

# Euler top, general Hamiltonian

yardan jumoorty

May 2022

For this exercise, we will work with a quantum version of the Lipkin-Meshkov-Glick (LMG) model with Hamiltonian  $H = J_x x^2 + J_y y^2$ . Note that the curly brackets we use are Poisson brackets

The Hamiltonian equation of motion are given as follows:

$$\begin{aligned}\frac{dx}{dt} &= \{x, H\} = \{x, J_x x^2 + J_y y^2\} \\ &= J_x \{x, x^2\} + J_y \{x, y^2\} \\ &= -J_y \{y^2, x\} \\ &= -J_y (\{y, x\}y + y\{y, x\}) \\ &= -J_y (-2zy) \\ &= 2J_y yz\end{aligned}$$

$$\begin{aligned}\frac{dy}{dt} &= \{y, H\} = \{y, J_x x^2 + J_y y^2\} \\ &= J_x \{y, x^2\} + J_y \{y, y^2\} \\ &= -J_x (\{x, y\}x + x\{x, y\}) \\ &= -2J_x xz\end{aligned}$$

$$\begin{aligned}\frac{dz}{dt} &= \{z, H\} = \{z, J_x x^2 + J_y y^2\} \\ &= -J_x \{x^2, z\} - J_y \{y^2, z\} \\ &= -J_x (\{x, z\}x + x\{x, z\}) - J_y (\{y, z\}y + y\{y, z\}) \\ &= (2J_x - 2J_y)xy\end{aligned}$$

The derivatives define a system of first order differential equations. The saddle points are located at (0,1,0) and (0,-1,0) The Jacobian is

$$\begin{bmatrix} 0 & 2J_y z & 2J_y y \\ -2J_x z & 0 & -2J_x x \\ 2(J_x - J_y)y & 2(J_x - J_y)x & 0 \end{bmatrix}$$

Evaluating the Jacobian at the saddle point (0,1,0) gives us

$$\begin{bmatrix} 0 & 0 & 2J_y \\ 0 & 0 & 0 \\ 2(J_x - J_y) & 0 & 0 \end{bmatrix}$$

The characteristic equation then gives the eigenvalues  $2\sqrt{J_y}\sqrt{J_x - J_y}$ ,  $0$ ,  $-2\sqrt{J_y}\sqrt{J_x - J_y}$ , implying that  $\lambda$  is  $2\sqrt{J_y}\sqrt{J_x - J_y}$