COM S 230 HW02

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1. Where x is all integers, and $P(x) =$ "x is odd" and $Q(x) =$ "x is even," Prove

$$(A)\,\forall x(P(x) \to Q(x))$$
$$\not\equiv$$
$$(B)\,\forall xP(x) \to \forall xQ(x),$$

   1) For proof by contradiction, Suppose (A) $\equiv$ (B)

   2) Since $\forall xP(x)$ and $\forall xQ(x)$ are both false, (B) is True

   3) (A) is false. Suppose x is odd. Then $P(x)$ is true while $Q(x)$ is false. P(x) does not imply Q(x) for all x. Alternatively, there exist some x where (A) is false.

   4) Because of (2) and (3), (A) $\not\equiv$ (B)

2. Translate

   (a) For all faculty, there exists two different students who both have asked the faculty a question.

   (b) $\exists x \exists y \forall z(F(x) \wedge F(y) \wedge (x \neq y)(S(z) \to \neg(A(x,z) \vee A(y,z))))$

3. True or False, Explain

   (a) False. Saying all freshmen take online classes does not give any information about not freshmen. Knowing Jim isn't a freshmen doesn't say anything about his classes.

   (b) False. Mary may like all her computer science classes, but that doesn't mean she doesn't like all other classes. 'Discrete Math' can still be liked without being a computer science class.

4. Define Predicates and formally prove the following:

(a) x is any person.
S(x) = x is a student in class; C(x) = x owns a personal computer; T(x) = x types up their homework
Prove $\exists x(S(x) \wedge T(x))$

Premises:

(a) J = Jim. $S(J) \wedge C(J)$
(b) $\forall x(C(x) \rightarrow T(x))$

Proof:

(1) $C(J)$ Simplification (a)
(2) $S(J)$ Simplification (a)
(3) $C(J) \rightarrow T(J)$ Universal Instantiation (b)
(4) $T(J)$ Modus Ponens (1) & (3)
(5) $S(J) \wedge T(J)$ Conjunction (2) & (4)
(6) $\exists x(S(x) \wedge T(x))$ Existential Generalization (5); QED

(b) x is all towns
C(x) = x is a college town; F(x) = x is fun; M(x) = x is in the Midwest
Prove $\exists x(M(x) \wedge F(x))$

Premises:

(a) $\exists x(C(x) \wedge M(x))$
(b) $\forall x(C(x) \rightarrow F(x))$

Proof:

(1) There exists a place Ames where $C(Ames) \wedge M(Ames)$ Existential Instantiation (a)
(2) $C(Ames)$ Simplification (1)
(3) $M(Ames)$ Simplification (1)
(4) $C(a) \rightarrow F(a)$ for arbitrary a Universal Instantiation (b)
(5) $F(Ames)$ Modus Ponens (4) & (2)
(6) $F(Ames) \wedge M(Ames)$ Conjunction (3) & (5)
(7) $\exists x(M(x) \wedge F(x))$ Existential Generalization (6); QED

5. Consider the argument

(a) x is any animal.
B(x) = x is a bear; S(x) = x is a good swimmer; H(x) = x goes hungry; F(x) = x can catch fish
Prove $\forall x(B(x) \to \neg H(x))$

Premises:

(a) $\forall x(B(x) \to S(x))$
(b) $\forall x(F(x) \to \neg H(x))$
(c) $\forall x(\neg F(x) \to \neg S(x))$

Proof:

(1) $B(a) \to S(a)$ for any a	Universal Instantiation (a)
(2) $\neg F(a) \to \neg S(a)$ for any a	Universal Instantiation (b)
(3) $S(a) \to F(a)$ for any a	Contrapositive of (2)
(4) $F(a) \to \neg H(a)$ for any a	Universal Instantiation (b)
(5) $B(a) \to F(a)$ for any a	Hypothetical Syllogism (1) & (3)
(6) $B(a) \to \neg H(a)$ for any a	Hypothetical Syllogism (5) & (4)
(7) $\forall x(B(x) \to \neg H(x))$	Universal Generalization (6); QED

(b) Prove *Universal Transitivity*:

(1) $\forall x(P(x) \to Q(x))$	Hypothesis 1
(2) $\forall x(Q(x) \to R(x))$	Hypothesis 2
(3) $P(a) \to Q(a)$ for any a	Universal Instantiation (1)
(4) $Q(a) \to R(a)$ for any a	Universal Instantiation (2)
(5) $P(a) \to R(a)$ for any a	Hypothetical Syllogism (3) & (4)
(6) $\forall x(P(x) \to R(x))$	Universal Generalization; QED

(c) Prove *Universal Contrapositive*:

(1) $\forall x(P(x) \to Q(x))$	Hypothesis 1
(2) $P(a) \to Q(a)$ for any a	Universal Instantiation (1)
(3) $\neg Q(a) \to \neg P(a)$ for any a	Contrapositive of (2)
(4) $\forall x(\neg Q(x) \to \neg P(x))$	Universal Generalization (3); QED

(d) Proof but with new rules, faster, and using same premises:

(1) $\forall x(S(x) \to F(x))$	Universal Contrapositive (c)
(2) $\forall x(B(x) \to F(x))$	Universal Transitivity (1) & (a)
(3) $\forall x(B(x) \to \neg H(x))$	Universal Transitivity (2) & (b); QED

6. x is all non-negative ints. Define using the given predicates:

(a) $Equal(m, n) : \neg(Greater(m, n) \lor Greater(n, m))$
(b) $One(n) : \forall k(M(k, k, n))$
(c) $Two(n) : \exists k(\neg Zero(n) \land A(k, n, n) \land M(k, n, n))$
(d) $Prime(p) : \exists i \, \exists j(M(p, i, j) \land One(i) \land Equal(p, j))$