Finding α for the Euler top

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Consider a system with Hamiltonian $H = J_x x^2 + J_y y^2$. We seek to analytically find the K-complexity growth rate α. We will assume that $J_x > J_y > 0$.

First consider the Hamiltonian. Rewriting the equation we get $y^2 = \frac{H - J_x x^2}{J_y}$

We know (from the 'Euler Top' paper) that $z'(t) = 2(J_x - J_y)xy$. Since $x^2 + y^2 + z^2 = 1$, we can use our equation for y^2 to get
$z'(t) = 2(-J_y + H + J_y z)\sqrt{\frac{H - J_x}{J_y}}$. Put $y_E(z) = 2(-J_y + H + J_y z)\sqrt{\frac{H - J_x}{J_y}}$

We obtain t $= \int \frac{1}{(-J_y + H + J_y z)\sqrt{\frac{H - J_x}{J_y}}} dz = \int \frac{1}{y_E(z)}$

The imaginary part,σ_*, of the singularity closest to the real plane is
$\int_{z_0}^{\infty} \frac{1}{i y_E(z)} dz$ where z_0 is the zero of $y_E(z)$ with the largest real part