N_k} \sum_{k < |\mathbf{k}| < k + \delta k} |F(\mathbf{k})|^2 - S(0)$$
(2.4.5)

where N_k is the number of modes in the shell.

In §2.3 we obtained the rather simple expressions 2.3.1, 2.3.2 for the variance in the power. These say that the rms fractional fluctuation in the power is just the square root of the number of 'coherence volumes' in k-space: $\sigma_P(k)/P(k) = \sqrt{V_c/V_k}$, where the coherence volume is defined by $V_c \equiv (2\pi)^3 \int d^3r \, \overline{n}^4 \, w^4 (1+1/\overline{n}P)$. However, these require that the shell be thick compared to the coherence length. This is fine at high frequencies $k \gg 1/D$, but at low frequency this would result in a loss of resolution. Instead, we use the analogue of 2.2.1

$$\sigma_P^2(k) = \frac{2}{N_k^2} \sum_{\mathbf{k'}} \sum_{\mathbf{k''}} |PQ(\mathbf{k'} - \mathbf{k''}) + S(\mathbf{k'} - \mathbf{k''})|^2$$
(2.4.6)

where \mathbf{k} and \mathbf{k}' are constrained to lie in the shell and which is valid for any shell thickness. If we make the shells thinner than or comparable to the coherence length then neighboring shells will be correlated, but the variance tends to a well defined finite value even as the shell tends to zero width.

2.5 Covariance Matrix

The error estimate 2.4.6 is adequate to give an estimate of the variance in the power spectrum, but for a more precise assessment of theoretical models it is necessary to quantify the degree of correlation of $\hat{P}(k)$ at differing wave numbers. The way to do this is via likelihood analysis.

Provided the fractional error is moderately small (i.e. the shell intercepts a sufficiently large number of coherence volumes), the fluctuations in the power will themselves tend to become Gaussian distributed, and the vector of estimates \hat{P}_i together with the correlation matrix allow one to evaluate the likelihood for any particular theory:

$$L[P_{\rm th}(k)] = p[P_i|P_{\rm th}(k)] = \frac{e^{-C_{ij}^{-1}[\hat{P}_i - P_{\rm th}(k_i)][\hat{P}_j - P_{\rm th}(k_j)]/2}}{(2\pi)^{N/2}|c|} . \tag{2.5.1}$$

This provides a quantitative way to compare theories.

This is rather similar to other applications of likelihood analysis to cosmological data sets (Kaiser 1992, Bond & Efstathiou 1984), but there is a slight difference here in that the correlation matrix for the binned estimates of \hat{P} actually depends on P(k) itself:

$$C_{ij} \equiv \langle \delta \hat{P}(k_i) \delta \hat{P}(k_j) \rangle = \frac{2}{N_k N_{k'}} \sum_{\mathbf{k}} \sum_{\mathbf{k'}} |PQ(\mathbf{k} - \mathbf{k'}) + S(\mathbf{k} - \mathbf{k'})|^2$$
 (2.5.2)

where \mathbf{k} and \mathbf{k}' lie in the shells around k_i and k_j respectively.

Equation 2.5.2 is derived, like equation 2.2.6, under the assumption that P(k) is effectively constant over a coherence scale. With this assumption we can write

$$C_{ij} = C_{ij}^{(0)} + C_{ij}^{(1)} \sqrt{P_{\text{th}}(k_i)P_{\text{th}}(k_j)} + C_{ij}^{(2)} P_{\text{th}}(k_i)P_{\text{th}}(k_j)$$
 (2.5.3)

The use of this will be illustrated in $\S 4$.

3. THE QDOT SURVEY

3.1 The Survey.

We apply the analysis to the updated (A. Lawrence 1993, private communication) IRAS QDOT survey (Efstathiou et al. 1990). The QDOT survey chooses at random one in six galaxies from the IRAS point source catalogue with a flux limit at 60μ m $f_{60} > 0.6$ Jy. In this sample there are 1824 galaxies above galactic latitude $|b| > 10^{\circ}$ with redshifts that corresponds to radii $20 h^{-1}$ Mpc $< R < 500 h^{-1}$ Mpc. We have used a revised version of the QDOT database in which a redshift error that afflicted approximately 200 southern galaxies has been corrected. We have converted all redshifts to the local group frame.

The survey provides us with an angular mask. The sky is divided into 41167 bins, each $\approx 1^{\circ} \times 1^{\circ}$, some of which are masked. Since most of the bins below galactic latitude $|b| < 10^{\circ}$ are masked because of obscuration due to the galactic plane, we masked off all bins that had $|b| < 10^{\circ}$. There are 30444 unmasked bins, a coverage of $\approx 74\%$ of the sky. The QDOT galaxy distribution and mask are shown in figure 1. A more complete description of the sample is given in Efstathiou *et al.* (1990).

To apply the above formalism to the QDOT survey, we wrote two independent codes that utilized different grids in k-space and different forms for the radial selection function. We obtained very good agreement between the results for all values of P(k) in the weight function (Eqn. 2.3.5).

3.2 Construction of the Synthetic Catalogue

Once we have the radial galaxy distribution from the survey, we bin the galaxies in bins of width dr and then fit, using a χ^2 technique, a number function

$$n(r) = A \left(\frac{r}{r_{\text{max}}}\right)^x \left[1 + \left(\frac{r}{r_{\text{max}}}\right)^y\right]^{-\frac{y+x}{y}}$$
(3.2.1)

where

$$A = 2^{\frac{y+x}{y}} n_{\text{max}} , (3.2.2)$$

 $n_{\text{max}} \equiv n(r = r_{\text{max}})$ and r_{max} is the radius where n(r) peaks (see figure 2 for the radial distribution of the IRAS QDOT galaxies and a best fit). We integrated n(r) and divided the function into n_n bins, each of which has the same number of galaxies. To check the dependence on the exact choice of the radial selection function, we also tried log-normal bins and varied the parameters in the fitting function Eqn. 3.2.1 to give us a 1σ effect. The effect of these choices on the final results is negligible. We constructed the synthetic catalogue by distributing galaxies with a Poisson distribution in all unmasked bins given the above selection function.

3.3 The Power spectrum.

We have seen that the optimum weight function depends on k (Eqn. 2.3.5), so that a different weighting should in principle be used at each wavenumber. This would be rather cumbersome, and so we have chosen in practice to evaluate the weight assuming a given constant level of power (i.e. a n=0 white noise power spectrum). By varying this assumed power over values that cover the observed power, it is easy to see what effect the exact choice of weight would have. We shall consider four different assumed power levels, which produce the different effective survey depths shown in figure 3. In figure 4 we show the power spectrum results for the full IRAS QDOT survey using these different weight function parameterizations. We see that the larger the power in the weight function is (i.e. the greater the effective depth of the survey) the more power we get. The effect is most marked at the shallow end: we gain roughly a factor 1.5 in power when the assumed power changes from $2000 (h^{-1}\text{Mpc})^3$ to $4000 (h^{-1}\text{Mpc})^3$, but things change relatively little thereafter. This suggests that the effect is a local one that represents true sampling fluctuations, but that the correct average power is detected for greater depths.

We shall generally adopt the results with $P = 8000 (h^{-1}\text{Mpc})^3$ as our 'standard' set of values, since this power is closest to the average level in the k range of interest.

We attempted to choose different selection criteria that might help us disentangle clustering effects that arise because the more distant galaxies are intrinsically more luminous than those which are nearby. We divided the data into six samples with different flux limits, and for each we chose a number of weight functions by changing the power P(k) in $w(\mathbf{r})$. We found no systematic effects that appeared to depend primarily on luminosity, rather than distance.

The most interesting feature of the power-spectrum results is that the function is relatively smooth, with no significant sharp features. As is discussed in more detail below, there is an excess at 0.03 < k < 0.07 when comparing to the CDM power spectrum with $\Omega h = 0.5$, normalized around k = 0.1. This occurs for all weight functions we used as well as for most sub-samples. At the largest wavelengths, our data indicate a turnover in the power spectrum, with reduced power for $k \lesssim 0.04$ corresponding to $\lambda \gtrsim 150 \; h^{-1} \; \mathrm{Mpc}$. Although the error bars are large, this also is a feature which appears to be robust with respect to changing the depth of the survey. There is the worry that the turnover may be a normalization effect: our power spectra must vanish at k=0 since we obtain the mean density from the data. Whether this is a problem depends on the convolution function G(k)(Eqn. 2.1.7), and is an issue discussed by Peacock & Nicholson (1991). If we approximate the function by a Gaussian $|G(k)|^2 = \exp{-k^2R^2}$, then the power–spectrum estimate is sensitive to normalization effects only for $k \leq 1.5/R$. The appropriate values of R that fit our G function vary from $R=65\,h^{-1}{\rm Mpc}$ for $P=2000\,(h^{-1}{\rm Mpc})^3$ to $R=105\,h^{-1}{\rm Mpc}$ for $P = 16000 (h^{-1}{\rm Mpc})^3$. The turnover in the QDOT power spectrum is thus not an artifact of self-normalization. Furthermore, the location of the turnover agrees well with the position of the same feature as seen in other data sets (e.g. Peacock & West 1992).

4. DISCUSSION OF POWER-SPECTRUM RESULTS

In §4.1 we compare our results to other probes and in §4.2 we present theoretical model power spectra and compare them to the QDOT results. In §4.3 we present the results of CDM likelihood analysis in redshift space and in §4.4 we transform the results to real space.

4.1 Comparison With Other Probes

In figure 5 we show a comparison between the shallow $[P(k) = 2000 \ (h^{-1} \text{ Mpc})^3]$ IRAS QDOT power spectrum and the 1.2–Jy Berkeley survey (Fisher *et al.* 1993). The error bars on the 1.2–Jy survey are derived from the standard deviation of 10 'mock' IRAS standard CDM simulations (see Fisher *et al.* 1993 for details). The analysis method used on the 1.2–Jy data differs somewhat from that used here; they weighted volumes equally within some cylindrical volume of varying size, up to a maximum of length $180h^{-1}$ Mpc by radius $90h^{-1}$ Mpc. This is equivalent in volume to a sphere of radius $103h^{-1}$ Mpc. Comparing with figure 3, we see that our shallowest weight does give roughly constant weighting to volumes out to this radius, which is why we have presented the comparison in this way.

Once scaled to the same depth, the surveys agree reasonably well. The 1.2–Jy data lie slightly below our shallow results; however, there would be a much more marked discrepancy if we had chosen to compare to the 'standard' $P = 8000(h^{-1}\text{Mpc})^3$ QDOT results — about a factor of 1.5 in power. Also, because of their choice of sampling in k–space, the 1.2–Jy resolution is not very high for large scales (small k) which perhaps led them to miss some of the structure of the power at scales $\approx 100 - -150 \, h^{-1}$ Mpc. As mentioned above, the survey geometry requires linear sampling of k–space, and so linear plotting of results.

In figure 6 we show a comparison with data from the CfA surveys (Vogeley et al. 1992). The power spectra are of galaxies from the CfA 1 and the CfA 2 surveys, divided into two categories: CfA 100 and the deeper CfA 145 (for details see Vogeley et al. 1992). The error bars are derived from estimating the 95% confidence level from the variation of 100 'mock' CfA surveys from open CDM simulations. As with QDOT, the CfA results appear to show some weak trend for the power to increase with increasing sample depth. The shapes of the CfA and QDOT power spectra appear to differ: similar levels of power are seen at large wavelength, but the CfA spectra show much more small—scale power. However, note the relative sizes of the large—wavelength error bars in the CfA data, plus the fact that their analysis gives no idea of the extent of any cross—correlation between different points. At present, it is probably not possible to rule out with great confidence the hypothesis that CfA and QDOT power spectra have the same shape, but an amplitude different by a factor of about 2. Improved data will be interesting here, as a difference in the shapes of the power spectra for optically—selected and IRAS galaxies must hold

information about the processes which created the differing spatial distributions for these objects.

4.2 Fitting Model Power Spectra

It is interesting to fit the power–spectrum data by some analytical model. Since the result appears to be a relatively smooth function, it is convenient to use for this purpose the CDM linear power spectrum. This allows for parameterization of the slope and amount of curvature, depending on the density. Initially, we shall use this simply as an empirical way of describing the data; the physical interpretation of the results will be given later.

The CDM power spectrum takes the form of a scale–invariant spectrum modified by some transfer function: $P(k) \propto k T_k^2(k)$. We shall use the BBKS approximation (Bardeen et al. 1986), which is the most accurate fitting formula:

$$T_k = \frac{\ln(1+2.34q)}{2.34q} [1+3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{-1/4}, \tag{4.2.1}$$

where $q \equiv k/[\Omega h^2 \text{ Mpc}^{-1}]$. Since observable wavenumbers are in units of $h \text{ Mpc}^{-1}$, the shape parameter is the apparent value of Ωh . [This should not be confused with the parameter Γ defined by Efstathiou, Bond & White (1992); they considered a transfer function which fits a CDM model with non–zero baryon density $\Omega_B = 0.03$. Empirically, the wavenumber scaling in the CDM model depends very nearly on $\Omega h^2 \exp[-2\Omega_B]$; Efstathiou et al. defined $\Gamma = 0.5$ to correspond to $\Omega h = 0.5$, and our values of Ωh are thus smaller than Γ by a factor of 0.94.] The normalization of the power spectrum can be expressed in various ways (e.g. Peacock 1991); here we shall use σ_8 — the linear rms in spheres of radius $8h^{-1}$ Mpc.

In figure 7 we show a comparison with a 'standard' linear CDM power spectrum having $\Omega h = 0.5$. The poor fit is apparent, although whether one regards this as the data having an excess of large—scale power or a lack of small—scale power is a matter both of taste and of where we choose to normalize. We further compare the results to the APM fitting formula of Peacock (1991):

$$P(k) = k^{-3} \frac{(k/k_0)^{1.6}}{1 + (k/k_c)^{-2.4}} . (4.2.2)$$

A reasonable fit to the IRAS QDOT power spectrum is achieved with $k_0 = 0.03 h \,\mathrm{Mpc}^{-1}$, and $k_c \in [0.025, 0.04] \, h \,\mathrm{Mpc}^{-1}$. The overall shape and position of the break agree reasonably well with the APM data; however, note that $k_c \simeq 0.015 h \,\mathrm{Mpc}^{-1}$ is required to best–fit the APM data, and such a large break wavelength appears to be excluded by our data.

We also compare our results with simulations of mixed [cold (70%) plus hot (30%)] dark matter (MDM) (Klypin et al. 1992). The 'red galaxies' power spectrum in the MDM simulations is the one of all the dark matter halos. The halos were defined to be at the maxima of the overdense regions with overdensity > 50. The 'galaxies' were displaced along the line of sight in accordance with there peculiar velocities to mimic redshift space and the density field was smoothed with a Gaussian filter of 1/2 a cell size radius to reduce shot noise (Klypin 1993, private communication). The overall shape of the power of the red galaxies in the MDM simulations agrees quite well with our power spectrum. As we shall see below, this is because the MDM scenario gives results that are rather similar to those of CDM models with low Ωh or 'tilted' spectra.

Rather than looking at a priori models any further, we shall now proceed to fit linear CDM power spectra to the QDOT redshift–space power–spectrum data. We emphasize that the resulting values of shape (Ωh) and normalization (σ_8) are apparent redshift–space values only. Before they can be interpreted, we shall need to consider the effects of nonlinearities and distortions caused by the mapping between real space and redshift space.

4.3 Results in Redshift Space

Since we have a procedure for constructing the power spectrum covariance matrix for any given model (equation 2.5.3), it is easy to fit the CDM models correctly to our data using maximum likelihood. To within a constant, the likelihood is

$$-\ln L = \chi^2/2 + [\ln \det C]/2, \tag{4.3.1}$$

and

$$\chi^2 = C_{ij}^{-1} [\hat{P} - P_{\text{CDM}}]_i [\hat{P} - P_{\text{CDM}}]_j, \tag{4.3.2}$$

where C_{ij} is the covariance matrix for our data.

Figure 8 shows contours of likelihood at $-\ln L = \text{minimum} + 0.5, 1, 2, \ldots$ On a Gaussian approximation, the 95% confidence level would be at $\Delta \ln L = 3$. This plot shows that the maximum–likelihood model is well defined (and is an excellent description of the data: $\chi^2 = 12$ on 18 degrees of freedom). The best–fitting parameters and their rms uncertainties are

$$\sigma_8 = 0.88 \pm 0.07$$
 (redshift space)
 $\Omega h = 0.25 \pm 0.08$ (redshift space) (4.3.3)

We may also consider the use of the CDM transfer function with a power spectrum which is not scale–invariant. Possibilities include 'tilted' models:

$$P \propto k^n; \ n \simeq 0.8 \tag{4.3.4}$$

(Cen et al. 1992) or an inflationary prediction for logarithmic corrections to a scale—invariant spectrum (Kofman, Gnedin & Bahcall 1993). The latter corrections have a negligible effect on our results, but the use of tilted models is important. The apparent value of σ_8 is insensitive to the assumed value of n, but the apparent redshift–space value of Ωh changes approximately as

$$\Omega h = 0.25 + 0.29([1/n] - 1). \tag{4.3.5}$$

Comparing these results with those from the Berkeley 1.2–Jy survey (Fisher et al. 1992), we find reasonable agreement. Their apparent value of σ_8 is 0.80, with a best-fitting $\Omega h = 0.2$. These figures are well within our confidence limits. This is perhaps a little surprising, since we have seen earlier that there appears to be a difference in power of about a factor 1.5 between the 1.2–Jy results and the deeper QDOT results which are used here. On this basis, we would have predicted a lower value of σ_8 for the Berkeley data; the reason for this discrepancy is unclear.

4.4 Results in real space

Our observed power spectrum in redshift space differs in three ways from the quantity of interest, which is the underlying linear—theory power spectrum of mass fluctuations. This is altered by nonlinear evolution, by redshift—space mapping and by bias.

The last of these is easily dealt with through ignorance: we shall assume scale—independent bias relating the *nonlinear* power spectra

$$P_{\text{real}} = b^2 P_{\text{mass}}. (4.4.1)$$

It is clearly reasonable to treat b as a constant if the galaxy distribution is close to being unbiased. For a significant degree of bias, one really needs a model; empirically, if our conclusions conflict with other data, this could be interpreted as saying that b must depend on scale.

The mapping between real and redshift space introduces two effects. On large scales, there is the linear increase of power described by Kaiser (1987):

$$P(k) \to P(k) \left[1 + \frac{2\Omega^{0.6}}{3b} + \frac{\Omega^{1.2}}{5b^2} \right].$$
 (4.4.2)

On small scales, there is the filtering effect of virialized peculiar velocities to deal with; measured redshifts will also be of limited precision. These effects can be treated in the same way: consider the simplest possible case in which these effective errors constitute a Gaussian scatter characterized by some spatial rms σ . In azimuthal average, this gives

$$P(k) \to P(k) \frac{\sqrt{\pi}}{2} \frac{\text{erf } (k\sigma)}{k\sigma}$$
 (4.4.3)

(Peacock 1992); modes at high k are thus only damped by one power of k, rather than exponentially. Fisher et~al.~(1992) show that the combination of these factors describes the relation between power spectra in real and redshift space quite accurately. Small–scale velocities of about 200 kms⁻¹ rms are observationally appropriate, and QDOT redshifts have a typical error of 300 kms⁻¹, making a total effective σ of $4.4h^{-1}{\rm Mpc}$. At the largest wavenumber considered here ($k=0.2h\,{\rm Mpc}^{-1}$), the corresponding correction factor to the power is 0.85. This latter correction is unavoidable, since it is based largely on uncertainties in the data. This effect alone alters the best–fitting values of the spectral parameters to $\sigma_8=0.92\pm0.07$ and $\Omega h=0.28\pm0.08$.

The nonlinear distortions of the spectrum can be dealt with analytically (assuming $\Omega = 1$) by using the remarkable formulae given by Hamilton *et al.* (1991). The result for CDM-like spectra is that power is removed from wavenumbers $k \simeq 0.1 h\,\mathrm{Mpc}^{-1}$, which lowers the apparent value of Ωh .

The result of applying all these corrections can be expressed in terms of the change in the best–fitting values of Ωh and σ_8 as a function of b:

$$b\,\sigma_8 \simeq 0.92 - 0.18/b^{0.8}$$
 (recovered linear value)
$$\Omega h \simeq 0.28 + 0.05/b^{1.3}$$
 (recovered linear value) (4.4.4)

Consistency with the values $\sigma_8 = 0.57$ deduced by White, Efstathiou & Frenk (1993) requires b = 1.37 and $\Omega h = 0.31 \pm 0.08$. This last figure is in remarkable agreement with the $\Omega h = 0.32 \pm 0.07$ deduced from the cluster–galaxy cross–correlation function by Mo, Peacock & Xia (1993).

If the density parameter is low, the linear amplification of power in redshift space does not occur, so that the nonlinear real–space value of $b \sigma_8$ will be higher: close to the observed redshift–space value of 0.92. The formulae of Hamilton *et al.* (1991) do not apply to the low–density case, so we cannot say so exactly what the effects of nonlinearities will be. However, experience with N–body simulations suggest that the recovered linear values of σ_8 and Ωh will still be altered by an amount of the order 0.1 (for b=1) — as in the $\Omega=1$ case.

5. TESTS FOR NON-GAUSSIANITY

The expectation value of the power that we have tried to estimate contains in itself no information about phase correlations or higher than two-point correlations. As we have seen however, the estimated power will inevitably fluctuate strongly from point to point in k-space, and these fluctuations about the mean power are rich in information about higher-order correlations.

One probe of non–Gaussianity is from the 1–point (in k–space that is) distribution of the power. According to the Gaussian hypothesis, the power should be exponentially distributed:

$$p(>\hat{P}) = \exp(-\hat{P}/\overline{P}) \tag{5.1}$$

(Kendall & Stewart 1977) and this has been exploited as a way to quantify evidence for periodicity in pencil—beam surveys for instance (Szalay et al. 1990). The pencil—beam analysis did indeed appear to show some evidence for non–Gaussianity: the distribution of the power was enhanced at high values of \hat{P} . However, it can plausibly be argued that what is happening here is that the distribution of power was calculated over a wide range of wave–numbers and that the high frequency components were suppressed by smoothing by redshift errors and by random motions (Kaiser & Peacock 1991). Thus what one sees in the pencil—beam case is a blend of exponential distributions with different length scales. One way to resolve this would be to restrict the range to low frequencies only, but unfortunately then the number of independent 'coherence cells' is rather small (even though the baseline for this sample is impressively long).

Because we work in three dimensions, the present work yields a much larger number of independent estimates of the low–frequency power. The distribution of the power for k < 0.1 (wavelengths $> 60h^{-1}$ Mpc) is shown in figure 9. The agreement with the exponential prediction is remarkably good. It should be kept in mind that what we are seeing here is the distribution of the 'raw' power which contains a superposition of the shot noise power, whose long wavelength Fourier components is essentially guaranteed to be Gaussian distributed by virtue of the central limit theorem. For the wavelengths employed in figure 9, the real long–wavelength power is roughly equal to the shot noise power. The fact that the exponential law is nevertheless exact out to $\hat{P}/\bar{P} \simeq 10$ is thus an extremely exacting test of the Gaussian hypothesis — and should provide an important constraint on non–Gaussian models such as those based on topological defects.

There are potentially many other statistics that one could construct based on the Fourier components which would measure interesting high order correlations. One particularly interesting one is the two-point function of the power $\chi(\delta k;k) \equiv \langle \hat{P}(k)\hat{P}(k+\delta k)\rangle$. As we saw in §2.4, if we assume Gaussian fluctuations then $\chi(\delta k;k)$ is simply determined from the geometry of the survey (it is essentially the Fourier transform of the survey volume) and will therefore have width in δK on the order of 1/D. It is not difficult to see however how this prediction might be modified for certain rather interesting non-Gaussian models. Consider, for instance, a density field which is the product of a Gaussian field with a 'modulating' field which has only long wavelength components. In such a model, the density field would look locally Gaussian, but seen on a larger scale one would have patches of greater or lesser amplitude. The micro-scale fluctuations in $\hat{P}(k)$ in such a model would be the same as in a pure Gaussian model but with a patchy survey volume; i.e. the two point function would be more extended than predicted by equation (2.4.6). The excess width of the χ statistic therefore measures the degree to which fluctuations of spatial frequency k are being modulated by frequencies $\sim \delta k$.

6. CONCLUSIONS

We have presented a formalism for power–spectrum analysis of fully three–dimensional deep redshift surveys. Our main new result is an analytical estimation of the statistical uncertainties in the power (including both sampling and galaxy counting statistics). We have also presented a rigorous analytical formulation of the optimal weight function for the data, assuming that the long–wavelength Fourier components are Gaussian distributed. Assuming that the power is smooth, we have shown how to derive the full covariance matrix for the power spectrum. This provides all the information necessary for a proper statistical comparison between power–spectrum data and theory.

We have applied the method to the updated 1-in-6 QDOT IRAS redshift survey. We find that survey depths in excess of $100h^{-1}$ Mpc are necessary in order to obtain a stable estimate of the power spectrum. Our results strengthen and quantify the impression that there is extra power on large scales as compared to the standard CDM model with $\Omega h \simeq 0.5$. Nevertheless, there appears to be a break in the power spectrum at wavelengths $\lambda \approx 150 - 200h^{-1}$ Mpc, with sharply reduced power for larger wavelengths. This is consistent with the picture emerging from a number of other studies.

We have applied likelihood analysis using the BBKS approximation of the CDM spectrum with Ωh as a free parameter as a phenomenological family of models: in redshift

space the best-fitting parameters are $\sigma_8 = 0.88 \pm 0.07$, $\Omega h = 0.25 \pm 0.08$. We have attempted to treat the distortions to the power spectrum introduced by nonlinear evolution and the redshift-space mapping, and so recover the parameters which describe the linear power spectrum. If the linear rms variance is taken to agree with White *et al.* (1993) $(\sigma_8 = 0.57)$, we find that a linear power spectrum with $\Omega h = 0.31 \pm 0.08$ is implied, in excellent agreement with the figure deduced from the cluster-galaxy cross-correlation function by Mo *et al.* (1993).

We have calculated the distribution of the estimated long-wavelength power, and searched for signs of non–Gaussianity in the 1–point (in k–space) distribution of the power. We found no trace of non–Gaussian behavior; rather, the distribution agreed exceptionally well with the exponential distribution expected for Gaussian fluctuations and we found no sign of periodicity or any particularly strong spatial frequencies. What is needed now is a well motivated non–Gaussian model with which to compare this strong observational constraint.

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Figure Captions

- Fig. 1) The IRAS QDOT galaxy distribution (open circles) in the range of $20h^{-1}{\rm Mpc} < R < 500h^{-1}{\rm Mpc}$ and the angular mask (solid squares). Each masked bin is $\approx 1^o \times 1^o$.
- Fig. 2) A histogram of the IRAS QDOT survey and a χ^2 fit of Eqn. 2.5.2. (solid line) used as the radial selection function that defines the synthetic catalogue and the mean space density of the galaxies.
- Fig. 3) The optimal weight function parameterized by the power P(k). By varying the assumed power over values that cover the observed power, we in effect produce different effective survey depths.
- Fig. 4) The power spectrum for the full IRAS QDOT survey using four weight function parameterizations. We see that the larger the power in the weight function is (i.e. the greater the effective depth of the survey) the more power we get. The effect is most marked at the shallow end: we gain roughly a factor 1.5 in power when the assumed power changes from $2000 (h^{-1}\text{Mpc})^3$ to $4000 (h^{-1}\text{Mpc})^3$, but things change relatively little thereafter. This suggests that the effect is a local one that represents true sampling fluctuations, but that the correct average power is detected for greater depths.
- Fig. 5) A comparison of the IRAS QDOT survey with $P(k) = 2000 \ (h^{-1} \text{ Mpc})^3$ in the weight function with the 1.2–Jy IRAS survey. Comparing with figure 3, we see that our shallowest weight gives roughly constant weighting to volumes corresponding to the 1.2–Jy survey, which is why we have presented the comparison in this way. Once scaled to the same depth, the surveys agree reasonably well. The 1.2–Jy data lie slightly below our shallow results; however, there would be a much more marked

- discrepancy if we had chosen to compare to the 'standard' $P = 8000(h^{-1}\text{Mpc})^3$ QDOT results about a factor of 1.5 in power.
- Fig. 6) A comparison the IRAS QDOT survey with $P(k) = 16000 \ (h^{-1} \text{ Mpc})^3$ in the weight function with the CfA surveys. As with QDOT, the CfA results appear to show some weak trend for the power to increase with increasing sample depth. The shapes of the CfA and QDOT power spectra appear to differ: similar levels of power are seen at large wavelength, but the CfA spectra show much more small—scale power. However, note the relative sizes of the large—wavelength error bars in the CfA data, plus the fact that their analysis gives no idea of the extent of any cross—correlation between different points.
- Fig. 7) A comparison of the IRAS QDOT survey with $P(k) = 8000 \ (h^{-1} \ \text{Mpc})^3$ in the weight function with some theoretical models normalized at $k = 0.1h \ \text{Mpc}^{-1}$. 1) For the BBKS linear CDM model with $\Omega h = 0.5$, the poor fit is apparent. Whether one regards this as suggesting an excess of large-scale power or a lack of small-scale power is a matter both of taste and of where we choose to normalize. 2) The APM fitting function gives a reasonable fit to the IRAS QDOT power spectrum. The overall shape agrees quite well with the APM data; however, the position of the break that is required to best-fit the APM data appears to be excluded by our results. 3) MDM simulations agree quite well with the IRAS QDOT data. The MDM scenario gives results that are rather similar to those of CDM models with low Ωh or 'tilted' spectra. 4) $P(k) \propto k^{-1.4}$ gives good agreement for $k > 0.04h \ \text{Mpc}^{-1}$.
- Fig. 8) Likelihood contours at $-\ln L = \text{minimum} + 0.5, 1, 2...$ The 95% confidence level would be at $\Delta \ln L = 3$. This plot shows that the maximum–likelihood model is well defined (and is an excellent description of the data: $\chi^2 = 12$ on 18 degrees of freedom).
- Fig. 9) The distribution of the power for k < 0.1 ($\lambda > 60h^{-1}$ Mpc). The agreement with the exponential prediction is remarkable. It should be kept in mind that what we are seeing here is the distribution of the 'raw' power which contains a superposition of the shot noise power, whose long wavelength Fourier components is essentially guaranteed to be Gaussian distributed by virtue of the central limit theorem. For the wavelengths employed here, the real long-wavelength power is roughly equal to the shot noise power. The fact that the exponential law is nevertheless exact

out to $\hat{P}/\bar{P} \simeq 10$ is thus an extremely exacting test of the Gaussian hypothesis – and should provide an important constraint on non–Gaussian models such as those based on topological defects.

APPENDIX A

Two-Point Function For a Poisson-Sample Point Process

If we have a point process $n(\mathbf{r})$ which is a "Poisson sample" of some continuous stochastic field $1 + f(\mathbf{r})$ with a given mean density of points $\overline{n}(\mathbf{r})$ (i.e. the probability that an infinitesimal volume element δV contains an object is $\overline{n}(\mathbf{r})[1+f(\mathbf{r})]\delta V$) then the two-point function of $n(\mathbf{r})$ is

$$\langle n(\mathbf{r})n(\mathbf{r}')\rangle = \overline{n}(\mathbf{r})\overline{n}(\mathbf{r}')[1 + \xi(\mathbf{r} - \mathbf{r}')] + \overline{n}(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}')$$
(A1)

where $\xi(\mathbf{r}) \equiv \langle f(\mathbf{r}')f(\mathbf{r}' + \mathbf{r}) \rangle$.

To prove this, consider the expectation value of

$$\int d^3r \int d^3r' g(\mathbf{r}, \mathbf{r}') n(\mathbf{r}) n(\mathbf{r}') \tag{A2}$$

where $g(\mathbf{r}, \mathbf{r}')$ is an arbitrary function. Using the standard procedure (Peebles 1980, §36) of converting such integrals to sums over infinitesimal microcells with occupation numbers $n_i = 0, 1$ and using $\langle n_i n_j \rangle = \overline{n}(\mathbf{r}_i) \overline{n}(\mathbf{r}_j) \delta V^2 [1 + \xi(\mathbf{r}_i - \mathbf{r}_j)]$ if $i \neq j$ and $\langle n_i^2 \rangle = \langle n_i \rangle = \overline{n}(\mathbf{r}_i) \delta V$ we find

$$\left\langle \int d^3r \int d^3r' g(\mathbf{r}, \mathbf{r}') n(\mathbf{r}) n(\mathbf{r}') \right\rangle = \int d^3r \int d^3r' g(\mathbf{r}, \mathbf{r}') \langle n(\mathbf{r}) n(\mathbf{r}') \rangle$$
$$= \sum_{i} \sum_{j} g(\mathbf{r}_i, \mathbf{r}_j) \langle n_i n_j \rangle$$

$$= \sum_{i} \sum_{j} g(\mathbf{r}_{i}, \mathbf{r}_{j}) \overline{n}(\mathbf{r}_{i}) \overline{n}(\mathbf{r}_{j}) [1 + \xi(\mathbf{r}_{i} - \mathbf{r}_{j})] \delta V^{2} + \sum_{i} g(\mathbf{r}_{i}, \mathbf{r}_{i}) \overline{n}(\mathbf{r}_{i}) \delta V$$

$$= \int d^{3}r \int d^{3}r' g(\mathbf{r}, \mathbf{r}') \overline{n}(\mathbf{r}) \overline{n}(\mathbf{r}') [1 + \xi(\mathbf{r} - \mathbf{r}')] + \int d^{3}r g(\mathbf{r}, \mathbf{r}) \overline{n}(\mathbf{r})$$

$$= \int d^{3}r \int d^{3}r' g(\mathbf{r}, \mathbf{r}') \{ \overline{n}(\mathbf{r}) \overline{n}(\mathbf{r}') [1 + \xi(\mathbf{r} - \mathbf{r}')] + \overline{n}(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') \}$$
(A3)

Since this must be true for an arbitrary function $g(\mathbf{r}, \mathbf{r}')$ then comparing the first and last lines of (A3) we obtain the identity (A1).

APPENDIX B

Fluctuations in power for a Gaussian field

We need to evaluate the two point function of fluctuations in the power $\langle \delta \hat{P}(\mathbf{k}) \delta \hat{P}(\mathbf{k}') \rangle$ [where $\delta \hat{P}(\mathbf{k}) \equiv \hat{P}(\mathbf{k}) - P(k)$]:

$$\langle \delta \hat{P}(\mathbf{k}) \delta \hat{P}(\mathbf{k}') \rangle = \langle \hat{P}(\mathbf{k}) \hat{P}(\mathbf{k}') \rangle - \langle P(\mathbf{k}) \rangle \langle P(\mathbf{k}') \rangle. \tag{B1}$$

To make progress, we shall assume that the large-wavelength portion of the power spectrum describes a Gaussian field. A possible way of proceeding would then be to consider separately the real and imaginary parts of the Fourier field $F(\mathbf{k}) = c_{\mathbf{k}} + i s_{\mathbf{k}}$, write down all relevant correlations ($\langle c_{\mathbf{k}} s_{\mathbf{k}'} \rangle$ etc.) and use the general relation for a bivariate Gaussian

$$\langle x^2 y^2 \rangle = \langle x^2 \rangle \langle y^2 \rangle + 3 \langle xy \rangle \sqrt{\langle x^2 \rangle \langle x^2 \rangle} - \langle xy \rangle^2. \tag{B2}$$

A less cumbersome method is to appeal to the idea that realizations of Gaussian processes in k space can be obtained by Fourier transforming a set of independent Gaussian random variables in real space:

$$F(\mathbf{k}) = \sum_{i} g_i \, e^{i\mathbf{k} \cdot \mathbf{r_i}}.\tag{B3}$$

In these terms, the power $\langle |F(\mathbf{k})|^2 \rangle = \sum_i g_i^2$ and the two-point function in k space is $\langle F(\mathbf{k})F^*(\mathbf{k}') \rangle = \sum_i g_i^2 e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r_i}}$. If we now write down the two-point function of the power in terms of this expansion, we obtain a fourfold product $\langle g_i g_j g_k g_\ell \rangle$, which only gives a nonzero result when two pairs of indices are equal – something that can happen in four distinct ways. The two-point function for the power is then

$$\langle \hat{P}(\mathbf{k})\hat{P}(\mathbf{k}')\rangle = \sum_{i} \langle g_{i}^{4}\rangle + \sum_{i} \sum_{j\neq i} \langle g_{i}^{2}\rangle \langle g_{j}^{2}\rangle \left[1 + e^{i(\mathbf{k} + \mathbf{k}') \cdot (\mathbf{r_{i}} - \mathbf{r_{j}})} + e^{i(\mathbf{k} - \mathbf{k}') \cdot (\mathbf{r_{i}} - \mathbf{r_{j}})}\right]. \quad (B4)$$

Since in the Gaussian case $\langle g_i^4 \rangle = 3 \langle g_i^2 \rangle^2$, this neatly allows the double sum to be made unrestricted. If the term involving $\mathbf{k} + \mathbf{k}'$ is ignored on the grounds that its rapid oscillations will give a result negligible by comparison with the one involving $\mathbf{k} - \mathbf{k}'$, then we obtain the desired result:

$$\langle \hat{P}(\mathbf{k})\hat{P}(\mathbf{k}')\rangle = \langle P(\mathbf{k})\rangle\langle P(\mathbf{k}')\rangle + |\langle F(\mathbf{k})F^*(\mathbf{k}')\rangle|^2. \tag{B5}$$