COM S 230 HW01

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1. Translations from english to logic

   F: Passing the final; A: Attending class regularly; P: Pass the class.

   a) The thing that is sufficient is first in implication $\boldsymbol{F \vee A \to P}$

   b) if P is true, then A is necessarily true $\boldsymbol{P \to A}$

   c) This is a straightforward translation $\boldsymbol{P \iff (A \wedge F)}$

2. Implication is not associative; $((p \to q) \to r) \iff (p \to (q \to r))$ is not a tautology.

p	q	r	$p \to q$	$(p \to q) \to r$	$q \to r$	$p \to (q \to r)$	$((p \to q) \to r) \iff (p \to (q \to r))$
0	0	0	1	0	1	1	0
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

3. Prove $(p \to \neg q) \wedge (q \to \neg r) \equiv (p \vee r) \to \neg q$

   - $(\neg p \vee \neg q) \wedge (\neg q \vee \neg r)$ Implication 1 x 2
   - $\neg\neg((\neg p \vee \neg q) \wedge (\neg q \vee \neg r))$ Double Negation
   - $\neg(\neg(\neg p \vee \neg q) \vee \neg(\neg q \vee \neg r))$ De Morgan's
   - $\neg((p \wedge q) \vee (q \wedge r))$ De Morgan's x 2
   - $\neg(((p \wedge q) \vee q) \wedge ((p \wedge q) \vee r))$ Distribution
   - $\neg(q \wedge ((p \wedge q) \vee r))$ Absorption
   - $\neg(q \wedge ((p \vee r) \wedge (q \vee r)))$ Distributive
   - $\neg(q \wedge (q \vee r) \wedge (p \vee r))$ Associative
   - $\neg((p \vee r) \wedge q)$ Absorption & Commutative

- $\neg(p \vee r) \vee \neg q$ De Morgan's
- $(p \vee r) \rightarrow \neg q$ Implication 1

4. Who's lying? T is Tom's claim, and S is Sue's claim. C: $T \oplus S \equiv TRUE$

   1) Suppose T is true.

   2) Then S is also true.

   3) (1) and (2) Contradicts C. T cannot be true.

   4) If Tom is the SE Major, and Sue is the CS Major, then T is false and S remains true.

   5) (4) Satisfies C. Tom is SE and Sue is CS.

5. Prove any compound proposition is logically equivalent to some proposition in CNF

   - For all compound propositions, there exists 2^n combinations of basic propositions, where n is the number of basic propositions.
   - For all combinations, the compound proposition is either true or false.
   - CNF can be written as "as long as none of the variable combinations that make the proposition false are satisfied, then the compound proposition is true."
   - Basically, $\neg((a_1 \wedge a_2 \wedge ... \wedge a_n) \vee (b_1 \wedge b_2 \wedge ... \wedge b_n) \vee ...)$ where a, b, ... are combinations of basic propositions that make the proposition false.
   - Applying De Morgan's, we get $\neg(a_1 \wedge a_2 \wedge ... \wedge a_n) \wedge \neg(b_1 \wedge b_2 \wedge ... \wedge b_n)...$
   - Again. $(\neg a_1 \vee \neg a_2 \vee ... \vee \neg a_n) \wedge (\neg b_1 \vee \neg b_2 \vee ... \vee \neg b_n) \wedge ...$
   - This is CNF. All conjugated clauses are the disjunction of the negation of basic propositions which make the compound proposition false.

6. Prove $\oplus, \wedge, TRUE$ is functionally complete

   - $\wedge$ is already present
   - $(p \oplus TRUE) \equiv \neg p$

     1) $(p \wedge \neg TRUE) \vee (\neg p \wedge TRUE)$ Definition of $\oplus$

     2) $(p \wedge FALSE) \vee \neg p$ Identity

     3) $FALSE \vee \neg p$ Domination

     4) $\neg p$ Identity
   - $\vee$ is constructed by $\neg$ and $\wedge$ via De Morgan's. $\neg$ is implemented by previous proof.

     1) $p \vee q$

     2) $\neg\neg(p \vee q)$ Double negation

     3) $\neg(\neg p \wedge \neg q)$ De Morgan's; All necessary components for this definition are present
   - $\neg, \wedge, \vee$ functionally complete, so $\oplus, \wedge, TRUE$ is complete aswell.