An Investigation on the Effect of Drop Height, Roll Car Mass, and Radius of Gryration on Roll Car Motion

Mechanical Engineering Laboratory (ME 107)
Spring 2018
University of California at Berkeley

Group Thursday 1C

Patrick Carlson
Salvador Gomez
James Martin
Yara Mubarak
Colin Um

TABLE OF CONTENTS

1 ABSTRACT

Dropping a roll car from a top of the hill is a seemingly simple physics problem. However, many variables will affect the motion of the roll car , such as friction, track deformation, slipping, aerodynamic drag, etc. Although a simplified physics model can be constructed to predict the roll car's dynamics, in some cases a more accurate model may be desired. Therefore, through experimental data, a model is developed that relates a roll car's drop height, mass, and radius of gyration to the position, velocity, acceleration, and number of complete passes over the center hill of a track.

2 INTRODUCTION

2.1 Objective

The objective of this experiment is to determine the effect of the roll car's drop height, mass, and radius of gyration on its position, velocity and acceleration, as well as the number of complete passes the over the track. Using the data gathered, a physics model will be used to fit a partial differential equation (PDE) to the dynamics of the car, and then a numerical classification algorithm will be used to predict the coefficients of the PDE from the input variables.

2.2 Physics Model

Let h be the drop height of the roll car, v be the instantaneous velocity of the roll car, r_g be the radius of gyration of the roll car, ω be the angular velocity of the roll car, y be the instantaneous height of the roll car, and W_f be the work done by friction. By conservation of energy:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mr_g^2\omega^2 + mgy + W_f \tag{1}$$

If the roll car is not slipping, then $v = R\omega$, where R is the outer radius of the wheel of the roll car.

3 EXPERIMENTAL METHODS

3.1 Experimental Design Matrix

For the initial stages of the experiment, a $2x2x2$ matrix of experimental configurations of the variables h, m, and r_g is tested. Each configuration is tested 4 times so that the random error of the sensors can be determined. Table 1 lists the minimum and maximum values of each variable. In addition to the 8 values required for the initial $2x2x2$ testing matrix, 6 additional configurations within the bounding $2x2x2$ configuration space will be tested.

Table 1: Bounds on Variables

Variable	Minimum Value	Maximum value
h	23.3 cm	55.9 cm
m	508.2 g	1759.1 g
r_g	42.76 mm	45.59 mm

Ideally, for the $2x2x2$ experimental matrix, all combinations of the minimum and maximum values of the variables would be tested, for a total of 8 testing cases. However, in practice it is not possible to achieve all combinations of minimum and maximum variable values. For example, the radius of gyration and the mass cannot be independently varied because the mass distribution and shape of the roll cars' components are predetermined. Table 7 in the appendix lists the bounding area of the $2x2x2$ testing matrix and Table 8 lists the configurations tested in addition to the $2x2x2$ testing matrix. This data is also displayed graphically in Figure 1.

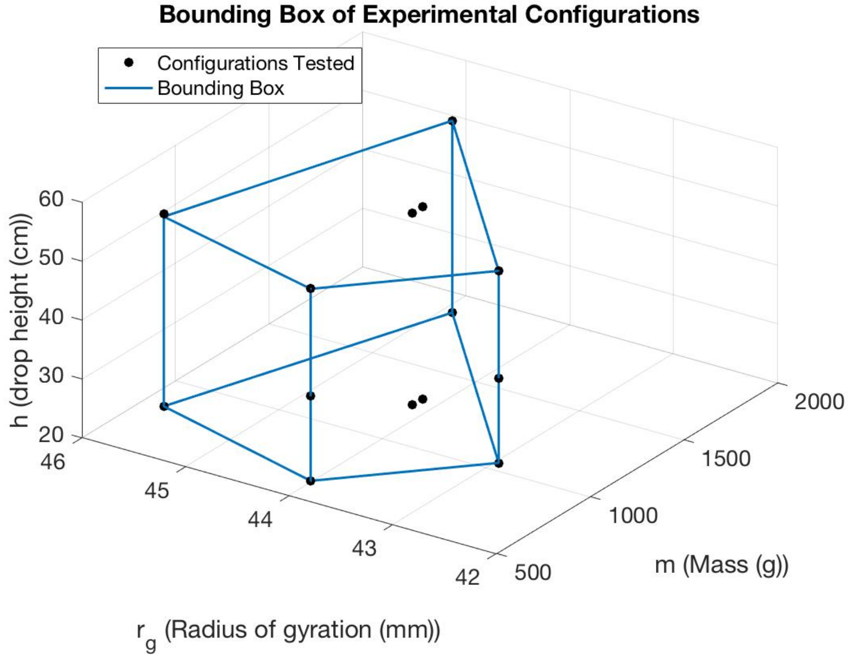

Figure 1: Plot of experimental configurations tested.

3.2 *Determining h, m, and r_g*

To determine the height at which the roll car is released, h, the positions of the release points of the roll car are measured with rulers. m is determined by using a weigh scale. To determine the radius of gyration r_g, each component of the roll car is first replicated in SolidWorks and the moment of inertia I is calculated using the built-in moment of inertia calculator. By definition of the radius of gyration, $I = mr_g^2$ and therefore $r_g = \sqrt{\frac{I}{m}}$. To find the overall radius of gyration of a configuration obtained by placing together N components on the same axle, the following equation is used.

$$r_g = \sqrt{\frac{\sum_{i=1}^{N} m_i r_{g,i}^2}{\sum_{i=1}^{N} m_i}} \tag{2}$$

where m_i is the mass of component i and $r_{g,i}$ is the radius of gyration of component i.

3.3 *Sensor List and Calibration*

Along with measuring the position of the roll car, the physical properties of the roll car need to be measured to be able to predict the position of the roll car. In order to measure the position of the car, 16 phototransistors are placed along the length of the track. The phototransistor responds to the intensity of light on it. When the roll car passes over the phototransistor, the voltage increases as a result of the change in intensity. A threshold voltage is chosen to filter between the roll car passing over a phototransistor and background noise. A digital acquisition system (DAQ) is used to convert the phototransistor voltages into digital signals.

A ruler is used to measure the track and develop a model of the roll car's path to analyze the forces on the roll car. A caliper is used to measure the geometry of the roll car's components to then develop CAD models of the various roll cars. A scale is used to measure the mass of the roll cars. Table 2 gives the list of the sensors used along with their respective resolutions.

Table 2: Sensor List

Sensor	Sensor Resolution
NI PCI-6251 DAQ	16 bit resolution
H21B1 Darlington Connected Phototransistor	Sampling Frequency of 250 Hz
Weigh Scale	0.1 g
Ruler	2 mm
Caliper	0.1 mm

Special care must be taken with the phototransistor because of the finite sampling frequency. A finite sampling frequency means that the phototransistor will not make an instantaneous measurement when the roll car is over the phototransistor. Because this introduces error in the time that the roll car passes over the phototransistor, that error is further propagated into the calculation of velocity and acceleration creating more noise in these measurements.

3.4 Converting Sensor Voltage to Position Data

The phototransistors are the sensors that translate the physical measurements of the position to a voltage which is converted to digital by a DAQ. Due to the limitations of the phototransistors, such as finite sampling rate and finite time which the roll car passes over the sensor, the peaks in the voltage of a sensor is not localized to a point in time, but rather it is distributed along a time interval. Because of this, the time in which the roll car passes directly over the sensor is not well defined, and prone to noise.

4 DIVISION OF LABOR AND PLANNING

Yara and Patrick are primarily responsible for developing and implementing the physics model. Salvador measured out the masses of the wheels and axles and used SolidWorks to compute the radius of gyration of the wheels and axles. Colin, Salvador, and James are in charge of processing raw data and writing the experimental methods, introduction, and results portion of the report. Everyone helped conduct the testing of the roll car. On March 22, 2018, Salvador Gomez missed lab. To make up for his absence, he will, in additional to his normal duties described in the above paragraph, help Yara and Patrick develop the regression model and spend more time working on the Planning Report and Intermediate Report.

Table 3: Proposed Schedule

Week #	Lab Date	In-lab Goal	Outside Lab Goal
1	03/15	Measure track, masses, and radii of gyration of roll car components.	Begin developing model. Start Planning Report.
2	03/22	Finish collecting data for $2x2x2$ matrix.	Determine most significant variable. Finish Planning Report.
3	04/05	Collect data for expanded experimental matrix.	Continue processing and analyzing data. Start Intermediate Report.
4	04/12	Collect data for expanded experimental testing matrix.	Continue processing and analyzing data. Finish Planning Report.
5	04/19	Collect data for expanded experimental testing matrix.	Finish model, prepare Presentation and start Final Report.
6	04/26	Conduct presentation and final configuration to test model.	Finish Final Report.

5 RESULTS AND DISCUSSION

5.1 Data Processing Procedure

5.2 Determination of Most Important Variable for Predicting Number of Passes

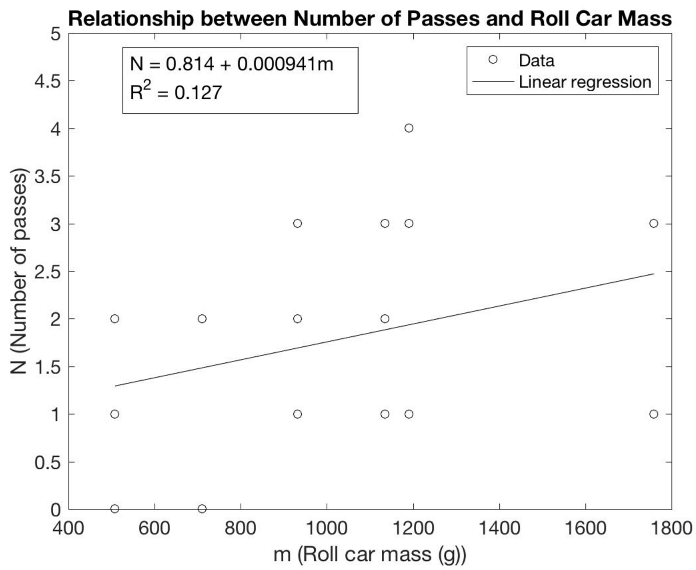

Figure 2: Relationship between number of passes completed and the roll car mass. The regression line is $N = 0.814 + 9.41 \times 10^{-3} m$, where N is the number of passes completed and m is the roll car mass, in g. The R^2 coefficient is 0.127.

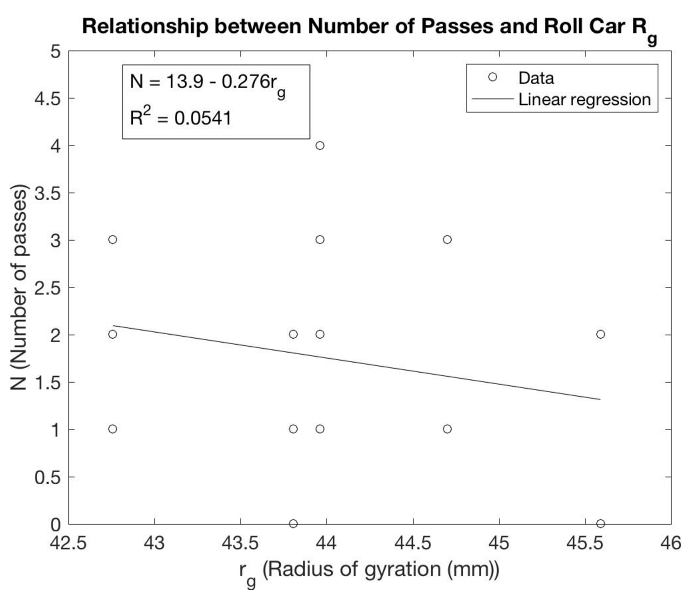

Figure 3: Relationship between number of passes completed and the roll car radius of gyration r_g. The regression line is $N = 13.9 - 0.276 r_g$, where N is the number of passes completed and r_g is the roll car mass, in mm. The R^2 coefficient is 0.0541.

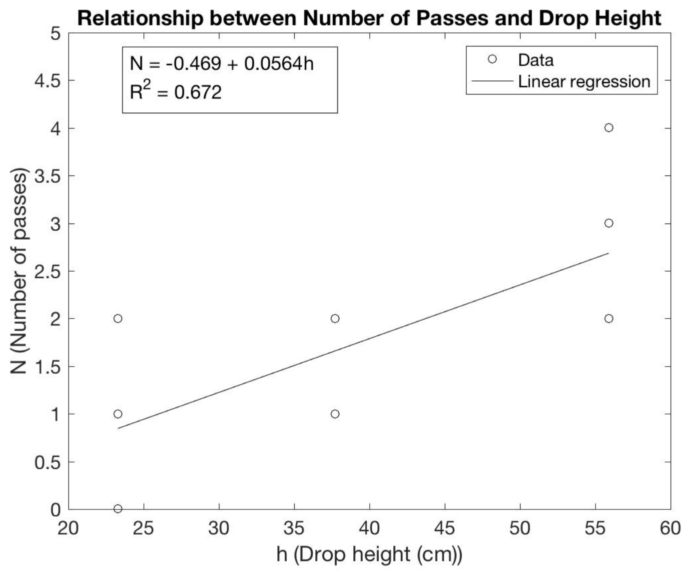

Figure 4: Relationship between number of passes completed and the roll car drop height h. The regression line is $N = -0.469 + 0.0546h$, where N is the number of passes completed and h is the roll car drop height, in cm. The R^2 coefficient is 0.672.

Figures 2,3,4 show that the number of passes N completed by the roll car most strongly depends on the drop height h of the roll car, since the R^2 coefficient between N and h (0.672) is significantly higher than the R^2 coefficients between N and r_g (0.0541) and N and m (0.127). In Figure 4, as h increases, N increases, which makes sense since if the height is larger, it takes more passes for the frictional forces to dissipate the gravitational potential energy. The second most important parameter is the roll car mass. Figure 2 shows that as m increases, N increases. Figure 3 shows that as r_g increases, N decreases. This is because as the radius of gyration increases, it is more difficult for the ball to slip; thus additional rotational kinetic energy is required to maintain the angular velocity of the roll car. However, r_g is not a very important parameter affecting N because r_g does not vary much.

To maximize the predictive ability of our model, it is important to conduct tests at more drop heights. Since mass is the second most important parameter, it is also important to test roll cars with different masses.

Therefore, the final testing matrix is $3 \times 4 \times 5$ for a total of 60 configurations. The heights that are tested are drop heights $2, 4, 6, 8, 10$, as listed in Table 4. To determine the different combinations of r_g and m, additional measurements need to be performed of the roll car components that were not used in the first portion of the experiment. Therefore, at this time it is not possible to tell exactly what values of r_g and m will be tested.

6 CONCLUSION

By performing a series of tests of the minimum and maximum values of the roll car drop height h, roll car mass m, and roll car radius of gyration r_g, it is determined that the number of passes N made by the roll car depended most strongly on h and most weakly depended on r_g. Based on this finding, additional experiments are proposed to test the roll car at more drop heights.

7 APPENDIX

Table 4: Drop Heights The reference point (x=0 cm,y=0 cm) is located at the bottom right edge of the taller hill (see Figure 5). Note that y=0 cm is level with the bottom of the plastic strip of the track.

Drop Height Number	x coordinate (cm)	y coordinate (cm)
1	41.0	23.3
2	37.6	26.6
3	32.7	29.9
4	29.0	33.5
5	25.0	37.7
6	20.2	41.3
7	17.7	45.2
8	13.5	48.7
9	9.9	52.8
10	5.9	55.9

Table 5: Masses and Radii of Gyration of Roll Car Components

Component	Mass of component (g)	Radius of gyration (mm)
Aluminum center piece (thick)	412.3	46.3
Aluminum center piece (thin)	140.7	48.6
Plastic center piece	154.1	47.4
Steel center piece	980.9	46.9
Steel axle	76.6	29.6
Aluminum axle	20.8	29.6
Wheel (1 hole)	701.6	42.9
Wheel (16 holes)	277.5	45.0

Table 6: Locations of sensors. The reference point (x=0 cm,y=0 cm) is located at the bottom right edge of the taller hill (see Figure 5). Note that y=0 cm is level with the bottom of the plastic strip of the track.

Sensor #	x coordinate (cm)	y coordinate (cm)
1	67.2	9.1
2	109.1	1.1
3	151.3	1.1
4	194.5	1.1
5	238.0	1.1
6	281.0	1.1
7	324.0	6.0
8	366.1	10.0
9	408.4	1.0
10	451.5	1.0
11	495.9	1.0
12	538.1	1.0
13	581.3	1.0
14	524.2	1.0
15	666.1	8.2
16	698.1	37.0

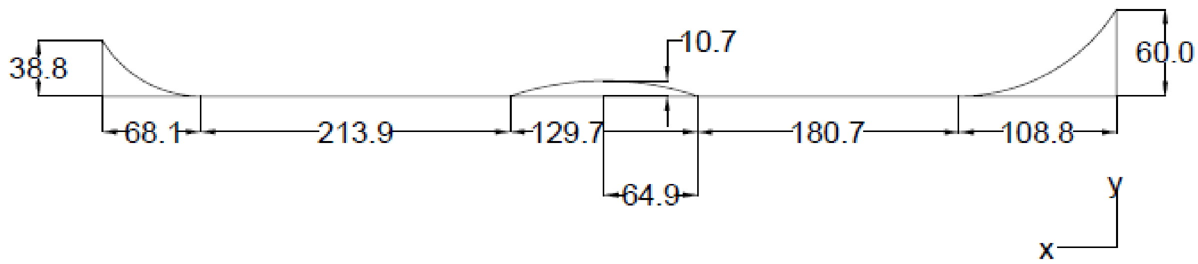

Figure 5: Track schematic.

Table 7: Bounding Configurations of $2x2x2$ Test Matrix

Configuration	Assembly	h (cm)	m (g)	r_g (mm)
1	Plastic Center Piece + Steel Axle + 16 Hole Wheel	23.3	508.2	43.8070
2	Plastic Center Piece + Steel Axle + 16 Hole Wheel	55.9	508.2	43.8070
3	Steel Center Piece + Steel Axle + 1 Hole Wheel	23.3	1759.1	44.7
4	Steel Center Piece + Steel Axle + 1 Hole Wheel	55.9	1759.1	44.7
5	Plastic Center Piece + Steel Axle + 1 Hole Wheel	23.3	932.3	42.7592
6	Plastic Center Piece + Steel Axle + 1 Hole Wheel	55.9	932.3	42.7592
7	Thin Aluminum Center Piece + Aluminum Axle + 16 Hole Wheel	23.3	710.6	45.59
8	Thin Aluminum Center Piece + Aluminum Axle + 16 Hole Wheel	55.5	710.6	45.59

Table 8: Additional Configurations Tested

Configuration	Assembly	h (cm)	m (g)	r_g (mm)
9	Plastic Center Piece + Steel Axle + 16 Hole Wheel	37.7	508.2	43.8070
10	Plastic Center Piece + Steel Axle + 1 Hole Wheel	37.7	932.3	42.7592
11	Thick Aluminum Center Piece + Steel Axle + 1 Hole Wheel	23.3	1190.5	43.959
12	Thick Aluminum Center Piece + Steel Axle + 1 Hole Wheel	55.9	1190.5	43.959
13	Thin Aluminum Center Piece + Aluminum Axle + 1 Hole Wheel	23.3	1134.7	43.959
14	Thin Aluminum Center Piece + Aluminum Axle + 1 Hole Wheel	55.9	1134.7	43.959