STAT 347 HW3

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3.1

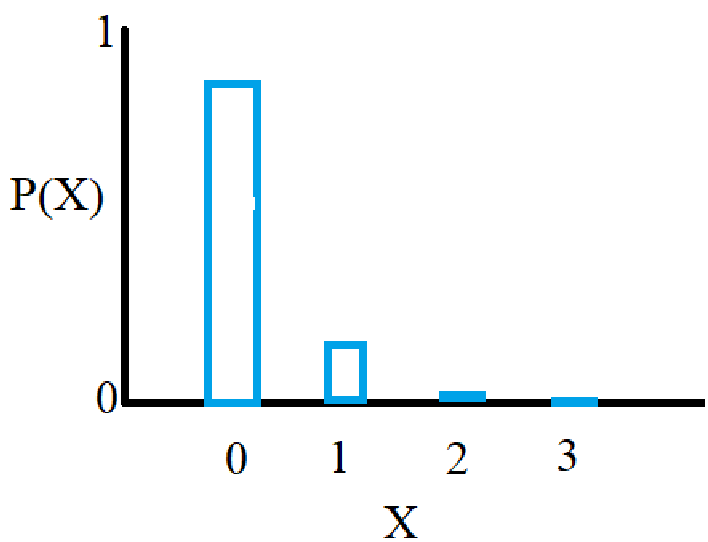
$$\begin{aligned}
P(0) &= 0.20 \\
P(1) &= (0.4 - 0.1) + (0.5 - 0.1) = 0.70 \\
P(2) &= 0.2 + 0.4 + 0.5 - 1 = 0.1
\end{aligned}$$

3.4

$$\begin{aligned}
P(0) &= P(v_1^c \cap (v_2^c \cup v_3^c)) = 0.2 * (0.2 + 0.2 - 0.04) = 0.072 \\
P(1) &= P(v_1 \cap (v_2^c \cup v_3^c)) + P(v_1^c \cap (v_2 \cap v_3)) \\
&= 0.8 * (0.2 + 0.2 - 0.04) + 0.2 * (0.64) = 0.416 \\
P(2) &= P(v_1 \cap v_2 \cap v_3) = 0.8^3 = 0.512
\end{aligned}$$

3.9 a

$$\begin{aligned}
P(0) &= C(3,0)(0.95)^3(0.05)^0 = 0.8574 \\
P(1) &= C(3,1)(0.95)^2(0.05)^1 = 0.1354 \\
P(2) &= C(3,2)(0.95)^1(0.05)^2 = 0.0071 \\
P(3) &= C(3,3)(0.95)^0(0.05)^3 = 0.0001
\end{aligned}$$



b

c $P(e > 1) = P(2) + P(3) = 0.0072$

3.12

$$\begin{aligned}
E(Y) &= 1*0.4 + 2*0.3 + 3*0.2 + 4*0.1 = 2.0 \\
E(\frac{1}{Y}) &= \sum \frac{1}{Y}p(Y) \\
&= \frac{1}{1}0.4 + \frac{1}{2}0.3 + \frac{1}{3}0.2 + \frac{1}{4}0.1 \\
&= 0.642 \\
E(Y^2 - 1) &= 1^2*0.4 + 2^2*0.3 + 3^2*0.2 + 4^2*0.1 - 1 \\
&= 4 \text{ (Corollary:} E(Y^2) = 5) \\
V(Y) &= E(Y^2) - E(Y)^2 \\
&= 5 - 2^2 \\
&= 1
\end{aligned}$$

3.14 **a** $3*0.03+4*0.05+5*0.07+6*0.1+7*0.14+8*0.2+9*0.18+10*0.12+11*0.07+12*0.03+13*0.01 = 7.9$

b $(3-7.9)^2*0.3+(4-7.9)^2*0.05+(5-7.9)^2*0.07+(6-7.9)^2*0.1+(7-7.9)^2*0.14+(8-7.9)^2*0.2+(9-7.9)^2*0.18+(10-7.9)^2*0.12+(11-7.9)^2*0.07+(12-7.9)^2*0.03+(13-7.9)^2*0.01 = 3.349^2$

c $\mu \pm 2\sigma = [4.551, 11.249]$, $P(5)+P(6)+P(7)+P(8)+P(9)+P(10)+P(11) = 0.88$

3.21 $E(8\pi R^2) = 8\pi(21^2*0.5+22^2*0.2+23^2*0.3+24^2*0.25+25*0.15+26^2*0.05) = 18788$

3.23 $P(15) = 8/52$, $P(5) = 8/52$, $P(4) = (52-16)/52$
$15*(8/52)+5*(8/52)+4*(36/52) = 5.85$

3.30 **a** The mean of x will be larger because all values of X are larger than their sibling values in Y

b $E(X) = E(Y) + 1$. Yes

c It should be the same. Shifting all values by 1 does not change the spread.

d

$$\begin{aligned}
V(X) &= E((X - E(X))^2) = \sigma^2 \\
V(X) &= E((Y + 1 - E(Y+1))^2) \\
V(X) &= E((Y + 1 - (E(Y)+1))^2) \\
V(X) &= E((Y + 1 - E(Y) - 1)^2) \\
V(X) &= E((Y - E(Y))^2) \\
V(Y) &= E((Y - E(Y))^2) = \sigma^2
\end{aligned}$$