CS 230 : Discrete Computational Structures

Spring Semester, 2021
HOMEWORK ASSIGNMENT #2
Due Date: Monday, February 15

Suggested Reading: Rosen Sections 1.4 - 1.6; LLM Chapter 3

For the problems below, explain your answers and show your reasoning.

1. [**4 Pts**] Prove, by counterexample, that the propositions $\forall x(P(x) \rightarrow Q(x))$ and $\forall x P(x) \rightarrow \forall x Q(x)$ are not logically equivalent. *Hint*: Let your domain be the set of integers. Define $P(x)$ to be 'x is odd' and $Q(x)$ to be 'x is even'.

2. [**8 Pts**] For the following problems, let $S(x)$, $F(x)$ and $A(x, y)$ be the statements "x is a student", "x is a faculty member" and "x has asked y a question". Let the domain be all people at ISU. Translate into English or logic, as appropriate.

   (a) $\exists x \exists y \forall z(S(x) \wedge S(y) \wedge (x \neq y) \wedge (F(z) \rightarrow (A(x, z) \leftrightarrow A(y, z))))$

   (b) There are at least two faculty members who have not asked questions to any students.

3. [**4 Pts**] State whether the following arguments are correct. Explain your answer briefly.

   (a) All freshmen take online classes. Jim is not a freshman. Therefore, Jim does not take online classes.

   (b) Mary likes all her computer science classes. Mary likes 'Discrete Math'. Therefore, 'Discrete Math' is a computer science class.

4. [**8 Pts**] Define predicates and prove the following using the appropriate rules of inference:

   (a) [**4 Pts**] Jim, a student in class, owns a personal computer. Everyone who owns a personal computer types up their homework. Therefore, someone in class types up their homework.

   (b) [**4 Pts**] There are college towns in the midwest. All college towns are fun places to live. There is a town in the midwest that is a fun to live in.

5. [**14 Pts**] Consider the following argument:
   All bears are good swimmers. If you can catch fish, you will not go hungry. If you can't catch fish, you are not a good swimmer. Therefore, no bears go hungry.

   (a) [**4 Pts**] Define the predicates $B(x)$, $S(x)$, $H(x)$ and $F(x)$ to describe the sentences above using predicate logic. Then, prove the argument using the rules of inference you have learned.

(b) [**3 Pts**] Prove the **universal transitivity** rule, which states:
if $\forall x(P(x) \to Q(x))$ and $\forall x(Q(x) \to R(x))$, then $\forall x(P(x) \to R(x))$.

(c) [**3 Pts**] Prove the **universal contrapositive** rule, which states:
if $\forall x(P(x) \to Q(x))$ then $\forall x(\neg Q(x) \to \neg P(x))$.

(d) [**4 Pts**] Now, prove the previous argument using the **universal transitivity** rule and the **universal contrapositive** rule. Is your proof shorter? Notice that you no longer have to use instantiation and generalization rules.

6. [**12 Pts**] Given the universe of all non-negative integers, we have the two predicates $A(k, m, n) : (k = m + n)$ and $M(k, m, n) : (k = mn)$. For example $A(4, 3, 1)$ is true while $A(5, 2, 2)$ is false. We can use A and M to define other predicates. For example, $Zero(n) : A(n, n, n)$. Note that $n = 0$ is the only integer such that $n = n + n$. We can now use $Zero$ to define $Greater(m, n) : \exists k(\neg Zero(k) \wedge A(m, n, k))$. Define the following predicates. You can use the predicates defined earlier in subsequent definitions.

   (a) [**4 Pts**] $Equal(m, n)$

   (b) [**4 Pts**] $One(n)$

   (c) [**4 Pts**] $Two(n)$

   (d) **Extra Credit** [**4 Pts**] $Prime(p)$

For more practice, work on the problems from Rosen Sections 1.4 - 1.6 and LLM Chapter 3.