Image Mosaics

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Overview

Steps

1. Getting Correspondences
2. Compute the Homography Parameters
3. Warping Between Image Planes
4. Create the output mosaic
5. Bonus

1) Getting Correspondences

- Using OpenCV we use "setMouseCallback()" which sets mouse handler for the specified image
- Then we create a function "mousePoints" for each image which has the following parameters:

  Event : EVENT_LBUTTONDOWN indicates that the left mouse button is pressed.

  X : X-coordinate of the mouse event

  Y : Y-coordinate of the mouse event.

- We save The X and Y each in a different "num_clicks x 2" matrix, num_clicks is the number of clicks the user enters at the beginning of the program; num_clicks can't be less than 4

And Voila ! we now have the correspondences we need for the next step!

2) Computing the Homography parameters

Given a set of matching points between image 1 and 2, we want to find the 3x3 homography matrix H that best agrees with the matches

2.1) When is homography valid?

It's always valid if:

1. we're capturing images of any 3D scene But must be from the same viewpoint
2. or when the scene is planar as well as when the scene is very far or has small (relative) depth variation⟶ scene is approximately planar

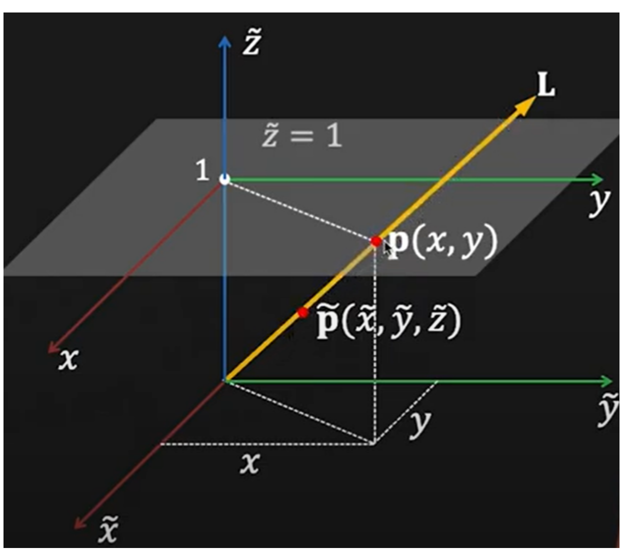
2.2) Why is the homography matrix 3x3

- Originally, any point p in an image has 2 coordinates x and y but we add to these 2 coordinate another fictitious coordinate z

P = (x,y) -> P' = (x' , y' , z')

Where x= $x' \div z'$ and y= $y' \div z'$

Figure 1



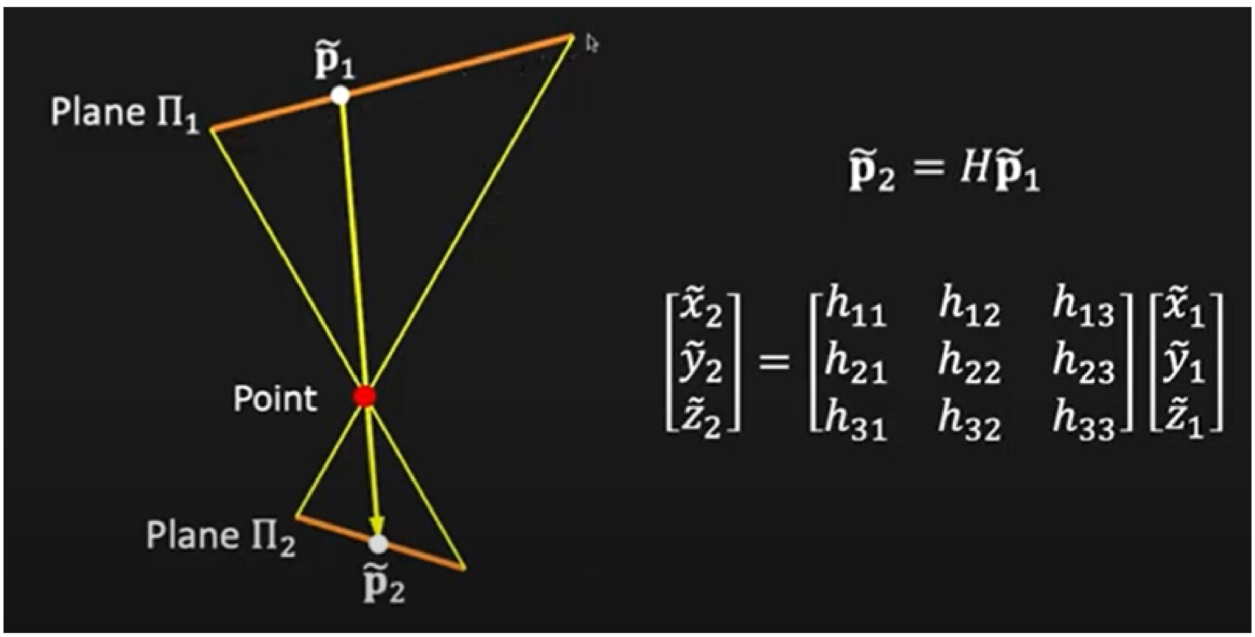
Every point on line L (except the origin) represents the homogenous coordinates of p(x,y)

- And since we will multiply the homogenous coordinates of p with H , therefore we need H to be 3x3 :

P’2 = H P’1

A projective transformation is an invertible linear mapping

2.3) What does the homography matrix 3x3 do?



Mapping of one plane to another through a point same as what camera does

2.4) What does it mean that homography can only be defined up to scale

Definition of homogeneous coordinates : the representation of a geometric object x is homogeneous if x and Ax represent the same object for A!= 0 not like the Euclidean space

And that What is meant by "defined only up to a scale vector " , therefore, Any point on line L represents the point p as mentioned if figure 1.

Why do we use homogeneous coordinates and when do we

Means that when we multiply H by any scaling factor , it is not going to make a difference to the outcome

The homography 8 Degrees of freedom not 9 , why?

As the homography matrix is estimated up to a scale. It is generally normalized with $H_{3,3}=1$ or $h^2_{1,1}+h^2_{1,2}+h^2_{1,3}+h^2_{2,1}+h^2_{2,2}+h^2_{2,1}+h^2_{3,1}+h^2_{3,2}+h^2_{3,3}=1$.

2.4) How Do we Compute H

First we write out linear equation for each correspondence:

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} \equiv k \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

We then Expand matrix multiplication and then divide out unknown scale factor then rearrange the terms to be like :

$$\begin{aligned} h_7xx' + h_8yx' + h_9x' - h_1x - h_2y - h_3 = 0 \\ h_7xy' + h_8yy' + h_9y' - h_4x - h_5y - h_6 = 0 \end{aligned}$$

This means that every 1 point correspondence gives us 2 equations , since we have 8 unknowns we need at least 4 point correspondence.

This is what we want A x = b:

$$\begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \\ -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \\ & & & & \vdots & & & & \\ -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3) What is bilinear Interpolation

Weighted mean [edit]

The solution can also be written as a weighted mean of the $f(Q)$:

$$f(x,y) \approx w_{11} f(Q_{11}) + w_{12} f(Q_{12}) + w_{21} f(Q_{21}) + w_{22} f(Q_{22}),$$

where the weights sum to 1 and satisfy the transposed linear system

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_1 & x_2 & x_2 \\ y_1 & y_2 & y_1 & y_2 \\ x_1 y_1 & x_1 y_2 & x_2 y_1 & x_2 y_2 \end{bmatrix} \begin{bmatrix} w_{11} \\ w_{12} \\ w_{21} \\ w_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ x \\ y \\ xy \end{bmatrix},$$

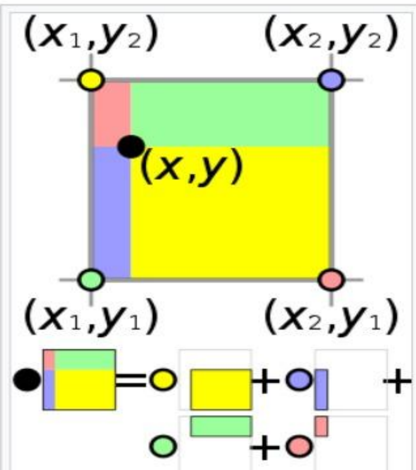
yielding the result

$$\begin{bmatrix} w_{11} \\ w_{21} \\ w_{12} \\ w_{22} \end{bmatrix} = \frac{1}{(x_2 - x_1)(y_2 - y_1)} \begin{bmatrix} x_2 y_2 & -y_2 & -x_2 & 1 \\ -x_2 y_1 & y_1 & x_2 & -1 \\ -x_1 y_2 & y_2 & x_1 & -1 \\ x_1 y_1 & -y_1 & -x_1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ y \\ xy \end{bmatrix},$$

which simplifies to

$$\begin{aligned} w_{11} &= (x_2 - x)(y_2 - y)/((x_2 - x_1)(y_2 - y_1)), \\ w_{12} &= (x_2 - x)(y - y_1)/((x_2 - x_1)(y_2 - y_1)), \\ w_{21} &= (x - x_1)(y_2 - y)/((x_2 - x_1)(y_2 - y_1)), \\ w_{22} &= (x - x_1)(y - y_1)/((x_2 - x_1)(y_2 - y_1)), \end{aligned}$$

in agreement with the result obtained by repeated linear interpolation. The set of weights can also be interpreted as a set of generalized barycentric coordinates for a rectangle.



A geometric visualisation of bilinear interpolation. The product of the value at the desired point (black) and the entire area is equal to the sum of the products of the value at each corner and the partial area diagonally opposite the corner (corresponding colours).

4) The rest

Forward warping before splatter

Forward warping after splatter

Inverse Warping

Final Stitching

5) Bonus

First Inverse warping

First Stitch

Second Inverse warping

Final Stitch