

# Predicting Roll Car Dynamics with Mass, Radius of Gyration, and Drop Height

ME 107  
Lab 105 (Thursday 1C)  
UC Berkeley

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# Motivation

- Develop a predictive model for kinematics of a roll car
- Model can be used to predict other variables

# Theory

- Mechanical Energy is not conserved!
- Energy losses ( $W_f$ ) due to
  - friction, drag, track deformation, etc.
- Total Energy can be modeled in terms of initial energy

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mr_g^2\omega^2 + mgy + W_f$$

$m$  = mass    $h$  = drop height    $v$  = speed    $r_g$  = radius of gyration  
 $w$  = angular velocity    $g$  = gravity,  $y$  = height

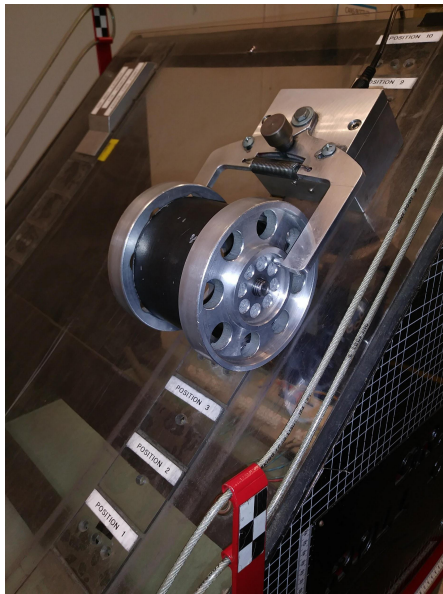


# Experimental Design: Measurements Before Data Collection

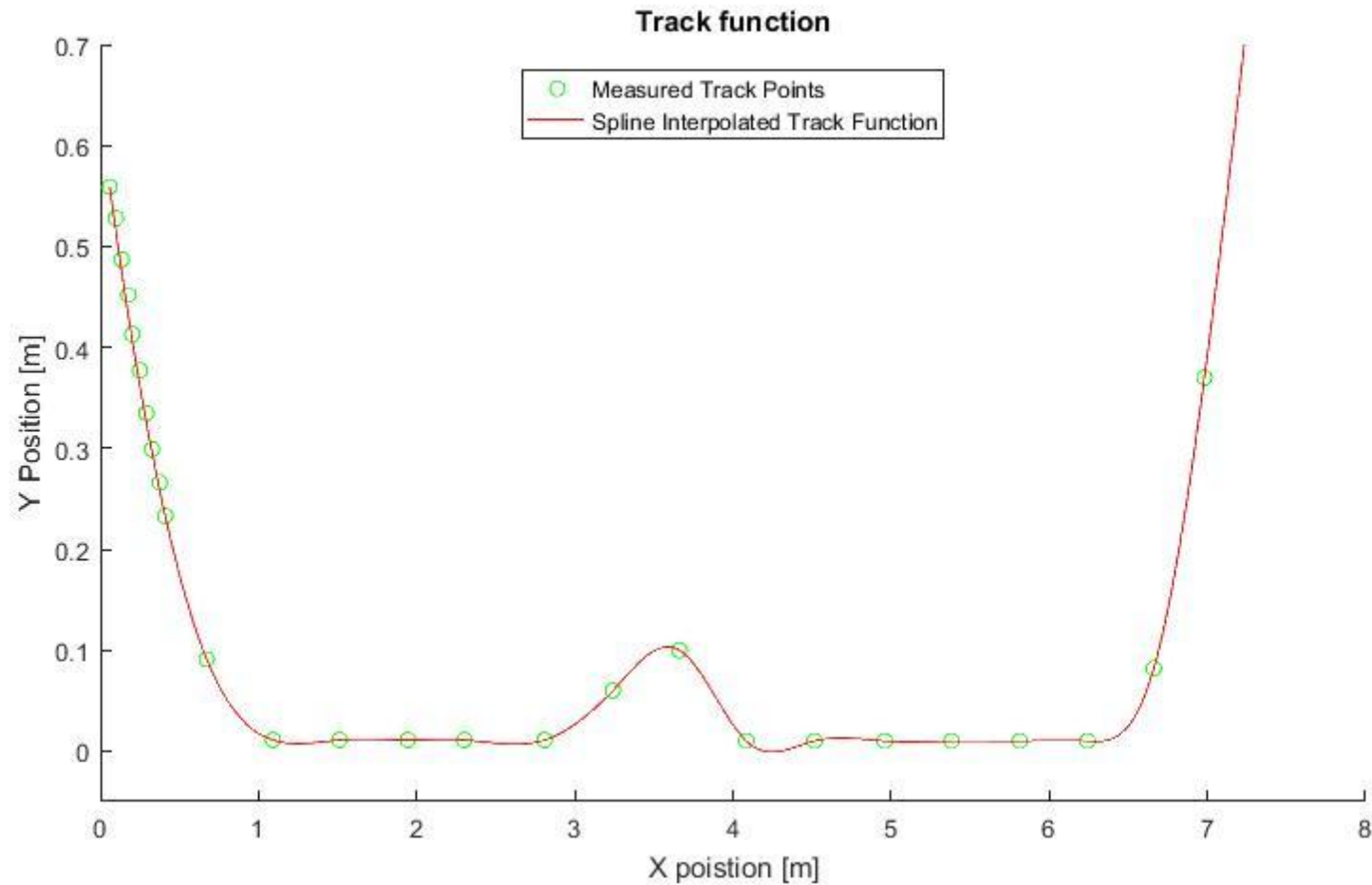
- Measure Track Shape and Curvature
- Weigh All Parts
- CAD All Parts

In order to obtain masses, radii of gyration and drop heights.

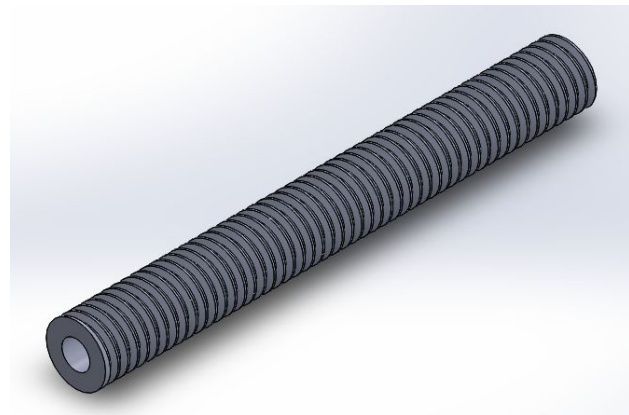
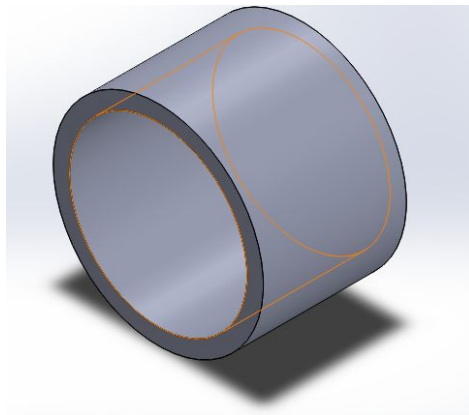
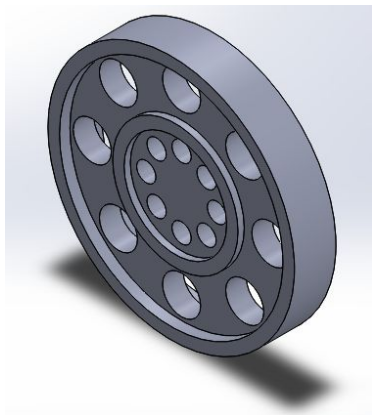
# Experimental Setup



# Track MATLAB Visualization

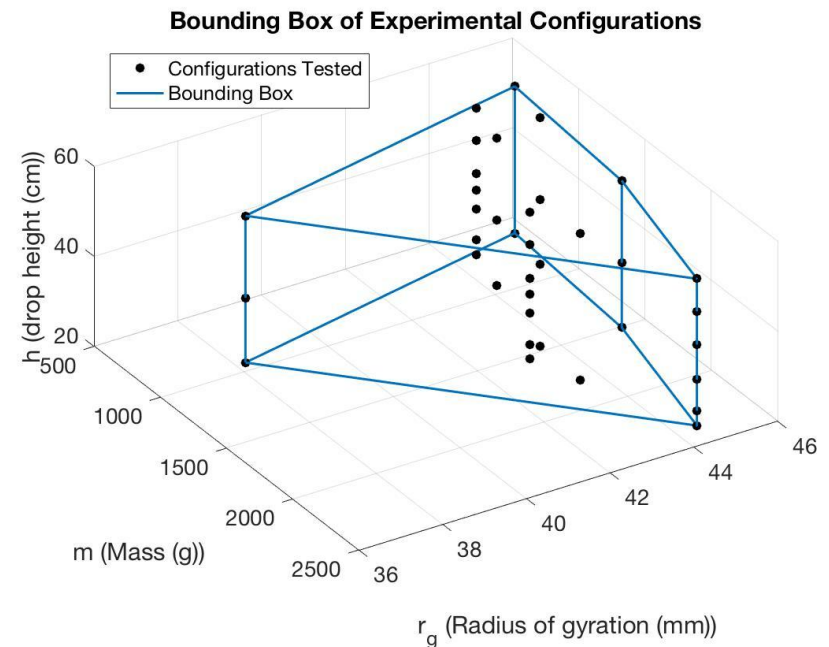


# Screenshots of CADed Objects



# Experimental Design

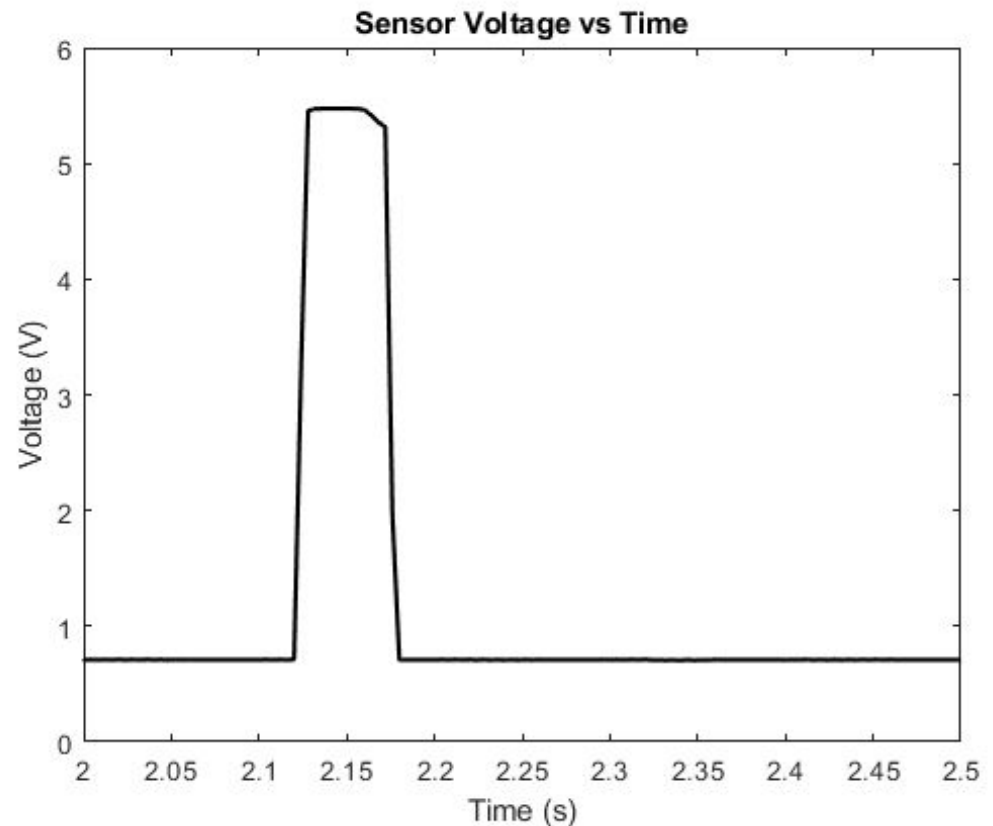
| Variable           | Minimum Value | Maximum Value |
|--------------------|---------------|---------------|
| Drop Height        | 23.3 cm       | 55.9 cm       |
| Mass               | 716.5 g       | 2461 g        |
| Radius of Gyration | 37.9 mm       | 45.4 mm       |



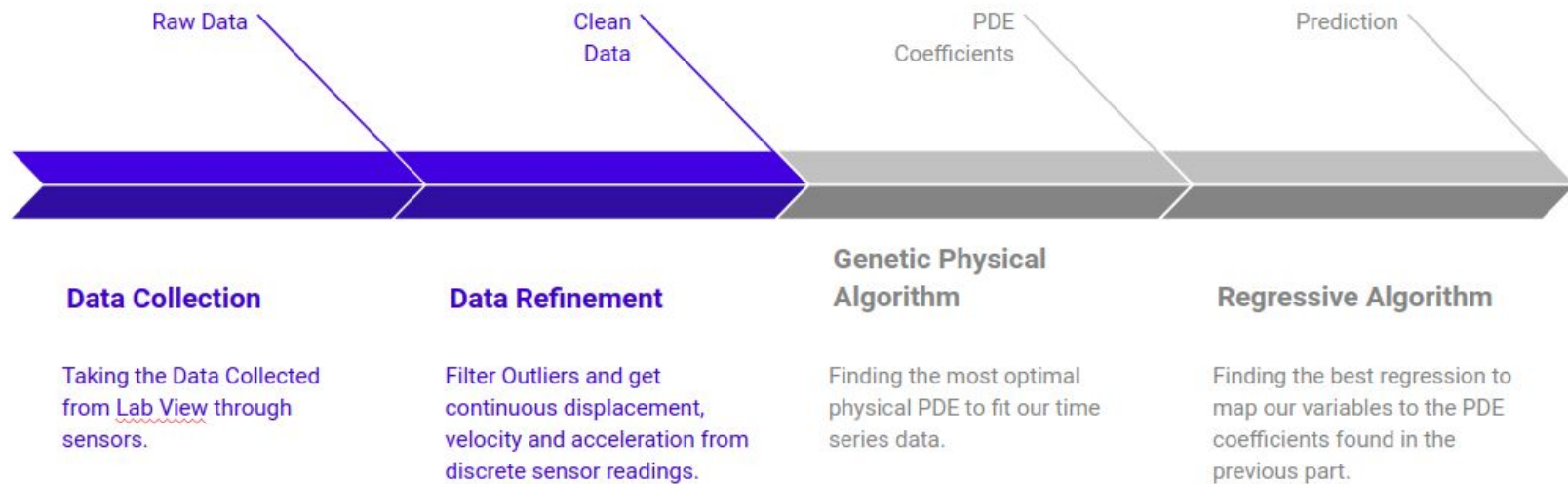


# Calibration of Sensor Signal

- 16 phototransistors
- Output based on light intensity
- Threshold voltage and average time used



# Data Pipeline



# Genetic Physical Algorithm

## Physical Model

NO SLIP Condition:

$$\begin{aligned}\ddot{S} &= \frac{\vec{F}_r \cdot \vec{e}_t}{m(1 + \frac{r_g^2}{r_w^2})} \\ \ddot{\theta} &= \frac{\ddot{S}}{r_w} \\ \vec{F}_n &= mK\dot{S}^2 - \vec{F}_r \cdot \vec{e}_n \\ \vec{F}_f &= m\ddot{S} - \vec{F}_r \cdot \vec{e}_t\end{aligned}$$

If SLIP occurs :

$$\begin{aligned}\ddot{S} &= \frac{\vec{F}_r \cdot \vec{e}_t + \vec{F}_f}{m} \\ \ddot{\theta} &= \frac{-\vec{F}_f r_w}{mr_g^2} \\ \vec{F}_n &= mK\dot{S}^2 - \vec{F}_r \cdot \vec{e}_n \\ \vec{F}_f &= \frac{\dot{S}}{|\dot{S}|} \mu_k \vec{F}_n\end{aligned}$$

$$\vec{F}_r = \frac{\dot{S}}{abs(\dot{S})} (F_{IDK} + C_{RF}F_n + 0.5A_{CS}\rho_a * |\dot{S}|^2 C_d) - mg\vec{E}_2$$

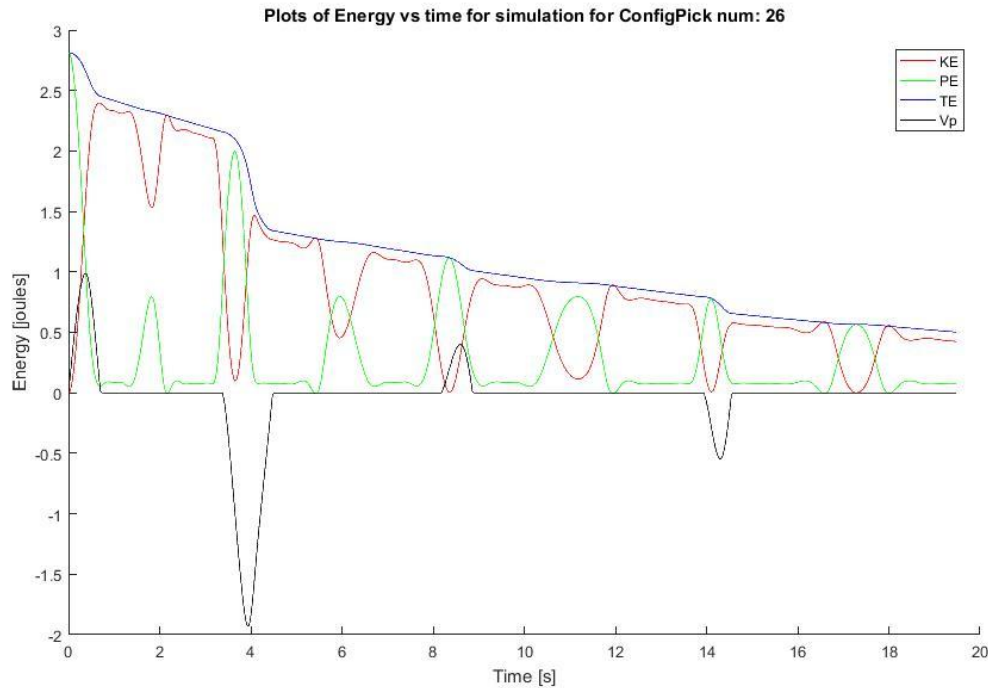
# PDE Curve Fitting

- Optimize Over The Following PDE Coefficients :

$$C_d, C_{RF}, F_{IDK}, \mu_s, \text{ and } S_{init}$$

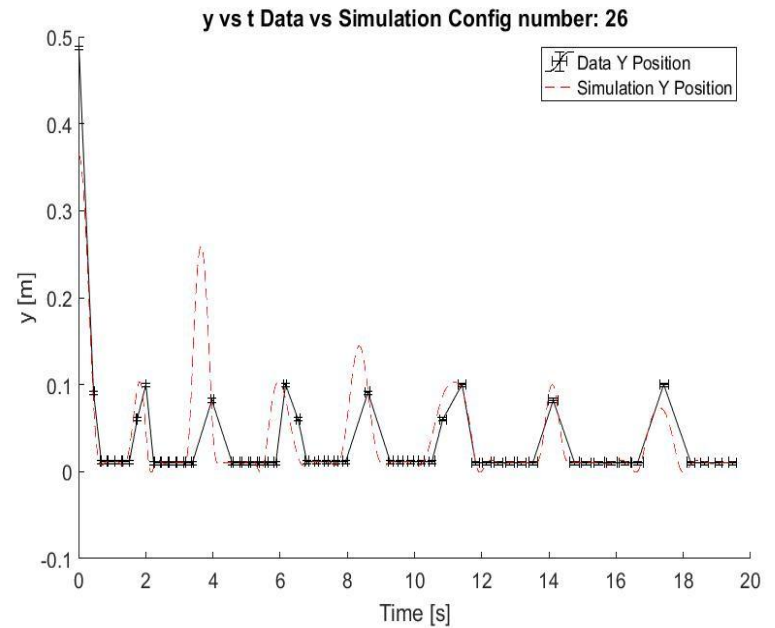
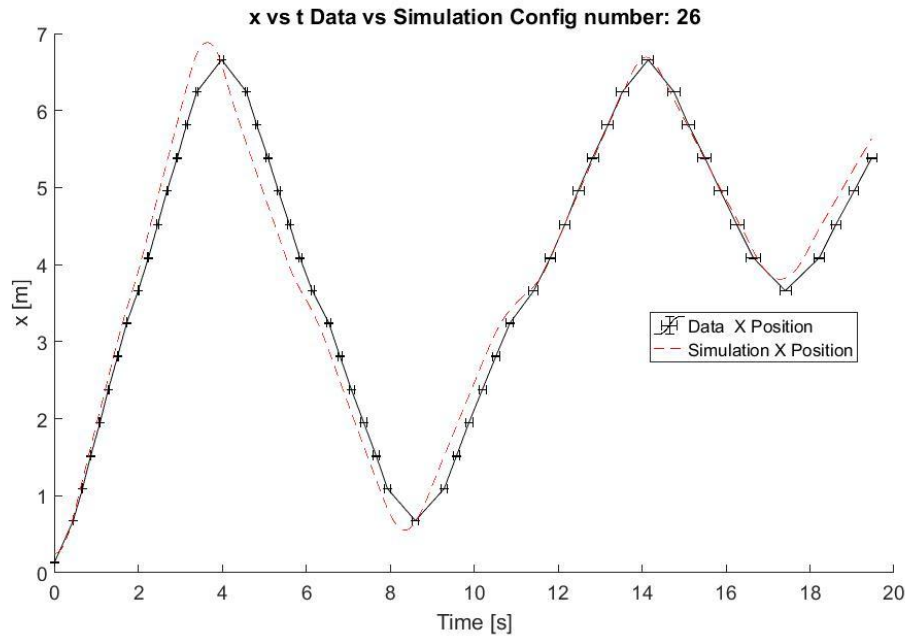
- Run random parameter sweep over bounds in order to get starting guess.
- Further refine with lsqnonlin

# PDE Curve Fitting

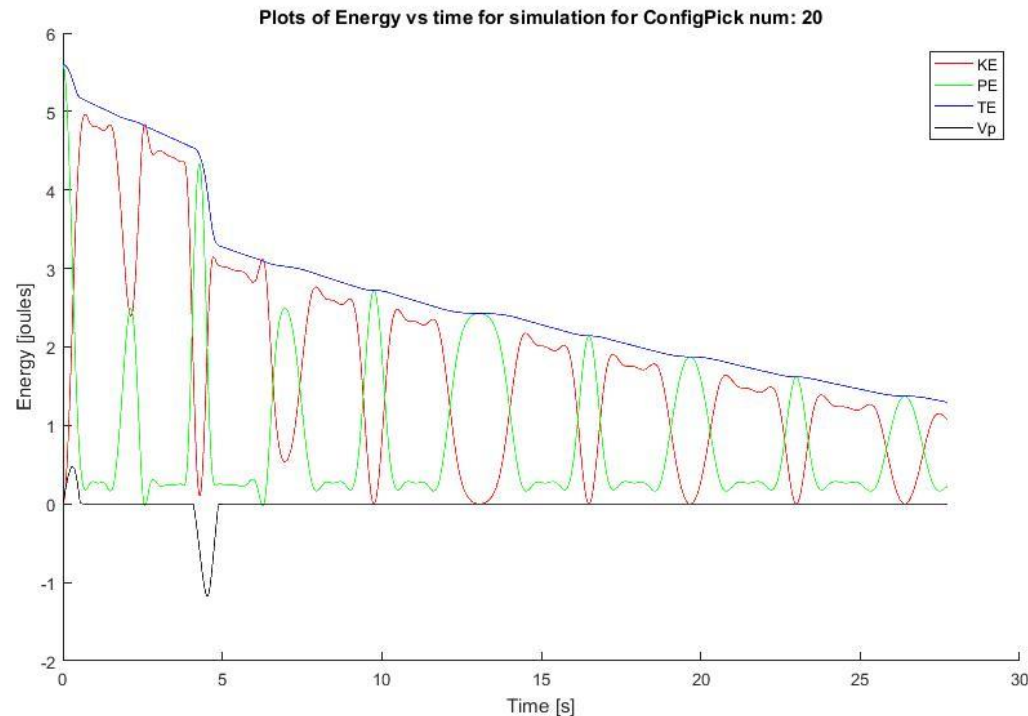


Config 26:  $m = .7857$  kg;  $rg = .04423$  m; Drop = #8  
 $CD = .7487$ ;  $CRF = .0014$ ;  $FIDK = .0187$ ;  $Muk = .0814$  ;  
 $MuS = .0979$ ;  $DeltaS = .0032$ ;

# PDE Curve Fitting



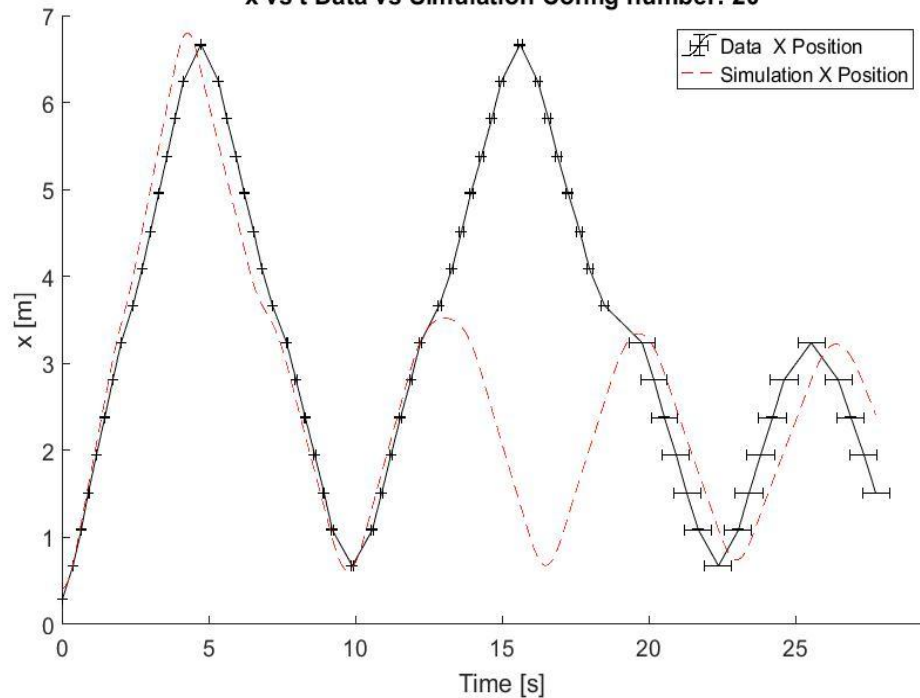
# PDE Curve Fitting



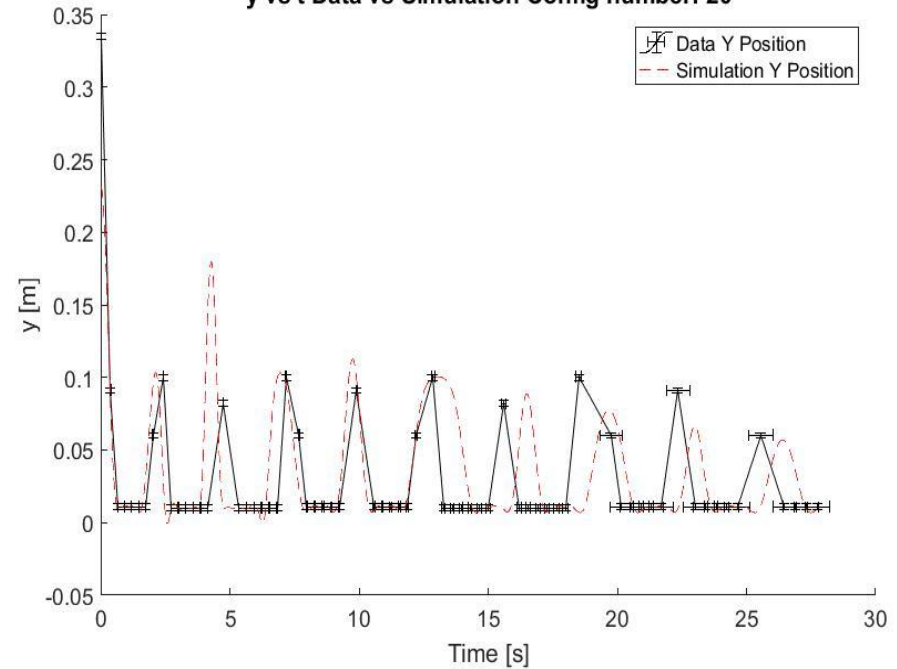
Config 20:  $m = 2.4607$  kg;  $rg = .04420$  m; Drop = #8  
 $CD = .6515$ ;  $CRF = .0028$ ;  $FIDK = .0255$ ;  $Muk = .0878$  ;  
 $MuS = .1207$   $\Delta S = .0043$ ;

# PDE Curve Fitting

x vs t Data vs Simulation Config number: 20



y vs t Data vs Simulation Config number: 20





# Regression Algorithms

Feature Matrix :

$$\vec{x}_i = [m_i, h_i, r_{gi}]$$

$$X = \begin{bmatrix} \dots & \vec{x}_1^T & \dots \\ \dots & \vec{x}_2^T & \dots \\ \vdots & \ddots & \vdots \\ \dots & \vec{x}_n^T & \dots \end{bmatrix}$$

Label Matrix :

Optimal Values Found for each Configuration:

$$\vec{y}_i = [C_{di}, C_{RFi}, F_{IDKi}, \mu_{si}, S_{initi}, R_i]$$

$$Y = \begin{bmatrix} \dots & \vec{y}_1^T & \dots \\ \dots & \vec{y}_2^T & \dots \\ \vdots & \ddots & \vdots \\ \dots & \vec{y}_n^T & \dots \end{bmatrix}$$

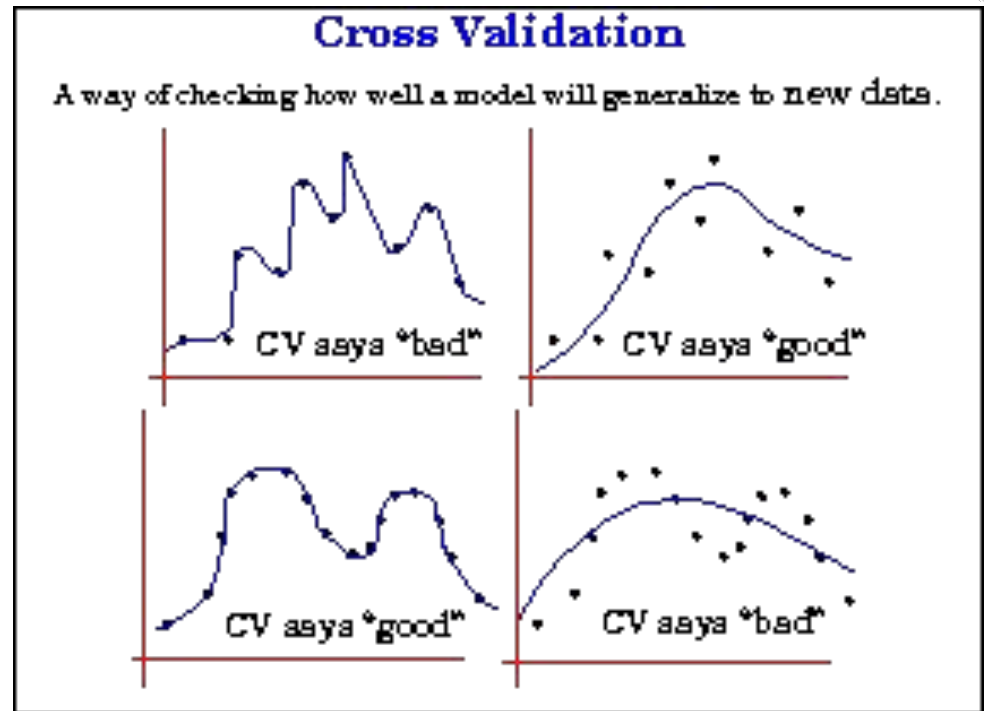


# Available Regression Algorithms

- Linear Algorithms :
  - Linear : Least Squares  $Xw = y$  (fit  $w$ )
  - Poly2 : Least Squares  $Xw = y$  (fit  $w$ ) but with  $X$  containing a 2nd degree polynomial of the Features
- Nonlinear Algorithms :
  - Neural Net with Gradient Descent Optimizer

# Validation Technique

- Leave One Out Cross Validation
- K-fold cross Validation
- Mean Squared Error



# Sample Equation

-Sample polynomial equation

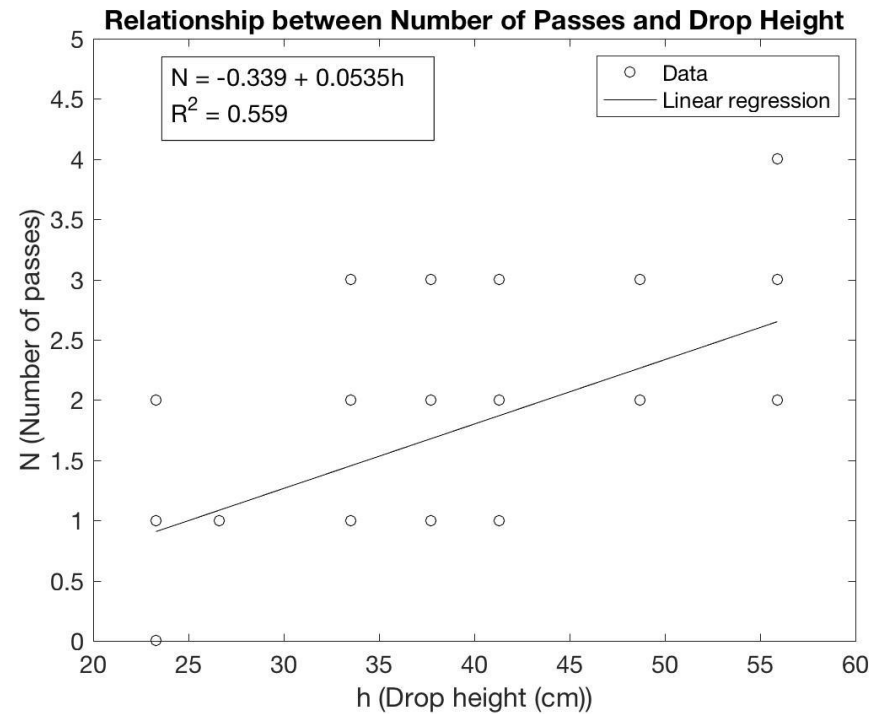
$$x_1 = h, x_2 = m, x_3 = r_g$$

$$(7 * 10^{-3})x_1 + (-3.4 * 10^{-5})x_2 + 0.012x_3 = C_d$$

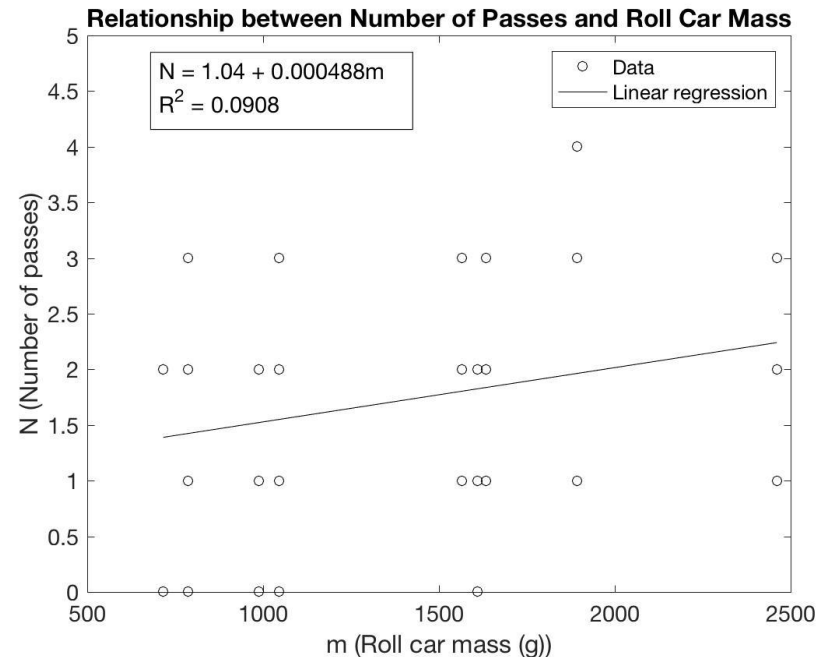
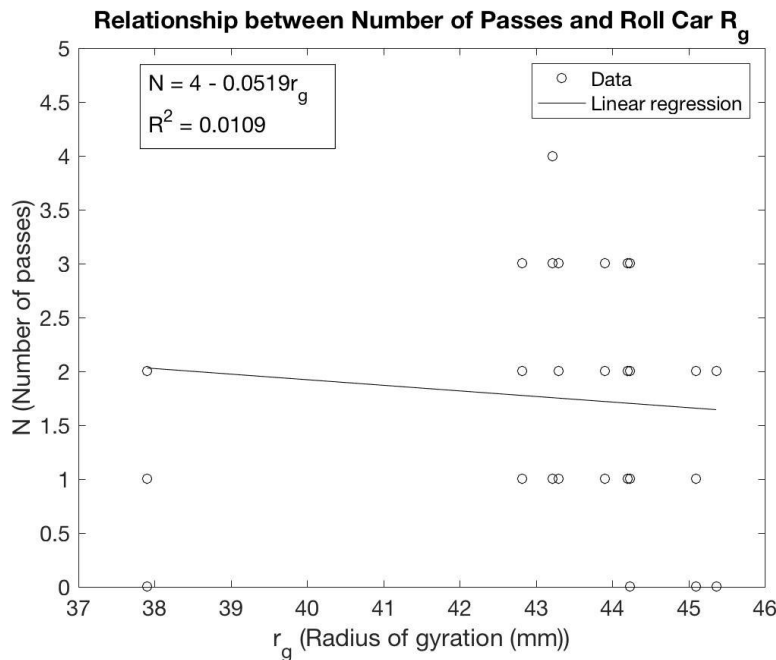
$$6.17x_1^2 + 0.00603x_2^2 + 0.00081x_3^2 + O(x_1x_2 + x_2x_3.....) = C_d$$

# Most Important Parameter

- The drop height most correlated to number of passes.

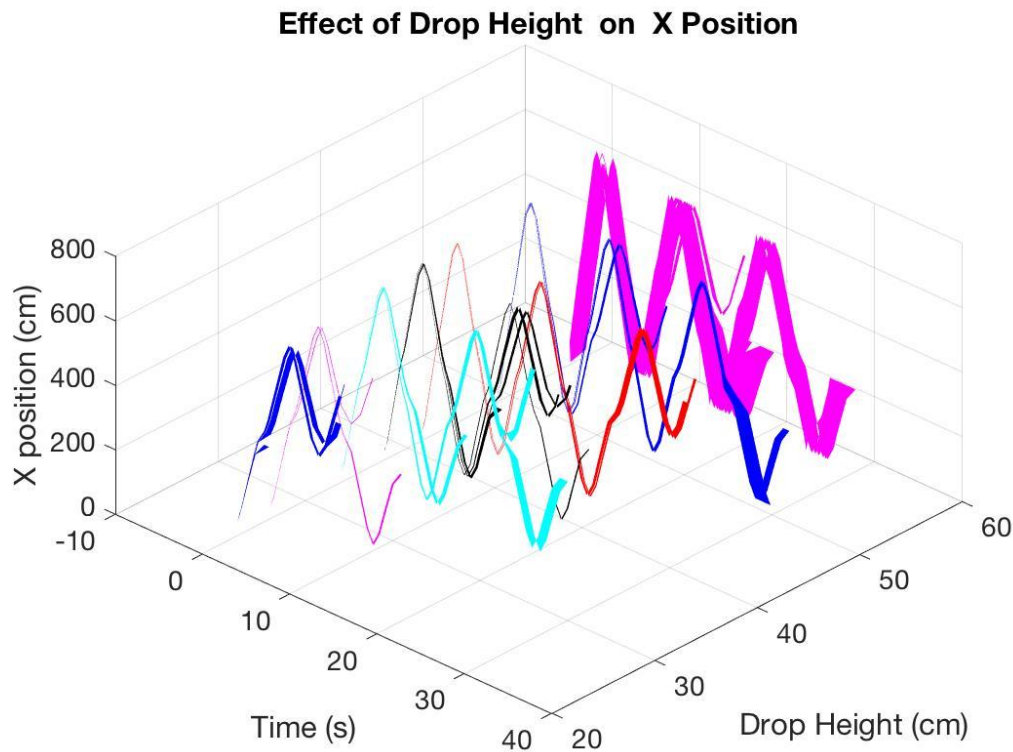


# Most Important Parameter (cont)



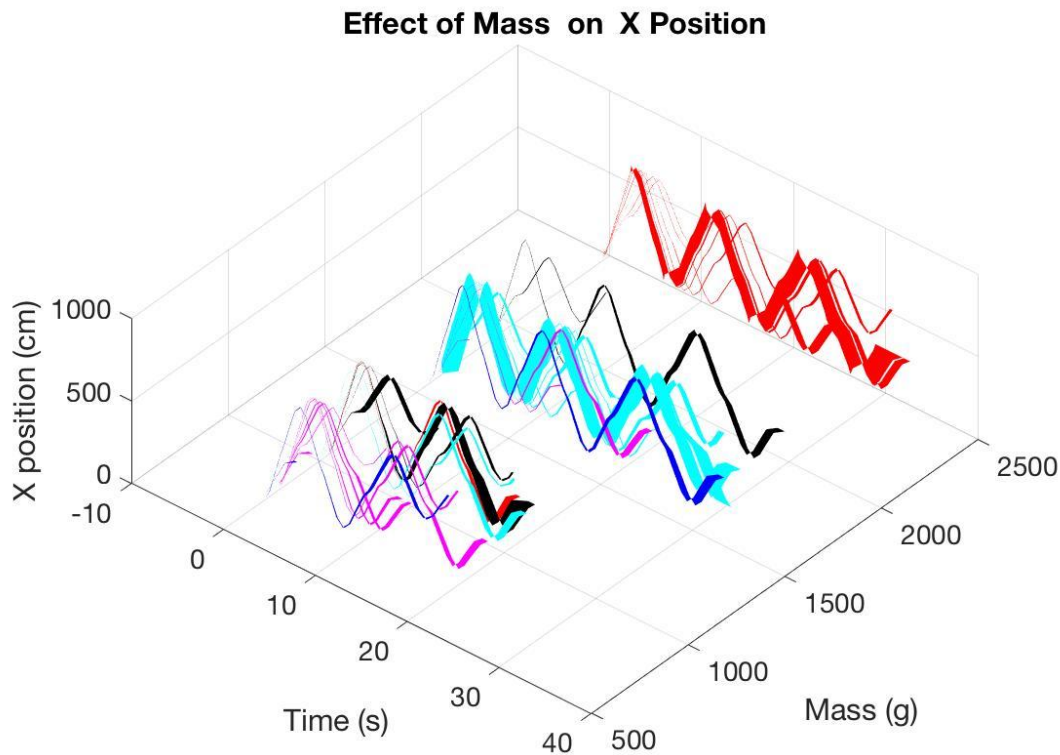
- Radius of gyration and mass don't have much effect.

# Results



- General shape of curve is same across all trials.
- Run time is most affected by drop height.

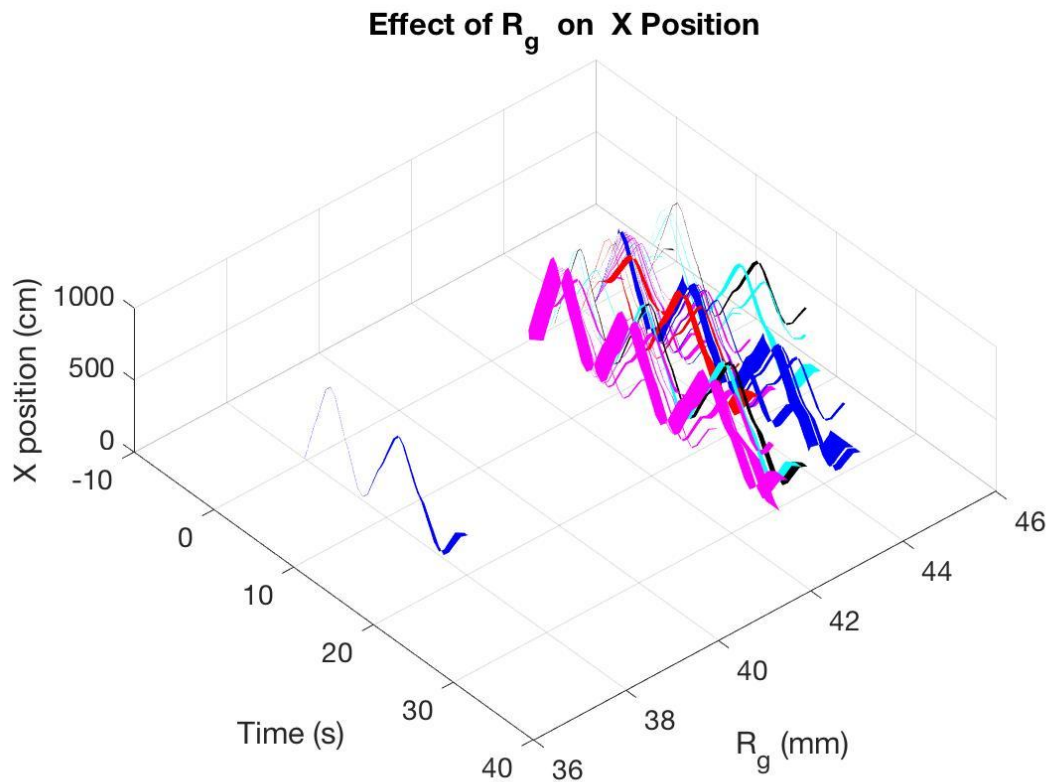
# Results (cont)



- No clear trend of mass of roll car on roll car run time.



# Results (cont)



- No clear trend of radius of gyration on the roll car run time.

# Model Performance



## Model Part 1 :

- Drag coefficient should not change, but changes in our model.
- Not picking up all physical properties.
- Can account for factors that we did not account for.

## Model Part 2 :

- 95% accuracy for  $s$  and  $\Delta s$  coefficient.
- 70% accuracy for rest of coefficients.
- Need more configurations.

# Conclusions

- No clear trend of mass or radius of gyration on the roll car run time or number of passes.
- Increased drop height tends to result in increased number of passes and roll car run time.
- Successful two-part model: PDE and regression.
  - Validated with real data.
  - Model can be used to calculate system parameters.