

## CS 230 : Discrete Computational Structures

**Spring Semester, 2021**

**HOMEWORK ASSIGNMENT #3**

**Due Date:** Monday, February 22

**Suggested Reading:** Rosen Sections 1.7 - 1.8; Lehman et al. Chapter 1

For the problems below, explain your answers and show your reasoning.

1. **[5 Pts]** Prove, using a direct proof that  $p$  is odd if and only if  $p^3$  is odd.
2. **[6 Pts]** Let  $x$  and  $y$  be non-zero rational numbers and let  $z$  be an irrational number. Prove that  $x + yz$  is irrational. Can you use a direct proof? Why or why not?
3. **[6 Pts]** Let  $m$  and  $n$  be positive integers. Prove, by contrapositive, that if  $mn > 35$ , then  $m \geq 6$  or  $n \geq 8$ .
4. **[6 Pts]** Suppose your college organization has 32 students. Prove that it has at least 5 freshmen or at least 8 sophomores or at least 10 juniors or at least 7 seniors.
5. **[6 Pts]** Prove by cases that if  $p \geq 3$  or  $p \leq -7$  then  $(p + 2)^2 \geq 25$ .
6. **[6 Pts]** Prove that the square root of 5 is irrational.
7. **[5 Pts]** Prove that there exist rational numbers  $x$  and  $y$  where  $x^y$  is irrational. Is your proof constructive or non-constructive? Explain.

For more practice, work on the problems from Rosen Sections 1.7 - 1.8 and LLM Chapter 1.