Modern Robotics by Lynch and Park

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Abstract

These are my study notes on Robotics to prepare myself for PhD applications. The notes follow the structure of the Northwestern Coursera course and the book by Lynch and Park.

1 Chapter 2 - Configuration Space, Module 2 - Foundations of Robot Motion

This module covers the ideas behind degrees of freedom and robot's configuration.

Configuration is a specification of the positions of all points of the robot.

Definition of Configuration - is a complete specification of the position of every point of the robot. The minimum number n of real-valued coordinates needed to represent the configuration is the number of degrees of freedom (dof) of the robot. **The n-dimensional space containing all possible configurations of the robot is called the configuration space (C-space)**. The configuration of a robot is represented by a point in its C-space.

Degrees of freedom = (sum of freedoms of the points) - (number of independent constraints)

A rigid body moving in three-dimensional space, which we call a **spatial rigid body**, has six degrees of freedom.

A rigid body moving in a two-dimensional plane, which we henceforth call a **planar rigid body**.

This book only describes the robotics of rigid bodies.

1.1 Degrees of Freedom of a Robot

Joints constrain the motion of a rigid body. Types of joints include:

The revolute joint (R), also called a hinge joint, allows rotational motion about the joint axis. The prismatic joint (P), also called a sliding or linear joint, allows translational (or rectilinear) motion along the direction of the joint axis. The helical joint (H), also called a screw joint, allows simultaneous rotation and translation about a screw axis. Revolute, prismatic, and helical joints all have one degree of freedom. Joints can also have multiple degrees of freedom. The cylindrical joint (C) has two degrees of freedom and allows independent translations and rotations about a single fixed joint axis. The universal joint (U) is another two-degreeof-freedom joint that consists of a pair of revolute joints arranged so that their joint axes are orthogonal. The spherical joint (S), also called a ball-and-socket joint, has three degrees of freedom and functions much like our shoulder joint.

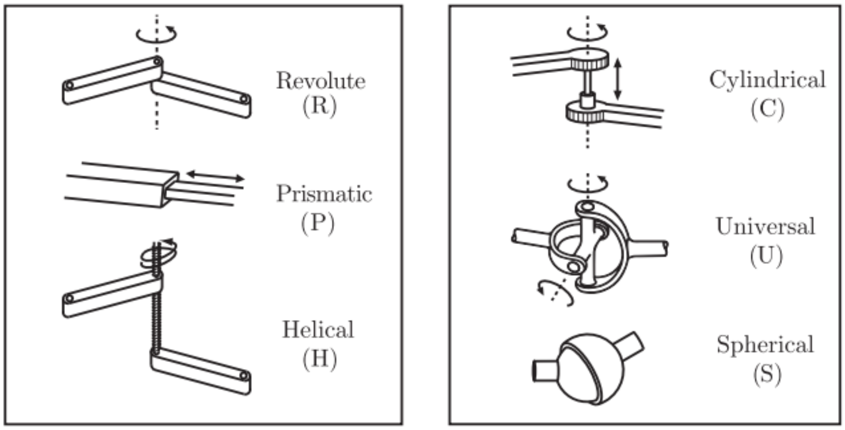


Figure 2.3: Typical robot joints.

Figure 1: Types of joints

Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

Table 2.1: The number of degrees of freedom f and constraints c provided by common joints.

Figure 2: DoF of joints

1.2 Grubler's Formula

Consider a mechanism consisting of N links, where ground is also regarded as a link. Let J be the number of joints, m be the number of degrees of freedom of a rigid body (m = 3 for planar mechanisms and m = 6 for spatial mechanisms), f_i be the number of freedoms provided by joint i, and c_i be the number of constraints provided by joint i, where $f_i + c_i =$ m for all i. Then Grubler's formula for the number of degrees of freedom of the robot is

$$dof = m(N - 1 - J) + \Sigma f_i$$

This formula holds only if all joint constraints are independent. If they are not independent then the formula provides a lower bound on the number of degrees of freedom.

1.3 Exercises from Module 2

1. The number of degrees of freedom of a robot is (select all that apply):
   (a) the dimension of its configuration space.
   (b) the number of real numbers needed to specify its configuration
   (c) the number of freedoms of the bodies minus the number of independent constraints between the bodies.

2. The number of degrees of freedom of a planar rigid body is: 3

3. A rigid body in n-dimensional space has m total degrees of freedom. How many of these m degrees of freedom are angular (not linear)? Select all that apply.

   (a) n(n-1)/2

   (b) m -n

4. Consider a mechanism consisting of three spatial rigid bodies (including ground, N=4) and four joints: one revolute, one prismatic, one universal, and one spherical. According to Grubler's formula, how many degrees of freedom does the mechanism have?

   (a) 1. Grubler's formula.

5. A mechanism that is incapable of motion has zero degrees of freedom. In some circumstances, Grubler's formula indicates that the number of degrees of freedom of a mechanism is negative. How should that result be interpreted?

   (a) The constraints implied by the joints must not be independent.

6. Assume your arm, from your shoulder to your palm, has 7 degrees of freedom. You are carrying a tray like a waiter, and you must keep the tray horizontal to avoid spilling drinks on the tray. How many degrees of freedom does your arm have while satisfying the constraint that the tray stays horizontal? Your answer should be an integer.

   (a) 5. The requirement that the tray be horizontal places two constraints on its orientation: the rotation of the tray about two axes defining the horizontal plane of the tray must be zero. (In other words, the roll and the pitch of the tray are zero.)

7. Find the number of degrees of freedom of this system while the grippers hold the object rigidly (no relative motion between the object and the last links of the SRS arms). Your answer should be an integer.

   (a) 10. No relative motion means the thing in the center is just one link. I.e. the object and the 4 other links that come from the spherical joints are just regartded as 1 link. Thus N = 10, J = 12. Sum of fi is 7*4.

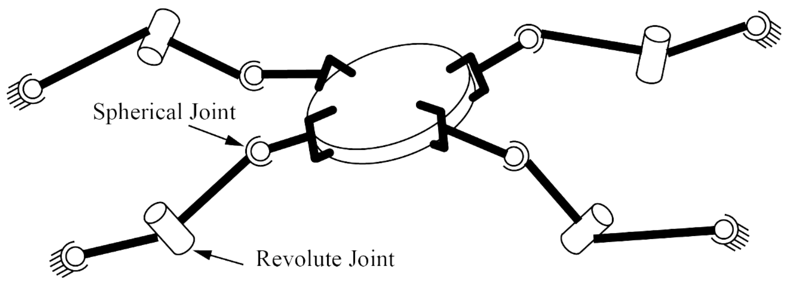
8. suppose there are now a total of n such arms grasping the object. What is the number of degrees of freedom of this system? Your answer should be a mathematical expression including n.

   (a) n + 6

9. Use the planar version of Grubler's formula to determine the number of degrees of freedom of the mechanism shown below. Your answer should be an integer. (Remember that a single joint can only connect two rigid bodies, so if you see more than two connecting at a single point, there must be more than one joint there. Also, the two blocks in the channels are only allowed to move prismatically in those channels, and one of the joints is labeled "P" for prismatic. You will need to identify all the other joints, and links.)

   (a) 3

Four identical SRS arms are grasping a common object as shown below. 1 / 1 poin



Find the number of degrees of freedom of this system while the grippers hold the object rigidly (no relative motion between the object and the last links of the SRS arms). Your answer should be an integer.

Figure 3: SRS for Question 7

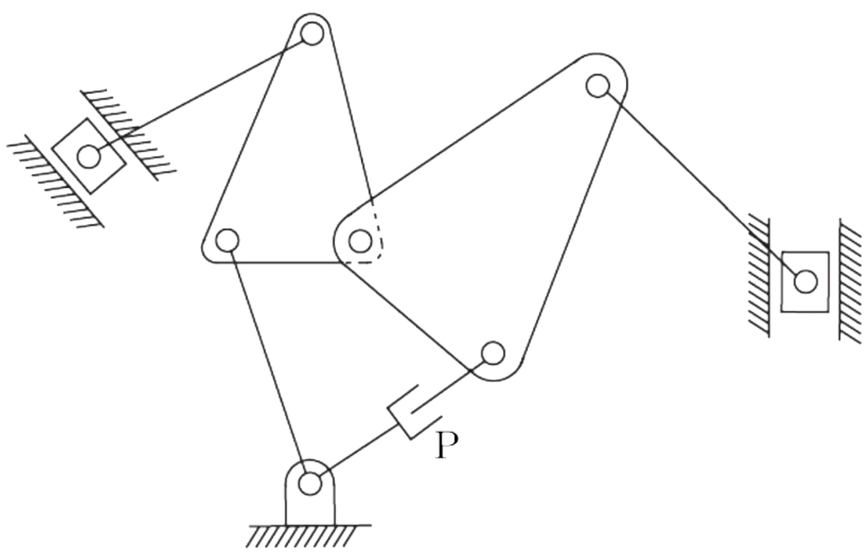


Figure 4: Joint Structure for Q9