MTE 204 Numerical Methods

Project 2

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1. Problem 1

1.1. Matrix Equations

$$F_i = [k] \times x_i = \begin{bmatrix} k_i & -k_i \\ -k_i & k_i \end{bmatrix} \times x_i$$

The k matrix in this problem is given by the stiffness of each spring. Thus, the element matrix equations for this problem are:

$$\text{Spring 1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.25 & 0.25 \end{bmatrix} \qquad \text{Spring 2} = \begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix} \qquad \text{Spring 3} = \begin{bmatrix} 1.5 & -1.5 \\ -1.5 & 1.5 \end{bmatrix}$$

$$\text{Spring 4} = \begin{bmatrix} 0.75 & -0.75 \\ -0.75 & 0.75 \end{bmatrix} \qquad \text{Spring 5} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Combining all of the above equations to create a system, we get the following global assembled matrix equations.

$$\begin{bmatrix} F_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.25 & -0.25 & 0 & 0 & 0 & 0 \\ -0.25 & 0.25+0.5 & -0.5 & 0 & 0 & 0 \\ 0 & -0.5 & 0.5+1.5 & -1.5 & 0 & 0 \\ 0 & 0 & -1.5 & 1.5+0.75 & -0.75 & 0 \\ 0 & 0 & 0 & -0.75 & 0.75+1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

Because $x_1 = 0$, we can simplify this system into:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.25+0.5 & -0.5 & 0 & 0 & 0 \\ -0.5 & 0.5+1.5 & -1.5 & 0 & 0 \\ 0 & -1.5 & 1.5+0.75 & -0.75 & 0 \\ 0 & 0 & -0.75 & 0.75+1 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

1.2. Problem 1 Answers

Solving this system with MATLAB, we get the following results. The node displacements (given by the x vector) for each node are:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 12 \\ \frac{40}{3} \\ 16 \\ 18 \end{bmatrix} m$$

The displacement at the final node is given by $x_6 = 18$m. The displacement vs position plot for all the nodes is given in figure 1.

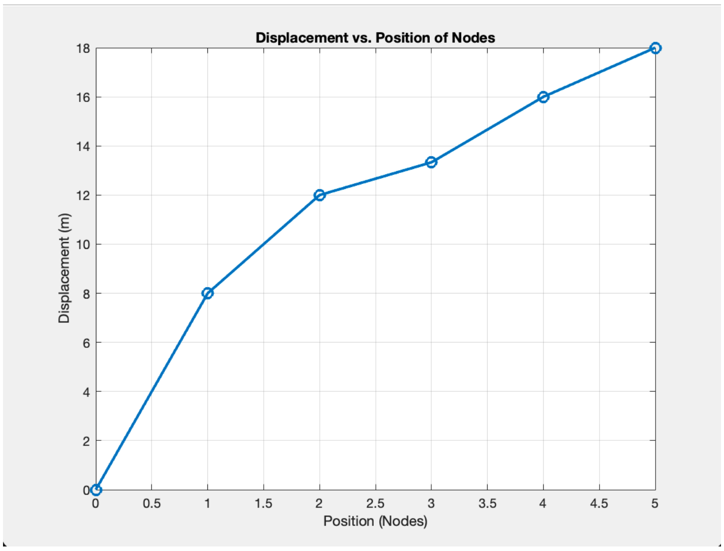

Figura 1. Displacement vs Position Plot of Each Node - P1

2. Problem 2

2.1. Matrix Equations

The k matrix in this problem is derived from the following equation.

$$\delta = \frac{PL}{EA}$$

Delta is the displacement, so we will call it x. P is our force. Thus, the remaining components–representing stiffness–are k. Therefore, the element matrix equations for this problem are:

$$\text{Cylinder 1} = 10^9 \cdot \begin{bmatrix} 1.4632 & -1.4632 \\ -1.4632 & 1.4632 \end{bmatrix} \qquad \text{Cylinder 2} = 10^9 \cdot \begin{bmatrix} 0.4335 & -0.4335 \\ -0.4335 & 0.4335 \end{bmatrix}$$

$$\text{Cylinder 3} = 10^9 \cdot \begin{bmatrix} 0.0813 & -0.0813 \\ -0.0813 & 0.0813 \end{bmatrix}$$

Combining all of the above equations to create a system, we get the following global assembled matrix equations.

$$\begin{bmatrix} F_1 \\ 0 \\ 0 \\ 1000 \end{bmatrix} = 10^9 \cdot \begin{bmatrix} 1.4632 & -1.4632 & 0 & 0 \\ -1.4632 & 1.8967 & -0.4335 & 0 \\ 0 & -0.4335 & 0.5148 & -0.0813 \\ 0 & 0 & -0.0813 & 0.0813 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Because $x_1 = 0$, we can simplify this system into:

$$\begin{bmatrix} 0 \\ 0 \\ 1000 \end{bmatrix} = 10^9 \cdot \begin{bmatrix} 1.897 & -0.4335 & 0 \\ -0.4335 & 0.5148 & -0.0813 \\ 0 & -0.0813 & -0.0813 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

2.2. Problem 2 Answers

Solving this system with MATLAB, we get the following results. The node displacements (given by the x vector) for each node are:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 10^{-5} \cdot \begin{bmatrix} 0 \\ 0.0683 \\ 0.299 \\ 1.5292 \end{bmatrix} m$$

The displacement at the final node is given by $x_4 = 10^{-5} \cdot 1.5292$m. The displacement vs position plot for all the nodes is given in figure 2.

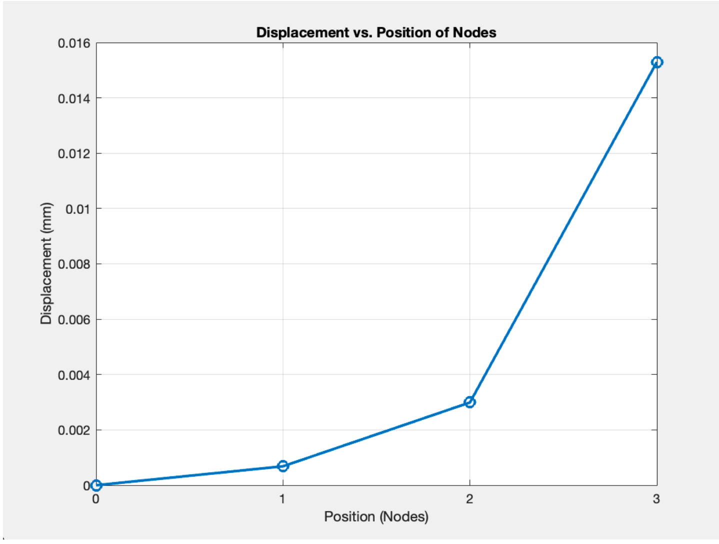

Figura 2. Displacement vs Position Plot of Each Node - P2

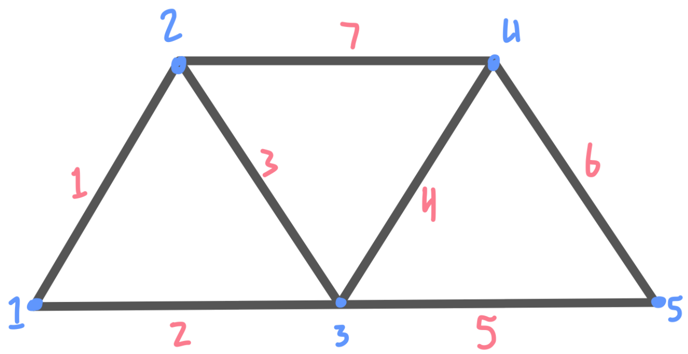

Figura 3. Truss Diagram with Element Labels

3. Problem 3

3.1. Matrix Equations

The truss diagram with element labels for this problem is given in figure 3. Much as in problem 2, the k value for this problem is given by $k = \frac{AE}{L}$. The k matrix must now account for the additional dimension, thus the general stiffness matrix is given by:

$$k = \frac{AE}{L}\begin{bmatrix} \cos\theta^2 & \cos\theta\sin\theta & -\cos\theta^2 & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin\theta^2 & -\cos\theta\sin\theta & -\sin\theta^2 \\ -\cos\theta^2 & -\cos\theta\sin\theta & \cos\theta^2 & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin\theta^2 & \cos\theta\sin\theta & \sin\theta^2 \end{bmatrix}$$

Thus, the element matrix equations for this problem are:

$$k_1 = 10^7 \times \begin{bmatrix} 1.6667 & 2.8868 & -1.6667 & -2.8868 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2.8868 & 5.0000 & -2.8868 & -5.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1.6667 & -2.8868 & 1.6667 & 2.8868 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2.8868 & -5.0000 & 2.8868 & 5.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k_2 = 10^7 \times \begin{bmatrix} 6.6667 & 0 & 0 & 0 & -6.6667 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -6.6667 & 0 & 0 & 0 & 6.6667 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k_3 = 10^7 \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.6667 & -2.8868 & -1.6667 & 2.8868 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.8868 & 5.0000 & 2.8868 & -5.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.6667 & 2.8868 & 1.6667 & -2.8868 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.8868 & -5.0000 & -2.8868 & 5.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k_4 = 10^7 \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.6667 & 2.8868 & -1.6667 & -2.8868 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.8868 & 5.0000 & -2.8868 & -5.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.6667 & -2.8868 & 1.6667 & 2.8868 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2.8868 & -5.0000 & 2.8868 & 5.0000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k_5 = 10^7 \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6.6667 & 0 & 0 & 0 & -6.6667 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6.6667 & 0 & 0 & 0 & 6.6667 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k_6 = 10^7 \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.6667 & -2.8868 & -1.6667 & 2.8868 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2.8868 & 5.0000 & 2.8868 & -5.0000 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1.6667 & 2.8868 & 1.6667 & -2.8868 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.8868 & -5.0000 & -2.8868 & 5.0000 \end{bmatrix}$$

$$k_7 = 10^7 \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6.6667 & 0 & 0 & 0 & -6.6667 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6.6667 & 0 & 0 & 0 & 6.6667 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Combining all of the above equations to create a system, we get the following global assembled stiffness matrix.

$$10^8 \cdot \begin{bmatrix} 0.8333 & 0.2887 & -0.1667 & -0.2887 & -0.6667 & 0 & 0 & 0 & 0 & 0 \\ 0.2887 & 0.5 & -0.2887 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.1667 & -0.2887 & 1 & 0 & -0.1667 & 0.2887 & -0.6667 & 0 & 0 & 0 \\ -0.2887 & -0.5 & 0 & 1 & 0.2887 & -0.5 & 0 & 0 & 0 & 0 \\ -0.6667 & 0 & -0.1667 & 0.2887 & 1.6667 & 0 & -0.1667 & -0.2887 & -0.6667 & 0 \\ 0 & 0 & 0.2887 & -0.5 & 0 & 1 & -0.2887 & -0.5 & 0 & 0 \\ 0 & 0 & -0.6667 & 0 & -0.1667 & -0.2887 & 1 & 0 & -0.1667 & 0.2887 \\ 0 & 0 & 0 & 0 & -0.2887 & -0 & 0 & 1.0000 & 0.2887 & -0.5 \\ 0 & 0 & 0 & 0 & -0.6667 & 0 & -0.1667 & 0.2887 & 0.8333 & -0.2887 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2887 & -0.5 & -0.2887 & 0.5 \end{bmatrix}$$

Recognizing that y is constrained at nodes 1 and 5 by the roller, we get the following simplified equation by removing rows and columns 2 and 10, corresponding to x_2 and x_{10}, which must be 0 as displacement is 0.

$$\begin{bmatrix} 0 \\ 0 \\ -2000 \\ 0 \\ 0 \\ 0 \\ -5000 \\ 0 \end{bmatrix} = 10^8 \times \begin{bmatrix} 0.833 & -0.167 & -0.289 & -0.667 & 0 & 0 & 0 & 0 \\ -0.167 & 1 & 0 & -0.167 & 0.289 & -0.667 & 0 & 0 \\ -0.289 & 0 & 1 & 0.289 & -0.5 & 0 & 0 & 0 \\ -0.667 & -0.167 & 0.289 & 1.667 & 0 & -0.167 & -0.289 & -0.667 \\ 0 & 0.289 & -0.5 & 0 & 1 & -0.289 & -0.5 & 0 \\ 0 & -0.667 & 0 & -0.167 & -0.289 & 1 & 0 & -0.167 \\ 0 & 0 & 0 & -0.289 & -0.5 & 0 & 1 & 0.289 \\ 0 & 0 & 0 & -0.667 & 0 & -0.167 & 0.289 & 0.833 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix}$$

3.2. Problem 3 Answers

Solving this system with MATLAB we get that the x and y components of displacement for each node is:

$$\text{x component} = 10^{-5} \times \begin{bmatrix} -2.419 \\ 1.803 \\ -0.03693 \\ -1.228 \\ 3.644 \end{bmatrix} \text{ m}$$

$$\text{y component} = 10^{-5} \times \begin{bmatrix} 0 \\ -7.938 \\ -10.50 \\ -11.31 \\ 0 \end{bmatrix} \text{ m}$$

The magnitude of each displacement vector calculated by $\sqrt{x^2 + y^2}$ is:

$$\sqrt{x^2 + y^2} = 10^{-5} \times \begin{bmatrix} 2.419 \\ 8.140 \\ 10.50 \\ 11.38 \\ 3.644 \end{bmatrix} \text{ m}$$

The normal stresses in each member are:

$$\text{Normal Stresses in Each Member} = \begin{bmatrix} -7.939 \\ 3.969 \\ 2.165 \\ -2.165 \\ 6.134 \\ -12.269 \\ -5.052 \end{bmatrix} \text{MPa}$$