STAT 347 HW2

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2.71

a $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.1}{0.3} = \frac{1}{3}$

b $P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{0.1}{0.5} = 0.2$

c $P(A|A\cup B) = \frac{P(A\cap(A\cup B))}{P(A\cup B)} = \frac{P(A)}{P(A\cup B)}$ By Absorption $= \frac{P(A)}{P(A)+P(B)-P(A\cap B)} = \frac{0.5}{0.5+0.3-0.1} = \frac{5}{7}$

d $P(A|A\cap B) = \frac{P(A\cap A\cap B)}{P(A\cap B)} = \frac{P(A\cap B)}{P(A\cap B)} = 1$

e $P(A\cap B|A\cup B) = \frac{P((A\cap B)\cap(A\cup B))}{P(A\cup B)} = \frac{P((A\cap B\cap A)\cup(A\cap B\cap B))}{P(A\cup B)} = \frac{P(A\cap B)}{P(A\cup B)} = \frac{0.1}{0.5+0.3-0.1} = \frac{1}{7}$

2.75

a 2 spade cards already removed. Next 3: $\frac{11}{50} * \frac{10}{49} * \frac{9}{48}$

b $\frac{10}{49} * \frac{9}{48}$

c $\frac{9}{48}$

2.83

$$\begin{aligned} P(A|A\cup B) &= \frac{P(A\cap(A\cup B))}{P(A\cup B)} \text{ Absorption law for sets} \\ &= \frac{P(A)}{P(A\cup B)} \text{Probability of the union of mutually exclusive sets gives:} \\ &= \frac{P(A)}{P(A)+P(B)} \end{aligned}$$

2.86

a Suppose $P(A\cap B) = 0.1$.
$1 \geq P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.8 + 0.7 - 0.1 = 1.4 > 1$.
Contradiction! The supposition breaks the laws of probability.

b $1 \geq P(A\cup B) = P(A) + P(B) - P(A\cap B)$
$P(A) + P(A) - 1 \geq P(A\cap B)$
$0.5 \geq P(A\cap B)$ This probability must be greater than or equal to 0.5.

c In b, we determined the maximum value is 0.7. So, 0.77 exceeds this, hence not possible.

d The largest possible value for $P(A\cap B)$ is 0.7 and this occurs when $B\subset A$.

2.91 The sum of probabilities of mutually exclusive events must be less than or equal to 1.
Therefore, A and B cannot be mutually exclusive because $P(A) + P(B) = 1.1 > 1$
The second scenario does not break this rule, and is therefore possible.

2.98 The series system needs both relays to be closed properly to work, which has a probability of $0.9 * 0.9 = 0.81$
Atleast one relay needs to work in the parallel system. $P(A\cup B) = 0.9 + 0.9 - 0.81 = 0.99$

2.124 Using Bayes' Theorem:
$P(Dem|Favor) = \frac{P(Favor|Dem)*P(Dem)}{P(Favor|GOP)*P(GOP)+P(Favor|Dem)*P(Dem)} = \frac{0.7*0.6}{0.4*0.3+0.7*0.6} = 7/9$

2.130

$$
\begin{aligned}
P(Ship|Cancer) &= 0.22,\ P(Ship|Cancer^c) = 0.14,\ P(Cancer) = 0.0004 \\
P(Cancer|Ship) &= \frac{P(Ship|Cancer) * P(Cancer)}{P(Ship|Cancer) * P(Cancer) + P(Ship|Cancer^c) * P(Cancer^c)} \\
&= \frac{0.22 * 0.0004}{0.22 * 0.0004 + 0.14 * 0.9996} \\
&= 0.0006284
\end{aligned}
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2.134

$$
\begin{aligned}
P(Fail|A) &= 0.2,\ P(Fail|B) = 0.1,\ P(B) = 0.3,\ P(A) = 0.7 \\
P(A|Fail) &= \frac{P(Fail|A) * P(A)}{P(Fail|A) * P(A) + P(Fail|B) * P(B)} \\
&= \frac{0.2 * 0.7}{0.2 * 0.7 + 0.1 * 0.3} \\
&= .8235
\end{aligned}
$$

2.135 $P(Major) = 0.6$, $P(Private) = 0.3$, $P(Other) = 0.1$,
$P(Busy|Major) = 0.5$, $P(Busy|Private) = 0.6$, $P(Busy|Other) = 0.9$

$0.5 * 0.6 + 0.3 * 0.6 + 0.1 * 0.9 = 0.57$

b $P(Busy \cap Private) = P(Busy|Private) * P(Private) = 0.6 * 0.3 = 0.18$

c $P(Private|Busy) = \frac{P(Private \cap Busy)}{P(Busy)} = \frac{0.18}{0.57} = 0.3158$

d Airline types are mutually exclusive. $P(Com) = P(Major \cup Other) = P(Major) + P(Other) = 0.7$
Can calculate Commercial and Business as all Business - Business and Private
$P(Com \cap Busy) = P(Busy) - P(Private \cap Busy) = 0.57 - 0.18 = 0.39$
$P(Busy|Com) = P(Com \cap Busy)P(Com) = 0.39 * 0.7 = 0.273$