ML THW 2 Yaropolov MAE 21

Problem 1

$\bar y_t = y(R_t)$. н.д. $Q(R_\ell)+Q(R_r) \le Q(R_m) \iff \sum_{x_i\in R_\ell}(y_i-\bar y_\ell)^2 + \sum_{x_i\in R_r}(y_i-\bar y_r)^2 \le \sum_{x_i \in R_m}(y_i - y_m)^2$

$\ell = |R_\ell|$, $r=|R_r|$

$m=|R_m|$

$$\sum_{R_\ell}(y_i^2 - 2y_i\bar y_\ell + \bar y_\ell^2) + \sum_{R_r}(y_i^2-2y_i\bar y_r+\bar y_r^2) \le \sum_{R_m}(y_i^2 - 2y_i\bar y_m + \bar y_m^2)$$

Problem 2

$I_k \equiv \{[y(x_i)=k] : x_i\in R_n\}$, $I_{i,k}$ - элементы I_k. Тогда дисперсия класса k:

$$Var_k = \frac1n \sum_{i\in I_k}(I_{i,k}-p_k)^2 = \frac1n\sum_i(I_{i,k}^2 - 2I_{i,k}p_k + p_k^2) = \frac1n\sum_i I_{i,k}^2 - \frac2n p_k\sum I_{i,k} + p_k^2 = p_k - 2p_k^2 + p_k^2 = p_k(1-p_k)$$

среднее I_k

Тогда $\sum_k Var_k = \sum_k p_k(1-p_k)$ — ч.т.д.

Problem 3

1. $\hat s_i = -\frac{\partial L(y_i,z)}{\partial z}\Big|_{z=a_{n-1}(x_i)} = y_i \cdot e^{-y_i a_{n-1}(x_i)}$, $L=e^{-yz}$

2. $\hat s_i = -2\sigma(-y_i\cdot a_{n-1}(x_i))(1-\sigma(-y_i a_{n-1}(x_i)))\cdot y_i$

Problem 4

$\gamma_{N_j} = \arg\min_\gamma \sum_{x_i\in R_j} L(y_i, a_{n-1}(x_i)+\gamma)$ - можно искать отдельно для $\forall j$.

1. $\gamma_{N_j} = \arg\min_\gamma \sum_{x_i\in R_j}(y_i - a_{n-1}(x_i)-\gamma)^2 = \arg\min_\gamma \sum_{R_j}[(y_i - a_{n-1}(x_i))^2 - 2\gamma(y_i - a_{n-1}(x_i)) + \gamma^2]$

FOC: $\sum_{R_j}[-2(y_i - a_{n-1}(x_i)) + 2\gamma] = 0$

$\gamma = \frac{1}{|R_j|}\sum_{R_j}(y_i - a_{n-1}(x_i)) = \gamma_{N_j}\ \forall j$

2. $\gamma_{N_j} = \arg\min \sum_{R_j} e^{-y_i(a_{n-1}(x_i)+\gamma)}$ FOC: $\sum_{R_j} y_i\cdot e^{-y_i(a_{n-1}(x_i)+\gamma)} = 0$ ~~нет аналитического решения~~ :(

Т.е. γ_{N_j}: $\sum_{R_j} y_i e^{-y_i(a_{n-1}(x_i)+\gamma)} = 0$ ~~можно решать одним шагом Ньютона-Рафсона~~

$$\sum_{где\ y_i=1} e^{-a_{n-1}(x_i)-\gamma} - \sum_{где\ y_i=-1} e^{a_{n-1}(x_i)-\gamma} = 0 \iff e^{-\gamma}\sum_{y_i=1}e^{-a_{n-1}(x_i)} = e^{\gamma}\sum_{y_i=-1}e^{+a_{n-1}(x_i)}$$

$$\gamma_{N_j} = \frac{\ln\sum_{y_i=1}e^{-a_{n-1}(x_i)} - \ln\sum_{y_i=-1}e^{a_{n-1}(x_i)}}{2}$$

Problem 1 продолжение

$\sum_{R_\ell}(y_i^2 - y_i\bar y_\ell) + \sum_{R_r}(y_i^2 - y_i\bar y_r) \le \sum_{R_m}(y_i^2 - y_i\bar y_m)$ т.к. $m\bar y_m = \ell\bar y_\ell + r\bar y_r$ и $r = m-\ell$

$$-\ell\bar y_\ell^2 - r\bar y_r^2 \le -m\bar y_m^2$$
$$\ell\bar y_\ell^2 + r\bar y_r^2 \ge m\bar y_m^2 = m\cdot\left(\frac{\ell\bar y_\ell + r\bar y_r}{m}\right)^2$$
$$m\ell\bar y_\ell^2 + mr\bar y_r^2 \ge \ell^2\bar y_\ell^2 + r^2\bar y_r^2 + 2\ell r\bar y_\ell\bar y_r$$
$$r\ell\bar y_\ell^2 + r\ell\bar y_r^2 \ge 2\ell r\bar y_\ell\bar y_r$$
$$r\ell(\bar y_\ell - \bar y_r)^2 \ge 0$$ ч.т.д.