STAT 347 HW1

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2.5a $A = A \cap S$ and $S = B \cup \bar{B}$ = The Universal Set, so

1 $A = (A \cap B) \cup (A \cap \bar{B})$

2 $A = (A \cap B \cup A) \cap (A \cap B \cup \bar{B})$

3 $A = (A \cap B \cup A) \cap (A \cap S)$

4 $A = A \cap B \cup A \cap A$

5 $A = A \cap B \cup A$ and $A \cap B \subset A$ by definition of intersection

6 So, $A = A$

2.5b $B \subset A$

1 $A = B \cup (A \cap \bar{B})$

2 $A = (B \cup A) \cap (B \cup \bar{B})$

3 $A = A \cap (B \cup \bar{B})$ by definition of subset

4 $A = A \cap S = A$

2.5c Show that $(A \cap B)$ and $(A \cap \bar{B})$ are mutually exclusive

1 $(A \cap B) \cap (A \cap \bar{B}) = \emptyset$ by definition of mutually exclusive

2 $(A \cap B \cap A) \cap (A \cap B \cap \bar{B}) = \emptyset$

3 $(A \cap B \cap A) \cap (A \cap \emptyset) = \emptyset$

4 $(A \cap B \cap A) \cap \emptyset = \emptyset$

5 $\emptyset = \emptyset$

2.5d Show that b and $(A \cap \bar{B})$ are mutually exclusive

1 $B \cap (A \cap \bar{B}) = \emptyset$ by definition of mutually exclusive

2 $(B \cap A) \cap (B \cap \bar{B}) = \emptyset$

3 $(B \cap A) \cap \emptyset = \emptyset$

4 $\emptyset = \emptyset$. And then:

5 $A = B \cup (A \cap \bar{B})$

6 $A = (B \cup A) \cap (B \cup \bar{B})$

7 $A = A \cap (B \cup \bar{B})$ by definition of subset

8 $A = A \cap S = A$

2.8a $9 + 36 + 3 = 48$

2.8b $36 - 3 = 33$

2.8c Graduates $g = 60 - 36 = 24$, Grad off campus $go = 9 - 3 = 6$
So, Graduates on campus $g - go = 24 - 6 = 18$

2.15a $1 - 0.01 - 0.09 - .81 = .09$

2.15b $1 - .81 = .19$

2.23 If $B \subset A$, then elements in B are also in A, but B does not contain all elements of A. After summing the probability of each element, $P(B) \leq P(A)$.
(unless A contains an element with negative probability, which is impossible)

2.33a L = income $\leq \$43,318$, and H = income $> \$43,318$.
$\{LLLL, LLLH, LLHL, LLHH, LHLL, LHLH, LHHL, LHHH,$
$HLLL, HLLH, HLHL, HLHH, HHLL, HHLH, HHHL, HHHH\}$
If order doesn't matter: $\{4L : 0H, 3L : 1H, 2L : 2H, 1L : 3H, 0L : 4H\}$

2.33b Simple events

A: $\{LLHH, LHLH, LHHL, LHHH,$
$HLLH, HLHL, HLHH, HHLL, HHLH, HHHL, HHHH\}$

B: $\{LLHH, LHLH, LHHL,$
$HLLH, HLHL, HHLL\}$

C: $\{LHHH,$
$HLHH, HHLH, HHHL\}$

2.33c Since half of the sampling population is below or above the median,
I can assume that the probability of selecting H or L are equal.

$$P(A) = \frac{11}{16}$$
$$P(B) = \frac{6}{16}$$
$$P(C) = \frac{4}{16}$$

2.39a Multiplication principle: $6 * 6 = 36$

2.39b By enumeration: $|\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}| = 6$.
So, $\frac{6}{36} = \frac{1}{6}$

2.51 Sample space $= C(50, 3) = 19600$

a $\frac{C(4,3)}{19600} = \frac{4}{19600}$

b $\frac{C(4,2)*C(46,1)}{19600} = \frac{276}{19600}$

b $\frac{C(4,1)*C(46,2)}{19600} = \frac{4140}{19600}$

b $\frac{C(46,3)}{19600} = \frac{15180}{19600}$

2.41 select 6 from 10 digits: $10^6 = 1,000,000$

2.64 $6^6 = 46656$ possibilities, $6! = 720$ desirable outcomes. $\frac{720}{46656}$

2.69 Prove $C(n+1,k) = C(n,k) + C(n,k-1)$

$$C(n+1,k) = C(n,k) + C(n,k-1)$$
$$\frac{(n+1)!}{k!(n+1-k)!} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-k+1)!}$$
$$\frac{(n+1)n!}{k!(n+1-k)!} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-k+1)!}$$
$$\frac{(n+1)}{k!(n+1-k)!} = \frac{1}{k!(n-k)!} + \frac{1}{(k-1)!(n-k+1)!}$$
$$\frac{n+1}{k(k-1)!(n+1-k)!} = \frac{1}{k(k-1)!(n-k)!} + \frac{1}{(k-1)!(n-k+1)!}$$
$$\frac{n+1}{k(n+1-k)!} = \frac{1}{k(n-k)!} + \frac{1}{(n-k+1)!}$$
$$\frac{n+1}{k(n+1-k)(n-k)!} = \frac{1}{k(n-k)!} + \frac{1}{(n-k+1)(n-k)!}$$
$$\frac{n+1}{k(n+1-k)} = \frac{1}{k} + \frac{1}{n-k+1}$$
$$\frac{n+1}{k} = \frac{(n+1-k)}{k} + 1$$
$$\frac{n+1}{k} = \frac{n+1-k}{k} + \frac{k}{k}$$
$$\frac{n+1}{k} = \frac{n+1}{k}$$