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Stat 347 Hw12

9.19. $E(Y) = \int_0^1 \theta y^\theta \, dy = \frac{\theta}{\theta+1} y^{\theta+1} \Big|_0^1 = \frac{\theta}{\theta+1}$

Unbiased $E(\bar{Y}) = E\left(\frac{1}{n}\sum Y\right) = \frac{1}{n} \frac{\theta n}{\theta+1} = \frac{\theta}{\theta+1}$

$E(Y_i^2) = \int_0^1 \theta y^{\theta+1} \, dy = \frac{\theta}{\theta+2}$

$V(Y) = \frac{\theta}{\theta+2} - \frac{\theta^2}{(\theta+1)^2} = \frac{\theta}{(\theta+1)^2(\theta+2)}$

$V(\bar{Y}) = V\left(\frac{1}{n}\sum Y\right) = \frac{1}{n^2} \frac{\theta n}{(\theta+1)^2(\theta+2)} = \frac{\theta}{n(\theta+1)^2(\theta+2)}$

$\lim_{n \to \infty} V(\bar{Y}) = 0$

So, $\bar{Y}$ is a consistent estimator of $\theta/(\theta+1)$

9.37 $\prod p^{x_i}(1-p)^{1-x_i} = p^{\sum x_i}(1-p)^{n-\sum x_i}$

$= \frac{p^{\sum x_i}}{(1-p)^{\sum x_i}}(1-p)^n = \left(\frac{p}{1-p}\right)^{\boxed{\sum x_i}} \cdot (1-p)^n$

9.39

$$f(y_1, y_2 \dots y_n \mid \Sigma Y) = \frac{f(y_1, y_2, \dots, y_n, \Sigma Y = t)}{P(\Sigma Y = t)}$$

$$P(\Sigma Y = t) = \frac{e^{-n\lambda} \lambda^t}{t!}$$

$$= f(y_1, y_2, \dots, y_{n-1}, y_n + \Sigma^{n-1} y = t)$$

$$= f(\dots, t - \Sigma^{n-1} y)$$

$$= \left(\prod^{n-1} \frac{e^{-\lambda} \lambda^y}{y!} \right) \cdot \frac{e^{-\lambda} \lambda^{t - \Sigma^{n-1} y}}{(t - \Sigma^{n-1} y)!} = \frac{e^{-n\lambda} \lambda^t}{\prod^{n-1} y_i! \, (t - \Sigma^{n-1} y_i)!}$$

$$f(\dots \mid \Sigma y) = \frac{t!}{\prod^{n-1} y_i! \, (t - \Sigma^{n-1} y_i)!}$$ Independent of λ

Therefore $\boxed{\Sigma y \text{ is sufficient}}$

9.62 $f(y|\theta) = e^{-(y-\theta)} \quad y \geq \theta$

$F_{Y_{(1)}} = 1-(e^{-(y-\theta)})^n = 1-e^{-n(y-\theta)}$

$f_{Y_{(1)}} = n(e^{-n(y-\theta)})$

$E(Y_{(1)}) = \int_\theta^\infty y n e^{-n(y-\theta)} dy$

	u	dv
$+$	y	$ne^{-n(y-\theta)}$
$-$	1	$-e^{-n(y-\theta)}$
$+$	0	$\frac{e^{-n(y-\theta)}}{n}$

$= \theta + -\frac{e^{-n(y-\theta)}}{n}\Big|_\theta^\infty = \theta + \frac{1}{n}$

So, $E(Y_{(1)} - \frac{1}{n}) = \theta$, $\boxed{Y_{(1)} - \frac{1}{n} \text{ is MVUE}}$

9.63 $f(y|\theta) = \frac{3y^2}{\theta^3} \quad 0 \leq y \leq \theta$

a. $F(Y|\theta) = y^3/\theta^3$

$F(Y_{(n)}) = F(Y|\theta)^n = y^{3n}/\theta^{3n}$

So, $\boxed{f(Y_{(n)}) = \frac{3ny^{3n-1}}{\theta^{3n}}}$

b. $\int_0^\theta \frac{3ny^{3n}}{\theta^{3n}} dy = \frac{3n\, y^{3n+1}}{(3n+1)\,\theta^{3n}}\Big|_0^\theta = \frac{3n\theta}{3n+1}$

So, $\boxed{E(Y_{(n)} \frac{3n+1}{3n\theta}) = \theta}$

9.71 $\boxed{\frac{1}{n}\sum_{i=1}^{n} Y_i^2 = \sigma^2}$

9.77 $f(y) = \frac{1}{3\theta} \quad 0 \le y \le 3\theta$

$E(Y) = \frac{3\theta}{2} = \bar{Y}$

$\boxed{\hat{\theta} = \frac{\bar{Y}2}{3}}$

9.81 $f(y) = \frac{1}{\theta}e^{-y/\theta}$

$L(\theta) = \prod_{i=1}^{\theta} \frac{1}{\theta}e^{-y_i/\theta} = \frac{1}{\theta^n}e^{-\frac{1}{\theta}\sum y}$

$l(\theta) = \ln L(\theta) = -n\log\theta - \frac{1}{\theta}\sum y$

$l'(\theta) = \frac{-n}{\theta} + \frac{1}{\theta^2}\sum y = 0$

$= -n\theta + \sum y = 0 \rightarrow \hat{\theta} = \frac{\sum y}{n} = \bar{Y}$

So. $\boxed{\hat{\theta}^2 = \bar{Y}^2}$

9.97 $p(y|p) = p(1-p)^{y-1}$

a. $\mu_1' = \frac{1}{p}$, so $\hat{p} = \frac{1}{m_1'} = \boxed{\frac{1}{\bar{y}}}$

b.

$$L(p) = \prod_{i=1}^{n} p(1-p)^{y-1} = p^n (1-p)^{\sum(y-1)}$$

$$l(p) = \ln L(p) = \cancel{n \ln p + \sum(y-1)\ln(1-p)}$$

$$\cancel{n \ln p}$$

$$n \ln p + (\sum y - n)\ln(1-p)$$

$$l'(p) = \frac{n}{p} - \frac{\sum y - n}{1-p} = 0$$

$$\frac{\sum y - n}{1-p} = \frac{n}{p}$$

$$\sum y - n = \frac{n(1-p)}{p}$$

$$(\sum y - n)p = n - np$$

$$p \sum y = n$$

$$p = n / \sum y = \boxed{\frac{1}{\bar{y}}}$$

8.40 a $z = 1.96$ by empirical rule

$\boxed{(\bar{x}-1.96,\ \bar{x}+1.96)}$

b. See z table for 95th percentile

$\boxed{(-\infty,\ \bar{x}+1.64)}$

c. Inverse is $\boxed{(\bar{x}-1.64,\ \infty)}$

8.56 a $p = 0.45$

z for .98 gives 2.33

$2.33\sqrt{\frac{.45 \cdot .55}{800}} \approx 0.04$

$\boxed{(0.41,\ 0.49)}$

b. No, .5 is outside of a 98% confidence interval of this proportion. Highly unlikely

8.60 a $z : 2.58$ $Se = 0.73/\sqrt{130} = 0.064$

$\bar{x} = 98.25$ $\boxed{(98.08,\ 98.42)}$

b. The interval does not include 98.6. There is a 99% chance that the interval captures the true population mean.