Accelerating Spike-by-Spike Neural Networks on FPGA with Hybrid Custom Floating-Point and Logarithmic Dot-Product Approximation

YARIB NEVAREZ[1], DAVID ROTERMUND[2], KLAUS R. PAWELZIK[3], ALBERTO GARCIA-ORTIZ[4] (Member, IEEE),

[1]Institute of Electrodynamics and Microelectronics, University of Bremen, Bremen 28359, Germany (e-mail: nevarez@item.uni-bremen.de)
[2]Institute for Theoretical Physics, University of Bremen, Bremen 28359, Germany (e-mail: davrot@@neuro.uni-bremen.de)
[3]Institute for Theoretical Physics, University of Bremen, Bremen 28359, Germany (e-mail: pawelzik@@neuro.uni-bremen.de)
[4]Institute of Electrodynamics and Microelectronics, University of Bremen, Bremen 28359, Germany (e-mail: agaracia@item.uni-bremen.de)

Corresponding author: Yarib Nevarez (e-mail: nevarez@item.uni-bremen.de).

ABSTRACT Spiking neural networks (SNNs) represent a promising alternative to conventional neural networks. In particular, the so called Spike-by-Spike (SbS) neural networks provide an exceptional noise robustness and a reduced complexity. However, deep SbS networks require a memory footprint and a computational cost unsuitable for embedded applications. To address this problem, this work exploits the intrinsic error-resilience of neural networks to improve performance and to reduce the hardware complexity. More precisely, we design a dot-product hardware unit based on approximate computing with configurable quality using hybrid custom floating-point and logarithmic number representation. This approach reduces computational latency, memory footprint, and power dissipation while preserving inference accuracy. To demonstrate our approach, we address a design exploration flow using high-level synthesis and a Xilinx SoC-FPGA. The proposed design reduces $20.5\times$ computational latency and $8\times$ weight memory footprint, with less than 0.5% of accuracy degradation on a handwritten digit recognition task.

INDEX TERMS Artificial intelligence, spiking neural networks, approximate computing, logarithmic, parameterisable floating-point, optimization, hardware accelerator, embedded systems, FPGA

I. INTRODUCTION

THE exponential improvement in computing performance and the availability of large amounts of data are boosting the use of artificial intelligence (AI) applications in our daily lives. Among the various algorithms developed over the years, neural networks (NNs) have demonstrated remarkable performance in a variety of image, video, audio, and text analytics tasks [1], [2]. Historically, artificial neural networks (ANNs) can be classified into three different generations [3]: the first one is represented by the classical McCulloch and Pitts neuron model using discrete binary values as outputs; the second one is represented by more complex architectures as multi-layer perceptrons (MLPs) and convolutional neural networks (CNNs) using continuous activation functions; while the third generation is represented

by spiking neural networks using spikes as means for information exchange between groups of neurons. Although the AI research is currently dominated by deep neural networks (DNNs) from the second generation, the SNNs belonging to the third generation are receiving considerable attention [3]–[6]. SNNs offer advantageous robustness and the potential to achieve a power efficiency closer to that of the human brain (see section III-A for more details).

Among the family of SNNs, the SbS neural network [5] is remarkable. Inspired by the natural information processing of the mammalian brain, it is a biologically plausible approach although with less complexity than other SNNs. The SbS model differs fundamentally from conventional ANNs since (a) the building blocks of the network are inference populations (IP) which are an optimized generative representation

with non-negative values, (b) time progresses from one spike to the next, preserving the property of stochastically firing neurons, and (c) a network has only a small number of parameters, which is an advantageous noise-robust stochastic version of Non-Negative Matrix Factorization (NNMF). In regard to biological realism and computational effort to simulate neural networks, these properties place the SbS network in between non-spiking NN and stochastically spiking NN, offering advantages from both worlds [7].

Although SbS networks provide numerous advantages over traditional ANNs and CNNs, deep SbS networks are highly compute and data intensive, representing a challenge for efficient deployment in resource-limited devices. As an alternative, based on the relaxed need for fully precise or deterministic computation of neural networks, approximate computing techniques allow substantial enhancement in processing efficiency with moderated accuracy degradation. Some research papers have shown the feasibility of applying approximate computing to the inference stage of neural networks [8]–[11]. Such techniques usually demonstrated small inference accuracy degradation, but significant enhancement in computational performance, resource utilization, and energy consumption. Hence, by taking advantage of the intrinsic error-tolerance of neural networks, approximate computing is positioned as a promising approach for inference on resource-limited devices.

In this paper, we accelerate SbS neural networks with a dot-product hardware design based on approximate computing with hybrid custom floating-point and logarithmic number representation. This hardware unit has a quality configurable scheme based on the bit truncation of the synaptic-weight vector. **Fig.** 1 illustrates the standard floating-point precise computation and our approach for dot-product approximation with hybrid custom floating-point and logarithmic number representation. As a design parameter, the mantissa bit-width of the weight vector provides a tunable knob to trade-off between efficiency and quality of result (QoR) [12], [13]. Since the lower-order bits have smaller significance than the higher-order bits, removing them may have only a minor impact on QoR [14], [15]. Further on, we can truncate completely the mantissa bits in order to use only the exponent of a floating-point representation. Therefore, the most efficient setup and yet the worst-case quality configuration becomes a logarithmic representation, which consequently leads to significant architectural-level optimizations using only adders and shifters for dot-product approximation in hardware. Moreover, since approximations and noise have qualitatively the same effect [16], we apply noise tolerance plots as an intuitive visual measure to provide insights into the quality degradation of SbS networks under approximate processing effects.

Our main contributions are as follows:

- We develop a hardware component for dot-product approximation. To perform the sum of pairwise products of two vectors, this hardware module has the following three design features: (1) the pairwise product is approx-
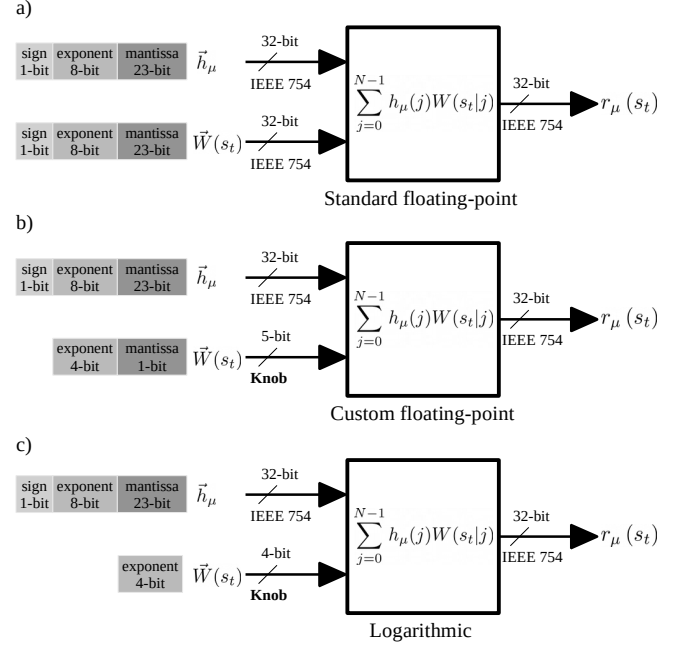

FIGURE 1. Dot-product hardware module with (a) standard floating-point computation, (b) hybrid custom floating-point approximation, and (c) hybrid logarithmic approximation.

imated by adding integer exponents and multiplying truncated mantissas, and the sum of products is done by accumulating denormalized integer products with barrel shifters, which increases computational throughput; (2) the synaptic weight vector uses either reduced custom floating-point or logarithmic representation, which reduces memory footprint; and (3) the neuron vector uses either standard or custom floating-point representation, which preserves QoR and overall inference accuracy.

- We address a design exploration with the proposed dot-product approximation using synaptic weight vectors with 1-bit mantissa as well as completely truncated. We evaluate inference latency, accuracy degradation, resource utilization and power dissipation. Experimental results demonstrate $20.5\times$ latency enhancement versus embedded CPU (ARM Cortex-A9 at 666MHz), and less than 0.5% of accuracy degradation on a hand written digit recognition task.

- We propose a noise tolerance plot as quality monitor, which serves as an intuitive visual model to provide insights into the accuracy degradation of SbS networks under approximate processing effects.

- Our proposed design for dot-product approximation is adaptable as a building block for other error-resilient applications (e.g., image/video processing).

The rest of the paper is organized as follows. Section II covers the related work; Section III introduces the background to SbS networks; Section IV describes the system design and the approximate dot-product hardware module; Section V presents the experimental results thorough a design exploration flow; Section VI concludes the paper.

To promote the research on SbS networks, our design exploration framework is made available to the public as an open-source project at http://www.ids.uni-bremen.de/sbs-framework.html

II. RELATED WORK

A. APPROXIMATE COMPUTING IN NEURAL NETWORKS

Approximate computing has been used in a wide range of applications to increase the computational efficiency in hardware [13]. For neural network applications, two main approximation strategies are used, namely network compression and classical approximate computing [17].

1) Network compression

Researchers focusing on embedded applications started lowering the precision of weights and activation maps to shrink the memory footprint of the large number of parameters representing ANNs, a method known as network compression or quantization. This practice takes advantage of the intrinsic error-tolerance of neural networks, as well as their ability to compensate for approximation while training. In this way, reduced bit precision causes a small accuracy loss [18]–[21].

In hardware development, weight quantization (WQ) has shown up to $2\times$ improvement in energy consumption with an accuracy degradation of less than 1% [22], [23]. Some advanced quantization methods yield to binary neural networks (BNNs) allowing the use of XNORs instead of the conventional costly multiply-accumulate circuits (MACs) [21]. In [24], Sun et al. report an accuracy of 98.43% on handwritten digit classification with a simple BNN. Hence, quantization is a powerful tool for improving the energy efficiency and memory requirements of ANN accelerators, with limited accuracy degradation.

In addition to quantization, network pruning reduces the model size by removing structural portions of the parameters and its associated computations [25]–[27]. This method has been identified as an effective technique to improve the efficiency of DNN for applications with limited computational budget [28]–[30].

These methods can be used for SNNs as well. In [31], Rathi et al. report up to $3.1\times$ improvement in energy consumption with an accuracy loss of around 3%. Weight quantization allows the designer to realize a trade-off between the accuracy of the SNN application and efficiency of resources. Approximate computing can also be applied at the neuron level, where irrelevant units are deactivated to reduce the computation cost of the SNNs [32]. This computation skipping can be applied randomly on synapses, training ANNs with stochastic synapses improves generalization, resulting in a better accuracy [33], [34]. Such methods are compatible with SNNs and have been tested both during training [35], [36] and operation [37], and even to define the connectivity between layers [38], [39]. Implementations of spiking neuromorphic systems in FPGA [40] and hardware [41] demonstrated that synaptic stochasticity allows to increase the final accuracy of the networks while reducing memory footprint.

Quantization is therefore a powerful technique to improve energy efficiency and memory requirements of ANN and SNN accelerators, with small accuracy degradation; however, this approach requires quantization-aware training methods that, in some cases, are problematic or even inaccessible, particularly in emerging deep SNN algorithms [42].

2) Classical approximate computing

This approach consists of designing processing elements that approximate their computation by employing modified algorithmic logic units [13]. In [43], Kim et al. have shown SNNs using carry skip adders achieving $2.4\times$ latency enhancement and 43% more energy efficiency, with an accuracy degradation of 0.97% on MNIST classification task. Therefore, approximate computing provides important enhancement in energy efficiency and processing speed.

However, as the complexity of the dataset increases, as well as the depth of the network topology, such as ResNet [44] on ImageNet [45], the accuracy degradation becomes more important and may not be negligible anymore [21], especially for critical applications such as autonomous driving. Therefore, it is not certain that network compression techniques and approximate computing are suitable for all applications.

B. SPIKE-BY-SPIKE NEURAL NETWORKS ACCELERATORS

Recently, Rotermund et al. demonstrated the feasibility of a neuromorphic SbS IP on a Xilinx Virtex 6 FPGA [46]. It provides a massively parallel architecture, optimized to reduce memory access and suitable for ASIC implementations. Nonetheless, this design is considerably resource-demanding if implemented as a full SbS network in today's embedded technology. In Ref. [47], we presented a cross-platform accelerator framework for design exploration and testing of fully functional SbS network models in embedded systems. As a hardware/software (HW/SW) co-design solution, this framework offers a comprehensive high level software API that allows the construction of scalable sequential SbS networks with configurable hardware acceleration. In this article, we will use this design exploration framework to investigate approximate computing for efficient deployment of deep SbS networks on resource-limited devices.

III. BACKGROUND

A. SPIKE-BY-SPIKE NEURAL NETWORKS

As a generative model [5], the SbS model iteratively finds an estimate of its input probability distribution $p(s)$ (i.e. the probability of input node s to stochastically send a spike) by its latent variables via $r(s) = \sum_i h(i)W(s|i)$, where $\vec{h}$ is an inference population composed of a group of neurons that compete with each other. An inference population sees only the spikes s_t (i.e. the index identifying the input neuron s which generated that spike at time t) produced by its input

neurons, not the underlying input probability distribution $p(s)$ itself. By counting the spikes arriving at a group of SbS neurons, $p(s)$ is estimated by $\hat{p}(s) = 1/T \sum_t \delta_{s,s^t}$ after T spikes have been observed in total. The goal is to generate an internal representation $r(s)$ from the string of incoming spikes s_t such that the negative logarithm of the likelihood $L = C - \sum_\mu \sum_s \hat{p}_\mu(s) log(r_\mu(s))$ is minimized. C is a constant which is independent of the internal representation $r_\mu(s)$ and μ denotes one input pattern from an ensemble of input patterns. Applying a multiplicative gradient descent method on L, an algorithm for iteratively updating $h_\mu(i)$ with every observed input spike s_t could be derived [5]

$$ h_\mu^{new}(i) = \frac{1}{1+\epsilon} \left(h_\mu(i) + \epsilon \frac{h_\mu(i)W(s_t|i)}{\sum_j h_\mu(j)W(s_t|j)} \right) \quad (1) $$

where ϵ is a parameter that also controls the strength of sparseness of the distribution of latent variables $h_\mu(i)$. Furthermore, L can also be used to derive online and batch learning rules for optimizing the weights $W(s|i)$.

Fundamentally, SbS is a stochastic gradient descent dynamics consistent with Non-Negative Matrix Factorization (NNMF). The stochasticity of gradient descent could in principle overcome local minima. Furthermore, it favors sparse solutions with little fluctuations (which is the case for over-complete representations). Finally this specific mechanism for inducing sparseness selects those sparse solutions that are robust against noise in the inputs.

In SbS, the expected change at a given h-state (i.e. $\Delta h_i^{s_t} \propto \left\langle \frac{p(s_t|i)h_i}{\sum_j p(s_t|j)h_j} - 1 \right\rangle_{p(s_t)}$ for all $i \in (1,...,N)$ is exactly the same we would have in a low pass version of NNMF ($\Delta h_i = \sum_s \frac{p(s)p(s|i)h_i}{\sum_j p(s|j)h_j} - 1$). Then, for each given h-state, the changes of h induced by SbS consist of the expected vector Δh plus fluctuations $\eta_i(s_t)$ with $<\eta_i(s_t)> p(s_t) = 0$ (i.e. $\Delta h_i^{s_t} = \sum_s \frac{p(s)p(s|i)h_i}{\sum_j p(s|j)h_j} + \eta_i(s_t)$). Thus, SbS performs a random walk with mean Δh and some variance, where we have a stochastic process in h-space with the correct drift (Δh) and diffusion. Such processes drift towards states where the drift vanishes except for remaining fluctuations. Thus, it produces a Brownian motion finally leading to a probability density for h-states centered around the fixed point.

An example of the noise tolerance of SbS is presented in **Fig.** 2. It compares the classification performance of a SbS network and a tensor flow network, with the same amount of neurons per layer as well as the same layer structure. We trained on MNIST dataset [48] without noise (see [7] for details). The figure shows the correctness for the MNIST test set with its 10000 patterns in dependency of the noise level for positive additive uniformly distributed noise. The blue curve shows the performance for the tensor flow network, while the red curve shows the performance for the SbS network with 1200 spikes per inference population. Beginning with a noise level of 0.1, the respective performances are different with a p - level of at least 10^{-6} (tested with the Fisher exact test). Increasing the number of spikes per SbS population to 6000
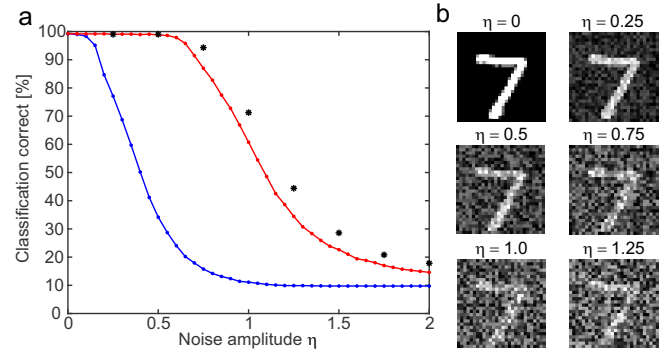

FIGURE 2. (a) Performance classification of SbS NN versus equivalent CNN, and (b) Example of the first pattern in the MNIST test data set with different amounts of noise.
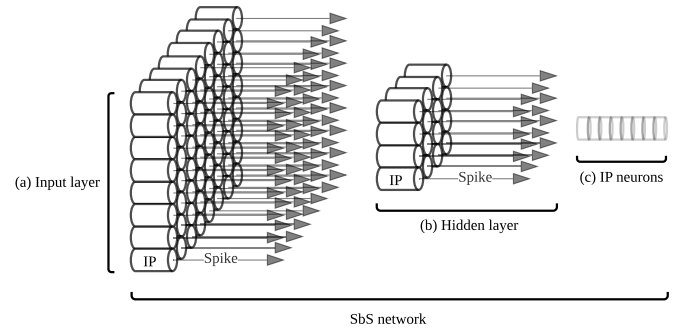

FIGURE 3. SbS IPs as independent computational entities, (a) illustrates an input layer with a massive amount of IPs operating as independent computational entities, (b) shows a hidden layer with an arbitrary amount of IPs as independent computational entities, (c) exhibits a set of neurons grouped in an IP.

(performance values shown as black stars), shows that more spike can improve the performance under noise even more.

The SbS network algorithm is highly suitable for parallelization. SbS network models are constructed in sequential layered structures, each layer consists of many IPs which can be simulated independently while the communication between the IPs is organized by a low bandwidth signal – the spikes [49]. Technically, each IP is an independent computational entity (see **Fig.** 3). This allows to design specialized hardware architectures that can be massively parallelized.

IV. SYSTEM DESIGN
In this section, we revisit the system design of [47], as an accelerator framework of SbS networks for inference and incremental learning in embedded systems. In principle, this architecture is a hardware/software cross-platform for design exploration and deployment of scalable SbS networks on FPGA.

Regarding the software architecture, it is structured as a layered object oriented application framework written in C programming language. This offers a comprehensive high level embedded software application programming interface (API) that allows the construction of scalable sequential SbS networks with configurable hardware acceleration. Conceptually this design is modular, reusable, and extensible. The
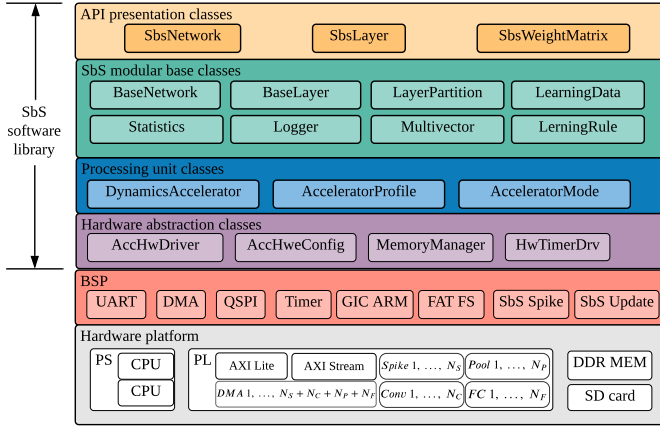
FIGURE 4. System-level overview of the software architecture.

FIGURE 5. System-level hardware architecture with scalable number of heterogeneous PUs: *Spike*, *Conv*, *Pool*, and *FC*

overall structure is depicted in **Fig.** 4.

A. HARDWARE ARCHITECTURE

As a hardware/software co-design, the system architecture is an embedded CPU+FPGA-based platform, where the acceleration of SbS network computation is based on asynchronous[1] execution in parallel heterogeneous processing units (PUs): *Spike* (input layer), *Conv* (convolution), *Pool* (pooling), and *FC* (fully connected). **Fig.** 5 illustrates the system hardware architecture as a scalable structure. For hardware configuration, each PU uses AXI-Lite interface; for data transfer, each PU uses AXI-Stream interfaces via Direct Memory Access (DMA) allowing data movement with high transfer rate. Each PU asserts an interrupt flag once the task or transaction is complete. This interrupt event is handled by the embedded CPU to collect results and start a new transaction.

The hardware architecture can resize its resource utilization by changing the number of PUs instances, this provides a good trade-off between area and throughput. The dedicated PUs for *Conv*, and *FC* implement the proposed dot-product approximation as a system component. The PUs are written in C using Vivado HLS (High-Level Synthesis) tool. In this publication, we illustrate the integration of the approximate dot-product component on the *Conv* processing unit.

B. CONV PROCESSING UNIT

This hardware module computes the IP dynamics defined by **Eq.** (1) and offers two modes of operation: *configuration* and *computation*.

1) Configuration mode

In this mode of operation, the PU receives and stores in on-chip memory (BRAM) the parameters to compute the IP dynamics: ϵ as the epsilon, N as the length of $\vec{h}_\mu \in \mathbb{R}^N$, $K \in \mathbb{N}$ as the size of the convolution kernel, and $H \in \mathbb{N}$ as

[1] The system is synchronous at the circuit level, but the execution is asynchronous in terms of jobs.
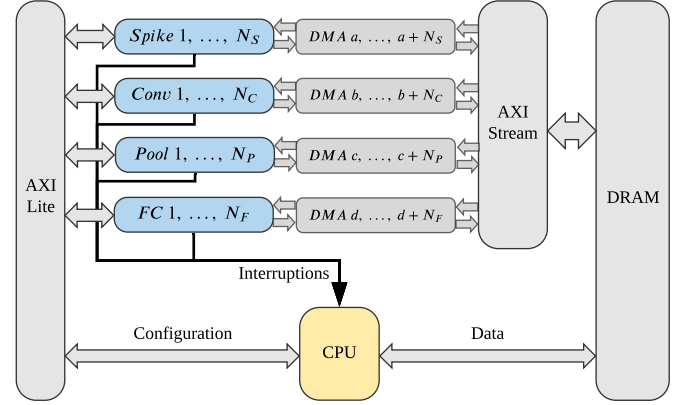
the number of IPs to process per transaction. H is the number of IPs forming a layer or a partition.

Additionally, the processing unit also stores in on-chip memory (BRAM) the synaptic weight matrix using a number representation with a reduced memory footprint. Fundamentally, the synaptic weight matrix is defined by $W \in \mathbb{R}^{K \times K \times M \times N}$ with $0 \leq W(s_t|j) \leq 1$ and $\sum_{j=0}^{N-1} W(s_t|j) = 1$ [7]. Hence, W employs only positive normalized real numbers. Therefore, W is deployed using a reduced floating-point or logarithmic representation as follows:

- Custom floating-point representation. In this case, W is deployed with a reduced floating-point representation using the user defined bit width for the exponent and for the mantissa. For example, 4-bit exponent, 1-bit mantissa; as a result: 5-bit custom floating-point.
- Logarithmic representation. In this case, the synaptic weight matrix is $W \in \mathbb{N}^{K \times K \times M \times N}$ with positive natural numbers. Since $0 \leq W(s_t|j) \leq 1$ and $\sum_{j=0}^{N-1} W(s_t|j) = 1$, W has only negative values in the logarithmic domain. Hence, the sign bit is omitted, and the values are represented in its positive form. Therefore, W is deployed with a representation using the necessary bit width for the exponent according to the given application. For example, 4-bit exponent.

In order to deploy different SbS network models, the *Conv* processing units can be set with different synaptic weight matrices and parameters as required through the embedded software.

2) Computation mode

In this mode of operation, the PU executes a transaction to process a group of IPs using the previously given parameters and synaptic weight matrix. This process operates in six stages as shown in **Fig.** 6. In the first two stages, the PU receives $\vec{h}_\mu \in \mathbb{R}^N$, then the PU calculates the emitted spike, and stores it in $S^{new} \in \mathbb{N}^H$ (output spike vector). From the third to the fifth stage, the PU receives $S_t \in \mathbb{N}^{K \times K}$ (input spike matrix), then it computes the update dynamics, and then it dispatches $\vec{h}_\mu^{new} \in \mathbb{R}^N$ (updated IP). This process repeats
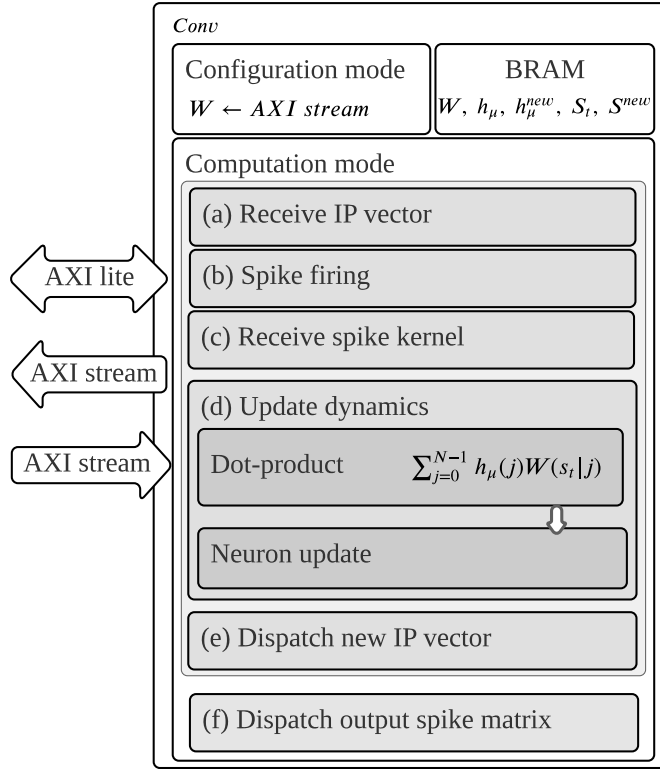
FIGURE 6. The *Conv* processing unit and its six stages: (a) receive IP vector, (b) spike firing, (c) receive spike kernel, (d) update dynamics, (e) dispatch new IP vector, (f) dispatch output spike matrix.

for H number of loops (for each IP of the layer or partition). Finally, the S^{new} is dispatched.

The computation of the update dynamics (see **Fig.** 6(d)) operates in two modular stages: *dot-product* and *neuron update*. First, the *dot-product* module calculates the sum of pairwise products of $\vec{h}_\mu$ and $\vec{W}(s_t)$, while storing each pairwise product as intermediate results. Subsequently, the *neuron update* module calculates **Eq.** (1) reusing previous results and parameters.

The computation of the dot-product of **Eq.** (1) represents a considerable computational cost using standard floating-point in non-quantized network models. Fortunately, the pair product of $h_\mu(j)$ and $W(s_t|j)$ was defined by us as an approximable factor in the dot-product of **Eq.** (1). In the following section, we focus on an optimized dot-product hardware design based on approximate computing.

C. DOT-PRODUCT HARDWARE MODULE
This dot-product hardware module is part of an application-specific architecture optimized to approximate the dot-product of arbitrary length, see **Eq.** (2). For quality configurability, we parameterized the mantissa bit-width of $\vec{W}(s_t)$, which provides a tunable trade-off between resource utilization and QoR. Since the lower-order bits have smaller significance than the higher-order bits, removing them may have only a minor impact on QoR. We designate this as hybrid custom floating-point approximation.

$$r_\mu(s_t) = \sum_{j=0}^{N-1} h_\mu(j)W(s_t|j) \qquad (2)$$

Further on, we remove the mantissa bits completely in order to use only the exponent of a floating-point representation. Hence, the worst-case quality and yet the most efficient configuration becomes a logarithmic representation. Consequently, this configuration leads to advantageous architectural optimizations using only adders and barrel shifters for dot-product approximation in hardware. We designate this as hybrid logarithmic approximation.

In this section, we will present three pipelined hardware modules with standard floating-point computation, hybrid custom floating-point approximation, and hybrid logarithmic approximation.

1) Dot-product with standard floating-point computation
The hardware module to calculate the dot-product with standard floating-point computation is shown in **Fig.** 7. This diagram presents the hardware blocks and their clock cycle schedule. This module loads both $h_\mu(j)$ and $W(s|j)$ from BRAM, then the PU executes the pairwise product (**Fig.** 7(c)) and accumulation (**Fig.** 7(d)). The intermediate results of $h_\mu(j)W(s_t|j)$ are stored in BRAM for reuse in the neuron update. The latency in clock cycles of this hardware module is defined by **Eq.** (3), where N is the dot-product length. This latency equation is obtained from the general pipelined hardware latency formula: $L = (N-1)II + IL$, where II is the initiation interval (**Fig.** 7(a)), and IL is the iteration latency (**Fig.** 7(b)). Both II and IL are obtained from the high-level synthesis analysis. The equation for the latency with standard 32-bit floating-point is:

$$L_{f32} = 10N + 9 \qquad (3)$$

In this design, the high level synthesis tool infers computational blocks with considerable latency cost for standard floating-point. In the case of floating-point multiplication (**Fig.** 7(c)), the synthesis infers a hardware block with a latency cost of 5 clock cycles. Theoretically, this block would handle exponents addition, mantissas multiplication, and mantissa correction if needed. Moreover, in the case of floating-point addition (**Fig.** 7(d)), the synthesis infers a hardware block with a latency cost of 9 clock cycles. Seemingly, this block would handle mantissas alignment, addition, and correction if needed. Therefore, the use of standard floating-point in high-level synthesis results in high computational cost, which represents unnecessary overhead in error-tolerant applications.

2) Dot-product with hybrid custom floating-point and logarithmic approximation
The hardware module to calculate dot-product with hybrid custom floating-point approximation is shown in **Fig.** 8. In this design, $h_\mu(j)$ uses standard 32-bit floating-point number
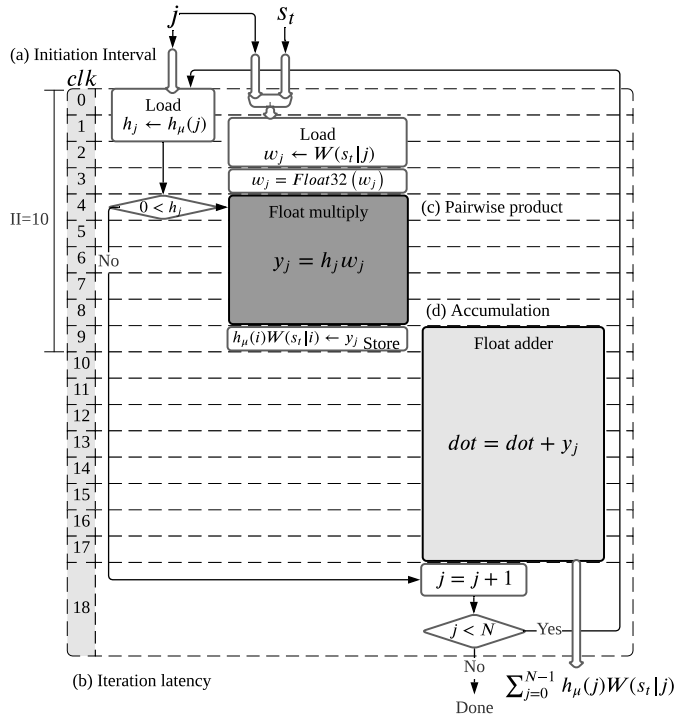
FIGURE 7. Dot-product hardware module with standard floating-point computation, (a) exhibits the initiation interval of 10 clock cycles, (b) presents the iteration latency of 19 clock cycles, (c) shows the pairwise product block in dark-gray, and (d) illustrates the accumulation block in light-gray.

representation, and $W(s|j)$ uses a positive reduced custom floating-point number representation, where the mantissa bit width is the quality configurability knob. This parameter is tuned by the designer to trade-off between QoR and resource utilization, thus, energy consumption.

As the most efficient setup and yet the worst-case quality configuration, by completely truncating the mantissa of $W(s|j)$ leads to a slightly different hardware architecture using only adders and shifters, which computes the dot-product with hybrid logarithmic approximation. This is shown in **Fig.** 9.

Additionally, the exponent bit width of $W(s|j)$ is a design parameter for efficient resource utilization and it is defined based on the application or deployment needs.

The hybrid custom floating-point and logarithmic approximation designs work in three phases: *Computation*, *Threshold-test*, and *Result normalization*.

- Phase I, *Computation*:
  This phase approximates the magnitude of the dot-product in a denormalized representation. This is calculated in two iterative steps over each vector element: *pairwise product* and *accumulation*, where *pairwise product* is executed either in hybrid custom floating-point or hybrid logarithmic approximation described below.
  - Pairwise product.
    - Hybrid custom floating-point approximation. As

shown in **Fig.** 8(c) in dark-gray, the pairwise product is approximated by adding exponents and multiplying mantissas of both $W(s|i)$ and $h_\mu(i)$. If the mantissa multiplication results in an overflow, then it is corrected by increasing the exponent and shifting the resulting mantissa by one position to the right. Then we get $h_\mu(j)W(s_t|j)$ as an intermediate result which is stored for future reuse in the neuron update calculation. In this design the pairwise product has a latency of 5 clock cycles.
    - Hybrid logarithmic approximation. As shown in **Fig.** 9(c) in dark-gray, the pairwise product is approximated by adding $W(s|i)$ to the exponent of $h_\mu(i)$, since $W(s|j)$ values are represented in the logarithmic domain and $h_\mu(j)$ in standard floating-point. In this design the pairwise product has a latency of one clock cycle.
  - Accumulation. As shown in both **Fig.** 8(d) and **Fig.** 9(d) in light-gray, first, it is obtained the denormalized representation of $h_\mu(j)W(s_t|j)$ by shifting its mantissa using its exponent as shifting parameter (barrel shifter). Then, this denormalized representation is accumulated to obtain the approximated magnitude of the dot-product.

The process of pairwise product and accumulation iterates over each element of the vectors. The computation latency is given by **Eq.** (4) for hybrid custom floating-point, and **Eq.** (5) for hybrid logarithmic, where N is the length of the vectors. Both pipelined hardware modules have the same throughput, since both have two clock cycles as initiation interval.

$$L_{custom} = 2N + 11 \qquad (4)$$

$$L_{log} = 2N + 7 \qquad (5)$$

- Phase II, *Threshold-test*:
  The accumulated denormalized magnitude is tested to be above of a predefined threshold, it must be above zero, since the dot-product is the denominator in **Eq.** (1). If passing the threshold, then the next phase is executed. Otherwise the rest of update dynamics is skipped. The threshold-test takes one clock cycle.
- Phase III, *Result-normalization*:
  In this phase, the dot-product is normalized to obtain the exponent and mantissa in order to convert it to standard floating-point for later use in the neuron update. The normalization is obtained by shifting the approximated dot-product magnitude in a loop until it is in the form of a normalized mantissa where the iteration count represents the exponent of the dot-product. Each iteration takes one clock cycle.

The total latency of the hardware module with hybrid custom floating-point and hybrid logarithmic approximation is the accumulated latency of the three phases.
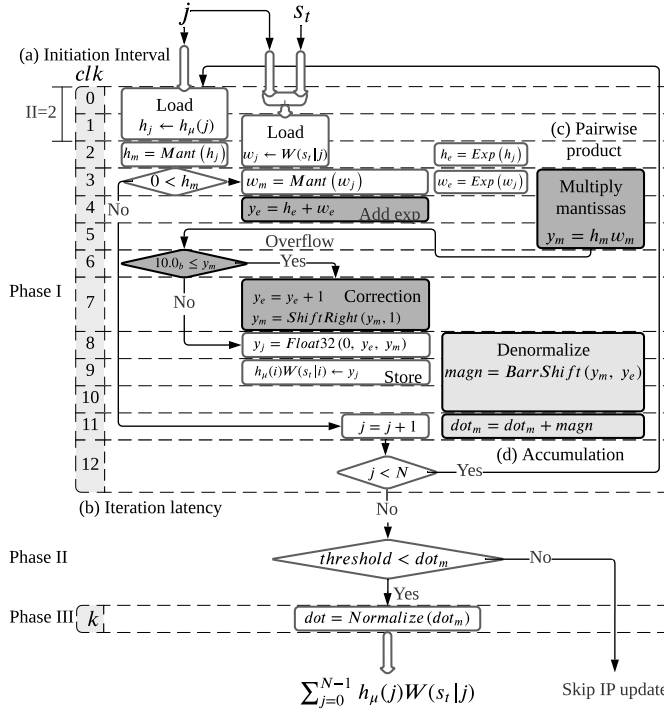
FIGURE 8. Dot-product hardware module with hybrid custom floating-point approximation, (a) exhibits the initiation interval of 2 clock cycles, (b) presents the iteration latency of 13 clock cycles, (c) shows the pairwise product blocks in dark-gray, and (d) illustrates the accumulation blocks in light-gray.

FIGURE 9. Dot-product hardware module with hybrid logarithmic approximation, (a) exhibits the initiation interval of 2 clock cycles, (b) presents the iteration latency of 9 clock cycles, (c) shows the pairwise product block in dark-gray, and (d) illustrates the accumulation blocks in light-gray.

The proposed architectures with approximation approach exceeds the performance of the design with standard floating-point. This performance enhancement is achieved by decomposing the floating-point computation into an advantageous handling of exponent and mantissa using intermediate accumulation in a denormalized representation and only one final normalization.

V. EXPERIMENTAL RESULTS

The proposed architecture is demonstrated on a Xilinx Zynq-7020. This device integrates a dual ARM Cortex-A9 based processing system (PS) and programmable logic (PL) equivalent to Xilinx Artix-7 (FPGA) in a single chip [50]. The Zynq-7020 architecture conveniently maps the custom logic and software in the PL and PS respectively as an embedded system.

In this platform, we implement the proposed hardware architecture to deploy the SbS network structure shown in **Fig.** 10 for handwritten digit classification task using MNIST data set. The SbS model is trained in Matlab without any quantization method, using standard floating-point. The resulting synaptic weight matrices are deployed on the embedded system. There, the SbS network is built as a sequential model using the API from the SbS embedded software framework [47]. This API allows to configure the computational workload of the neural network, which can be distributed among the hardware processing units and the embedded CPU.
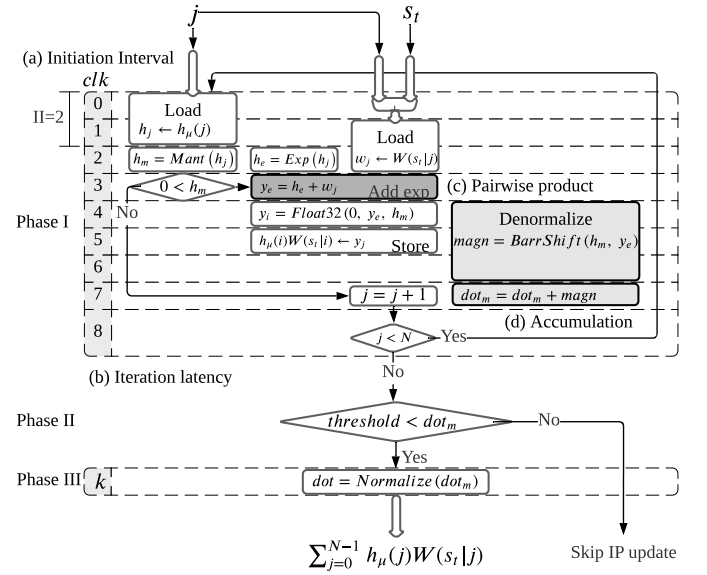
For the evaluation of our approach, we address a design exploration by reviewing the computational latency, inference accuracy, resource utilization, and power dissipation. First, we benchmark the performance of SbS network simulation on the embedded CPU, and then repeat the measurements on hardware processing units with standard floating-point computation. Afterwards, we evaluate our dot-product architecture, addressing a design exploration with hybrid custom floating-point approximation, as well as the hybrid logarithmic approximation. Finally, we present a discussion of the presented results.

A. PERFORMANCE BENCHMARK

1) Benchmark on embedded CPU

We examine the performance of the CPU for SbS network simulation with no hardware coprocessing. In this case, the embedded software builds the SbS network as a sequential model mapping the entire computation to the CPU (ARM Cortex-A9) at 666 MHz and a power dissipation of $520mW$.

The SbS network computation on the CPU achieves a latency of $34.28ms$ per spike with an accuracy of 99.3% correct classification on the $10,000$ image test set with 1000 spikes. The latency and schedule of the SbS network computation are displayed in **Tab.** 1 and **Fig.** 11 respectively.

2) Benchmark on processing units with standard floating-point computation

To benchmark the computation on hardware PUs with standard floating-point, we implement the system architecture shown in **Fig.** 12. In this case, the embedded software builds the SbS network as a sequential model mapping the network
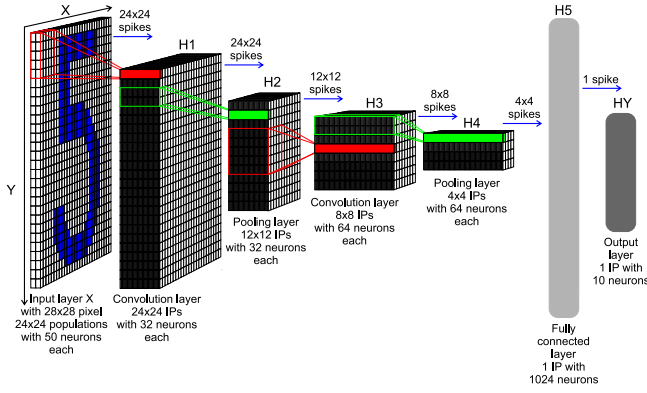
FIGURE 10. SbS network structure for MNIST classification task. Input X: Input layer with 28×28 normalization modules for 28×28 input pixel. From this layer spikes are send to layer $H1$. $H1$: Convolution layer $H1$ with 24×24 IPs with 32 neurons each. Every IP processes the spikes from 5×5 spatial patches of the input pattern (x and y stride is 1). $H2$: 2×2 pooling layer $H2$ (x and y stride is 2) with 12×12 IPs with 32 neurons each. The weights between $H1$ and $H2$ are not learned but set to a fixed weight matrix that creates a competition between the 32 features of $H1$. $H3$: 5×5 convolution layer $H3$ (x and y stride is 1) with 8×8 IPs. Similar to $H1$ but with 64 neuron for each IP. $H4$: 2×2 pooling layer $H4$ (x and y stride is 2) with 4×4 IPs with 64 neurons each. This layer is similar to layer $H2$. $H5$: Fully connected layer $H5$. $1,024$ neurons in one big IP which are fully connected to layer $H4$ and output layer HY. HY: Output layer HY with 10 neurons for the 10 types of digits. selected.

TABLE 1. Computation on embedded CPU.

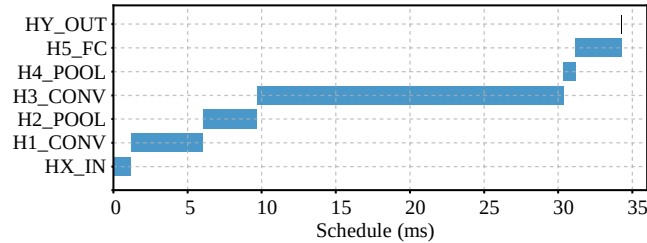| Layer	Latency (ms)
HX_IN	1.184
H1_CONV	4.865
H2_POOL	3.656
H3_CONV	20.643
H4_POOL	0.828
H5_FC	3.099
HY_OUT	0.004
TOTAL	34.279

FIGURE 11. Computation on embedded CPU.

computation to the hardware processing units at 200 MHz as clock frequency.

The layers of the neural network with the most neurons are partitioned for asynchronous parallel processing. Since *H2_POOL* and *H3_CONV* are the layers with the most neurons, the computational workload is distributed between two PUs for each one of these layers. The output layer *HY_OUT* is fully processed by the CPU, since it is the layer with fewest neurons. The hardware mapping and the computation schedule of this deployment are displayed in **Tab.** 2 and **Fig.** 13.

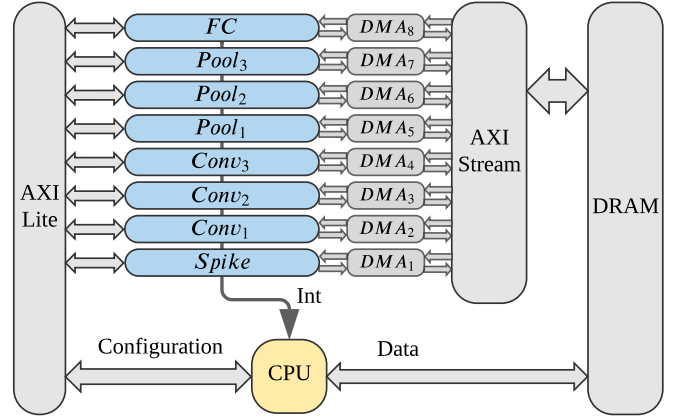In the computation schedule, the following terms are de-

FIGURE 12. System overview of the top-level architecture with 8 processing units.

TABLE 2. Performance of processing units with standard floating-point computation.

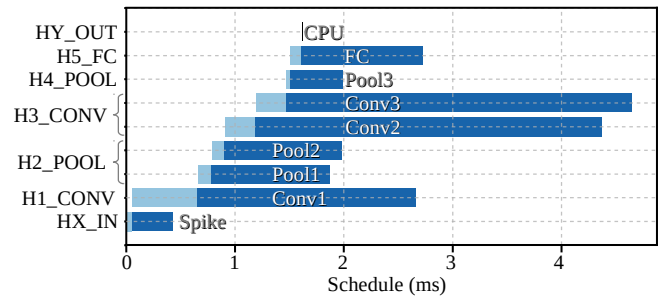| Hardware mapping		Computation schedule (ms)			
Layer	PU	t_s	t_{CPU}	t_{PU}	t_f
HX_IN	Spike	0	0.056	0.370	0.426
H1_CONV	Conv1	0.058	0.598	2.002	2.658
H2_POOL	Pool1	0.658	0.126	1.091	1.875
	Pool2	0.785	0.125	1.075	1.985
H3_CONV	Conv2	0.911	0.280	3.183	4.374
	Conv3	1.193	0.279	3.176	4.648
H4_POOL	Pool3	1.473	0.037	0.481	1.991
H5_FC	FC	1.512	0.101	1.118	2.731
HY_OUT	CPU	1.615	0.004	0	1.619

FIGURE 13. Performance of processing units with standard floating-point computation.

fined as follows: $t_s(n)$ as the start time for the processing of the neural network layer (as a compute node) $n \in L$ where L represents the set of layers; $t_{CPU}(n)$ as the CPU preprocessing time; $t_{PU}(n)$ as the PU latency; and $t_f(n)$ as the finish time. For data preparation, the $t_{CPU}(n)$ is the duration in which the CPU writes a DRAM buffer with $\vec{h_\mu}$ (vector of neuron latent variables) of the current processing layer and S_t (input spike matrix) from its preceding layer. This buffer is streamed to the PU via DMA.

The total execution time of the CPU is defined by **Eq.** (6). In a cyclic spiking inference, the execution time of the network computation is the longest path among the processing units including the CPU. This is denoted as the latency of an
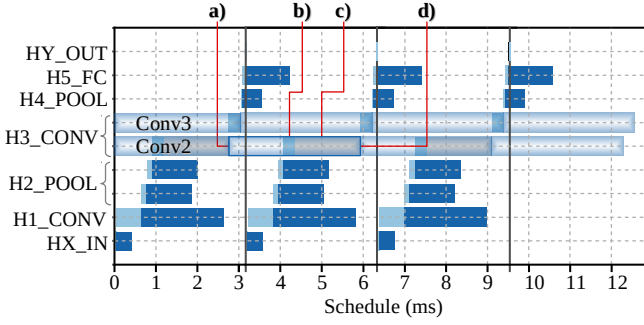
FIGURE 14. Performance bottleneck of cyclic computation on processing units with standard floating-point computation, (a) exhibits the starting of t_{PU} of *Conv2* on a previous computation cycle, (b) presents t_{CPU} of *Conv2* on the current computation cycle, (c) shows the CPU waiting time (in gray color) for *Conv2* as a busy resource (awaiting for *Conv2* interruption), and (d) illustrates the t_f from the previous computation cycle, the starting of t_{PU} on the current computation cycle (*Conv2* interruption on completion, and start current computation cycle).

spike cycle and it is defined by **Eq.** (8). The total execution time of the network computation is the last finish time (t_f) in the schedule defined by **Eq.** (9).

$$T_{CPU} = \sum_{n \in L} t_{CPU}(n) \tag{6}$$

$$T_{PU} = \max_{n \in L}(t_{PU}(n)) \tag{7}$$

$$T_{SC} = \begin{cases} T_{PU}, & \text{if } T_{CPU} \leq T_{PU} \\ T_{CPU}, & \text{otherwise} \end{cases} \tag{8}$$

$$T_f = \max_{n \in L}(t_f(n)) \tag{9}$$

Using standard floating-point requires a high computational cost. As the largest layer, the computational workload of *H3_CONV* is evenly partitioned among two PUs: *Conv2* and *Conv3*. However, in the cyclic schedule, *Conv2* causes the performance bottleneck as shown in **Fig.** 14. In this case, the CPU has to await for *Conv2* to finish the computation of the previous cycle in order to start the current computation cycle. Applying **Eq.** (8), we obtain a latency of $3.18ms$ per spike cycle. This deployment achieves an accuracy of 98.98% correct classification on the $10,000$ image test set with 1000 spikes.

The post-implementation resource utilization and power dissipation are shown in **Tab.** 3. Each *Conv* PU instantiates an on-chip stationary weight matrix of $52,000$ entries, wish is sufficient to store $W \in \mathbb{R}^{5 \times 5 \times 2 \times 32}$ and $W \in \mathbb{R}^{5 \times 5 \times 32 \times 64}$ for *H1_CONV* and *H3_CONV*, respectively. In order to reduce BRAM utilization, we use a custom floating-point representation composed of 4-exponent and 4-bit mantissa. Each 8-bit entry is promoted to its standard floating-point representation for the dot-product computation. The methodology to find the appropriate bit width parameters for custom floating-point representation is presented in Section V-B1.

TABLE 3. Resource utilization and power dissipation of processing units with standard floating-point computation.

PU	LUT	FF	DSP	BRAM 18K	Power (mW)
Spike	2,640	4,903	2	2	38
Conv	2,765	4,366	19	37	89
Pool	2,273	3,762	5	3	59
FC	2,649	4,189	8	9	66

3) Benchmark on noise tolerance plot

The noise tolerance plot serves as an intuitive visual model used to provide insights into accuracy degradation under approximate processing effects. This plot revels inherent error-resilience, and hence, approximation-resilience. As an application-specific quality metric, this plot offers an effective method to estimate the overall quality degradation of the SbS network under different approximate processing effects, since both approximations and noise have qualitatively the same effect [16].

In order to experimentally obtain the noise tolerance plot, we measure the inference accuracy of the neural network with increasing number of spikes. Then we repeat the measurements with uniformly distributed noise applied on the input images. We gradually ascend the levels of the noise amplitude, until accuracy degradation is detected. **Fig.** 15 demonstrates this method using 100 sample images.

As benchmark, the tolerance plot in **Fig.** 15 revels accuracy degradation having 50% noise and convergence with 400 spikes. In this case, the particular SbS network with precise processing demonstrates a remarkable inherent error resilience, hence, a great opportunity for approximate processing.

B. DESIGN EXPLORATION WITH HYBRID CUSTOM FLOATING-POINT AND LOGARITHMIC APPROXIMATION
In this section, we address a design exploration to evaluate our approach for SbS neural network simulation using hybrid custom floating-point and logarithmic approximation. First, we examine the synaptic weight matrix of each SbS network layer in order to determine the minimum requirements for numeric representation and memory storage. Second, we implement the proposed dot-product architecture using the minimal floating-point and logarithmic representation as design parameters. Finally, we evaluate the overall performance, the inference accuracy, the resource utilization, and the power dissipation.

1) Parameters for numeric representation of synaptic weight matrix

We obtain the parameters for numeric representation from the $\log_2$-histograms of the synaptic weight matrix for each layer as shown in **Fig.** 13. Since $0 \leq W(s_t|j) \leq 1$ and $\sum_{j=0}^{N-1} W(s_t|j) = 1$, the W elements have only negative values in the logarithmic domain. Hence, the sign bit is omitted and the values are stored in its positive numbers, as detailed in Section IV-A. The smallest floating-point entry of
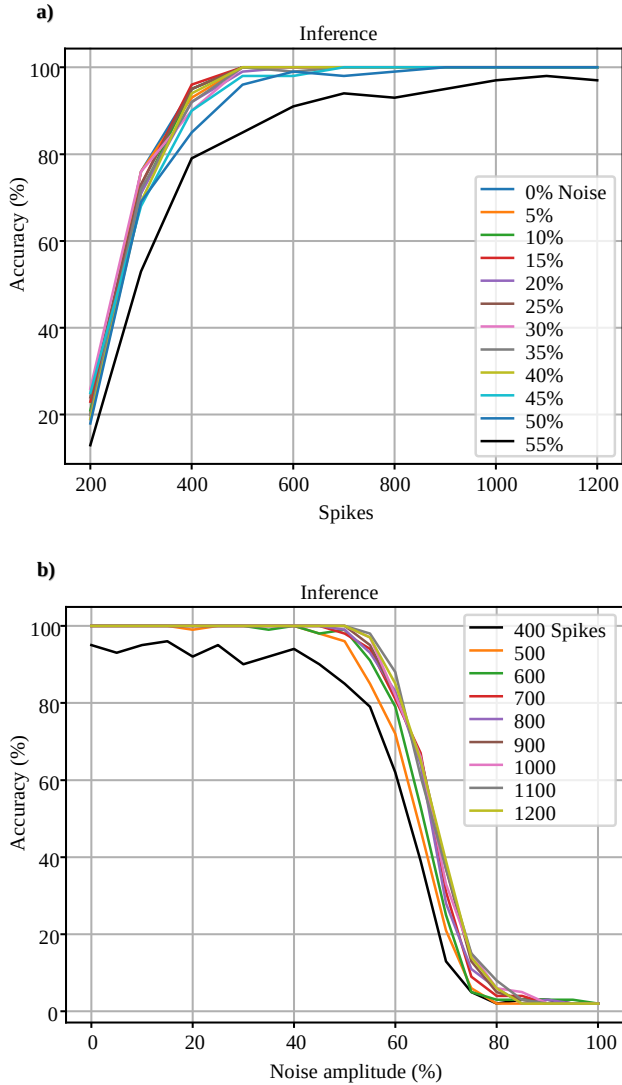
a)

b)

FIGURE 16. $\log_2$-histogram of each synaptic weight matrix showing the percentage of matrix elements with given integer exponent.

W represents the minimum exponent value, as defined by **Eq.** (10), and the bit width needed for its absolute binary representation is defined by **Eq.** (11).

$$E_{\min} = \log_2(\min_{\forall i}(W(i))) \qquad (10)$$

$$N_E = \lceil \log_2(|E_{\min}|) \rceil \qquad (11)$$

Applying **Eq.** (10) and **Eq.** (11) to the given SbS network, we obtain $E_{\min} = -13$ and $N_E = 4$. Thus, for absolute binary representation of the exponents, it requires 4-bits.

For quality configurability, the mantissa bit width is a knob parameter that is tuned by the designer. This procedure leverages the builtin error-tolerance of neural networks and performs a trade-off between resource utilization and QoR.
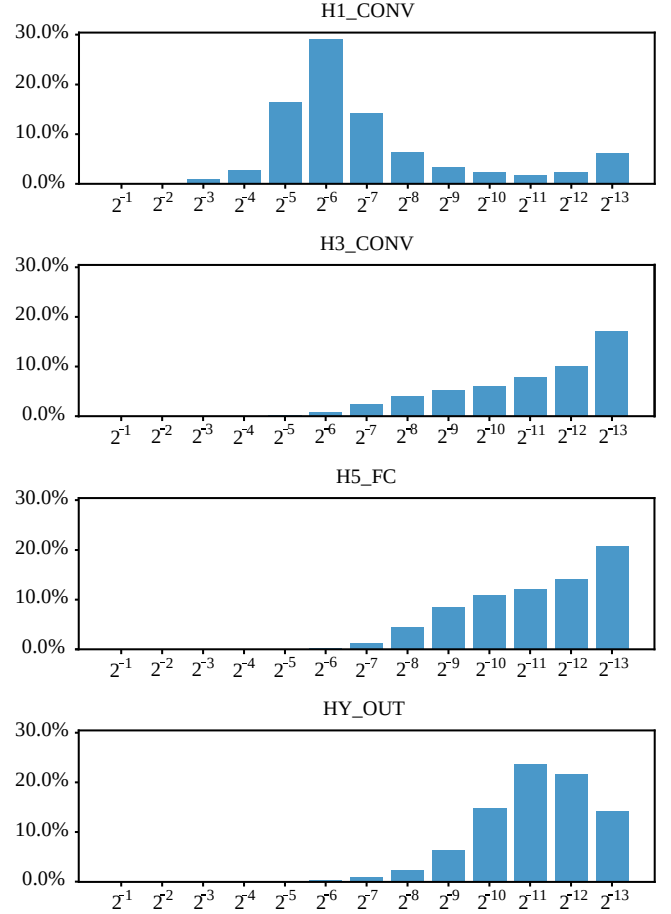
In the following subsection, we present a case study with 1-bit mantissa corresponding to the custom floating-point approximation.

2) Design exploration for dot-product with hybrid custom floating-point approximation

For this design exploration, we use a custom floating-point representation composed of 4-bit exponent and 1-bit mantissa. This format is used for the synaptic weight vector on the proposed dot-product architecture. Each *Conv* PU instantiates an on-chip stationary weight matrix for $52,000$ entries of 5-bit. The available memory size is large enough to store $W \in \mathbb{R}^{5 \times 5 \times 2 \times 32}$ and $W \in \mathbb{R}^{5 \times 5 \times 32 \times 64}$ for *H1_CONV* and *H3_CONV*, respectively. The same dot-product architecture is implemented in the processing unit of the fully connected layer (*FC*). However, due to lack of BRAM resources, this PU can not instantiate on-chip stationary synaptic weight matrix. Instead, *FC* receives the $\overrightarrow{W(s_t)}$ (weight vectors) during operation as well as $\overrightarrow{h_\mu}$ and S_t. The hardware mapping and the computation schedule of this implementation are displayed in **Tab.** 5 and **Fig.** 17.

As shown in the computation schedule in **Tab.** 5 and **Fig.** 17, this implementation achieves a maximum hardware
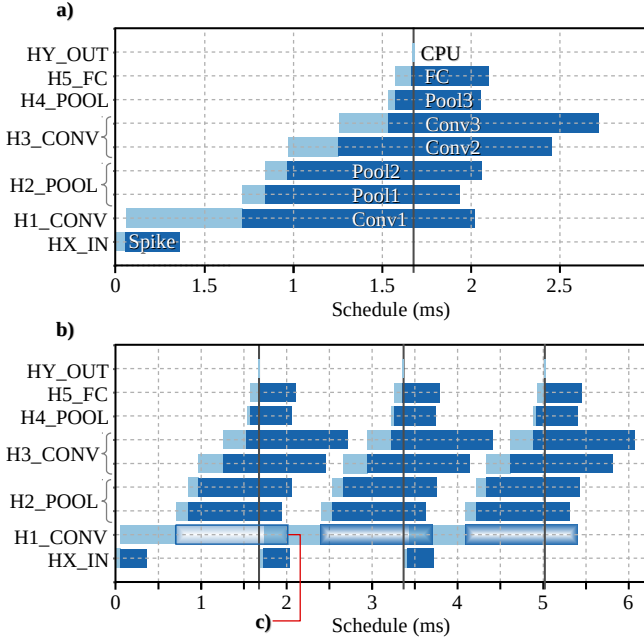
a)

b)

c)

PU latency of $1.30ms$ according to **Eq.** (7), and a CPU latency of $1.67ms$. Therefore, applying **Eq.** (8), we obtain a latency of $1.67ms$ per spike cycle as shown in **Fig.** 17. In this case, the cyclic bottleneck is in the performance of the CPU.

This configuration achieves an accuracy of 98.97% correct classification on the $10,000$ image test set with 1000 spikes. This indicates an accuracy degradation of 0.33%. For output quality monitoring, the noise tolerance plot in **Fig.** 18 revels accuracy degradation for noise higher than 50% on the input images, and convergence of inference with 400 spikes. Thus, the particular SbS network implementation under approximate processing effects demonstrates a minimal impact on the overall accuracy. This proves an inherent error resilience, and hence, remaining approximation budget.

The post-implementation resource utilization and power dissipation are shown in **Tab.** 4.

TABLE 4. Resource utilization and power dissipation of processing units with hybrid custom floating-point approximation.

PU	LUT	FF	DSP	BRAM 18K	Power (mW)
Conv	3,139	4,850	19	25	82
FC	3,265	5,188	8	9	66

3) Design exploration for dot-product whit hybrid logarithmic approximation

As the most efficient setup and yet the worst-case quality configuration, we use a 4-bit integer exponent for logarithmic

TABLE 5. Performance of hardware processing units with hybrid custom floating-point approximation.

Hardware mapping		Computation schedule (ms)			
Layer	PU	t_s	t_{CPU}	t_{PU}	t_f
HX_IN	Spike	0	0.055	0.307	0.362
H1_CONV	Conv1	0.057	0.654	1.309	2.020
H2_POOL	Pool1	0.713	0.131	1.098	1.942
	Pool2	0.845	0.125	1.098	2.068
H3_CONV	Conv2	0.972	0.285	1.199	2.456
	Conv3	1.258	0.279	1.184	2.721
H4_POOL	Pool3	1.538	0.037	0.484	2.059
H5_FC	FC	1.577	0.091	0.438	2.106
HY_OUT	CPU	1.669	0.004	0	1.673
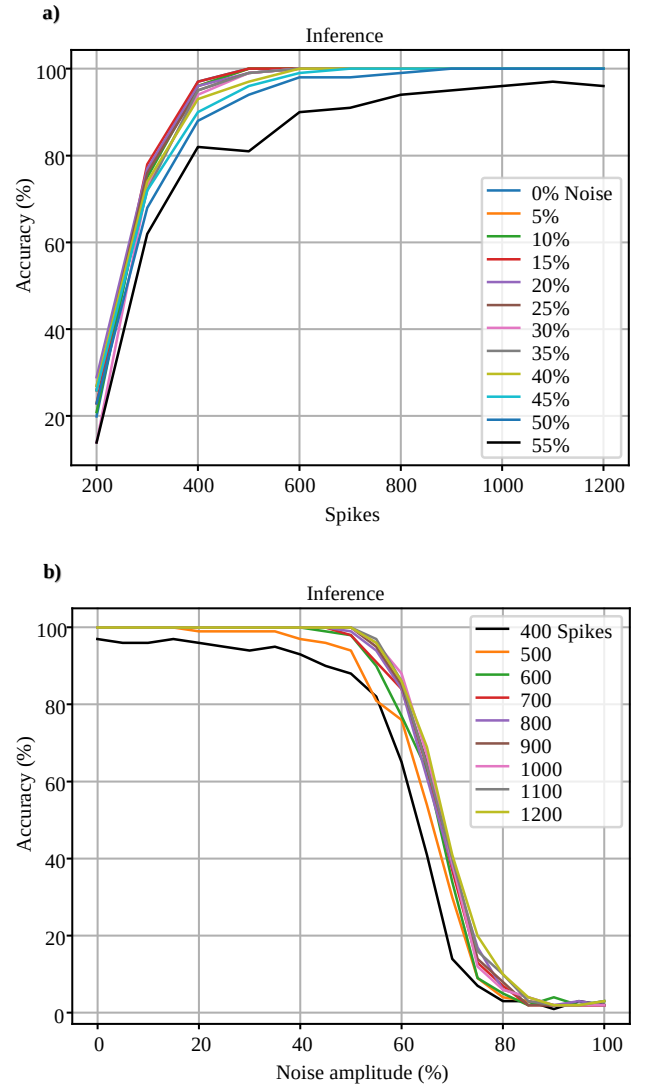
a)

b)

representation of the synaptic weight matrix. Each *Conv* processing unit implements the proposed dot-product architecture including an on-chip stationary weight matrix for $52,000$ entries of 4-bit integer each one to store $W \in \mathbb{N}^{5 \times 5 \times 2 \times 32}$

TABLE 6. Performance of hardware processing units with hybrid logarithmic approximation.

Hardware mapping		Computation schedule (ms)			
Layer	PU	t_s	t_{CPU}	t_{PU}	t_f
HX_IN	Spike	0	0.055	0.264	0.319
H1_CONV	Conv1	0.057	0.655	1.271	1.983
H2_POOL	Pool1	0.714	0.130	1.074	1.918
	Pool2	0.845	0.126	1.106	2.077
H3_CONV	Conv2	0.973	0.285	1.179	2.437
	Conv3	1.258	0.278	1.176	2.712
H4_POOL	Pool3	1.538	0.037	0.488	2.063
H5_FC	FC	1.577	0.091	0.388	2.056
HY_OUT	CPU	1.669	0.004	0	1.673

and $W \in \mathbb{N}^{5\times5\times32\times64}$ for *H1_CONV* and *H3_CONV*, respectively. The same dot-product architecture is implemented in the *FC* processing unit without stationary synaptic weight matrix. The hardware mapping and the computation schedule of this implementation are displayed in **Tab.** 6 and **Fig.** 19.

As shown in the computation schedule in **Tab.** 6 and **Fig.** 19, this implementation achieves a maximum hardware PU latency of $1.27ms$ according to **Eq.** (7), and a CPU latency of $1.67ms$. Therefore, applying **Eq.** (8), we obtain a latency of $1.67ms$ per spike cycle as shown in **Fig.** 19. In this case, the cyclic bottleneck is in the CPU performance.

This quality configuration achieves an accuracy of 98.84% correct classification on the $10,000$ image test set with 1000 spikes. This indicates an accuracy degradation of 0.46%. For output quality monitoring, the noise tolerance plot in **Fig.** 20 revels accuracy degradation having 40% noise on the input images, and convergence of inference with 600 spikes. The particular SbS network implementation under approximate processing demonstrates a minor impact on the overall accuracy. This exhibits remaining budget for further approximate processing approaches.

The post-implementation resource utilization and power dissipation are shown in **Tab.** 7.

TABLE 7. Resource utilization and power dissipation of processing units with hybrid logarithmic approximation.

PU	LUT	FF	DSP	BRAM 18K	Power (mW)
Conv	3,086	4,804	19	21	78
FC	3,046	4,873	8	8	66

C. RESULTS AND DISCUSSION

As a reference, the SbS network simulation on embedded CPU using standard 32-bit floating-point achieves an accuracy of 99.3% with a latency of $T_{SC} = 34.28ms$. As a second reference point, the network simulation on hardware processing units with standard floating-point achieves an accuracy of 98.98% with a latency $T_{SC} = 3.18ms$. As result we get a $10.7\times$ latency enhancement and an accuracy degradation of 0.32%. The tolerance plot in **Fig.** 15 reveals accuracy degradation having 50% noise on the input images, and convergence of inference with 400 spikes. In this case,
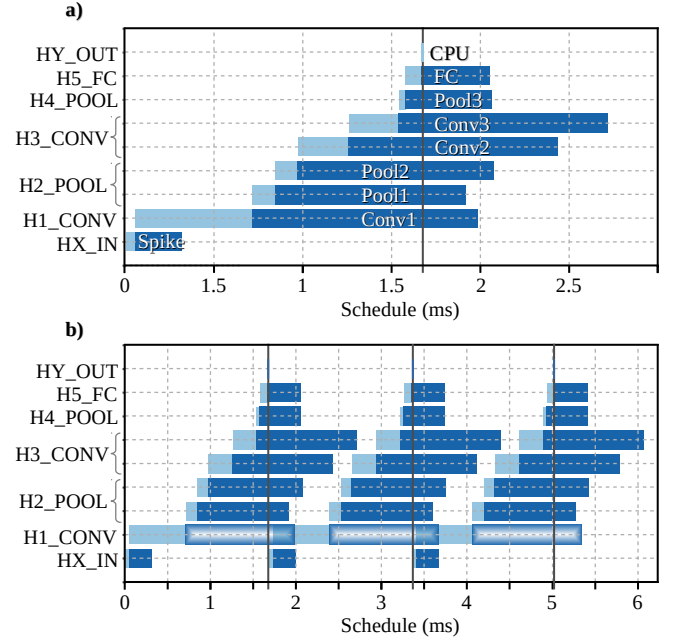
FIGURE 19. Performance of processing units with hybrid logarithmic approximation, (a) exhibits computation schedule, and (b) illustrates cyclic computation schedule.

the SbS network deployment with precise computing proves extraordinary inherent error resilience, and hence, this represents a great potential for approximate processing.

As a demonstration of the proposed dot-product architecture, the SbS network simulation on hardware PUs with synaptic representation using 5-bit custom floating-point (4-bit exponent, 1-bit mantissa) and 4-bit logarithmic (4-bit exponent) achieve $20.5\times$ latency enhancement and accuracy of 98.97% and 98.84%, respectively. This results in an accuracy degradation of 0.33% and 0.46%, respectively. For output quality monitoring, the noise tolerance plot in **Fig.** 18 and **Fig.** 20 revel accuracy degradation when having 50% and 40% noise on the input images, and convergence of inference with 400 and 600 spikes, respectively. Therefore, the design exploration under the proposed approximate computing approach indicates sufficient inherent error resilience for further and more aggressive approximate processing approaches.

Regarding resource utilization and power dissipation with the proposed approach, the *Conv* processing units have a 43.24% reduction of BRAM, and a 12.35% of improvement in energy efficiency over the standard floating-point implementation. The experimental results of the design exploration are summarized in **Tab.** 8.

VI. CONCLUSIONS

In this work, we accelerate SbS neural networks with a dot-product functional unit based on approximate computing that combines the advantages of custom floating-point and logarithmic representations. This approach reduces computational latency, memory footprint, and power dissipation while preserving classification accuracy. For output quality

TABLE 8. Experimental results.

| Dot-product implementation	PU	Post-implementation resource utilization				Power (mW)	Latency		Accuracy (%)[e]	
		LUT	**FF**	**DSP**	**BRAM 18K**		T_{SC} (ms)	**Gain**[d]	**Noise 0%**	**50%**
Standard floating-point computation[a]	Conv	2,765	4,366	19	37	89	3.18	10.7x	98.98	98.63
	FC	2,649	4,189	8	9	66				
Hybrid custom floating-point approx[b]	Conv	3,139	4,850	19	25	82	1.67	20.5x	98.97	98.47
	FC	3,265	5,188	8	9	66				
Hybrid logarithmic approximation[c]	Conv	3,086	4,804	19	21	78	1.67	20.5x	98.84	95.22
	FC	3,046	4,873	8	8	66				

[a] Reference with standard floating-point arithmetic (IEEE 754).
[b] Synaptic weight with number representation composed of 4-bit exponent and 1-bit mantissa.
[c] Synaptic weight with number representation composed of 4-bit exponent.
[d] Acceleration with respect to the computation on embedded CPU (ARM Cortex-A9 at 666 MHz) with latency $T_{SC} = 34.28 ms$.
[e] Accuracy on 10,000 image test set with 1000 spikes.

a)

b)
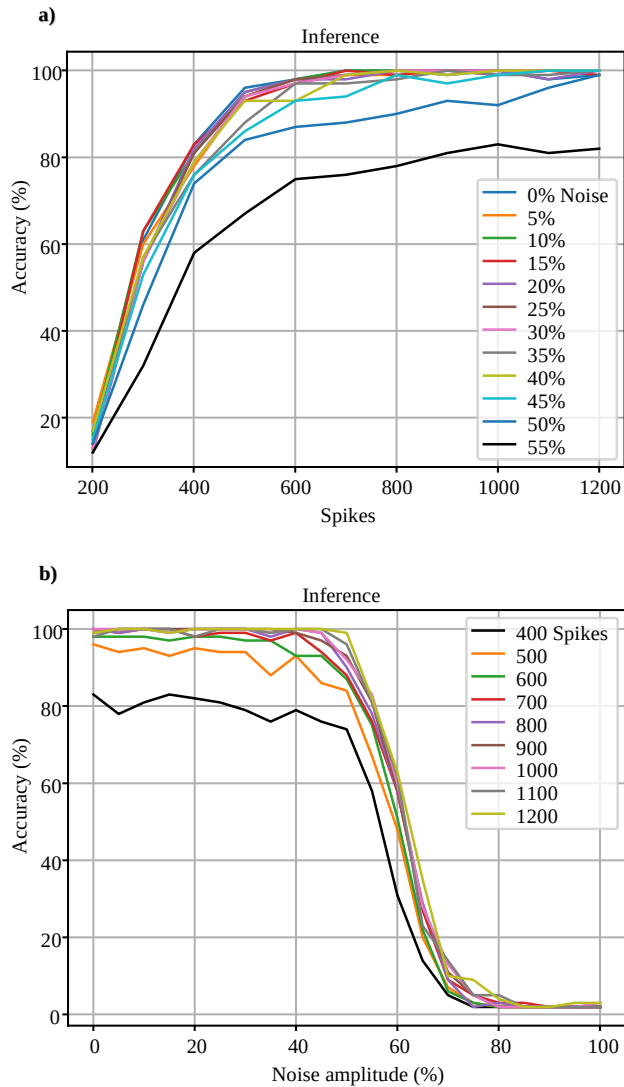
FIGURE 20. Noise tolerance on hardware PU with hybrid logarithmic approximation, (a) exhibits accuracy degradation applying 40% of noise amplitude, (b) illustrates convergence of inference with 600 spikes.

monitoring, we applied noise tolerance plots as an intuitive visual measure to provide insights into the accuracy degrada-

tion of SbS networks under different approximate processing effects. This plot revels inherent error resilience, hence, the possibilities for approximate processing.

We demonstrate our approach using a design exploration flow on a Xilinx Zynq-7020 with a deployment of SbS network for the MNIST classification task. This implementation achieves up to $20.5\times$ latency enhancement, $8\times$ weight memory footprint reduction, and 12.35% of energy efficiency improvement over the standard floating-point hardware implementation, and incurs in less than 0.5% of accuracy degradation. Furthermore, with a noise amplitude of 50% added on top of the input images, the SbS network presents an accuracy degradation of less than 5%. As output quality monitor, the resulting noise tolerance plots demonstrate a sufficient QoR for minimal impact on the overall accuracy of the neural network under the effects of the proposed approximation technique. These results suggest available room for further and more aggressive approximate processing approaches.

In summary, based on the relaxed need for fully accurate or deterministic computation of SbS neural networks, approximate computing techniques allow substantial enhancement in processing efficiency with moderated accuracy degradation.

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YARIB NEVAREZ received the B.E. (Hons) degree in electronics from the Durango Institute of Technology, Durango, Mexico, in 2009, and the M.Sc. degree in Embedded Systems Design from the University of Applied Sciences Bremerhaven, Bremen, Germany, in 2017. He is currently pursuing a Ph.D. degree with the Institute of Electrodynamics and Microelectronics, University of Bremen, Germany. His research interest is focused mainly on System-on-Chip architectures and hardware implementation for deep learning accelerators in Embedded Systems. During his professional experience, he served as a Senior Embedded Software Engineer at Texas Instruments, IBM, Continental Automotive, TOSHIBA, and Carbon Robotics. He has designed and developed software architectures for graphic calculators, automotive systems, robotic drivers, and more.

DAVID ROTERMUND started his scientific career as a chemical technical assistant in 1992 and received a pre-diploma in electrical engineering at the Hochschule Bremen (City University for Applied Science) in 1996. In 2002, he finished his studies of physics at the University of Bremen with a diploma (specialization in neuroscience and solid state physics). In 2007 he received his PhD "Extraction of information from the dynamical activities of neural networks". Among other neuroscience projects, he participated in several project in the field of neuro-prosthetics like the German-Israeli joint project "Models and Experiments towards Adaptive Control of Motor Prostheses" (METACOMP), the research focus Neurotechnology at the University of Bremen, and the Creative Unit "I-See: The artificial eye – chronic wireless interface to the visual cortex". In the BMBF project KALOMED, where the goal was to design a fully wireless recording system that can be implanted under the skull of an user, he worked as project organizer and hardware/ software/ firmware designer as well as data miner. He will be the co-organizer of the upcoming Era-Net Neuron (a joint Canadian / EU project) for the development of advanced techniques in the field of visual cortex prosthesis. Beside his research in the field of neuro-prosthetics, he is keenly interested in information processing using spiking neuronal networks.

KLAUS R. PAWELZIK received his PhD in the field of Nonlinear Dynamics in 1990. From 1991 till March 1998 he was research assistant at several well-known institutes in Germany and the US. Since April 1998 he is a tenured professor for Theoretical Physics and Theoretical Biology at the University of Bremen. He works mainly on topics in Theoretical Neuroscience, but also on problems in Neuro-technology and studies models of other complex adaptive systems. His many publications underline his expertise in these fields. Currently he is the director of the Center of Cognitive Sciences at the University of Bremen and has raised a number of third-party funds, among them several in the field of Neuro-technology. There he recently filed a patent with the title "Artificial neural network data processing apparatus and data processing method".

ALBERTO GARCIA-ORTIZ obtained the diploma degree in Telecommunication Systems from the Polytechnic University of Valencia (Spain) in 1998. After working for two years at Newlogic in Austria, he started the Ph.D. at the Institute of Microelectronic Systems, Darmstadt University of Technology, Germany. In 2003, he received from the Department of Electrical Engineering and Information Technology of the university the Ph.D. degree with "summa cum laude."

From 2003 to 2005, he worked as a Senior Hardware Design Engineer at IBM Deutschland Development and Research in Böblingen. After that he joint the start-up AnaFocus (Spain), where he was responsible for the design and integration of AnaFocus" next generation Vision Systems-on-Chip. He is currently full professor for the chair of integrated digital systems at the university of Bremen. Dr. Garcia-Ortiz received the "Outstanding dissertation award" in 2004 from the European Design and Automation Association. In 2005, he received from IBM an innovation award for contributions to leakage estimation. Two patents are issued with that work. He serves as editor of JOLPE and is reviewer of several conferences, journals, and European projects.

His interests include low-power design and estimation, communication-centric design, SoC integration, and variations-aware design.

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