Sheet two Solution

[12] A Transmitter transmit an AM Signal with a Carrier Frequency of 1500 KHZ. When an inexpensive radio receiver (which has a Poor selectivity in its RF-stage band pass filter) is tuned to 1500 KHZ, The Signal is heard loud and clear. This same signal is also heard (not as strong) at another dial setting. State with reasons, at what Frequency you will hear this station.
the IF frequency is 455 KHZ.

Solution

Intermediate Freq. = 455

Image Freq. = 1500

$$F_{cm} = F_c \pm 2F_{IF}$$

$$\therefore F_c = F_{cm} \pm 2F_{IF}$$

$$= 1500 \pm 910$$

$$\therefore F_c = 2410 \;\&\; 590$$

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Section 5

Angle Modulation :-

- previously, we studied amplitude modulation which means that the amplitude of the carrier is function in the message

- However, amplitude Modulation is Sensitive to noise. Any noise added to the modulated signal affects the received message. Noise doesnot affect the frequency of the modulated signal. So, how about changing the carrier frequency/phase based on the message? That is Angle Modulation.

Angle Modulation: The angle of the carrier is function in the message.

* If the carrier phase changes with respect to the message, that is phase Modulation [PM]

* If the carrier frequency changes w.r.t. the message, that is Frequency Modulation [FM].

→ How to perform Angle Modulation?

Let the message be $m(t)$ & the carrier be $A_c \cos(\omega_c t)$.

Also, Let the modulated signal be $S(t)$.

(PM) * In PM, we add the message to the carrier phase.

$$\therefore \quad S(t) = A_c \cos(w_c t + \overset{\text{Constant}}{K_p}\, m(t))$$

Why K_p? $\underset{\text{rad/sec} \quad \text{sec}}{w_c t}$ = rad & $m(t)$ is volt. So, we need some constant to be able to perform the addition.

* So, the unit of K_p is rad/volt. [So that $K_p\, m(t)$ unit becomes rad]

(FM) * In FM, we add the message to the carrier frequency

Recall:
* The frequency is the differentiation of phase.

$$\omega = \frac{d\,\phi(t)}{dt}$$

EX: $\cos(w_c t)$

$$\frac{d}{dt}(w_c t) = w_c$$

$\phi(t)$ = Carrier phase
$= w_c t.$

$$\text{Carrier freq} = \frac{d\phi(t)}{dt} = w_c$$

$$\therefore \text{New Carrier freq} = w_c + \overset{\text{constant}}{K_v}\, m(t)$$

$$\therefore \text{New } \phi(t) = \int [w_c + K_v\, m(t)]\, dt$$

$$= w_c t + K_v \int m(t)\, dt$$

$$\therefore \quad S(t) = A_c \cos(w_c t + K_v \int m(t)\, dt)$$

$w_c t = \frac{\text{rad}}{\text{sec}} \cdot \text{sec} = \text{rad}.$

$\int m(t) \cdot dt$ = Volt . Sec

$\therefore K_v$ unit is rad / Sec.volt

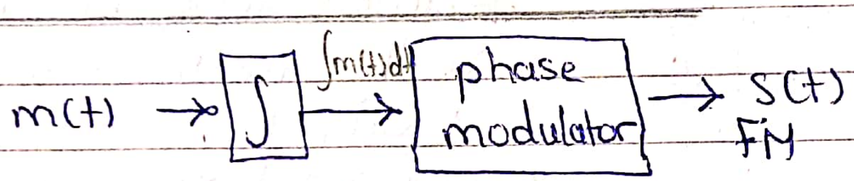
$\therefore K_v \int m(t) \cdot dt$ unit becomes rad

→ The relation between PM & FM:

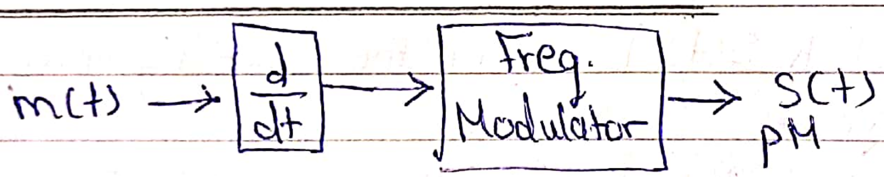
$$S_{PM}(t) = A_c \cos(\omega_c t + K_p m(t))$$
→ Multiply m(t) by Kp
→ Insert it in the phase

$$S_{FM}(t) = A_c \cos(\omega_c t + K_v \int m(t)\,dt)$$
→ Integrate m(t)
→ Multiply m(t) by Kv
→ Insert it in the phase

(a) Generating FM using PM modulator:



(b) Generating PM using FM modulator:



→ How to sketch the modulated signal?

(a) phase modulation: $S_{PM}(t) = A_c \cos(\omega_c t + K_p m(t))$

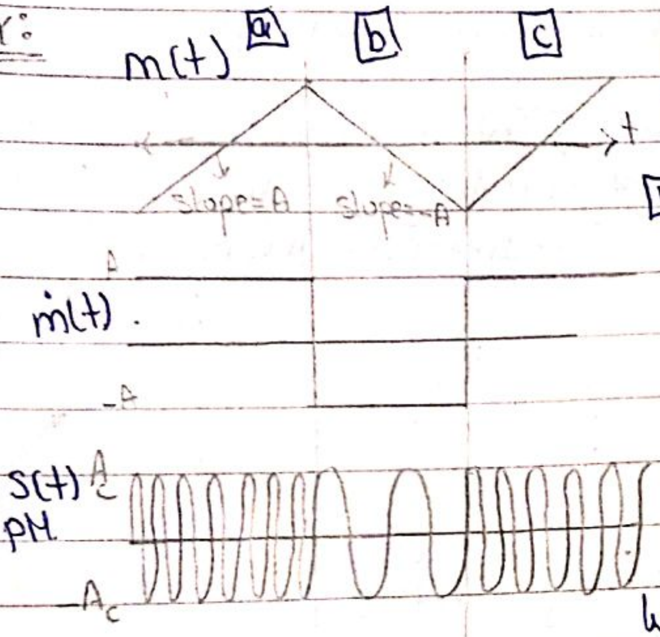
* The phase of the carrier varies at each time instance.
So, for ease of drawing, we will deduce the frequency of the carrier & draw our carrier based on this frequency.
[Remember: $\text{Freq} = \frac{d}{dt}\,\text{phase}$]

$\therefore$ Carrier Freq $= \omega_c + K_p \dot{m}(t)$ ← Message differentiation

Steps: [1] Differentiate the message $m(t)$ to obtain $\dot{m}(t)$

[2] Draw the carrier while changing its frequency based on $\dot{m}(t)$

Ex:



Important Notes:

[1] In time slots ⓐ & ⓒ, $\dot{m}(t)$ is +ve. So, the carrier frequency is $\omega_c + k_p A$. [Shrinks]

[2] In time slot ⓑ, $\dot{m}(t)$ is -ve. So, the carrier freq is $\omega_c - k_p A$ [widens].

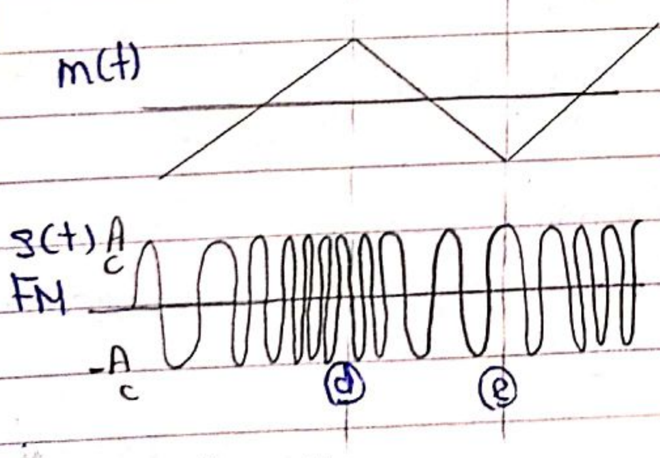
$\omega_{\substack{max \\ min}} = \omega_c + k_p [\dot{m}(t)]_{\substack{max \\ min}}$

(b) Frequency Modulation. $S_{FM}(t) = A_c [\cos(\omega_c t + k_f \int m(t)\, dt]$

Again, we will sketch $S(t)$ based on the carrier frequency.

$\therefore$ Carrier freq $= \omega_c + k_f m(t)$

Steps: [*] Draw the carrier while changing the frequency based on $m(t)$



Important notes:

[1] unlike PM, here the carrier freq. increases when $m(t)$ increases. So, the highest frequency is at time instant point ⓓ, while the lowest frequency is at time instant ⓔ

$\omega_{max} = \omega_c + k_f\, m(t)_{max}$

$\omega_{min} = \omega_c + k_f\, m(t)_{min}$

$\rightarrow m(t) = A_m \cos(\omega_m t)$

* For a single tone message [Sinusoidal Signals only],
We shall define some important parameters such as:

* → Phase deviation $\Delta\theta$ → Frequency deviation Δf or $\Delta\omega$
→ Modulation Index β

[1] phase deviation $\Delta\theta$: The max. amount by which the phase of the carrier wave is shifted from its unmodulated state.

PM: $s_{PM}(t) = A_c \cos(\underbrace{\omega_c t + K_p \cdot A_m \cos(\omega_m t)}_{\text{phase } \phi(t)})$.

$\phi(t)$ of $s(t)$ $= \omega_c t + K_p A_m \cos(\omega_m t)$ ← The phase of the modulated signal

Instantaneous phase deviation $= \phi(t)\big|_{s(t)} - \phi(t)\big|_{c(t)}$
fn in time

$= \omega_c t + K_p A_m \cos(\omega_m t) - \omega_c t$

$= K_p A_m \cos(\omega_m t)$

The instan. phase deviation is max. when $\cos(\omega_m t) = 1$.

∴ Max. phase deviation $\Delta\theta = K_p A_m$.

FM $\quad S(t)_{FM} = A_c \cos(\underbrace{w_c t + k_v \int m(t)\,dt}_{\text{phase } \phi(t)}) = A_c \cos(w_c t + \frac{k_v A_m}{w_m} \sin(w_m t))$

$m(t) = A_m \cos(w_m t)$

$$\phi(t)_{S(t)} = w_c t + k_v \int m(t)\,dt.$$

Instantaneous phase deviation $= \phi(t)|_{S(t)} - \phi(t)|_{c(t)}$

$$= w_c t + k_v \int m(t)\,dt - w_c t$$

$$= \frac{k_v A_m}{w_m} \sin(w_m t)$$

Max phase deviation $\Delta\theta = \dfrac{k_v A_m}{w_m}$

[2] Frequency deviation Δw or Δf : The max. amount by which the frequency of the carrier is shifted from its unmodulated state.

PM: Frequency$|_{S(t)} = w(t) = \dfrac{d\phi(t)}{dt} = w_c + k_p \dot{m}(t)$

Instantaneous Freq. ~~deviation~~ $= w_c - k_p A_m w_m \sin(w_m t)$

Instantaneous Freq. deviation $=$ Freq$|_{S(t)}$ $-$ Freq$|_{c(t)}$

$$= w_c - k_p A_m w_m \sin(w_m t) - w_c$$

$$= k_p A_m w_m \sin(w_m t)$$

Max Freq deviation $\Delta w = k_p A_m w_m$

$$\Delta f = \frac{k_p A_m w_m}{2\pi}$$

FM: Frequency$|_{s(t)} = \frac{d\phi(t)}{dt} = W_c + K_f\, m(t)$

$= W_c + K_f A_m \cos(\omega_m t)$

Instantaneous freq deviation = freq$|_{s(t)}$ − freq$|_{c(t)}$

$= K_f A_m \cos(\omega_m t)$

Max freq deviation: $\Delta\omega = K_f A_m$, $\Delta f = \frac{K_f A_m}{2\pi}$

3) Modulation Index β [Represents the number of harmonics found around the Carrier frequency]

* $\beta = \frac{\text{Max Freq. deviation}}{\text{Baseband BW of message}} = \frac{\Delta\omega}{\omega_m}$.

PM $\beta = \frac{\Delta\omega}{W_m} = \frac{K_p A_m \cancel{W_m}}{\cancel{W_m}} = K_p A_m$ [which is $\Delta\theta$]

FM $\beta = \frac{\Delta\omega}{\omega_m} = \frac{K_f A_m}{W_m}$ [which is $\Delta\theta$]

→ What is the power of an angle modulated signal?

Power of an angle modulated signal = $\frac{A_c^2}{2}$

no matter whether the signal is modulated using PM or FM.

Bandwidth of the angle modulated signals:

$$S(t) = Re\{A\, e^{j[\omega_c t + K\alpha(t)]}\}$$

$$= Re\{A\, e^{j\omega_c t}\, \underbrace{e^{jK\alpha(t)}}_{\text{Series expansion}}\}$$

$$= Re\{A\, e^{j\omega_c t}\, [1 + jK\alpha(t) - \frac{K_f^2 \alpha^2(t)}{2!} + \dots]\}$$

$$= Re\{A\, [\underbrace{\cos \omega_c t}_{\text{real}} + j \sin \omega_c t][\underbrace{1}_{\text{Real}} + jK\alpha(t) - \underbrace{\frac{K_f^2 \alpha^2(t)}{2!}}_{\text{Real}} + \dots]\}$$

Recall.

$e^{jx} = \cos(x) + j\sin(x)$

$\therefore Re\{e^{jx}\} = \cos(x)$

$$\therefore S(t) = A[\underbrace{\cos(\omega_c t)}_{\text{Carrier}} - \underbrace{K_f \alpha(t) \sin \omega_c t}_{\alpha(t) \xrightarrow[BW]{} W} - \underbrace{\frac{K^2 \alpha^2(t)}{2!} \cos(\omega_c t)}_{\alpha^2(t) \xrightarrow[BW]{} 2W} + \dots]$$

$\alpha^3(t)$, $\alpha^4(t)$
$3W$ $\quad$ $4W$

$\therefore$ BW is infinity !!!

$\alpha^2(t) = \alpha(t)\alpha(t)$

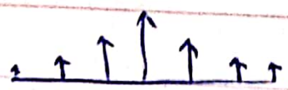
$\alpha^2(\omega) = \alpha(\omega) \circledast \alpha(\omega)$

$-\omega_m \leftrightarrow \omega_m$ $\quad$ $-\omega_m \leftrightarrow \omega_m$

$-2\omega_m \leftrightarrow 2\omega_m$

Start + Start $\quad$ end + end

→ To solve this problem, the bandwidth is obtained based on the value of β.
By increasing β, the bandwidth increases.

Ex:  @ β_1 &  @ β_2 where $\beta_2 > \beta_1$

* β Controls the amount of harmonics around the Carrier frequency.

[1] $\beta << 1$ $\therefore |K\alpha(t)| << 1$ $\therefore$ we can ignore $K_f^2 \alpha^2(t)$ & so on–

$\therefore S(t) \approx A[\underbrace{\cos(\omega_c t)}_{\text{Carrier}} - \underbrace{K\alpha(t)\sin(\omega_c t)}_{\text{Message} \times \text{Carrier}}]$

if $\alpha(t) = m(t) \rightarrow$ PM
if $\alpha(t) = \int m(t)\,dt \rightarrow$ FM

[Similar to AM-DSB-LC. The only diff. is that sideband spectrum has phase shift $\frac{\pi}{2} \rightarrow \sin(\omega_c t)$ instead of $\cos(\omega_c t)$]

$\boxed{\therefore \text{Bandwidth} = 2W_m}$ where W_m = Baseband BW of the message.

In this case, Modulation is denoted as Narrow band FM/PM

[2] $\beta >> 1$: In this case, Modulation is denoted as Wideband FM/PM.

* when β increases, new harmonics appear.

* We check the power values of the harmonics & we can neglect the small values.

* Using mathematics, we found out that the Bessel function controls the power distribution across the harmonics such that $\sum$ of all the harmonics power $\approx \frac{A_c^2}{2}$.

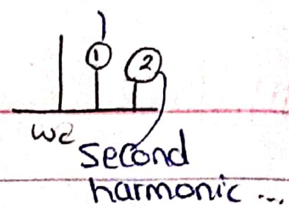
When $\beta >> 1$, there are two approximate ways to calculate the BW:

(a) using Bessel function table:

* Neglect values < 0.01 $\therefore$ take the values at which $|J_n(\beta)| \geq 0.01$

$\boxed{\therefore BW = 2nW_m}$

What is n? → Harmonics around the center frequency.



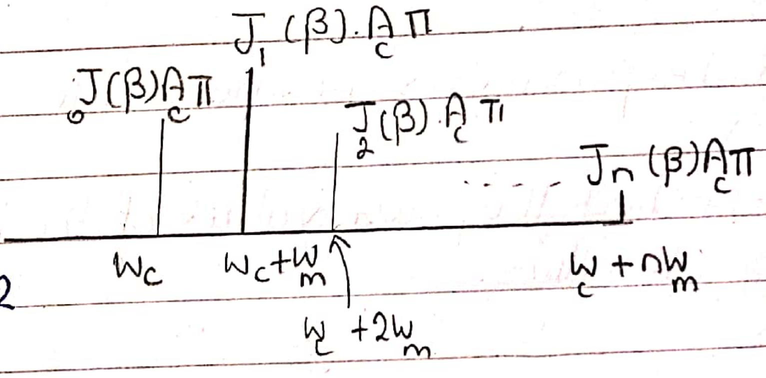
(b) Using Carson's rule:

$BW \approx 2W_m(\beta+1)$

$\therefore BW \approx 2W_m\left(\frac{\Delta W}{W_m}+1\right) \approx 2(\Delta W + W_m)$ → Function in Freq. deviation

* How to find the power of a certain harmonic? [WB FM]

Bessel function controls the power distributed over the harmonics



∴ Power of a certain harmonic

$$= 2\left(\frac{Amplitude}{2\pi}\right)^2 = \left(\frac{J_n(\beta).A_c\cancel{\pi}}{2\cancel{\pi}}\right)^2 \times 2 = \boxed{\frac{J_n^2(\beta)A_c^2}{2}}$$

2 → at the +ve side & the -ve side.

Sheet 3 Solution

1 (a) $10\cos\left(100\pi t + \frac{\pi}{3}\right)$

- $\phi_i(t) = 100\pi t + \frac{\pi}{3}$
- $\omega_i(t) = \frac{d\phi(t)}{dt} = 100\pi$ rad/sec.
- $\boxed{f_i(t) = \frac{\omega_i(t)}{2\pi} = 50 \text{ Hz}} \rightarrow \#$

(b) $10\cos(200\pi t + \sin(\pi t))$

- $\phi_i(t) = 200\pi t + \sin(\pi t)$
- $\omega_i(t) = 200\pi + \pi\cos(\pi t)$
- $\boxed{f_i(t) = 100 + \frac{1}{2}\cos(\pi t) \text{ Hz}} \#$

(c) $2\exp[j\underbrace{200\pi t(1+\sqrt{t})}_{\text{phase}}]$

Recall: $re^{j\theta \leftarrow \text{phase}}$

$\phi_i(t) = 200\pi t(1+\sqrt{t}) \rightarrow \cancel{\omega_i(t)=}$

$\omega_i(t) = 200\pi(1+\sqrt{t}) + 200\pi t\left(\frac{1}{2}\right).t^{-1/2}$

$= 200\pi(1+\sqrt{t}) + 100\pi\sqrt{t}$

$= 200\pi + 300\pi\sqrt{t}.$

$\boxed{f_i(t) = 100 + 150\sqrt{t}. \text{ Hz}} \rightarrow \#$

(d) $\cos(200\pi t)\cos(5\sin(2\pi t)) + \sin(200\pi t)\sin[5\sin(2\pi t)]$

$= \cos(200\pi t - 5\sin(2\pi t))$

$[\because \cos x \cos y + \sin x \sin y = \cos(x-y)]$

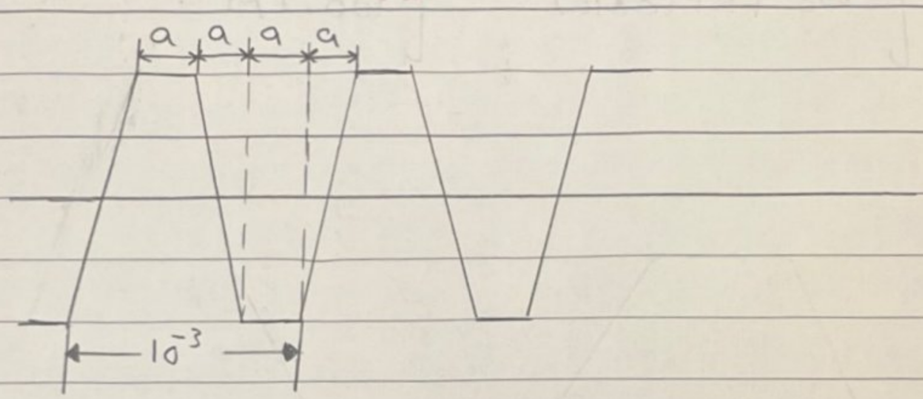
$\therefore \phi_i(t) = 200\pi t - 5\sin(2\pi t)$

$\omega_i(t) = 200\pi - 10\pi\cos(2\pi t)$

$\boxed{f_i(t) = 100 - 5\cos(2\pi t) \ Hz} \rightarrow$ #

Sheet 3 Solution

[2] Sketch $\varphi_{FM}(t)$ and $\varphi_{PM}(t)$ for the modulating signal $m(t)$ shown in the Figure, given $w_c = 10^8$, $K_f = 10^5$ and $K_P = 25$



Solution

$$a = \frac{1}{4} 10^{-3} \quad , w_c = 10^8 \quad , K_f = 10^5 \quad , K_P = 25$$

$$m(t) = \begin{cases} 1 \\ 1 - 8 * 10^3 t \\ -1 \\ -1 + 8 * 10^{-3} t \end{cases}$$

→ FM $\quad w_i = w_c + K_f\, m(t)$

$$w_i = \begin{cases} w_c + K_f \\ w_c + K_f(1 - 8*10^3 t) \\ w_c - K_f \\ w_c + K_f(-1 + 8 * 10^{-3} t) \end{cases} = \begin{cases} 100.1\ M \\ \text{tran} \\ 99.9\ M \\ \text{tran} \end{cases}$$

→ PM $\qquad \omega_i = \omega_c + K_p \dot{m}(t)$

$$\omega_i = \begin{cases} \omega_c \\ \omega_c - K_P(8*10^3) \\ \omega_c \\ \omega_c + K_P(8*10^3) \end{cases} = \begin{cases} 100M \\ 99.8M \\ 100M \\ 100.2M \end{cases}$$