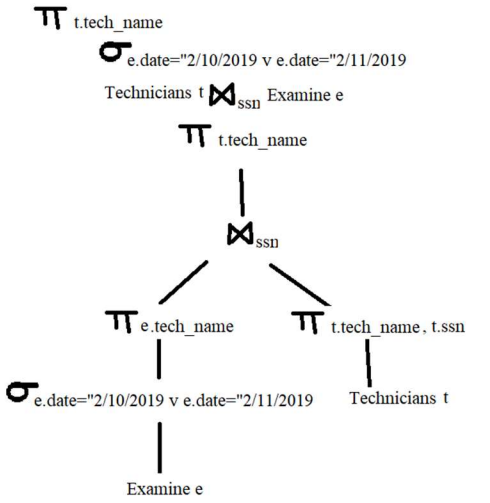
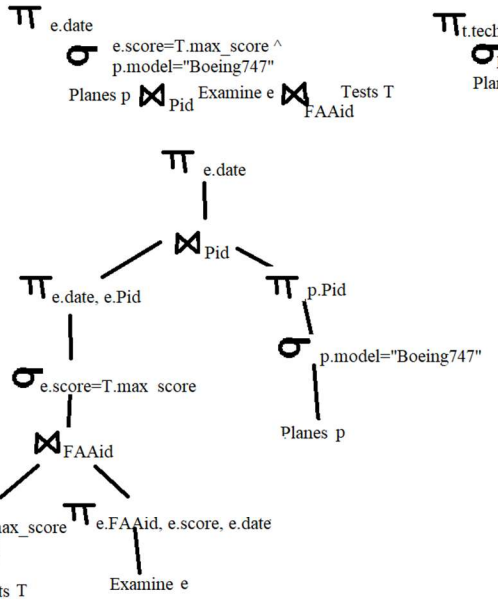
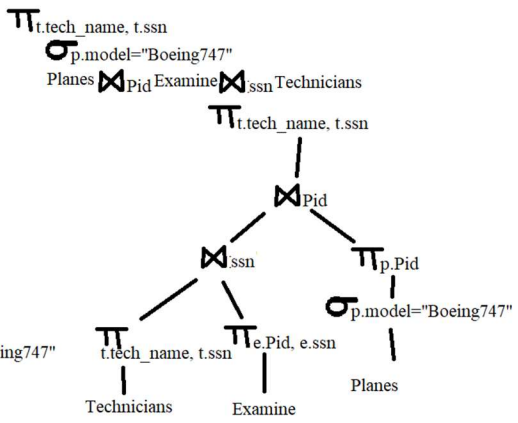
1)







1c) 12. Calculated with a Java program that I wrote.

```java
public static void main(String[] args) {
    ArrayList<RelationKeys> test = new ArrayList<>();
    /*int numRelations = 3; // for testing n! relations with common keys
    for (int i = 0; i < numRelations; i++) {
        test.add(new RelationKeys(new String[]{"id"}));
    }*/
    test.add(new RelationKeys(new String[]{"SSN"}));
    test.add(new RelationKeys(new String[]{"FAAid"}));
    test.add(new RelationKeys(new String[]{"Pid"}));
    test.add(new RelationKeys(new String[]{"SSN", "FAAid", "Pid"}));

    System.out.println(enumerateLeftDeepPlans(test));
}
```

```java
public static int enumerateLeftDeepPlans(ArrayList<RelationKeys> relations) {
    return recursiveEnumeration(relations, relations.size());
}
```

```java
private static int recurseOnNewRelations(ArrayList<RelationKeys> relations, int numTables, int count, int i, int j) {
    ArrayList<RelationKeys> newRelations = new ArrayList<>(relations);
    newRelations.add(new RelationKeys(relations.get(i), relations.get(j)));
    newRelations.remove(relations.get(i));
    newRelations.remove(relations.get(j));
    count += recursiveEnumeration(newRelations, numTables);
    return count;
}
```

```java
private static int recursiveEnumeration(ArrayList<RelationKeys> relations, int numTables) {
    // Base Case: all relations are joined successfully
    if (relations.size() == 1) return 1;

    int count = 0;
    if (relations.size() < numTables) {
        for (int i = 0; i < relations.size(); i++) {
            for (int j = i + 1; j < relations.size(); j++) {
                if (relations.get(i).hasCommonKey(relations.get(j))) {
                    count = recurseOnNewRelations(relations, numTables, count, i, j);
                }
            }
        }
    }
    else {
        for (int i = 0; i < relations.size(); i++) {
            for (int j = 0; j < relations.size(); j++) {
                if (i != j && relations.get(i).hasCommonKey(relations.get(j))) {
                    count = recurseOnNewRelations(relations, numTables, count, i, j);
                }
            }
        }
    }

    return count;
}
```

2)