CS230-HW9Sol

1. **Ying [4 Pts]**

   (a)By the product rule there are $6 \times 5 \times 2 \times 2 = 120$ different types of shirt.
   (b)There are $4 \times 5 \times 2 \times 2 = 80$ different types of shirts whose color is not black or yellow, and there are $2 \times 5 \times 1 \times 1 = 10$ different types of shirts whose color is black or yellow. So the total shirts is $80 + 10 = 90$.

2. **Ying [6 Pts]**

   There are $\lfloor \frac{99999}{3} \rfloor - \lfloor \frac{10000-1}{3} \rfloor = 33333 - 3333 = 30000$ numbers between 10000 and 99999 that are divisible by 3. Similarly there are $\lfloor \frac{99999}{5} \rfloor - \lfloor \frac{10000-1}{5} \rfloor = 19999 - 1999 = 18000$ numbers between 10000 and 99999 that are divisible by 5. Similarly, there are $\lfloor \frac{99999}{7} \rfloor - \lfloor \frac{10000-1}{7} \rfloor = 14285 - 1428 = 12857$ multiples of 7. We need to remove the numbers that are counted twice, which is any number that is a multiple of 3 and 5 or 3 and 7 or 5 and 7, or in other words, a multiple of 15, 21 and 35. There are $\lfloor \frac{99999}{15} \rfloor - \lfloor \frac{10000-1}{15} \rfloor = 6666 - 666 = 6000$ and $\lfloor \frac{99999}{21} \rfloor - \lfloor \frac{10000-1}{21} \rfloor = 4761 - 476 = 4285$ and $\lfloor \frac{99999}{35} \rfloor - \lfloor \frac{10000-1}{35} \rfloor = 2857 - 285 = 2572$ of these. But now we removed the numbers which are multiple of 3, 5 and 7. In other words we have to add the number of multiples of 105 which is $\lfloor \frac{99999}{105} \rfloor - \lfloor \frac{10000-1}{105} \rfloor = 952 - 95 = 857$ Thus, there are $30000 + 18000 + 12857 - 6000 - 4285 - 2572 + 857 = 48857$ integers between 10000 and 99999 that are divisible by 3, 5 or 7

3. **Ling [10 Pts]**

   a) Each function from A to B maps each of the 7 elements of A to one of the 10 elements of B. There are 10 choices for the first element in A, times 10 choices for the second, times 10 choices for the third, and so on, for a total of $10^7 = 10,000,000$ functions.
   By the same logic, there are $7 * 7 * 7 * \cdots * 7 = 7^{10} = 282,475,249$ functions from B to A.

   b) A relation from A to B is a subset of $A \times B$. The set of all subsets of a set S is the powerset of S. The size of the powerset of S is $2^{|S|}$. The size of $A \times B$ is 70. So the number of different relations from A to B is $|Pow(A \times B)| = 2^{|A \times B|} = 2^{70}$.

   c) If a function from A to B is one-to-one, each element of A gets sent to a different element of B. So, we have 10 choices for where to send the first element of A; wherever we send the first element, we can no longer send the remaining elements there, so we have 9 choices for the second element of A, then only 8 choices for the third, and so on down to 4 choices for the seventh, for $10 * 9 * 8 * \cdots * 4 = 604,800$ choices.

   d) Since each element of A must be mapped to, and there are three more elements in B than in A, there are three possibilities. (i) four elements in B mapped to the same element in A, (ii) three elements in B mapped to one element in A, and two mapped to another element in A, (iii) two elements in B mapped to one element in A, two more elements mapped to a second element in A, and two more elements mapped to a third element in A. In each case, the remaining elements in B are mapped to distinct elements in A.
   (i) We group 4 elements in B together, the other 6 are single. We then map the 7 groups

in B to elements in A. So, $\binom{10}{4}$ ways to select the group of 4, times 7! ways to assign the 7 groups of B to the 7 elements of A, for a total of $\frac{10!*7!}{6!*4!}$.
(ii) We group 3 elements in B together, 2 elements of B together, the other 5 are single. We then map the 7 groups in B to elements in A. So, $\binom{10}{3} \times \binom{7}{2}$ ways to select the groups, times 7! ways to assign the 7 groups of B to the 7 elements of A, for a total of $\frac{10!*7!*7!}{7!*3!*5!*2!}$.
(iii) We have 3 groups of 2 elements in B, the remaining 4 are single. We then map the 7 groups in B to elements in A. So, $\binom{10}{2} \times \binom{8}{2} \times \binom{6}{2}/3!$ ways to select the groups, times 7! ways to assign the 7 groups of B to the 7 elements of A, for a total of $\frac{10!*8!*6!*7!}{8!*2!*6!*2!*4!*2!*3!}$.

4. **Ling [6 Pts]**

   (a) First, we choose 5 people from 8 people (not including Mom and Dad). There are C(8,5) = 56 different ways to do this. Then, we arrange the 7 people (including Mom and Dad) in a row. There are 7! = 5,040 ways to do this. Therefore, the answer is 56 * 5,040 = 282,240.
   (b) For this case, we can treat Mom and Dad as one person, but there are 2! = 2 ways to arrange them. Then we choose 5 people from 8 people (not including Mom and Dad). There are C(8,5) = 56 ways to do this. Then, there are 6! = 720 ways to arrange the 6 people (Mom and Dad counted as one plus 5 more). So the total number of ways is 2 * 56 * 720 = 80,640.
   (c) First, we choose 1 either Mom or Dad. There are 2 ways to do this. Then we choose 6 people from 8 people (not including Mom and Dad). There are C(8,6) = 28 ways to do this. Then we arrange the 7 people we chose in a row. There are 7! == 5,040 ways. So the final answer is 2 * 28 * 5,040 = 282,240.

5. **Modeste [6 Pts]**

   The maximum number of tickets that can be drawn without having 5 of one kind is 4 of each kind, or 32 tickets. The 33rd ticket will guarantee that there will be at least 5 to the same movie. This uses the Pigeonhole Principle.

   If everyone wants to go to 'Godzilla vs Kong', then all 40 tickets must be removed, because it is possible to draw 39 tickets without getting all 5 'Godzilla vs Kong' tickets.

6. **Modeste [6 Pts]**

   a) We can count this by counting the number of ways to select three bits from seven - the selected bits are 1s, and the others 0s. This gives $\binom{7}{3} = 35$ strings.

   b) If a bit string has at most three 1s, then it has exactly 3, exactly 2, exactly 1, or exactly 0 1s. Since these possibilities don't overlap, we can add together $\binom{7}{3} + \binom{7}{2} + \binom{7}{1} + \binom{7}{0} = 35 + 21 + 7 + 1 = 64$ strings.

   c) At least three 1s, by the same technique, is $\binom{7}{3}+\binom{7}{4}+\binom{7}{5}+\binom{7}{6}+\binom{7}{7} = 35+35+21+7+1 = 99$ strings.

7. **Jonathan [6 Pts]**

   1) We can add together the possible outcomes with exactly 5, 6, 7, 8, and 9 heads: $\binom{9}{5} + \binom{9}{6} + \binom{9}{7} + \binom{9}{8} + \binom{9}{9} = 126 + 84 + 36 + 9 + 1 = 256$ strings with at least 5 heads.

2) We can count all bit strings of length 9 and subtract those less than 5 heads: $2^9 - \binom{9}{4} + \binom{9}{3} + \binom{9}{2} + \binom{9}{1} + \binom{9}{0} = 126 + 84 + 36 + 9 + 1 = 256$

3) Notice that exactly half the possible strings have at least 5 heads: if you flip all the heads to tails and vice versa, you get a string with less than 5 heads. (Mathematically, this corresponds to the fact that $\binom{9}{5} = \binom{9}{4}, \binom{9}{6} = \binom{9}{3}$, and so on.) There are 2^9 total bit strings of length 9, so there are $2^8 = 256$ strings with at least 5 heads.

8. **Jonathan [6 Pts]**

a) Number of ways to choose a committee - number of ways to choose a committee with both Claire and Jane: $\binom{18}{6} - \binom{16}{4}$.

Alternately, number of committees with only Claire + committees with only Jane + committees with neither: $\binom{16}{5} + \binom{16}{5} + \binom{16}{6}$.

b) Number of committees - number of man-only committees: $\binom{18}{6} - \binom{8}{6}$

Alternatively, add up the number of exactly-1-woman, 2-woman, 3-woman, 4-woman, 5-woman, and 6-woman committees: $\binom{10}{1}\binom{8}{5} + \binom{10}{2}\binom{8}{4} + \binom{10}{3}\binom{8}{3} + \binom{10}{4}\binom{8}{2} + \binom{10}{5}\binom{8}{1} + \binom{10}{6}$.

c) Number of committees - manless committees - womanless committees: $\binom{18}{6} - \binom{10}{6} - \binom{8}{6}$

Alternatively, add up the number of 1-man 5-woman, 2-man 4-woman, 3-man 3-woman, 4-man 2-woman, and 5-man 1-woman committees:$\binom{8}{1}\binom{10}{5} + \binom{8}{2}\binom{10}{4} + \binom{8}{3}\binom{10}{3} + \binom{8}{4}\binom{10}{2} + \binom{8}{5}\binom{10}{1}$