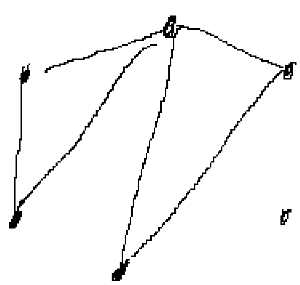
CS 230 : Discrete Computational Structures
**Spring Semester, 2021**
ASSIGNMENT #11 [**Extra Credit**]
**Due Date:** Friday, April 30

For the problems below, explain your answers and show your reasoning.

1. [**10 Pts**] If $G$ is a simple graph with $n$ vertices and $n$ edges, is $G$ connected? If *yes*, give a short justification. If *no*, give a counterexample.
   No. Consider this beautifully drawn graph as a counter example:



2. [**8 Pts**] Consider a graph $G$ that has 7 vertices with degrees of 5, 4, 3, 3, 2, 2, 1. How many edges does $G$ have? Explain.
   By the Handshake Theorem: $(5+4+3+3+2+2+1)/2 = 10$ edges.

3. [**12 Pts**] Prove by induction that a complete binary tree of height $h$ has $2^h$ leaves. Use the inductive definition of complete binary trees.
   (a) Base Case: A complete binary tree of height 0 has 1 leaf node $2^0$. The CBT of height $0+1=1$ will have 2 leaves because by def of CBT's, the CBT of the next height will fill the left and right subtrees of all the current leaves. The # of leaves will double for every increase in height by 1. So, $2^0 * 2 = 2^1$
   (b) IH: A CBT of height k has twice the leaves of a CBT with height $k-1$. So, $2^{k-1} * 2 = 2^k$ leaves
   (c) Prove: CBT's of height $(k+1)$ has twice the leaves of a CBT of height $(k+1)-1 = k$
   (d) By IH, tree of height k has $2^k$ leaves
   (e) $n^k * 2 = n^{k+1}$ QED

4. [**20 Pts**] Prove that a graph is a tree if and only if it is acyclic but adding any edge will create a cycle.