

Online ARIMA

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Short description

The ubiquity of time series is a fact of modern-day life: from stock market data to social media information; from search-engine metadata to webpage analytics, modern-day data exists as a continuous flow of information indexed by timestamps.

ARIMA models are the main time-series long-term forecasting models in many practical applications. In time series trends or dependencies usually change. This requires frequent refitting of the model from scratch or performing additional training. However, fitting ARIMA models to large-scale data (e.g. hourly time series for 10 years) is time consuming. In this case, the online learning approach helps.



Objectives

For a real-time application, online modeling is desirable — a self-generating and self-updating forecasting method is desirable for producing a timely forecast. The ARIMA method in time series is an effective means to address a forecast; it has a solid foundation in classical probability theory and mathematical statistics.

I. Implement the ARIMA online Gradient Descent algorithm.

II. Compare with common online learning approach:

1. complete refitting on data of steps $[1, t]$ each step t
2. additional fitting (warm_start) for every new observation
3. rolling-window refitting on $[t-h, t]$



A time series is defined as a sequence of quantitative observations at successive time. We assume time is a discrete variable, X_t denotes the observation at time t , and ϵ_t denotes the zero-mean random noise term at time t . The MA(q) (short for Moving Average) model considers the process: $X_t = \sum_{i=1}^q \beta_i \epsilon_{t-i} + \epsilon_t$, where β_i is a coefficient. Similar to MA(q) models, Autoregression model, denoted by AR(k), satisfies $X_t = \sum_{i=1}^k \alpha_i X_{t-i} + \epsilon_t$. In other words, it assumes each X_t is a noisy linear combination of the previous k observations. This is similar to traditional multiple regression model, but X_t is regressed on past values of X_t .

$$\text{ARMA: } X_t = \sum_{i=1}^q \beta_i \epsilon_{t-i} + \sum_{i=1}^k \alpha_i X_{t-i} + \epsilon_t \quad , \text{ where } \epsilon_t \text{ are zero-mean noise term.}$$

ARMA model describing stationary series.

$$\text{ARIMA: } \nabla^d X_t = \sum_{i=1}^q \beta_i \epsilon_{t-i} + \sum_{i=1}^k \alpha_i \nabla^d X_{t-i} + \epsilon_t \quad , \text{which are parameterized by three terms k, d, q}$$

and weights vector $\alpha \in R^k$ and $\beta \in R^q$.

ARIMA model describing non-stationary series.

Description of methods:

- **Complete refitting**

This method produces complete refitting at each step t of our model on the data of steps $[1, t]$.

- **Rolling-window refitting**

This method produces rolling-window refitting at each step t on the data of steps $[t - h, t]$.

- **Warm start**

This method uses built-in functions of statsmodel.sarimax by using params of the previously fitted model to speed up convergence of online fitting model at each step t .

- **Gradient descent. Real online**

We now implement a more general online convex optimization solver, Online Gradient Descent, which is applicable to any convex loss functions. The projection $\prod_{\mathcal{K}}^{(y)}$ refers to the Euclidean projection onto K , i.e., $\prod_{\mathcal{K}}^{(y)} = \arg \min_{x \in \mathcal{K}} \|y - x\|_2$

Input: parameter k, d, q ; learning rate η .

Set $m = \log_{\lambda_{max}}((TLM_{max}q)^{-1})$.

for $t = 1$ **to** $T - 1$ **do**

predict $\tilde{X}_t(\gamma^t) = \sum_{i=1}^{k+m} \gamma_i \nabla^d X_{t-i} + \sum_{i=0}^{d-1} \nabla^i X_{t-1}$;
receive X_t and incur loss $\ell_t^m(\gamma^t)$;

Let $\nabla_t = \nabla \ell_t^m(\gamma^t)$;

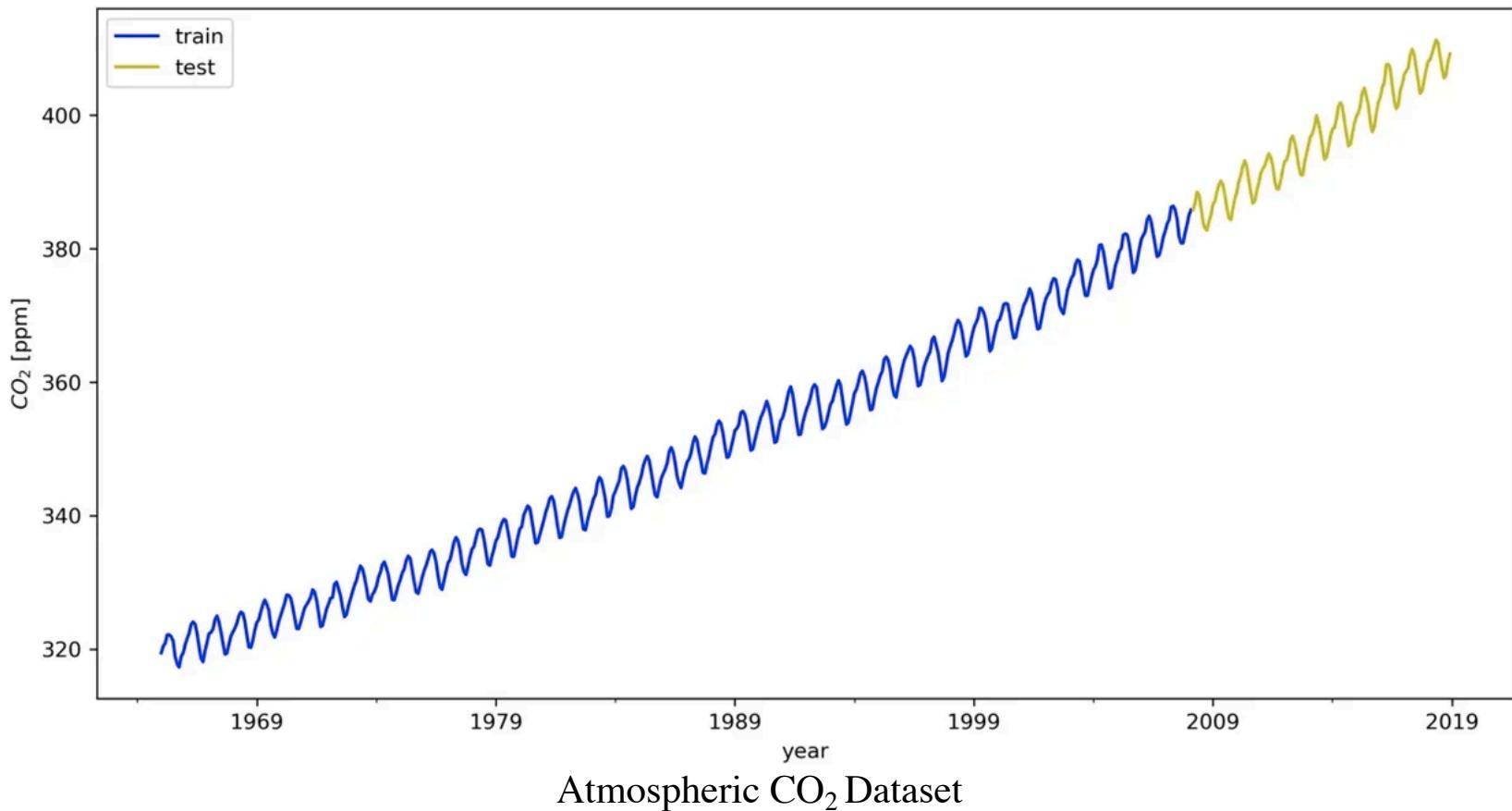
Set $\gamma^{t+1} \leftarrow \prod_{\mathcal{K}}(\gamma^t - \frac{1}{\eta} \nabla_t)$;

end for

ARIMA- OGD algorithm

First dataset

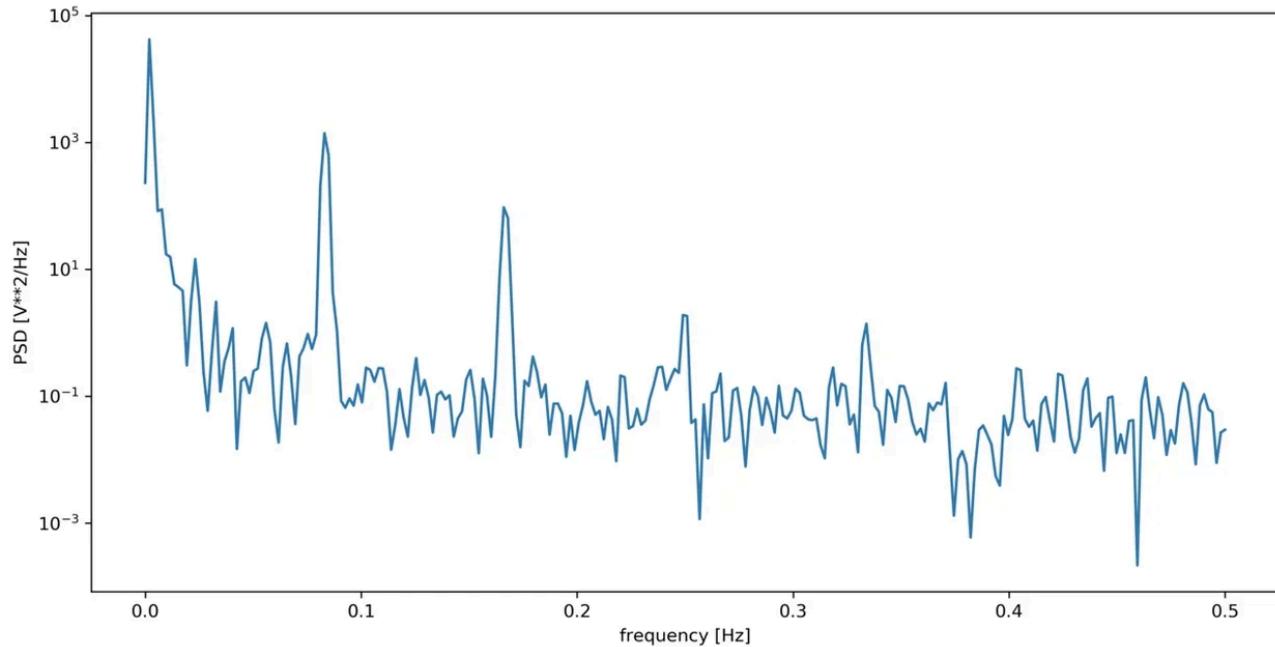
The first Dataset is Atmospheric CO₂ Data. After dropping NaNs and dividing Dataset for train and test we get following.



Data analysis

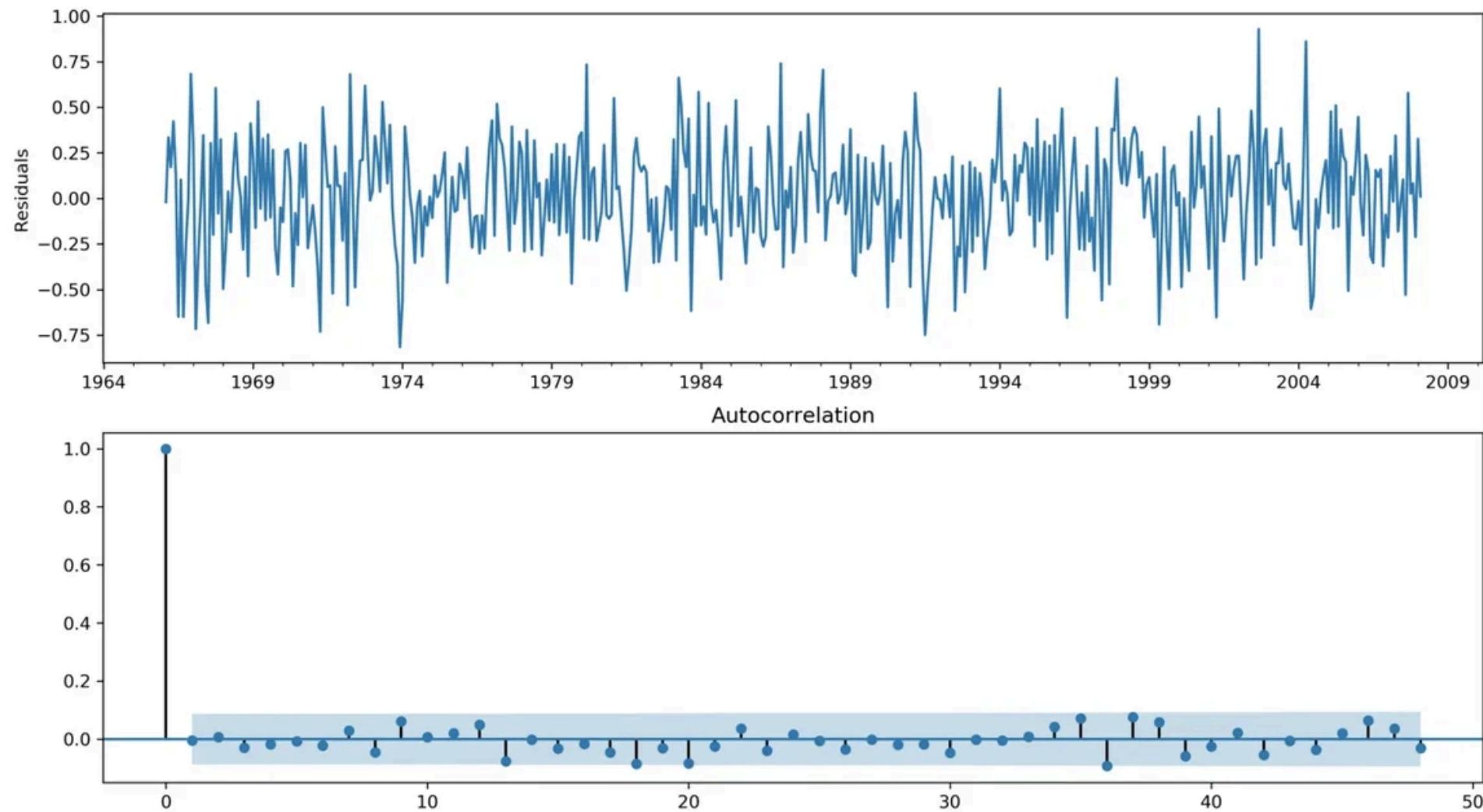
- Estimate spectral density using Welch's method
- Selection of the period for seasonal differences to delete seasonal component using SARIMA
- Provide Augmented Dickey-Fuller test
- Selection of the period for first differences to delete trend component
- Provide Augmented Dickey-Fuller test again to approve non-stationarity of the received Dataset

12 month - the main time period

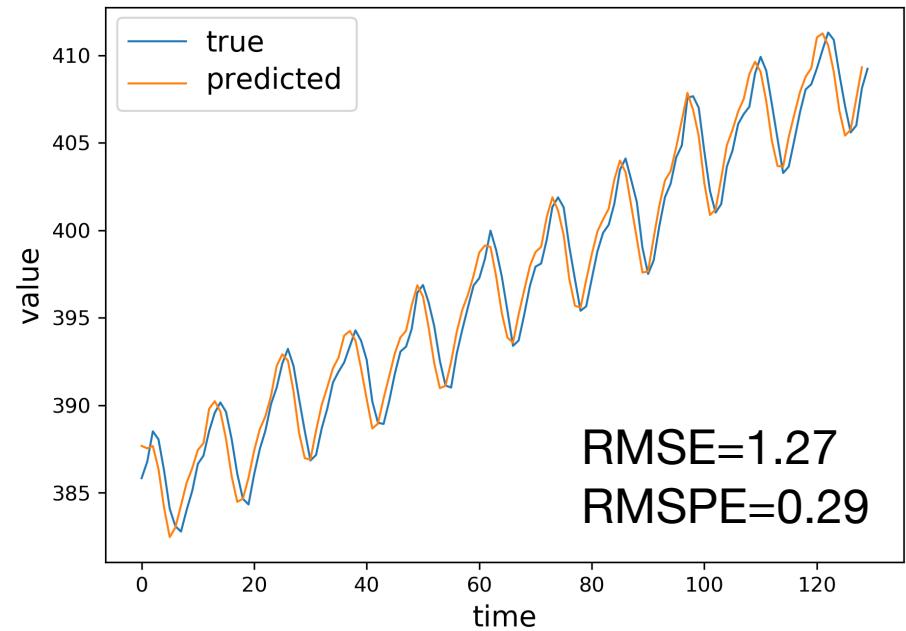


Spectral density estimation using Welch's method

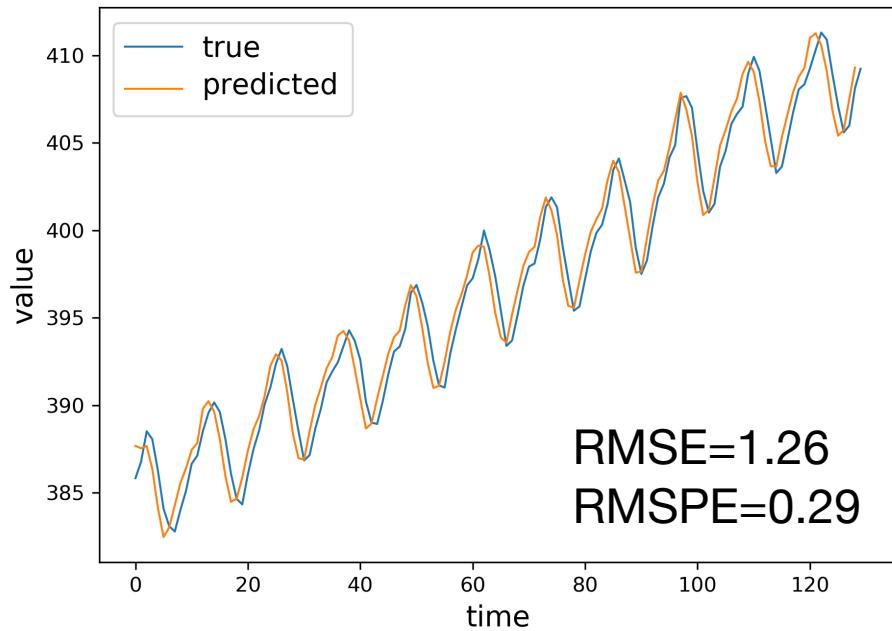
Residuals analysis



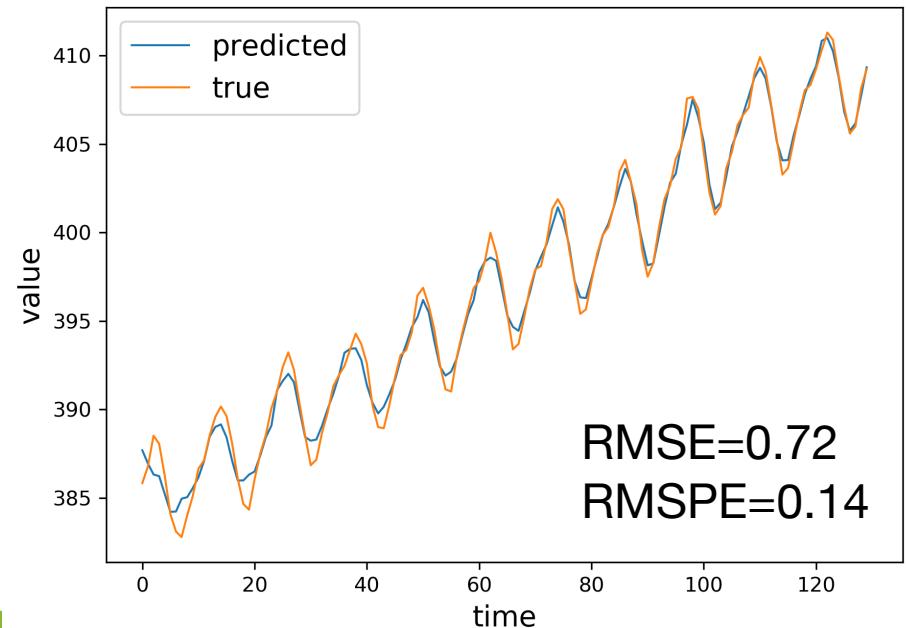
first online ARIMA short term forecast2



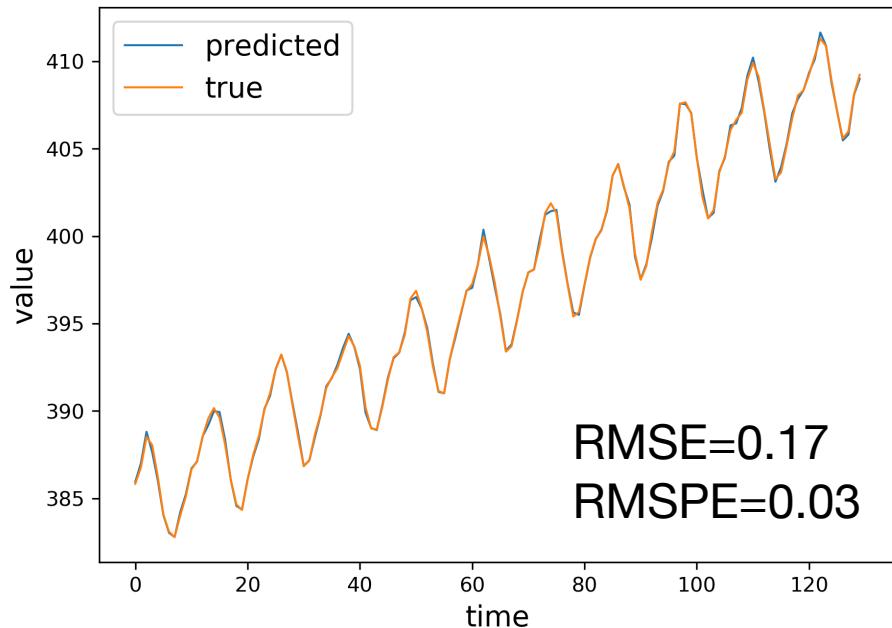
second online ARIMA short term forecast2



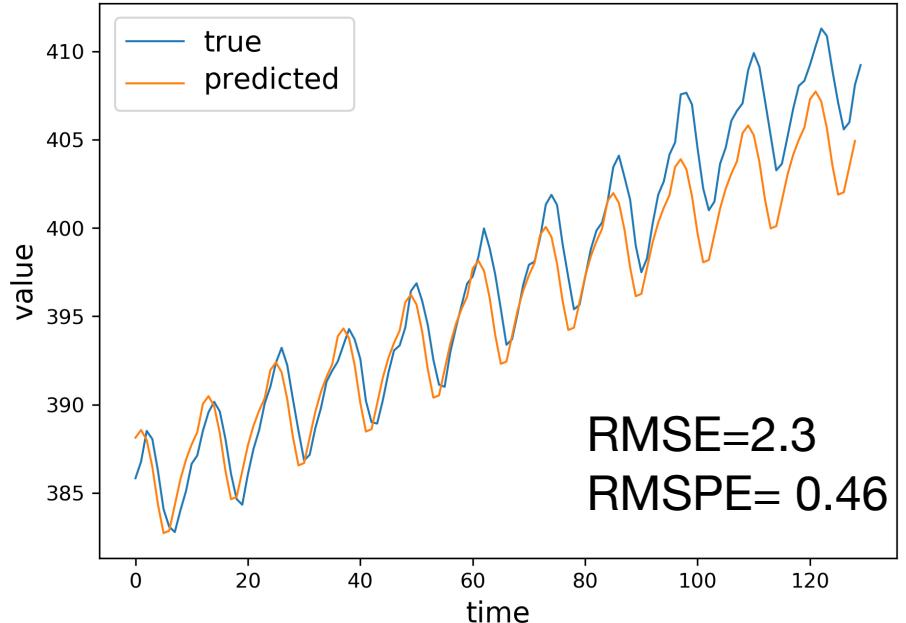
warm start online ARIMA short term forecast



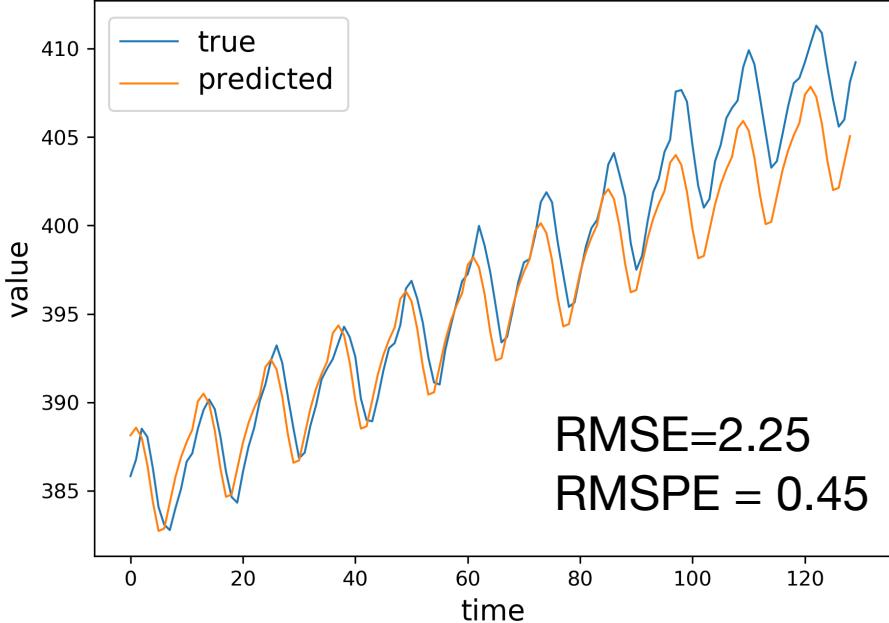
ONLINE ARIMA short term



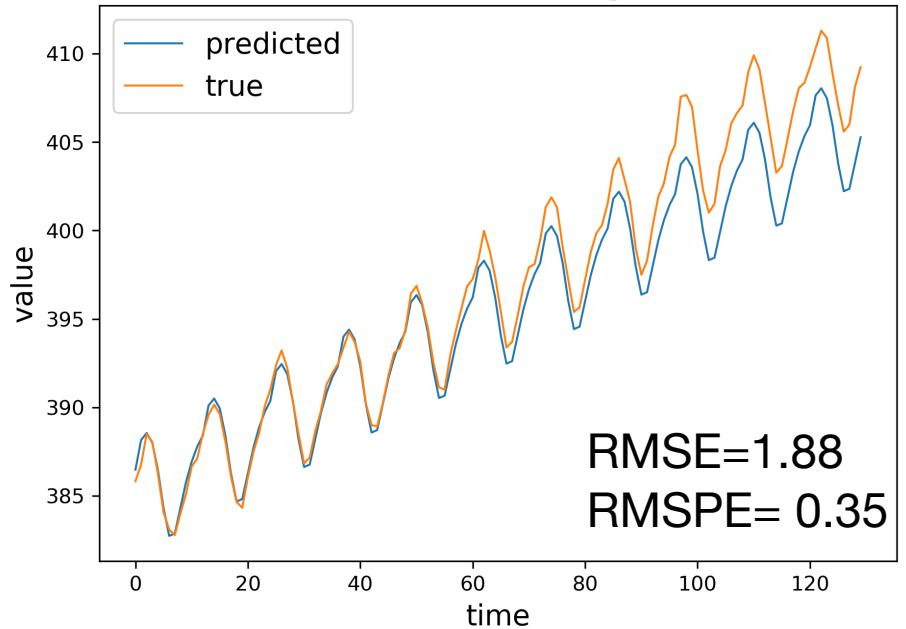
first online ARIMA long term forecast2



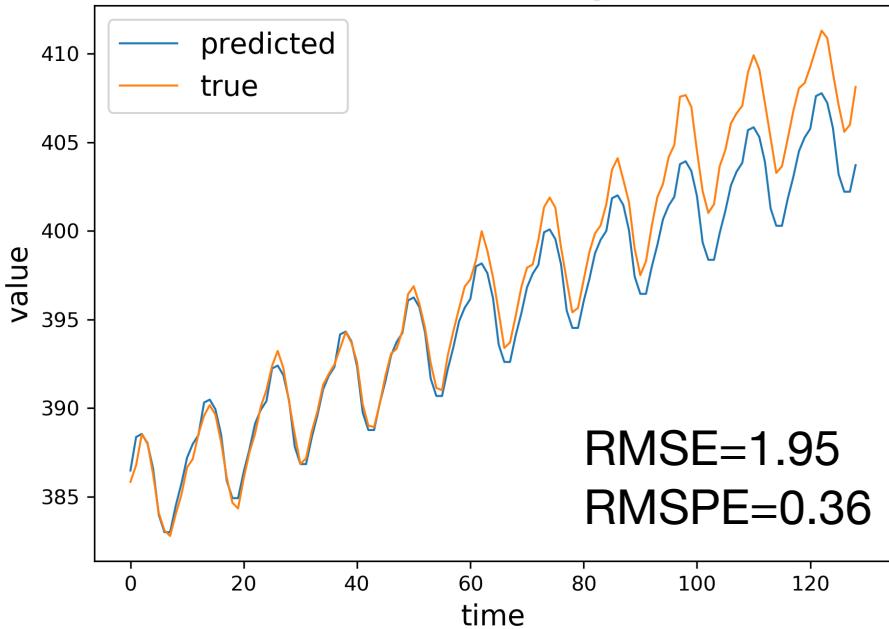
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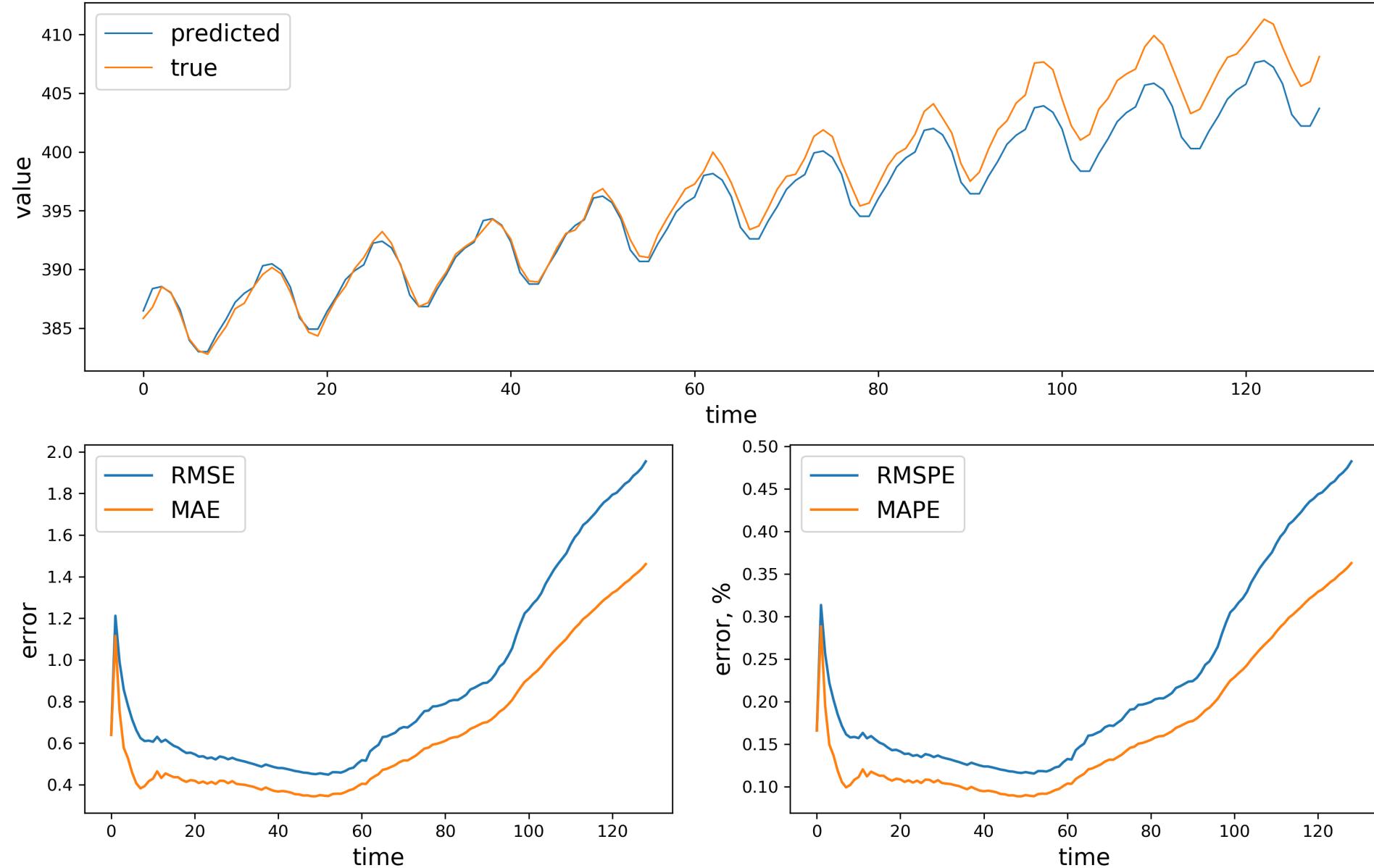
warm start online ARIMA long term forecast



ONLINE ARIMA long term

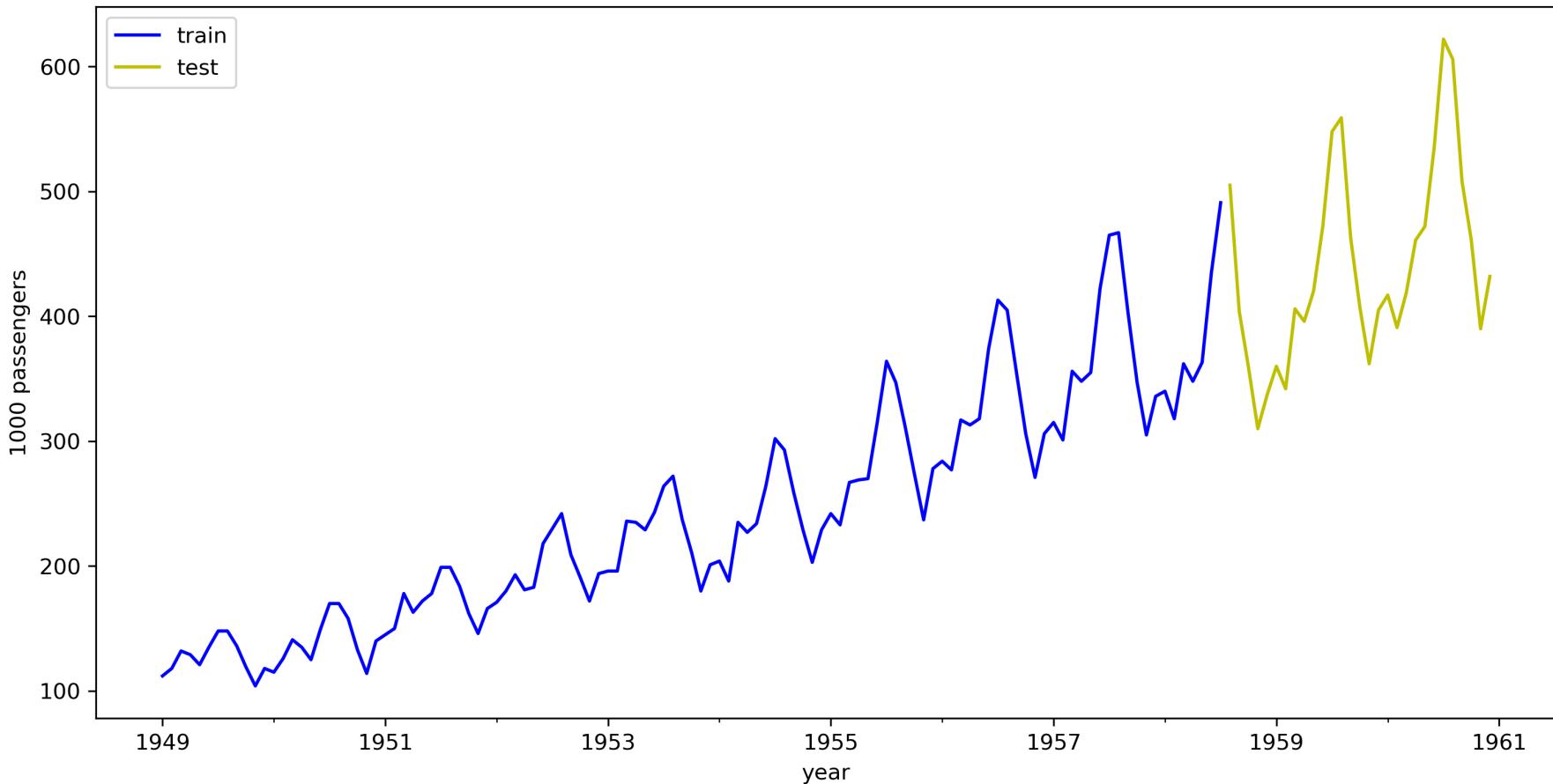


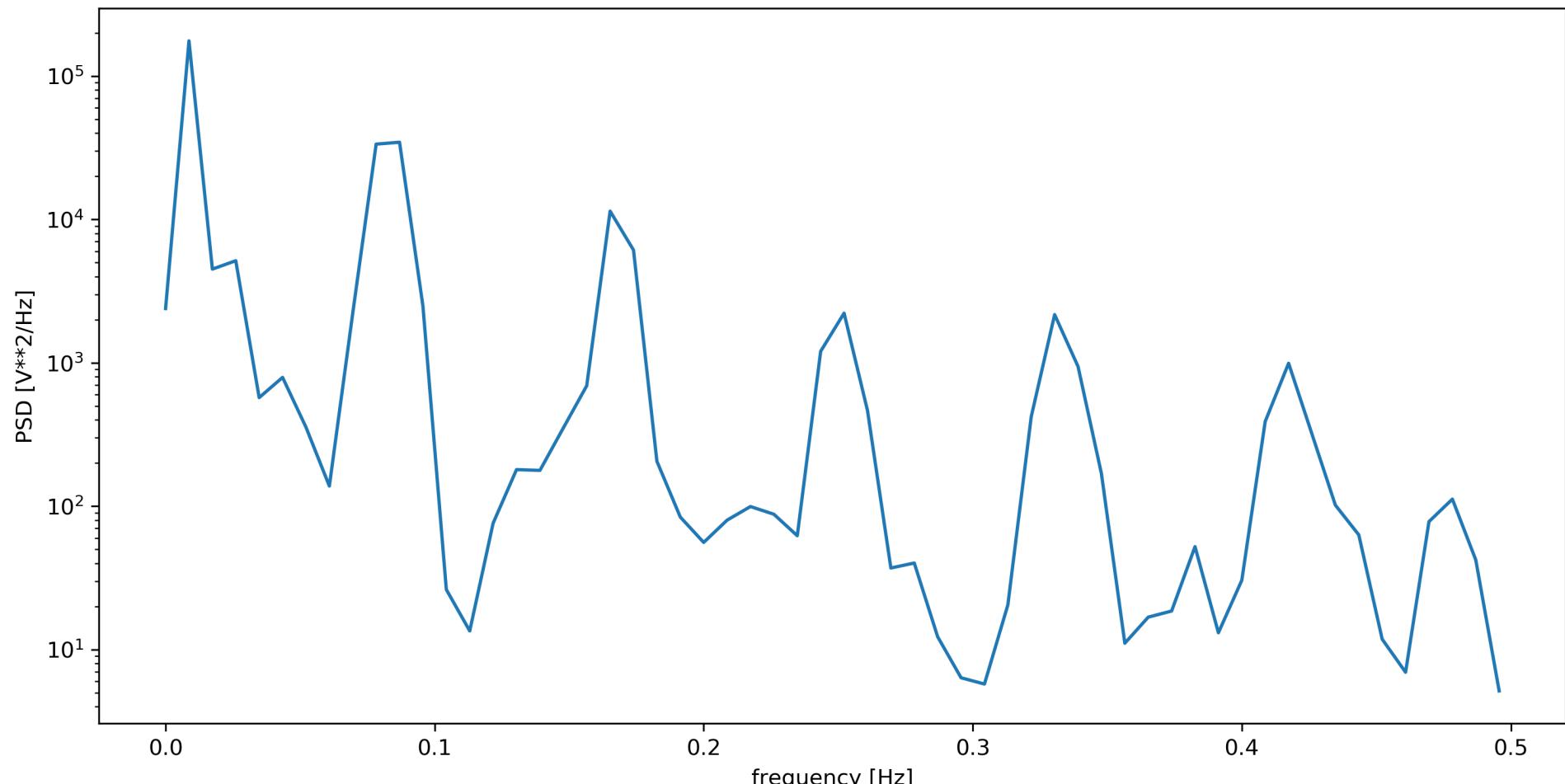
ONLINE ARIMA long term



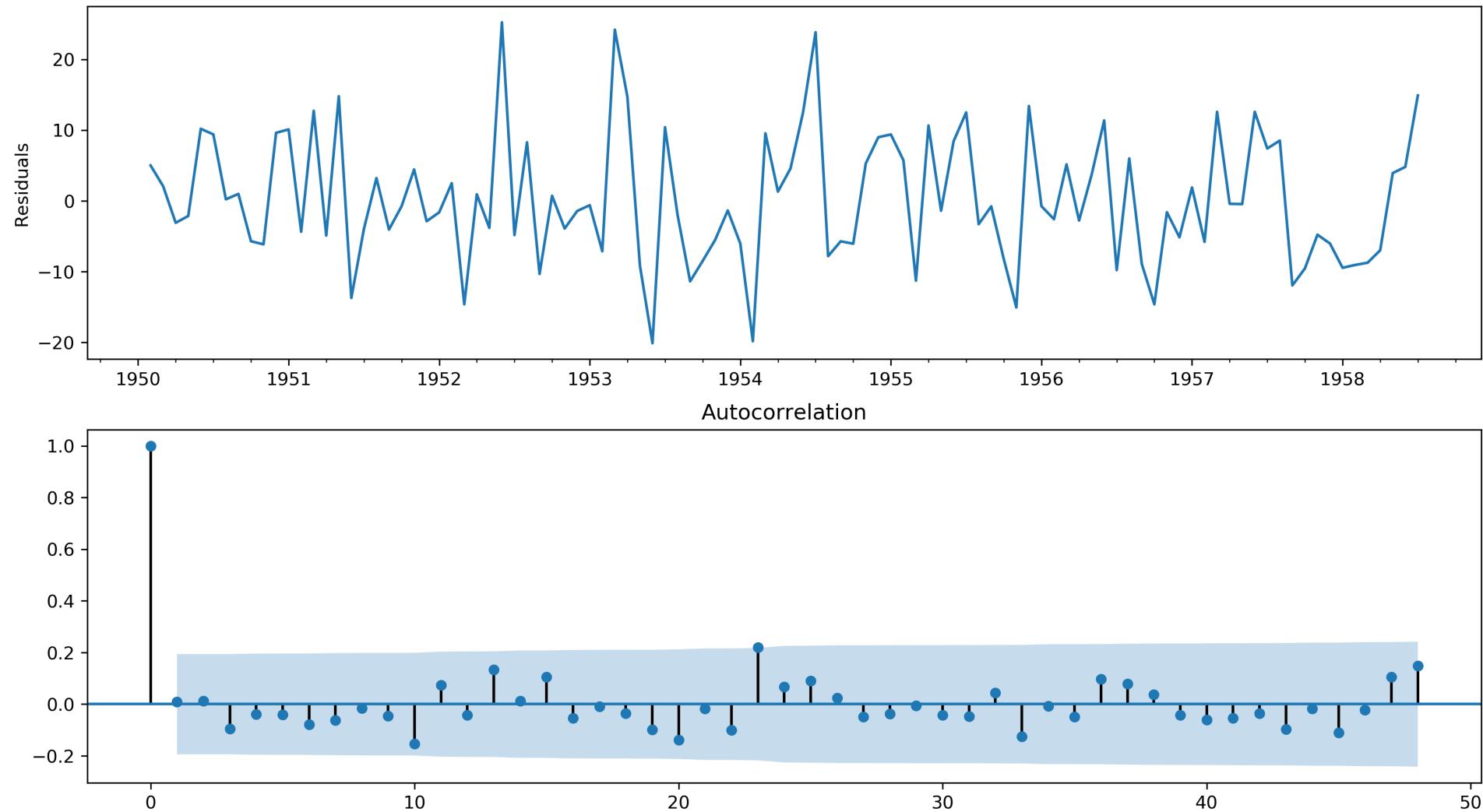
Second Dataset

The second Dataset is International airline passengers: monthly totals in thousands. After dropping NaNs and deviding Dataset for train and test second dataset looks like at the figure 3. Then we provide all the procedures described earlier for the first Dataset.

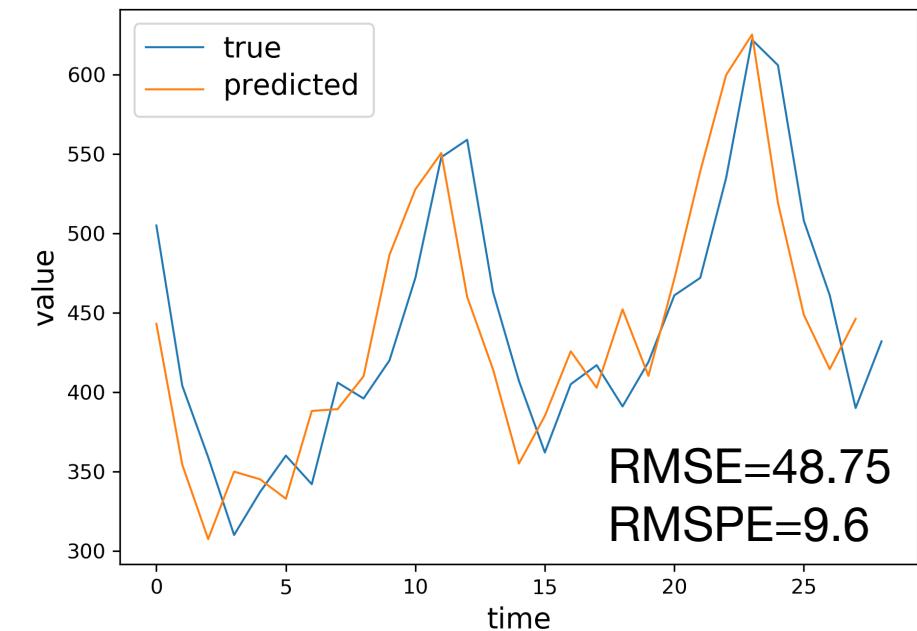




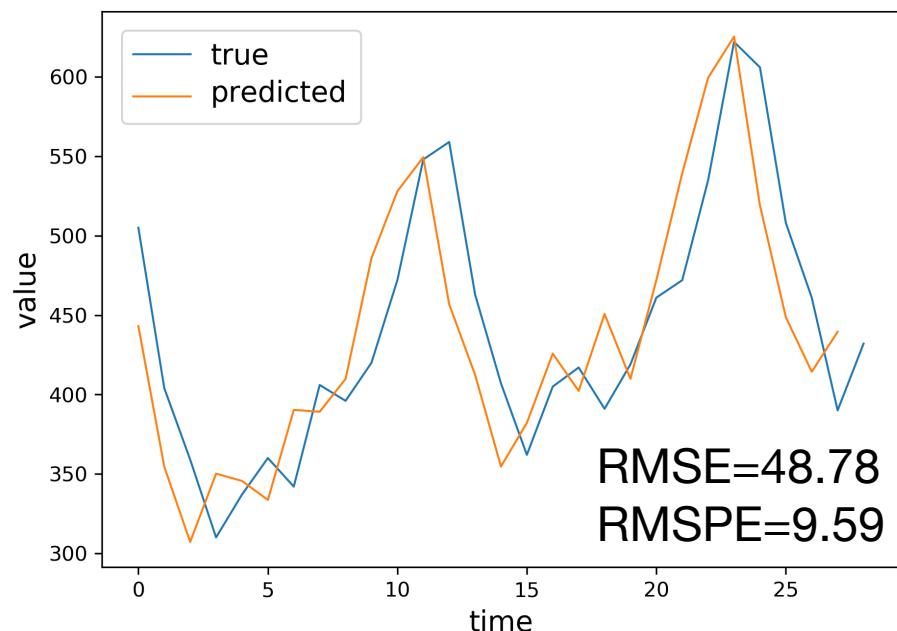
Spectral density estimation using Welch's method



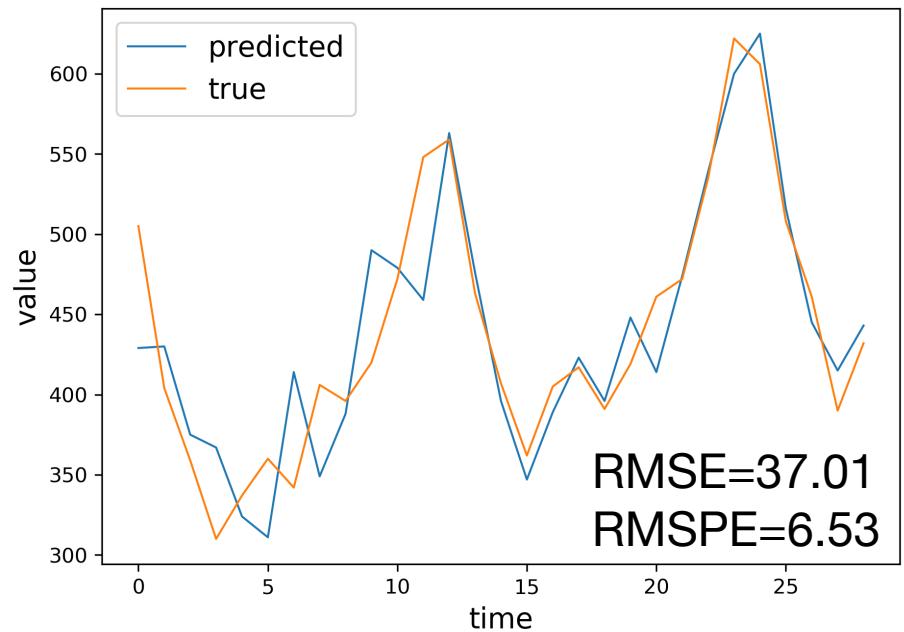
first online ARIMA short term forecast2



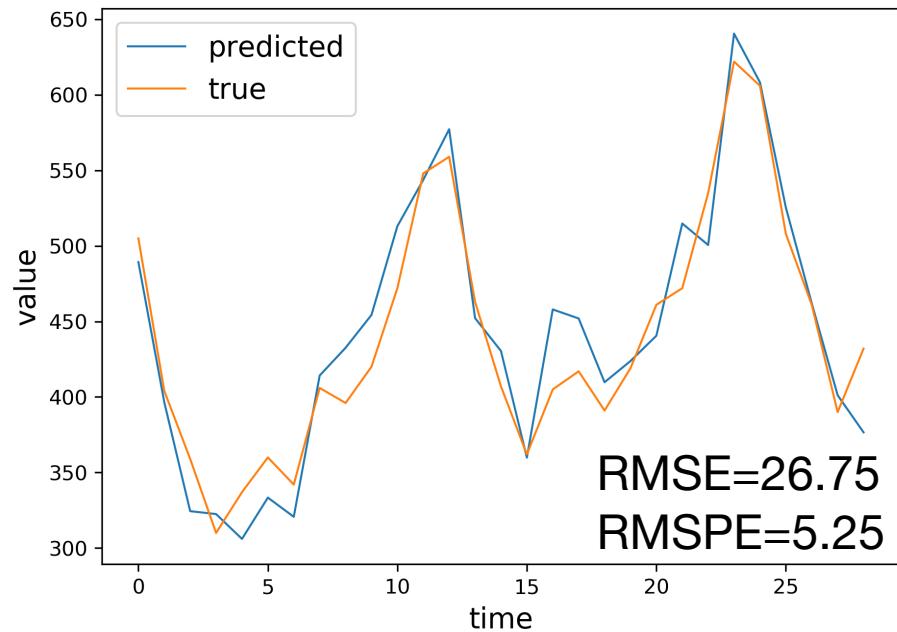
second online ARIMA short term forecast2



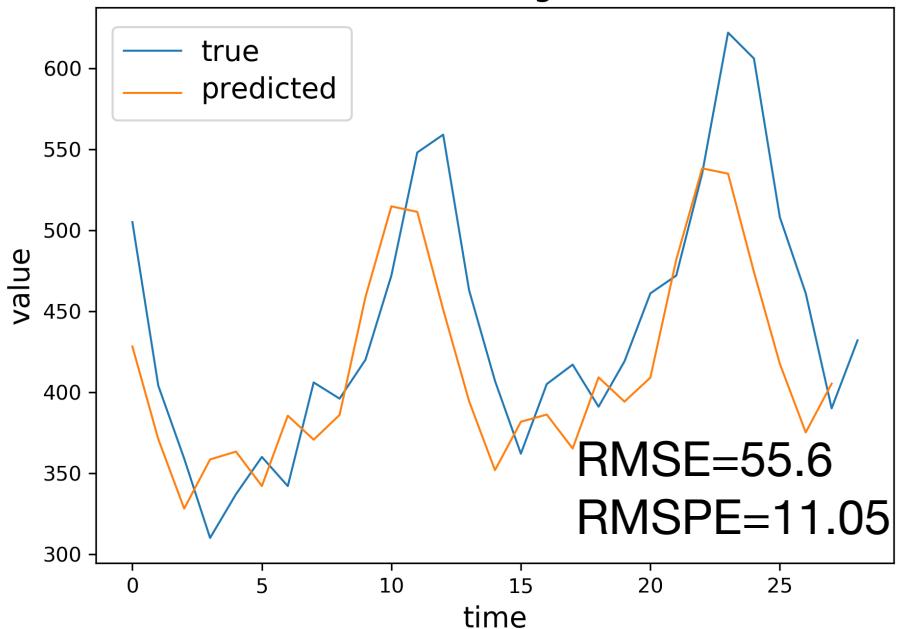
warm start online ARIMA short term forecast2



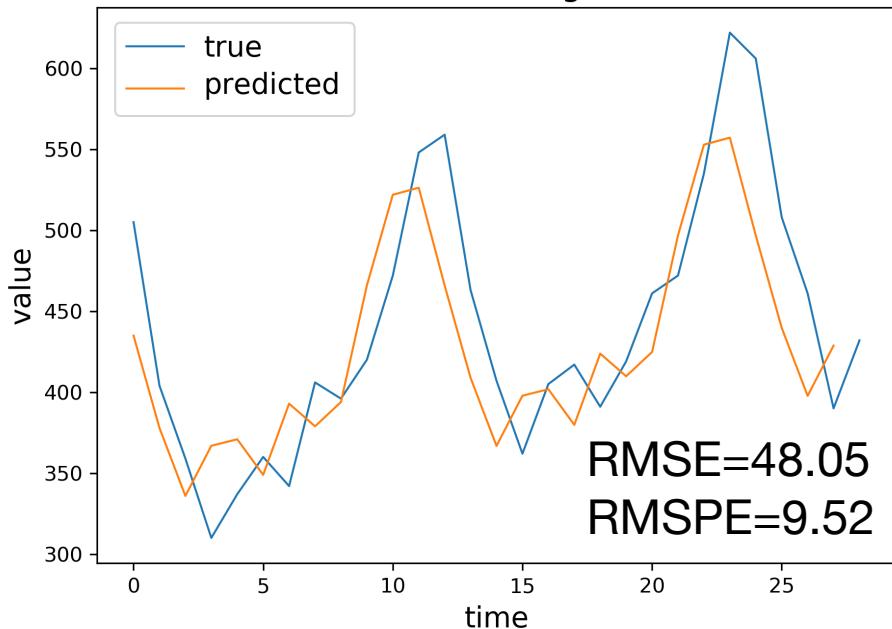
ONLINE ARIMA short term2



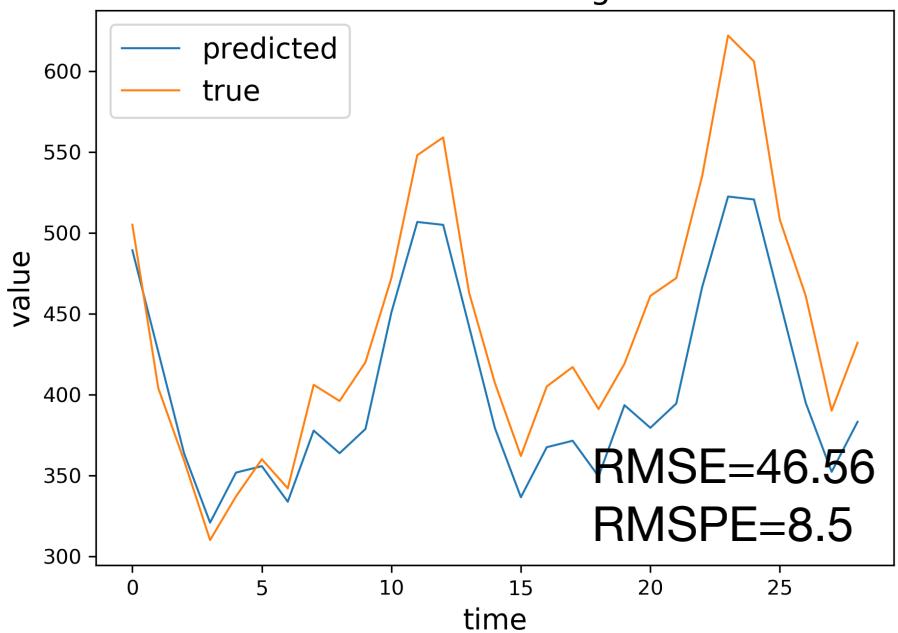
first online ARIMA long term forecast2



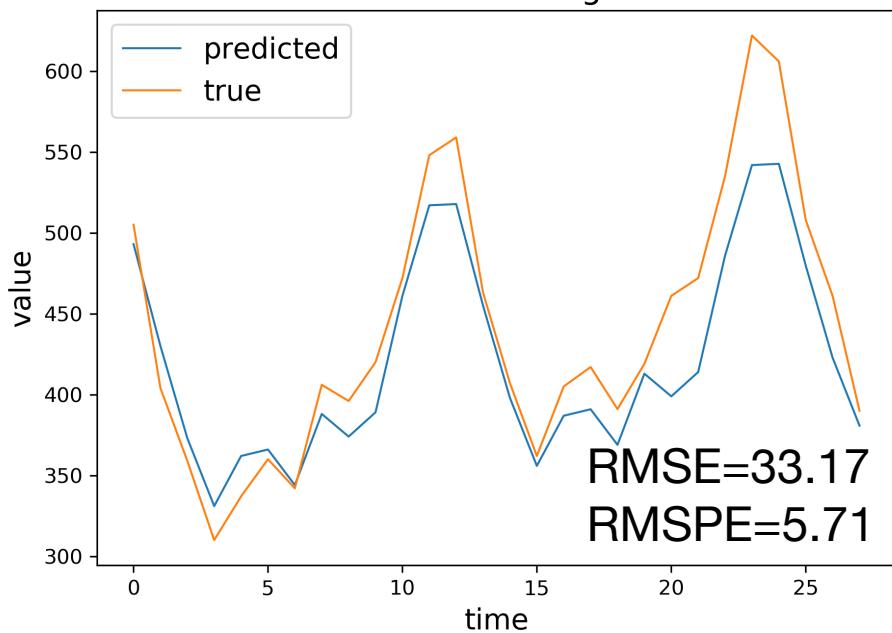
second online ARIMA long term forecast2



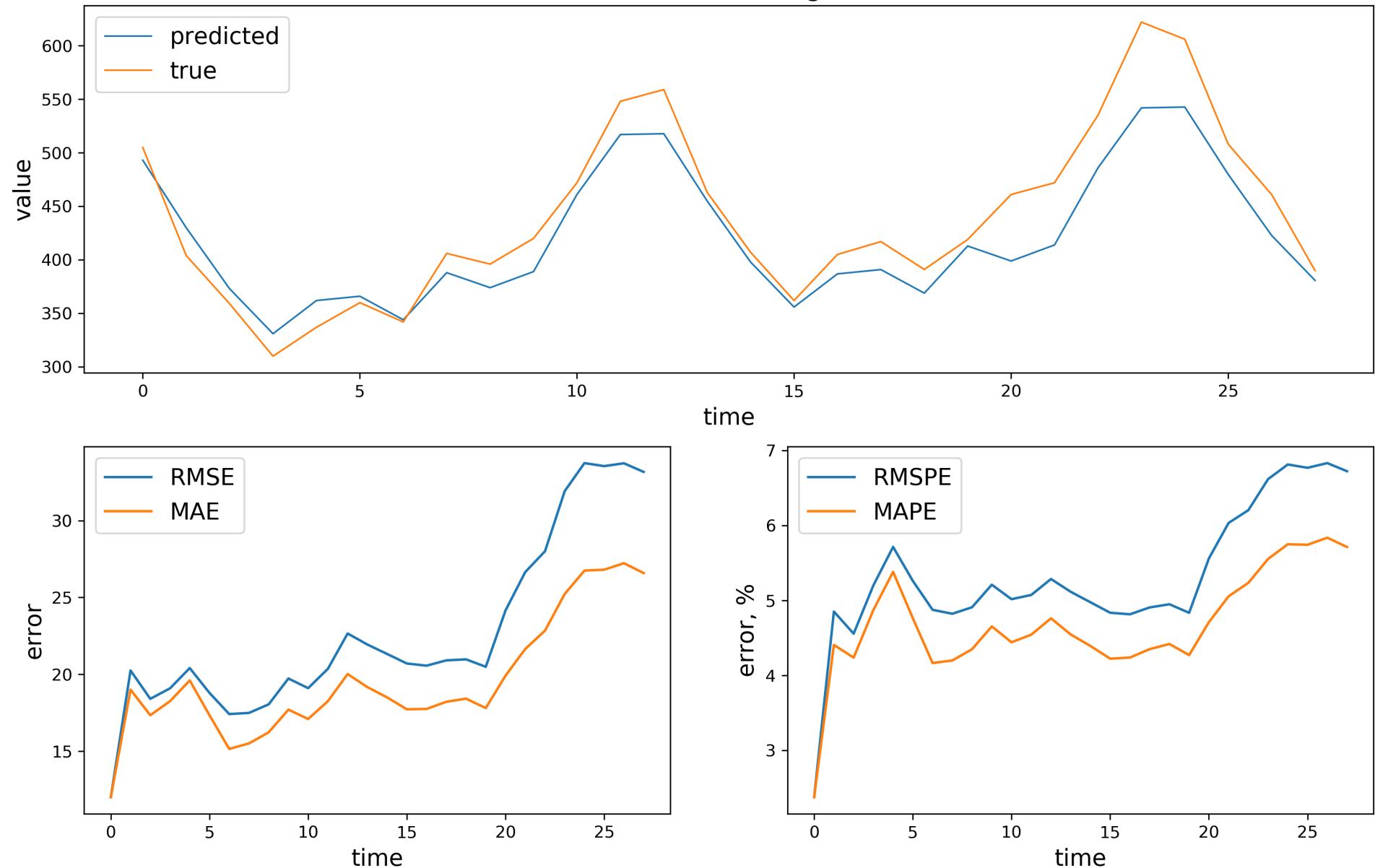
warm start online ARIMA long term forecast2



ONLINE ARIMA long term2



ONLINE ARIMA long term



Performance

Method	forecast	Dataset 1			
		RMSE	MAE	RMSPE	MAPE
Complete refit	short	1.27	1.15	0.32	0.29
Roll-window refit		1.26	1.15	0.32	0.29
Warm start		0.72	0.54	0.18	0.14
Online ARIMA-GD		0.17	0.13	0.04	0.03
Complete refit	long	2.30	1.82	0.58	0.46
Roll-window refit		2.25	1.79	0.56	0.45
Warm start		1.88	1.41	0.46	0.35
Online ARIMA-GD		1.95	1.46	0.48	0.36
Method	forecast	Dataset 2			
		RMSE	MAE	RMSPE	MAPE
Complete refit	short	48.7	41.5	11.1	9.6
Roll-window refit		48.8	41.4	11.1	9.6
Warm start		37.0	27.3	8.9	6.5
Online ARIMA-GD		26.7	22.2	6.4	5.3
Complete refit	long	55.6	45.7	13.1	11.0
Roll-window refit		48.1	41.0	10.9	9.5
Warm start		46.6	39.3	9.6	8.5
Online ARIMA-GD		33.2	26.6	6.7	5.7

Computational efficiency

Method	Time in msec/sample			
	short-term		long-term	
	fit	pred	fit	pred
Complete refit	180.9	14.3	134	13.2
Roll-window refit	130.3	12.4	127.9	10.7
Warm start	2.15	3.61	0.094	0.197
Online ARIMA-GD	0.0074	0.0065	0.0079	0.0086

Conclusion

We empirically compared our four online ARIMA algorithms, in which the promising results on two different datasets validate that all of the algorithms are effective and promising for time series prediction. Online-GD ARIMA method shows significantly decreasing in computational time results and lowest errors. Online-GD ARIMA is the best way to implement online time series forecasting for datasets with high frequency or, in other words, computationally complex problem.

Further research of this topic will include practical task analysis and improvements of implemented algorithms attempts for online: time reduction without reduced forecasting accuracy.

Thank you for attention!
Questions?

```
In [16]: 1 print("Dickey-Fuller criterion: p = {0}, used lag = {1}".format(round(sm.tsa.stattools.adfuller(train['CO2 [ppm]'])[1],4)
2 ,sm.tsa.stattools.adfuller(train['CO2 [ppm]'])[2]))
Dickey-Fuller criterion: p = 0.999, used lag = 14
```

ADF test confirms the non-stationarity hypothesis with a high degree of confidence. Let us find seasonal difference and conduct ADF test again:

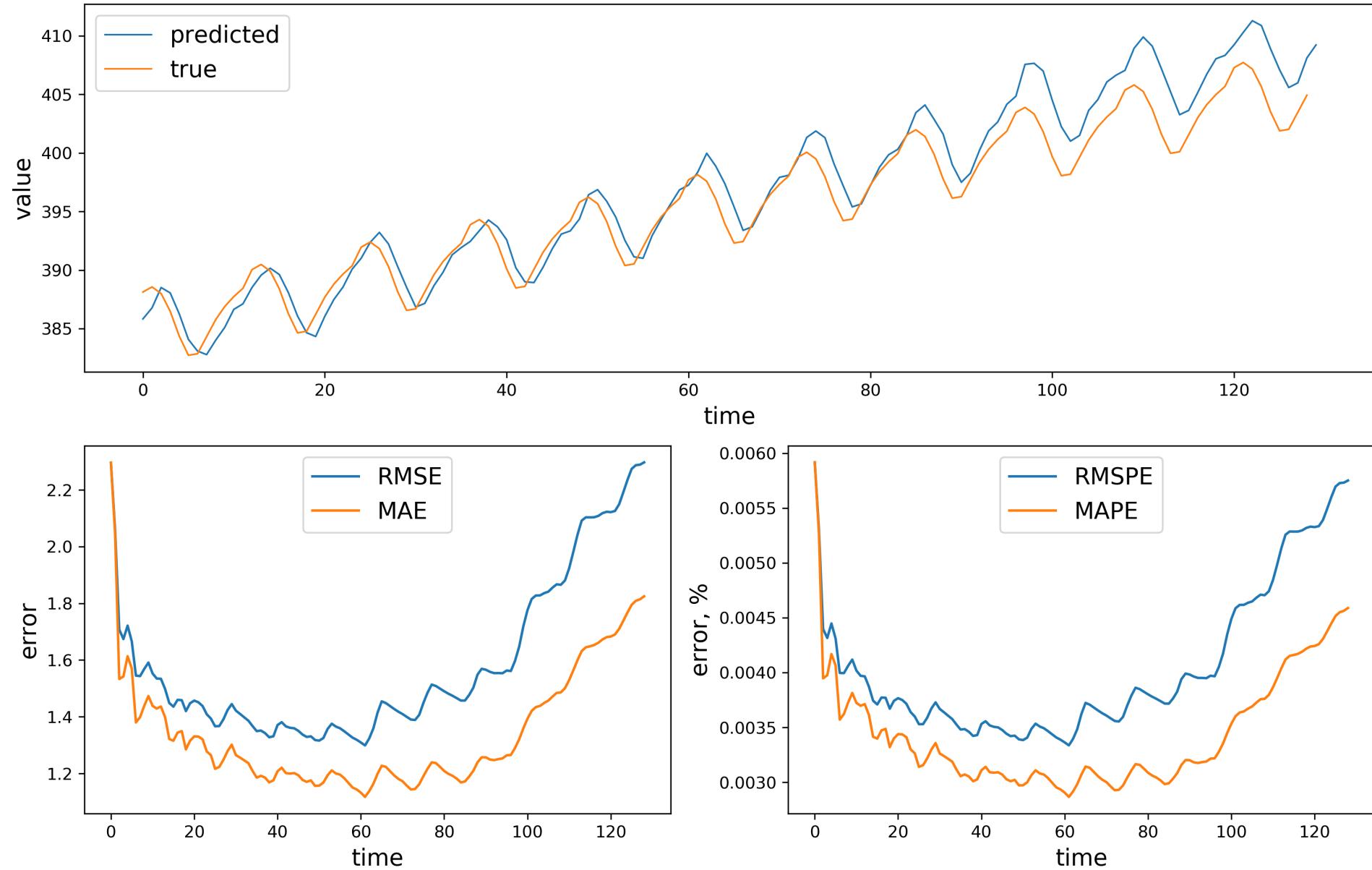
```
In [17]: 1 train['CO2 [ppm]_sdiff'] = train['CO2 [ppm]'] - train['CO2 [ppm]'].shift(12)
2 print("Dickey-Fuller criterion: p = {0}, used lag = {1}".format(round(sm.tsa.stattools.adfuller(train['CO2 [ppm]_sdiff'])[12],
3 ,sm.tsa.stattools.adfuller(train['CO2 [ppm]_sdiff'][12:])[2])))
Dickey-Fuller criterion: p = 0.0005, used lag = 13
```

ADF test confirms the non-stationarity hypothesis with a low degree of confidence. Let us find first difference of the time series to delete trend from the data:

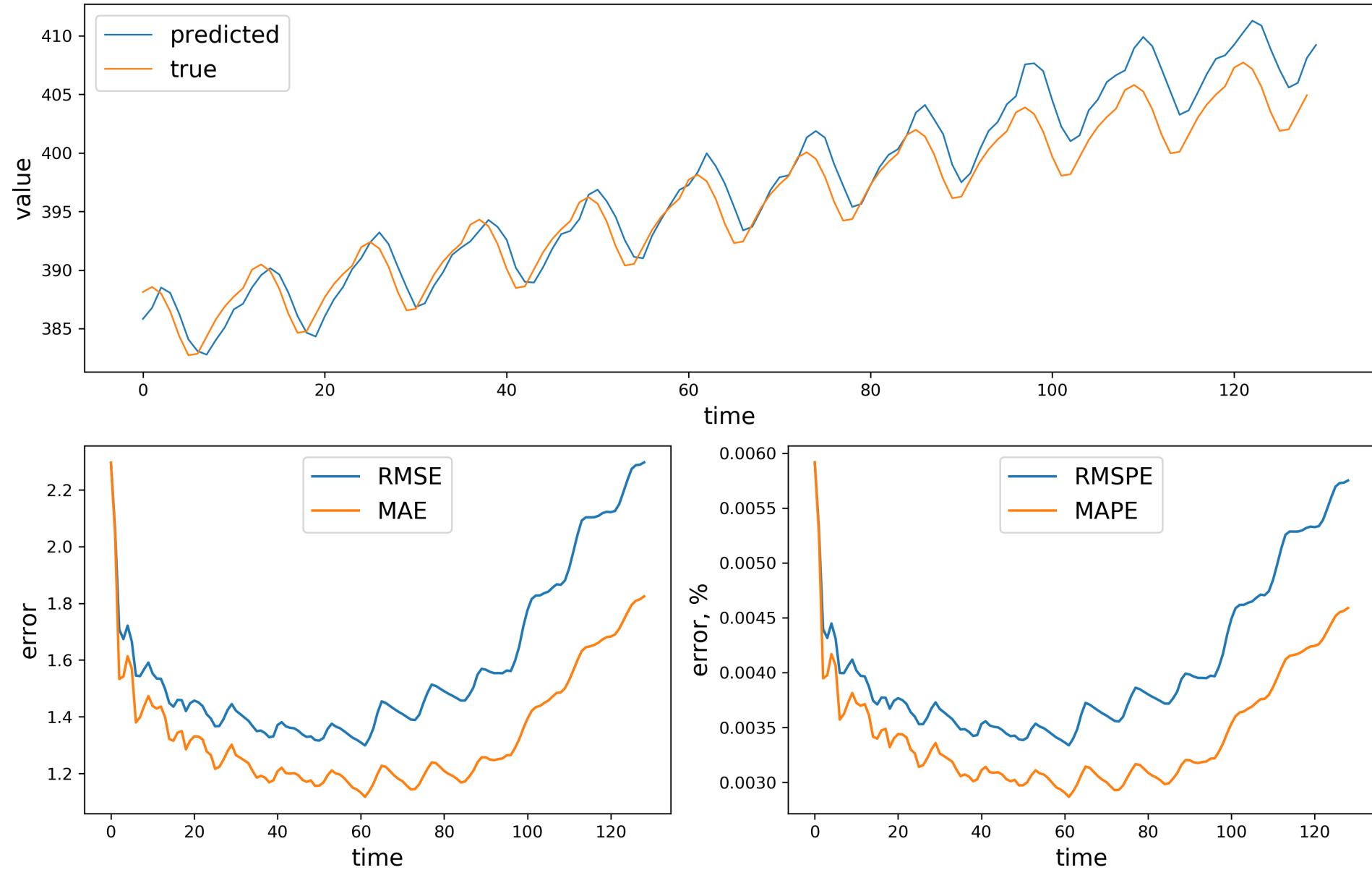
```
In [18]: 1 train['CO2 [ppm]_sdiff_diff'] = train['CO2 [ppm]_sdiff'] - train['CO2 [ppm]_sdiff'].shift(1)
2 print("Dickey-Fuller criterion: p = {0}, used lag = {1}".format(round(sm.tsa.stattools.adfuller(train['CO2 [ppm]_sdiff_diff'],
3 ,sm.tsa.stattools.adfuller(train['CO2 [ppm]_sdiff_diff'][13:]))
```

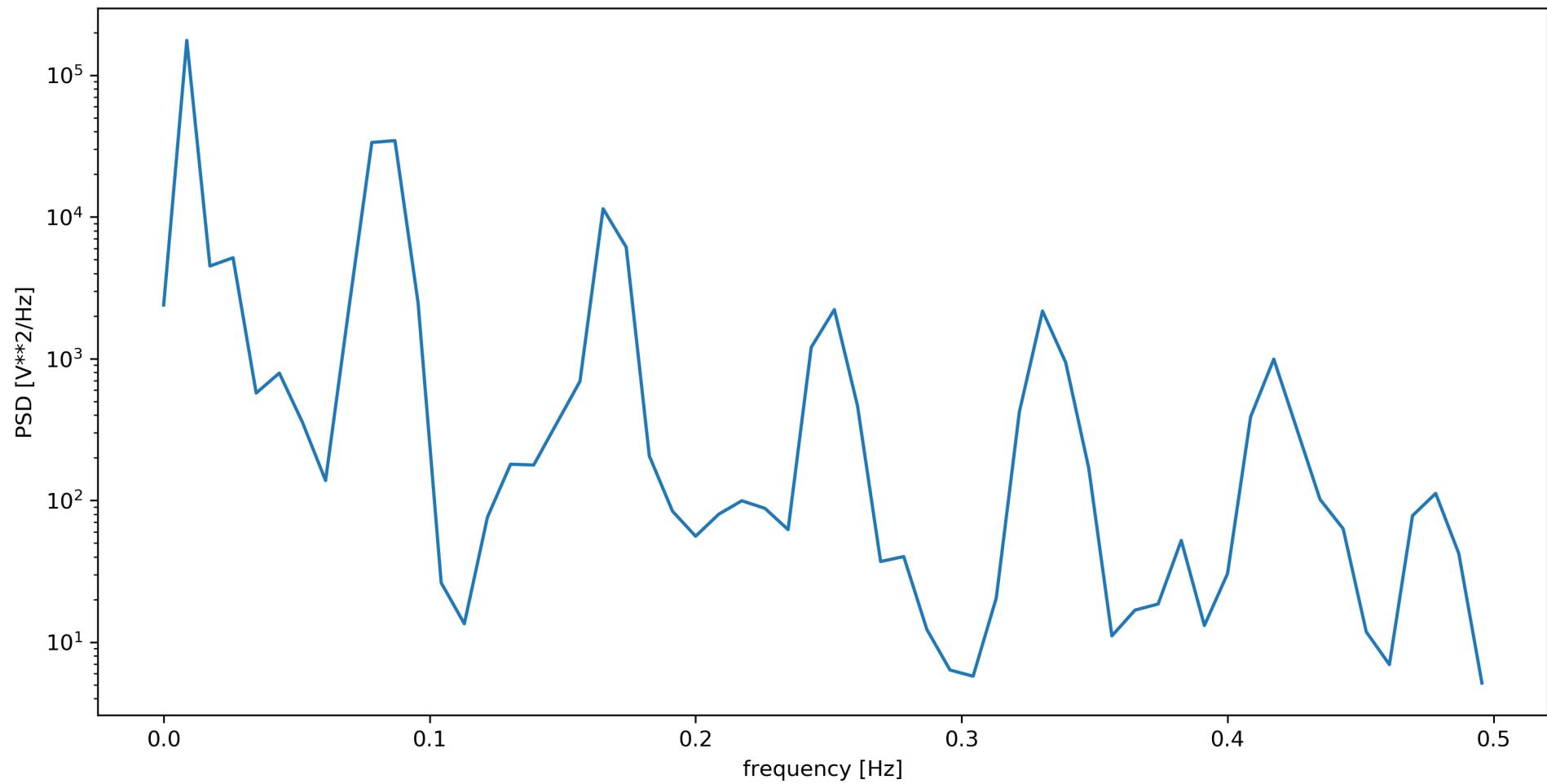
Dickey-Fuller criterion: p = 0.0, used lag = 12

simple ARIMA long term forecast



simple ARIMA long term forecast





ONLINE ARIMA long term

