STAT 347 HW5

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3.105

- **a** Y has a hypergeometric distribution because we are selecting candidates without replacement.
- **b** $P(Y \geq 2) = P(2) + P(3) = \frac{C(5,2)C(3,1)}{C(8,3)} + \frac{C(5,3)C(3,0)}{C(8,3)}$
- **c** $E(Y) = (3 * 5)/8 = 1.875$, $V(Y) = 8\frac{5}{8}\frac{3}{8}\frac{5}{7} = 0.502$

3.106 $E(X) = 5 * 4/10 = 2.\ 2 * 50 = 100$
$V(50X) = 50^2 V(X) = 2500 * 5 * (4/10) * (10 - 4)/10 * (10 - 5)/(10 - 1) = 1667$

3.121

- **a** $P(Y = 4) = \frac{2^4 * e^{-2}}{4!} = 0.9$
- **b** $P(Y \geq 4) = 1 - \sum_{i=0}^{3} \frac{2^i * e^{-2}}{i!} = 0.143$
- **c** $P(Y < 4) = 1 - P(Y \geq 4) = 0.857$
- **d** $P(Y \geq 4 | Y \geq 2) = \frac{P(Y \geq 4 \text{ \& } Y \geq 2)}{P(Y \geq 2)} = \frac{P(Y \geq 4)}{1 - P(Y < 2)}$
  $= \frac{0.143}{1 - P(0) - P(1)} = 0.241$

3.122

- **a** $P(Y \leq 3) = \sum_{i=0}^{3} \frac{7^i * e^{-7}}{i!} = 0.082$
- **b** $P(Y \geq 2) = 1 - P(0) - P(1) = 0.993$
- **c** $P(Y = 5) = \frac{7^5}{e^7 * 5!} = 0.128$

3.132 This is reasonable because we know the average given a certain time period,
and we want to calculate probability of a value during that time period.
A Poisson distribution is reasonable here.
$P(Y > 3) = 1 - \sum_{i=0}^{3} \frac{1^i * e^{-1}}{i!} = 1 - \sum_{i=0}^{3} \frac{1}{e * i!} = 0.019$

3.147 Geometric moment generating function

$$
\begin{aligned}
m_y(t) &= E(e^{ty}), \text{ and } P(y) = q^{y-1}p \\
&= \sum_{y=1}^{\infty} e^{ty} q^{y-1} p \\
&= \sum_{y=1}^{\infty} e^{ty} q^{y} q^{-1} p \\
&= \frac{p}{q} \sum_{y=1}^{\infty} e^{ty} q^{y} \\
&= \frac{p}{q} \sum_{y=1}^{\infty} (e^{t} q)^{y} \\
&= \frac{p}{q} \frac{qe^t}{1-qe^t} \\
&= \frac{pe^t}{1-qe^t}
\end{aligned}
$$

3.148

$$
\begin{aligned}
E(Y) &= \frac{d}{dt}[\frac{pe^t}{1-qe^t}] \\
&= p\frac{(1-qe^t)*e^t + e^t(qe^t)}{(1-qe^t)^2} \\
&= pe^t\frac{(1-qe^t) + (qe^t)}{(1-qe^t)^2} \\
&= \frac{pe^t}{(1-qe^t)^2} \\
&= \frac{p}{(1-(1-p))^2} \text{ at } t = 0 \\
&= \frac{p}{p^2} = \frac{1}{p}
\end{aligned}
$$

$$
\begin{aligned}
E(Y^2) &= (\frac{pe^t}{(1-qe^t)^2})' \\
&= p\frac{(1-qe^t)^2e^t + 2e^t(1-qe^t)qe^t}{(1-qe^t)^4} \\
&= pe^t\frac{(1-qe^t)^2 + 2(1-qe^t)qe^t}{(1-qe^t)^4} \\
&= pe^t\frac{1-qe^t+2qe^t}{(1-qe^t)^3} \\
&= pe^t\frac{1+qe^t}{(1-qe^t)^3} \\
&= p\frac{1+q}{(1-q)^3} \text{ at } t=0 \\
&= \frac{2-p}{p^2}
\end{aligned}
$$

$$
V(Y) = E(Y^2) - E(Y)^2 = \frac{2-p}{p^2} - \frac{1}{p^2} = \frac{1-p}{p^2}
$$

3.158 $E(e^{aY+b}) = E(e^{bt}e^{atY}) = e^{bt}E(e^{atY}) = e^{bt}m(at)$

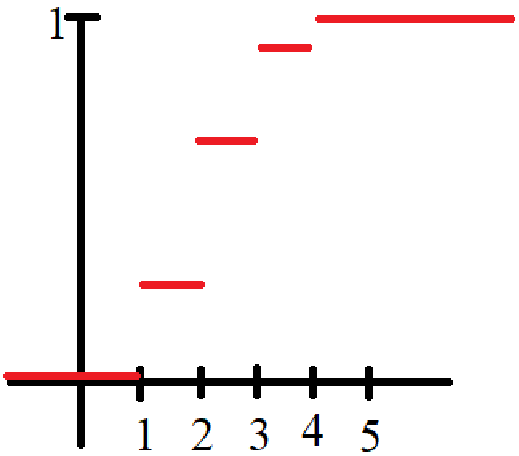
3.167 **a** $P(6-11 < Y-\mu < 16-11) = P(|Y-\mu| < 5) \geq 1 - \frac{3^2}{5^2} = 16/25$

b

$$
\begin{aligned}
1 - P(|Y-11| < C) &\leq 0.09 \\
P(|Y-11| < C) &\geq 0.91 \\
P(-C < Y-11 < C) &\geq 0.91 \\
P(11-C < Y-11 < 11+C) &\geq 0.91 \\
1 - \frac{1}{k^2} = 0.91 \implies k &= 10/3 \\
C = k\sigma = 3*10/3 &= 10
\end{aligned}
$$

4.1

$$
F(y) = \begin{cases} 0 & -\infty < y < 1 \\ 0.3 & 1 \leq y < 2 \\ 0.7 & 2 \leq y < 3 \\ 0.9 & 3 \leq y < 4 \\ 1 & 4 \leq y < \infty \end{cases}
$$

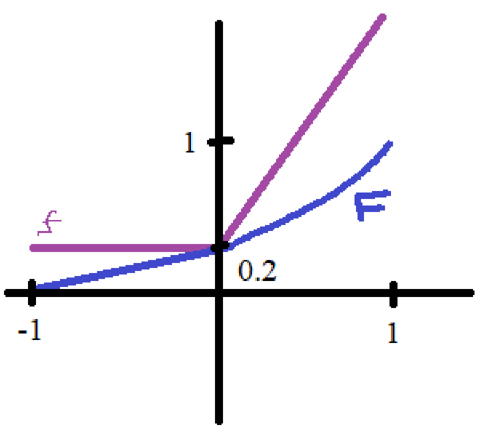


4.18 **a**

$$\int_{-1}^{0} 0.2dy + \int_{0}^{1} 0.2 + cydy = 1$$
$$0.2y|_{-1}^{0} + (0.2y + \frac{cy^2}{2})|_0^1 = 1$$
$$0.2 + 0.2 + \frac{c}{2} = 1$$
$$\frac{c}{2} = 0.6$$
$$c = 1.2$$

b Integrals done on paper... $F(y) = \int_{-\infty}^{y} f(t)dt$

$$F(y) = \begin{cases} 0 & y \leq -1 \\ 0.2 + 0.2y & -1 < y \leq 0 \\ 0.6y^2 + 0.2y + 0.2 & 0 < y \leq 1 \\ 1 & y > 1 \end{cases}$$



c

d $F(-1) = 0,\ F(0) = 0.2,\ F(1) = 0.6 + 0.2 + 0.2 = 1$

e $\int_0^{1/2} 0.2 + 0.2ydy = 0.25$

f

$$\begin{aligned}\frac{P(Y > 0.5 \ \& \ P > 0.1)}{P(Y > 0.1)} &= \frac{P(Y > 0.5)}{P(Y > 0.1)} \\ &= \frac{1 - P(Y \leq 0.5)}{1 - P(Y \leq 0.1)} \\ &= \frac{1 - F(0.5)}{1 - F(0.1)} \\ &= 0.7106\end{aligned}$$

4.22 $\int_{-1}^{0} 0.2y dy + \int_{0}^{1} 0.2y + 1.2y^2 dy = -0.1 + 0.7 = 0.6$
$\int_{-1}^{0} 0.2y^2 dy + \int_{0}^{1} 0.2y^2 + 1.2y^3 dy = 0.2/3 + 0.2/3 + 0.3 = 0.43333$
$V(Y) = 0.43333333333 - (0.6)^2 = 0.07333333333$

4.22

$$\begin{aligned}E(Y) &= \int_0^1 \frac{3}{2}y^3 + y^2 \\ &= \frac{3y^4}{8} + \frac{y^3}{3}|_0^1 \\ &= \frac{3}{8} + \frac{1}{3} = 17/24 \\ E(Y^2) &= \int_0^1 \frac{3}{2}y^4 + y^3 \\ &= \frac{3y^5}{10} + \frac{y^4}{4}|_0^1 \\ &= 3/10 + 1/4 = 11/20 \\ E(5 - \frac{Y}{2}) &= 5 - \frac{E(Y)}{2} = 4.646 \\ V(Y) &= 11/20 - (17/24)^2 = 0.0483 \\ V(5 - \frac{Y}{2}) &= 0.0483 * (-0.5)^2 = 0.012\end{aligned}$$