Supplementary Materials of ”Handling Dynamic Multi-objective Optimization Environments via Layered Prediction and Subspace-based Diversity Maintenance”

Yaru Hu, Jinhua Zheng, Shouyong Jiang, Shengxiang Yang, Juan Zou

I. Appendix A

A. Performance Metrics

1) *Inverted Generational Distance (IGD):* IGD metric is a comprehensive metric testing convergence and diversity of DMOEAs. It can be expressed by the following formula:

$$IGD(POF_t, P_t) = \frac{\sum_{v \in POF_t} d(v, P_t)}{|POF_t|} \tag{1}$$

where $d(v, P_t) = min_{u \in P_t} \sqrt{\sum_{j=1}^{m} (f_j^v - f_j^u)^2}$ is the minimum Euclidian distance between v and the point in P_t; P_t is the obtained solution set by the algorithm, and POF_t is a point set that is uniformly sampled from the true POF at time t. The lower IGD value, the better the convergence and distribution of the obtained solution set. For DMOPs, the MIGD metric is defined as the average of the IGD values for few timesteps in a run:

$$MIGD(POF_t, P_t) = \frac{1}{|T|} \sum_{t \in T} IGD(POF_t, P_t) \tag{2}$$

where T is a set of discrete-time points in a run and $|T|$ is represents the cardinality of T.

2) *Generational Distance (GD):* GD performance metric is used to measure the convergence of the discovered solutions by the algorithm. GD can be expressed by the following formula:

$$GD(POF_t, P_t) = \frac{\sum_{v \in P_t} d(POF_t, v)}{|P_t|} \tag{3}$$

where $d(POF_t, v) = min_{u \in POF_t} \sqrt{\sum_{j=1}^{m} (f_j^u - f_j^v)^2}$ is the minimum Euclidian distance between the point in POF_t and v; P_t is the obtained solution set by the algorithms and POF_t is a set of points evenly selected from the true POF at time t. The lower GD value is, the better convergence the algorithms have.
This metric is modified similar to IGD to act as a performance metric for evaluating DMOEAs:

$$MGD(POF_t, P_t) = \frac{1}{|T|} \sum_{t \in T} GD(POF_t, P_t) \tag{4}$$

Algorithm 1 Pseudo-codes for LPSDM

Input: current population, $\mathscr{P}$; population size, N; weight vectors, $\mathscr{W}$; archive, $\mathscr{V}$; time index, t.
Output: $\mathscr{P}$.

▶ **Initialization:**

Step 1. Initialize a population $\mathscr{P}$ by uniform random initialization within the decision space. Afterward, Evaluate all solutions in $\mathscr{P}$ in the objective space.
Step 2. Initialize K weight vectors $\mathscr{W} = w_1, \cdots, w_K$ by uniform initialization within the objective space. Generate K subspaces (i.e., $\triangle_1, \cdots, \triangle_K$.) by resorting to the vector set $\mathscr{W}$ on the objective space.
Step 3. Initialize an empty archive $\mathscr{V} \leftarrow \emptyset$ and an empty set $\mathscr{E} \leftarrow \emptyset$.
Step 4. Initialize a stopping criterion.
Step 5. Randomly initialize detector individuals in the decision space to change detection.

▶ **Update**:

Step 1. **Change Detection:** Assumes an environmental change if the objective values of random detectors are different before and after reevaluation.
Step 2. If an environmental change has been detected, the algorithm executes a layered prediction (LP) strategy (i.e., the following Step 3). Otherwise, the algorithm proceeds to a subspace-based diversity maintenance (SDM) strategy (i.e., the following Step 4).
Step 3. **Layered prediction (LP) strategy:**
- Step 3.1 Divide $\mathscr{P}$ into three subpopulations, i.e. $Sub1$, $Sub2$ and $Sub3$, by nondominated sorting.
- Step 3.2 Update solutions in $Sub1$, $Sub2$, and $Sub3$ by different strategies, respectively.
- Step 3.3 $\mathscr{P} \leftarrow \emptyset$ and $\mathscr{P} \leftarrow Sub1 \bigcup Sub2 \bigcup Sub3$.

%% Note that LPs' implementation process and key points have been introduced in detail in Section IV-A and Algorithm 1 of the main body. %%

Step 4. **The Subspace-based Diversity Maintenance (SDM) Strategy:**
- Step 4.1 Save the nondominated individuals of $\mathscr{P}$ to $\mathscr{V}$ and set $\mathscr{Q} \leftarrow \emptyset$.
- Step 4.2 Associate each individual of $\mathscr{P}$ with a unique subspace according to the minimum distance from the individual to the weight vectors.
- Step 4.3 Realize mating selection and generating Offspring $\mathscr{Q}$, which performs:
  - Step 4.3.1 gap filling;
  - Step 4.3.2 probability-based reproduction.
- Step 4.4 Update the population $\mathscr{P}$ on the combined population $\mathscr{P} \bigcup \mathscr{Q}$ by the environmental selection method.

%% Note that the implementation process and key points during SDM have been more introduced in detail in Section IV-B and Algorithm 2 of the main body.%%

▶ **Stopping Criteria:** If the stopping condition is met, then stop and output the final $\mathscr{P}$. Otherwise, go to ▶ **Update**.

3) *Schotts Spacing Metric (SP):* The SP performance metric is adopted to measure the distribution of the discovered

TABLE I: Mean and standard deviation values of MGD obtained by five algorithms

Problems	(n_t, τ_t)	LPSDM	DNSGA-II-A	DNSGA-II-B	SGEA	MOEA/D	PPS	MoE
FDA2	$(10, 10)$	5.18e-2(3.31e-4)	3.31e-1(2.08e-3)‡	6.11e-2(1.13e-3)‡	**4.36e-2(2.68e-4)**	5.09e-2(8.98e-4)‡	5.88e-2(1.13e-3)‡	2.78e-1(1.41e-3)‡
	$(10, 20)$	4.98e-2(3.30e-4)	3.27e-1(1.57e-3)‡	6.15e-2(8.44e-4)‡	**4.37e-2(2.14e-4)**	5.09e-2(5.50e-4)‡	5.66e-2(9.74e-4)‡	2.76e-1(5.76e-4)‡
	$(10, 30)$	4.91e-2(1.73e-4)	3.26e-1(1.10e-3)‡	6.17e-2(6.04e-4)‡	**4.38e-2(2.23e-4)**	5.04e-2(1.73e-4)‡	5.64e-2(1.15e-3)‡	2.76e-1(2.45e-4)‡
FDA3	$(10, 10)$	1.02e-1(5.00e-4)	5.37e-2(2.66e-3)	1.13e-1(4.64e-3)†	**4.05e-2(1.95e-3)**	7.85e-2(4.57e-3)	1.98e-1(1.88e-1)‡	7.00e-1(3.37e-2)‡
	$(10, 20)$	1.01e-1(5.00e-4)	4.88e-2(2.12e-4)	5.04e-2(1.44e-3)	**3.91e-2(1.23e-3)**	7.79e-2(1.26e-3)	5.66e-2(5.46e-3)	5.88e-1(4.95e-2)‡
	$(10, 30)$	1.03e-1(9.57e-4)	4.82e-2(2.29e-4)	4.22e-2(2.56e-3)	**4.25e-2(1.27e-3)**	7.60e-2(2.28e-3)	7.64e-2(1.01e-2)	5.02e-1(6.31e-2)‡
dMOP1	$(10, 10)$	**1.52e-3(3.76e-4)**	9.91e-2(2.73e-2)‡	1.31e-1(2.23e-2)‡	3.31e-3(6.96e-4)‡	5.59e-3(5.73e-3)‡	1.20e-1(7.89e-2)‡	5.97e-2(1.53e-2)‡
	$(10, 20)$	**1.25e-3(1.65e-4)**	2.01e-2(4.28e-3)‡	1.55e-2(4.45e-3)‡	1.34e-3(1.45e-4)‡	1.53e-3(5.65e-5)‡	6.63e-3(4.61e-3)‡	1.25e-2(2.94e-3)‡
	$(10, 30)$	**1.02e-3(4.09e-5)**	1.07e-2(2.27e-3)‡	2.57e-3(4.43e-4)‡	1.11e-3(3.62e-5)†	1.44e-3(2.80e-4)‡	3.23e-3(1.40e-4)‡	5.22e-3(9.85e-4)‡
dMOP2	$(10, 10)$	**1.54e-3(6.13e-5)**	3.05e-1(7.61e-2)‡	1.53e-1(1.36e-2)‡	1.08e-2(5.38e-4)‡	1.87e-2(2.63e-4)‡	2.81e-1(5.13e-2)‡	1.32e-1(1.90e-3)‡
	$(10, 20)$	**1.37e-3(3.00e-5)**	3.87e-2(5.00e-3)‡	2.02e-2(6.69e-4)‡	5.11e-3(1.35e-4)‡	7.45e-3(1.24e-4)‡	4.77e-2(4.60e-2)‡	3.76e-1(2.71e-3)‡
	$(10, 30)$	**1.33e-3(1.05e-4)**	1.70e-2(1.32e-3)‡	7.96e-3(3.49e-4)‡	3.32e-3(7.45e-5)‡	4.64e-3(2.03e-4)‡	1.67e-2(2.37e-2)‡	1.15e-1(5.98e-4)‡
dMOP3	$(10, 10)$	**1.59e-3(3.94e-5)**	2.03e-2(1.62e-3)‡	1.65e-1(1.00e-2)‡	9.25e-3(3.18e-4)‡	1.93e-2(4.71e-4)‡	1.26e-1(9.65e-2)‡	3.91e-1(5.10e-3)‡
	$(10, 20)$	**1.59e-3(2.44e-5)**	5.89e-3(1.48e-4)‡	1.64e-2(7.37e-4)‡	4.72e-3(1.45e-4)‡	6.78e-3(1.63e-4)‡	4.27e-2(1.94e-2)‡	3.76e-1(2.71e-3)‡
	$(10, 30)$	**1.53e-3(3.31e-5)**	3.72e-3(7.23e-5)‡	4.87e-3(1.82e-4)‡	3.22e-3(5.56e-5)‡	4.07e-3(2.84e-4)‡	1.65e-2(5.66e-3)‡	3.71e-1(1.58e-3)‡

‡ and † indicate LPSDM performs significantly better than and equivalently to the corresponding algorithm, respectively.

TABLE II: Mean and standard deviation values of MSP obtained by seven algorithms

Problems	(n_t, τ_t)	LPSDM	DNSGA-II-A	DNSGA-II-B	SGEA	MOEA/D	PPS	MoE
FDA2	$(10, 10)$	**6.69e-3(1.39e-3)**	6.74e-3(4.05e-4)‡	6.80e-3(5.05e-4)‡	1.13e-2(1.60e-3)‡	1.66e-2(3.02e-3)‡	6.86e-3(6.67e-4)†	8.92e-3(1.05e-4)‡
	$(10, 20)$	4.95e-3(8.32e-4)	5.51e-3(3.26e-4)‡	5.64e-3(2.87e-4)‡	8.98e-3(9.07e-4)‡	1.67e-2(2.39e-3)‡	**4.77e-3(1.44e-4)**	7.54e-3(5.77e-5)‡
	$(10, 30)$	5.11e-3(6.09e-4)	5.30e-3(1.20e-4)†	5.47e-3(9.04e-5)‡	6.61e-3(2.04e-3)‡	1.81e-2(4.27e-4)‡	**4.68e-3(7.32e-5)**	3.19e-3(2.64e-5)
FDA3	$(10, 10)$	**4.64e-3(2.86e-4)**	2.67e-2(1.01e-2)‡	8.68e-3(9.85e-4)‡	3.98e-2(2.86e-4)‡	7.31e-3(4.70e-4)‡	1.30e-2(2.80e-3)‡	6.30e-2(1.04e-2)‡
	$(10, 20)$	**5.15e-3(1.74e-4)**	2.67e-2(2.51e-3)‡	5.81e-3(2.38e-4)‡	3.31e-2(1.54e-3)‡	7.04e-3(2.57e-4)‡	6.00e-3(2.66e-4)‡	6.75e-2(1.92e-2)‡
	$(10, 30)$	**4.62e-3(4.10e-4)**	2.40e-2(4.27e-3)‡	5.79e-3(4.02e-4)‡	3.24e-2(1.01e-3)‡	7.01e-3(2.65e-4)‡	4.77e-3(3.56e-4)†	5.75e-2(1.76e-2)‡
dMOP1	$(10, 10)$	**3.21e-3(1.03e-4)**	2.05e-2(9.76e-3)‡	1.43e-2(3.80e-3)‡	4.17e-3(4.71e-4)‡	5.87e-3(7.02e-4)‡	2.06e-2(7.10e-3)‡	9.27e-3(1.42e-3)‡
	$(10, 20)$	**3.29e-3(2.37e-4)**	7.40e-3(2.25e-3)‡	7.90e-3(1.81e-3)‡	3.61e-3(5.06e-4)‡	5.02e-3(5.50e-5)‡	4.86e-3(2.13e-3)‡	4.55e-3(2.82e-4)‡
	$(10, 30)$	**3.14e-3(7.11e-5)**	4.61e-3(1.25e-3)‡	6.13e-3(9.11e-4)‡	3.19e-3(1.45e-4)†	4.31e-3(2.98e-5)‡	3.22e-3(8.57e-5)†	3.66e-3(8.63e-5)‡
dMOP2	$(10, 10)$	**3.02e-3(9.57e-6)**	4.72e-2(2.65e-2)‡	1.24e-2(9.72e-4)‡	8.29e-3(9.65e-4)‡	6.29e-3(1.84e-4)‡	2.20e-2(2.92e-3)‡	8.51e-3(4.17e-4)‡
	$(10, 20)$	**3.04e-3(2.06e-5)**	9.62e-3(9.62e-4)‡	6.80e-3(1.98e-4)‡	4.90e-3(6.81e-4)‡	6.43e-3(1.34e-4)‡	6.93e-3(1.61e-3)‡	6.50e-3(1.18e-4)‡
	$(10, 30)$	**3.02e-3(2.82e-5)**	8.15e-3(4.45e-3)‡	5.36e-3(1.79e-4)‡	3.79e-3(1.07e-4)‡	6.05e-3(8.96e-5)	3.84e-3(9.82e-4)‡	5.96e-3(3.28e-5)‡
dMOP3	$(10, 10)$	**3.33e-3(3.31e-5)**	9.89e-3(8.80e-4)‡	9.10e-3(4.68e-4)‡	7.11e-3(3.15e-4)‡	8.20e-3(6.18e-4)‡	1.61e-2(7.45e-3)‡	1.18e-2(5.12e-4)‡
	$(10, 20)$	**3.25e-3(2.36e-5)**	5.91e-3(5.38e-4)‡	4.82e-3(3.98e-4)‡	4.34e-3(3.42e-4)‡	7.32e-3(8.80e-5)‡	7.42e-3(3.65e-3)‡	1.08e-2(1.28e-4)‡
	$(10, 30)$	**3.28e-3(2.70e-5)**	5.20e-3(7.56e-5)‡	4.60e-3(3.72e-4)‡	3.57e-3(8.64e-5)‡	6.67e-3(1.37e-4)‡	5.06e-3(7.13e-4)‡	1.07e-2(1.00e-4)‡

‡ and † indicate LPSDM performs significantly better than and equivalently to the corresponding algorithm, respectively.

POF. The SP can be expressed by the following formula:

$$SP = \sqrt{\frac{1}{|P_t| - 1} \sum_{i=1}^{|P_t|} (D_i - \overline{D})^2} \quad (5)$$

where D_i is the Euclidean distance between the ith member in P_t and its nearest member in P_t, and $\overline{D}$ is the average value of D_i. SP measures how evenly the solutions in $|P_t|$ are distributed.

This metric is modified similar to IGD to act as a performance metric for evaluating DMOEAs:

$$MSP = \frac{1}{|T|} \sum_{t \in T} SP \quad (6)$$

4) *Hypervolume Difference(HVD):* The HVD measures the gap between the hypervolume of the obtained POF and that of the true POF.

$$HVD(POF_t, P_t) = HV(POF_t) - HV(P_t) \quad (7)$$

where P_t is the solution obtained by the algorithm at time t and POF_t is the solution of the true POF at t time. HV(S) is the hypervolume of a set S. The reference point for the computation of hypervolume is $(z_1^t + 0.5, z_2^t + 0.5, ..., z_M^t + 0.5)$, where z_j^t is the maximum value of the jth objective of the true POF at t time and M is the number of objectives.

For DMOPs, the MHVD metric is defined as the average of the HVD values for few timesteps in a run:

$$MHVD(POF_t, P_t) = \frac{1}{|T|} \sum_{t \in T} HVD(POF_t, P_t) \quad (8)$$

B. Performance Compared to Other DMOEAs

MOEA/D: MOEA/D has been widely used in most the literature for comparative studies. This is because MOEA/D maintains a good diversity of the population by means of the diversity of subproblems; in the meantime, it also accelerates the convergence of the population by defining a neighborhood for each subproblem and the corresponding mating selection and solution update within this neighborhood. In this paper, the tchebycheff approach is adopted as the aggregation function for MOEA/D because it has been recently proved to provide better distribution than its original version.

PPS: For dealing with DMOPs, Zhou et al. proposed a strategy to predict a whole population using historical

TABLE III: Mean and standard deviation values of MIGD obtained by seven algorithms

Problems	(n_t, τ_t)	LPSDM	DNSGA-II-A	DNSGA-II-B	SGEA	MOEA/D	PPS	MoE
FDA2	(10, 10)	**2.85e-2(5.77e-5)**	3.67e-1(1.35e-3)‡	2.99e-2(1.65e-4)‡	2.92e-2(5.92e-4)†	4.70e-2(1.72e-2)‡	3.48e-2(7.89e-4)‡	2.78e-1(1.41e-3)‡
	(10, 20)	2.88e-2(2.01e-4)	3.65e-1(7.06e-4)‡	2.90e-2(2.04e-4)†	**2.86e-2(4.08e-5)**	4.91e-2(1.27e-2)‡	3.19e-2(4.12e-4)‡	2.76e-1(5.76e-4)‡
	(10, 30)	2.89e-2(9.57e-5)	3.64e-1(3.28e-5)‡	2.88e-2(1.29e-4)†	**2.86e-2(5.48e-5)**	4.08e-2(1.56e-3)‡	3.10e-2(3.00e-4)‡	2.76e-1(2.45e-4)‡
FDA3	(10, 10)	**1.32e-2(3.38e-3)**	8.97e-2(8.66e-3)‡	8.82e-2(7.44e-3)‡	2.02e-2(5.09e-3)‡	5.72e-2(3.73e-3)‡	9.03e-2(1.05e-1)‡	7.00e-1(3.37e-2)‡
	(10, 20)	1.57e-2(1.93e-3)	8.17e-2(4.62e-3)‡	2.99e-2(7.35e-3)‡	**1.37e-2(2.04e-3)**	3.40e-2(5.76e-3)‡	1.46e-2(4.08e-4)	5.88e-1(4.95e-2)‡
	(10, 30)	1.87e-2(1.78e-3)	8.09e-2(3.50e-3)‡	2.12e-2(3.65e-3)‡	9.57e-3(1.51e-3)	1.93e-2(2.98e-3)‡	**8.33e-3(1.00e-3)**	5.02e-1(6.31e-2)‡
dMOP1	(10, 10)	**1.01e-2(4.01e-3)**	7.14e-2(6.47e-2)‡	7.91e-2(3.11e-2)‡	2.68e-2(8.52e-3)‡	3.40e-2(2.19e-2)‡	4.78e-2(4.09e-2)‡	5.97e-2(1.53e-2)‡
	(10, 20)	**5.10e-3(1.50e-3)**	3.44e-2(2.68e-2)‡	3.13e-2(1.29e-2)‡	1.29e-2(1.74e-3)‡	8.31e-3(4.19e-3)‡	7.88e-3(2.54e-3)‡	1.25e-2(2.94e-3)‡
	(10, 30)	**3.97e-3(3.94e-5)**	1.08e-1(1.44e-1)‡	1.63e-2(8.93e-3)‡	8.17e-3(2.17e-3)‡	5.51e-3(6.18e-4)‡	5.58e-3(5.41e-4)‡	5.22e-3(9.85e-4)‡
dMOP2	(10, 10)	**5.33e-3(2.06e-3)**	1.73e-1(2.74e-2)‡	1.25e-1(1.48e-2)‡	1.78e-2(4.03e-3)‡	3.35e-2(9.67e-3)‡	2.00e-1(5.77e-2)‡	1.32e-1(1.90e-3)‡
	(10, 20)	**4.12e-3(2.38e-5)**	6.44e-2(4.53e-2)‡	2.09e-2(7.17e-4)‡	8.33e-3(1.40e-3)‡	1.18e-2(3.83e-4)‡	3.62e-2(3.72e-2)‡	1.18e-1(1.46e-3)‡
	(10, 30)	**4.10e-3(8.38e-5)**	3.54e-2(3.42e-2)‡	9.85e-3(2.77e-4)‡	6.07e-3(8.38e-4)‡	8.24e-3(5.68e-4)‡	1.40e-2(1.47e-2)‡	1.15e-1(5.98e-4)‡
dMOP3	(10, 10)	**4.08e-3(2.21e-5)**	2.00e-2(2.65e-3)‡	2.71e-1(1.47e-2)‡	1.39e-2(3.69e-3)‡	3.91e-2(1.09e-2)‡	8.36e-2(5.07e-2)‡	3.91e-1(5.10e-3)‡
	(10, 20)	**4.09e-3(4.61e-5)**	7.73e-3(1.78e-4)‡	1.58e-1(7.22e-3)‡	7.85e-3(2.87e-3)‡	1.38e-2(2.14e-3)‡	2.80e-2(5.41e-3)‡	3.76e-1(2.71e-3)‡
	(10, 30)	**4.03e-3(2.44e-5)**	6.12e-3(6.02e-5)‡	1.20e-1(3.88e-3)‡	5.25e-3(1.46e-4)‡	8.23e-3(7.33e-4)‡	1.26e-2(2.37e-3)‡	3.71e-1(1.58e-3)‡

‡ and † indicate LPSDM performs significantly better than and equivalently to the corresponding algorithm, respectively.

TABLE IV: Mean and standard deviation values of HVD obtained by seven algorithms

Problems	(n_t, τ_t)	LPSDM	DNSGA-II-A	DNSGA-II-B	SGEA	MOEA/D	PPS	MoE
FDA2	$(10, 10)$	**3.27e-2(5.00e-5)**	5.650e-1(1.85e-3)‡	3.46e-2(1.92e-4)‡	3.34e-2(2.25e-4)†	3.66e-2(2.38e-3)‡	4.13e-2(1.10e-3)‡	5.35e-1(2.51e-3)‡
	$(10, 20)$	**3.29e-2(7.60e-5)**	5.590e-1(7.64e-4)‡	3.34e-2(8.96e-5)‡	3.30e-2(5.16e-5)†	3.53e-2(9.84e-4)‡	3.69e-2(5.74e-4)‡	5.28e-1(2.51e-3)‡
	$(10, 30)$	**3.30e-2(5.34e-5)**	5.570e-1(1.92e-5)‡	3.32e-2(7.17e-5)‡	3.30e-2(5.48e-5)†	3.47e-2(9.57e-5)‡	3.57e-2(4.69e-4)‡	5.22e-1(2.06e-4)‡
FDA3	$(10, 10)$	**5.13e-1(1.00e-3)**	5.620e-1(1.50e-2)‡	5.45e-1(2.14e-2)‡	6.01e-1(4.32e-3)‡	5.41e-1(2.59e-2)‡	5.42e-1(1.50e-1)‡	4.03e+0(4.37e-1)‡
	$(10, 20)$	5.14e-1(1.50e-3)	5.510e-1(6.97e-3)‡	5.12e-1(1.44e-3)‡	5.96e-1(4.04e-3)‡	5.25e-1(2.88e-2)‡	**4.92e-1(1.63e-2)**	2.60e+0(5.78e-1)‡
	$(10, 30)$	5.12e-1(1.15e-3)	5.420e-1(9.61e-3)‡	5.13e-1(6.94e-4)†	5.92e-1(6.35e-3)‡	5.27e-1(2.75e-2)‡	**4.96e-1(1.60e-2)**	1.76e+0(5.58e-1)‡
dMOP1	$(10, 10)$	**7.28e-3(9.19e-4)**	4.550e-2(9.00e-3)‡	7.07e-2(1.17e-2)‡	2.03e-2(9.28e-3)‡	4.35e-2(3.53e-2)‡	8.60e-2(5.46e-2)‡	1.19e-1(3.69e-2)‡
	$(10, 20)$	**7.52e-3(1.30e-3)**	2.680e-2(3.38e-3)‡	2.40e-2(3.78e-3)‡	1.36e-2(2.25e-3)‡	1.30e-2(4.07e-3)‡	1.75e-2(5.31e-3)‡	2.38e-2(1.96e-3)‡
	$(10, 30)$	**5.71e-3(1.74e-4)**	3.540e-2(2.21e-2)‡	1.44e-2(2.54e-3)‡	1.14e-2(1.00e-3)‡	1.07e-2(5.83e-4)‡	1.26e-2(9.43e-4)‡	1.54e-2(1.29e-3)‡
dMOP2	$(10, 10)$	**7.66e-3(1.18e-3)**	2.710e-1(1.93e-2)‡	2.86e-1(2.20e-2)‡	4.07e-2(3.00e-3)‡	8.52e-2(4.07e-3)‡	3.16e-1(6.07e-3)‡	2.70e-1(2.23e-3)‡
	$(10, 20)$	**6.70e-3(6.18e-5)**	8.690e-2(5.09e-3)‡	5.79e-2(3.01e-3)‡	2.06e-2(1.22e-3)‡	3.51e-2(4.71e-4)‡	7.70e-2(6.09e-2)‡	2.41e-1(1.27e-3)‡
	$(10, 30)$	**6.61e-3(3.38e-4)**	4.130e-2(3.41e-3)‡	2.70e-2(1.29e-3)‡	1.50e-2(1.27e-3)‡	2.31e-2(1.66e-3)‡	3.44e-2(3.53e-2)‡	2.35e-1(3.18e-4)‡
dMOP3	$(10, 10)$	**6.27e-3(8.24e-5)**	4.360e-2(3.93e-3)‡	3.99e-1(1.72e-2)‡	2.94e-2(2.12e-3)‡	6.81e-2(3.97e-3)‡	1.51e-1(9.46e-2)‡	4.97e-1(5.38e-3)‡
	$(10, 20)$	**6.28e-3(1.53e-4)**	1.630e-2(4.46e-4)‡	1.35e-1(3.54e-3)‡	1.58e-2(1.32e-3)‡	2.88e-2(1.38e-3)‡	6.19e-2(1.66e-2)‡	4.75e-1(6.97e-3)‡
	$(10, 30)$	**6.08e-3(7.22e-5)**	1.180e-2(2.01e-4)‡	1.01e-1(2.30e-3)‡	1.10e-2(4.32e-4)‡	1.87e-2(2.06e-4)‡	2.68e-2(5.45e-3)‡	4.68e-1(4.34e-3)‡

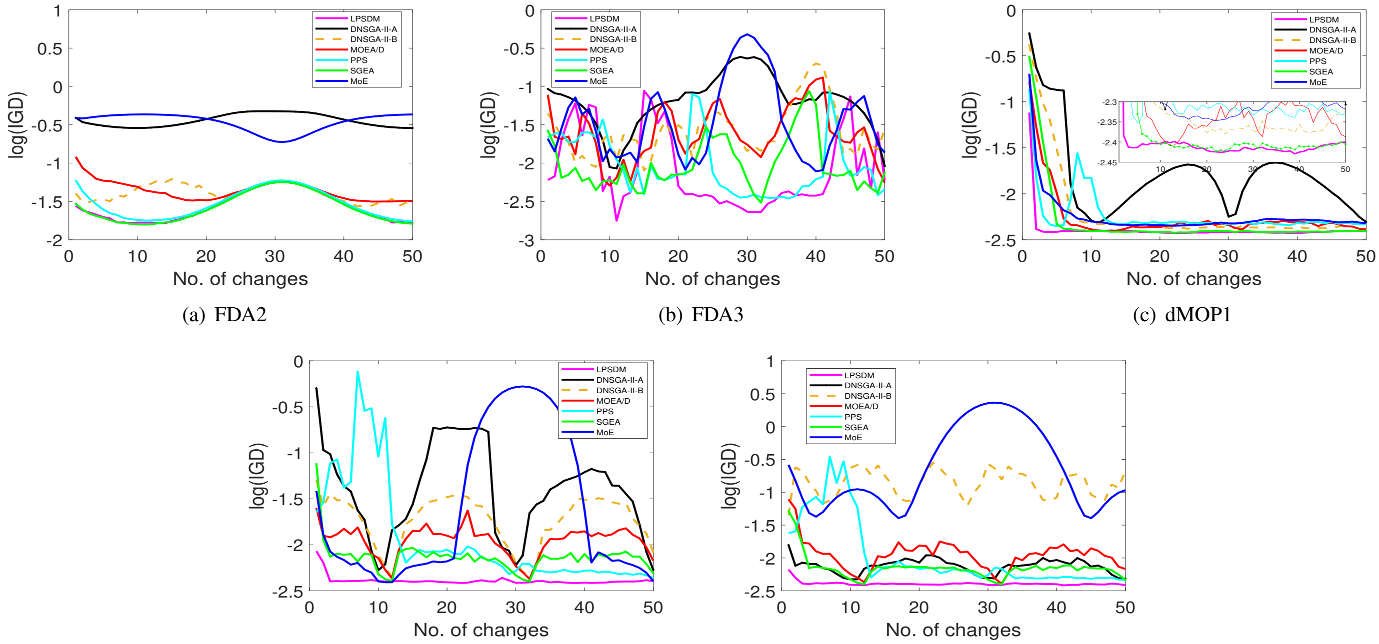
‡ and † indicate LPSDM performs significantly better than and equivalently to the corresponding algorithm, respectively.

information. The predicted population is composed of two parts: a center point and manifolds. A univariate autoregressive (AR) model learns some history information of the archived population centers over a number of continuous time series to be able to predict the next population center point. The next manifold is estimated by the last two approximated manifolds. Hence, the whole population is re-initialized by combining the current population with the anticipates center point and manifolds.

SGEA: SGEA detects environmental changes and responds to them by means of a steady-state manner. When an environmental change is detected, half of the outdated solutions with good distribution and close to the new Pareto front are relocated by the farthest first selection, and the other half of the solutions are reinitialized to provide good tracking ability.

DNSGA-II: DNSGA-II includes two versions of the dynamic NSGA-II (DNSGA-II) to effectively handle DMOPs with different characteristics. In this paper, Deb et al. presented two particular versions for DMOEAs to solve DMOPs, called NSGA-II-A and NSGA-II-B, respectively. The first type, NSGA-II-A introduces some new individuals by means of randomly creating solutions in proportion for handling DMOPs whenever there are changes. Some solutions of the current population are randomly replaced by those new individuals. The method may perform better when the degree of the environmental changes is large to address DMOPs. In contrast, the NSGA-II-B may have a pretty good effect when the degree of the change in the fitness function is small. In NSGA-II-B, some current solutions, chosen randomly, are replaced by their corresponding mutated solutions. In our experiment, the latter method (called DNSGA-II in experimental studies) is adopted as it shows slightly better performance than the former.

MoE: MoE utilizes multiple prediction mechanisms, i.e., LP, RND, KF and A-LKF, which has been introduced in detail in the literature [1], to improve the overall prediction. A gating network is applied to manage to switch among the various predictors based on performance of the predictors at different time intervals of the optimization process.



(a) FDA2 (b) FDA3 (c) dMOP1

(d) dMOP2 (e) dMOP3

Fig. 1: Evolution curves of average IGD values for seven problems with n_t =10 and τ_t=20.

II. Appendix B

This section provides some experimental data on classical FDA2, FDA3 and dMOP benchmark suites, which is trying to show that performance of LPSDM is not only fine on FDA1, FDA4 and JY benchmark suite up to date but also good at classical benchmark suites. Therefore, detailed descriptions of these experiences are given in the following, respectively. The obtained average SP, GD, IGD, and HVD results over a series of time windows and their standard deviation values are presented in Tables I, II, III and IV, respectively. Besides, the best values obtained by one of the five algorithms are highlighted in bold face. The Wilcoxon rank sum test [2] is carried out to indicate significance between different results at the 0.05 significance level. The parameters of this supplementary material considered in the experiment are referenced in Section V-D of the proposed papers except indicated parameter.

FDA2: Table I shows that the convergence of SGEA is the best across the whole process of evolution and the convergence of LPSDM and MOEA/D have no significance according to the Wilcoxon rank sum test, implying that LPSDM performs equivalently to MOEA/D. Then the metric SP is considered to detect the distribution of solutions obtained approximating the POF by different algorithms. In particular, under different frequencies of change, the SP metric of LPSDM is similar to PPS and distinctly better than DNSGA-II-A, DNSGA-II-B, SGEA and MOEA/D on FDA2, implying that LPSDM performs equivalently to PPS since there is no significance according to the Wilcoxon rank sum test. However, when dealing with FDA2, overall performance metrics IGD and HVD of LPSDM and SGEA perform best, and which have been verified by values from Tables III and IV at all frequencies of change, signifying that LPSDM and SGEA can deal with this kind of problem well in dynamic environments. In addition, we conclude that the distribution of the discovered solutions by algorithm is only a part of diversity to the corresponding algorithm, e.g., although the distribution throughout the optima obtained by PPS is the best result, its overall performance is not the best when $\tau_t = 20$ or 30.

FDA3: Table II shows that the metric SP of LPSDM is better than other algorithms across the whole process of evolution on dealing with FDA3; in the meantime, we also came to a fact that LPSDM is the worst on convergence out of all the algorithms from overall perspective. Therefore, to improve the performance of the LPSDM algorithm, convergence must be improved. On overall performance metrics of IGD and HVD (they can be gained by Tables III and IV), we conclude that the expression of LPSDM is not always the best along with decreasing the frequency of changes, signifying that LPSDM cannot handle the optimization problem FDA3 very well.

dMOP1, dMOP2 and dMOP3: Different from FDA1−FDA4, dMOP test suite is extended from the FDA suite. Tables I shows that the convergence of LPSDM is better than the other compared DMOEAs whenever the environment changes (i.e., the value of the GD metric which is obtained by LPSDM is smaller than those values that are gained by the other selected algorithms). What's more, the value of the SP metric obtained by LPSDM is smaller than those of the other selected algorithms according to Table II; therefore, the distribution of population that is received by LPSDM performs best in all compared algorithms. According to Tables III and IV, the experimental results suggested what the overall performance of LPSDM is the best since LPSDM achieves a better IGD and HVD than the other algorithms under any frequency of change.

Fig. 1 depicts the IGD profile averaged over 20 runs of

TABLE V: Mean and standard deviation values of MIGD and MHVD obtained by seven algorithms

Problems	(n_t, τ_t)	metric	LPSDM	DNSGA-II-A	DNSGA-II-B	SGEA	MOEA/D	PPS	MoE
FDA1	(5, 10)	HVD	**7.94e-3(7.32e-4)**	1.59e-1(6.98e-3)‡	1.09e+0(1.19e-1)‡	4.31e-2(2.99e-3)‡	9.64e-2(3.90e-3)‡	1.32e-1(4.49e-2)‡	2.39e-1(3.14e-3)‡
		IGD	**4.78e-3(3.44e-4)**	8.70e-2(3.86e-3)‡	9.62e-1(6.90e-2)‡	2.22e-2(6.49e-3)‡	7.59e-2(8.44e-3)‡	6.21e-2(2.30e-2)‡	1.16e-1(8.59e-4)‡
	(10, 10)	HVD	**6.14e-3(4.12e-5)**	1.64e-2(5.17e-4)‡	1.89e-1(1.67e-2)‡	1.51e-2(5.72e-4)‡	2.86e-2(6.92e-4)‡	2.31e-1(2.38e-3)‡	2.31e-1(2.38e-3)‡
		IGD	**4.14e-3(5.90e-5)**	1.83e-2(5.44e-4)‡	1.03e-1(1.75e-2)‡	1.29e-2(9.92e-4)‡	3.19e-2(3.03e-3)‡	8.24e-2(6.49e-2)‡	1.13e-1(7.68e-4)‡
	(30, 10)	HVD	**5.62e-3(1.81e-4)**	1.76e-2(1.33e-3)‡	3.94e-2(2.49e-3)‡	1.93e-2(1.20e-3)‡	3.32e-2(3.39e-3)‡	4.04e-2(1.75e-2)‡	2.63e-2(3.68e-3)‡
		IGD	**3.96e-3(4.04e-5)**	8.47e-3(4.99e-4)‡	1.77e-2(1.24e-3)‡	8.81e-3(9.82e-4)‡	1.65e-2(4.24e-3)‡	1.78e-2(7.58e-3)‡	1.02e-2(1.29e-3)‡
dMOP2	(5, 10)	HVD	**1.02e-2(1.16e-3)**	7.26e-1(4.41e-2)‡	1.01e+0(8.00e-2)‡	6.89e-2(6.58e-3)‡	1.27e-1(3.20e-3)‡	1.55e-1(7.07e-2)‡	2.00e-1(3.59e-3)‡
		IGD	**5.30e-3(4.33e-4)**	6.19e-1(4.48e-2)‡	1.24e+0(5.91e-2)‡	3.28e-2(1.16e-2)‡	7.66e-2(4.17e-2)‡	6.79e-2(3.16e-2)‡	1.32e-1(2.83e-3)‡
	(10, 10)	HVD	**6.70e-3(6.18e-5)**	8.69e-2(5.09e-3)‡	2.86e-1(2.20e-2)‡	2.06e-2(1.22e-3)‡	3.51e-2(4.71e-4)‡	7.70e-2(6.09e-2)‡	1.19e-1(3.69e-2)‡
		IGD	**5.33e-3(2.06e-3)**	1.73e-1(2.74e-2)‡	1.25e-1(1.48e-2)‡	1.78e-2(4.03e-3)‡	3.35e-2(9.67e-3)‡	2.00e-1(5.77e-2)‡	2.92e-2(7.49e-3)‡
	(30, 10)	HVD	**5.62e-3(1.30e-4)**	8.91e-2(5.51e-3)‡	6.69e-2(4.46e-3)‡	2.81e-2(1.93e-3)‡	4.84e-2(1.93e-3)‡	9.03e-2(2.99e-2)‡	3.54e-2(3.86e-3)‡
		IGD	**4.01e-3(2.50e-5)**	8.65e-2(2.23e-2)‡	2.62e-2(3.93e-3)‡	1.29e-2(3.76e-3)‡	1.69e-2(3.04e-3)‡	3.66e-2(1.51e-2)‡	1.24e-2(2.46e-3)‡
JY2	(5, 10)	HVD	**6.78e-3(9.85e-4)**	2.13e-1(2.33e-2)‡	3.46e+0(2.82e-1)‡	4.13e-2(2.21e-3)‡	3.97e-2(2.71e-3)‡	1.38e-1(2.34e-2)‡	3.24e-1(4.16e-3)‡
		IGD	**4.82e-2(3.31e-4)**	1.18e-1(9.44e-3)‡	8.95e-1(4.20e-2)‡	5.99e-2(1.57e-3)‡	5.78e-2(4.12e-4)‡	8.87e-2(6.17e-3)‡	1.76e-1(9.09e-4)‡
	(10, 10)	HVD	**6.13e-3(3.46e-5)**	2.46e-2(3.65e-4)‡	2.24e-1(2.50e-2)‡	1.36e-2(7.96e-4)‡	1.28e-2(1.13e-3)‡	4.16e-2(4.69e-2)‡	3.07e-1(5.61e-3)‡
		IGD	**5.02e-2(5.00e-5)**	5.71e-2(6.33e-4)‡	9.94e-2(6.07e-3)‡	5.66e-2(1.21e-3)‡	5.64e-2(6.35e-4)‡	9.41e-2(5.93e-2)‡	1.68e-1(1.63e-3)‡
	(30, 10)	HVD	**6.39e-3(7.22e-5)**	2.13e-2(7.03e-4)‡	3.47e-2(1.57e-3)‡	1.92e-2(1.73e-3)‡	1.91e-2(2.20e-3)‡	9.17e-2(2.33e-2)‡	2.19e-2(2.29e-3)‡
		IGD	**4.41e-2(2.01e-5)**	4.58e-2(2.93e-4)‡	6.22e-2(9.23e-4)‡	4.65e-2(9.67e-4)‡	4.66e-2(3.86e-4)‡	6.93e-2(7.84e-3)‡	4.60e-2(4.45e-4)‡
JY3	(5, 10)	HVD	2.17e-1(9.67e-2)	**4.29e-2(6.03e-3)**	4.00e-1(4.35e-3)‡	4.21e-1(1.01e-2)‡	4.01e-1(8.84e-3)‡	7.19e-1(4.04e-1)‡	2.69e-1(2.81e-1)‡
		IGD	3.16e-1(6.07e-3)	**2.95e-1(2.22e-3)**	3.26e-1(1.49e-3)‡	3.57e-1(1.07e-2)‡	3.40e-1(1.03e-2)‡	3.72e-1(5.82e-2)‡	3.17e-1(4.18e-3)†
	(10, 10)	HVD	1.13e-1(1.61e-1)	**2.80e-2(6.35e-3)**	3.86e-1(5.81e-3)‡	3.67e-1(4.76e-2)‡	3.58e-1(2.82e-2)‡	4.26e-1(4.17e-2)‡	2.40e-1(1.96e-1)‡
		IGD	3.15e-1(4.03e-4)	**2.92e-1(1.26e-2)**	3.15e-1(1.39e-3)†	3.47e-1(1.43e-2)‡	3.29e-1(1.86e-2)‡	3.39e-1(1.25e-2)‡	3.16e-1(6.93e-3)‡
	(30, 10)	HVD	1.91e-1(1.10e-1)	**3.21e-2(4.29e-3)**	3.86e-1(3.04e-3)‡	3.82e-1(3.05e-2)‡	3.78e-1(2.64e-2)‡	4.68e-1(2.04e-2)‡	2.20e-1(1.19e-1)‡
		IGD	3.26e-1(3.00e-3)	**3.00e-1(3.96e-3)**	3.45e-1(6.80e-4)‡	3.63e-1(3.44e-2)‡	3.49e-1(4.95e-2)‡	3.34e-1(4.69e-3)‡	4.27e-1(3.17e-3)‡
JY5	(5, 10)	HVD	**4.54e-3(2.38e-5)**	7.91e-2(1.09e-4)‡	9.00e-3(6.97e-4)‡	6.62e-3(7.64e-4)‡	1.04e-2(9.91e-4)‡	1.45e-2(4.85e-3)‡	1.99e-2(2.46e-3)‡
		IGD	**4.19e-3(5.77e-6)**	5.34e-2(1.80e-4)‡	6.61e-3(3.79e-4)‡	5.14e-3(4.16e-4)‡	1.10e-2(1.34e-3)‡	8.40e-3(2.73e-3)‡	1.01e-2(4.72e-4)‡
	(10, 10)	HVD	**4.69e-3(5.00e-6)**	4.24e-2(8.86e-4)‡	8.77e-3(4.49e-4)‡	5.07e-3(3.67e-4)‡	6.23e-3(1.12e-4)‡	7.77e-3(3.20e-5)‡	1.92e-2(2.54e-3)‡
		IGD	**4.20e-3(5.00e-6)**	3.06e-2(3.62e-4)‡	6.37e-3(3.24e-4)‡	5.04e-3(4.17e-4)‡	9.97e-3(1.84e-3)‡	7.24e-3(2.15e-4)‡	9.89e-3(4.54e-4)‡
	(30, 10)	HVD	**4.80e-3(3.77e-5)**	2.53e-2(8.93e-4)‡	8.55e-3(6.01e-4)‡	6.44e-3(7.63e-4)‡	8.06e-3(5.94e-4)‡	1.54e-2(3.52e-3)‡	2.02e-2(1.83e-3)‡
		IGD	**4.15e-3(9.57e-6)**	1.49e-2(6.44e-4)‡	6.29e-3(2.71e-4)‡	4.95e-3(4.31e-4)‡	8.62e-3(2.57e-4)‡	8.82e-3(1.81e-3)‡	9.45e-3(3.21e-4)‡
JY6	(5, 10)	HVD	**8.77e-1(4.51e-1)**	1.26e+1(1.45e+0)‡	1.15e+2(1.15e+1)‡	2.65e+0(2.00e-1)‡	3.96e+0(1.42e+0)‡	3.57e+1(5.31e+0)‡	1.92e+1(1.59e+0)‡
		IGD	**1.75e-1(7.10e-2)**	1.76e+0(1.25e-1)‡	6.00e+0(3.15e-1)‡	6.86e-1(3.83e-2)‡	8.16e-1(7.01e-2)‡	3.17e+0(1.70e-1)‡	2.03e+0(1.30e-1)‡
	(10, 10)	HVD	**4.10e-2(4.86e-2)**	3.97e+0(7.10e-1)‡	3.19e+1(3.67e+0)‡	3.44e-1(2.32e-2)‡	2.80e-1(4.86e-2)‡	1.79e+1(8.99e-1)‡	1.42e+1(1.08e+0)‡
		IGD	**3.72e-2(2.72e-3)**	1.70e+0(1.24e-1)‡	3.21e+0(1.81e-1)‡	5.34e-1(4.15e-2)‡	4.15e-1(3.81e-2)‡	2.67e+0(4.69e-2)‡	1.64e+0(1.35e-1)‡
	(30, 10)	HVD	**6.14e-3(4.40e-4)**	4.45e+0(6.97e-1)‡	5.09e+0(3.58e-1)‡	8.35e-1(7.59e-2)‡	5.90e-1(1.36e-1)‡	2.18e+1(1.83e+0)‡	2.79e+0(4.48e-1)‡
		IGD	**5.66e-3(2.39e-4)**	8.25e-1(6.91e-2)‡	1.03e+0(5.48e-2)‡	3.27e-1(1.93e-2)‡	2.80e-1(1.92e-2)‡	2.50e+0(1.08e-1)‡	4.18e-1(7.63e-2)‡
JY9	(5, 10)	HVD	**7.23e-3(4.54e-4)**	6.99e-1(4.21e-1)‡	5.86e-1(6.36e-2)‡	1.75e+1(3.05e+0)‡	4.66e-2(2.28e-3)‡	1.03e+0(1.13e-1)‡	1.30e-1(2.03e-2)‡
		IGD	**5.59e-3(2.34e-4)**	2.96e-1(7.47e-2)‡	2.33e-1(1.55e-2)‡	1.03e+0(9.08e-2)‡	3.30e-2(8.61e-4)‡	2.90e-1(3.94e-3)‡	4.64e-2(6.20e-3)‡
	(10, 10)	HVD	**3.97e-3(1.17e-4)**	3.55e-1(1.70e-1)‡	1.38e-1(1.45e-2)‡	4.06e-2(7.42e-3)‡	1.16e-2(1.30e-3)‡	3.92e-2(8.74e-3)‡	6.92e-2(2.44e-2)‡
		IGD	**6.45e-3(1.71e-3)**	4.71e-1(2.10e-1)‡	7.87e-2(4.79e-3)‡	3.94e-1(5.02e-2)‡	3.18e-2(1.02e-2)‡	2.61e-1(2.26e-1)‡	2.32e-2(4.19e-3)‡
	(30, 10)	HVD	**4.12e-3(4.68e-4)**	1.15e-1(6.07e-2)‡	7.00e-2(9.74e-3)‡	9.38e-1(3.69e-1)‡	2.25e-2(7.33e-3)‡	8.06e-2(2.10e-2)‡	8.00e-2(3.41e-2)‡
		IGD	**4.83e-3(2.29e-4)**	6.86e-2(3.37e-2)‡	3.56e-2(2.95e-3)‡	1.48e-1(3.94e-2)‡	1.93e-2(4.90e-3)‡	2.81e-2(6.14e-3)‡	2.46e-2(7.41e-3)‡

‡ and † indicate LPSDM performs significantly better than and equivalently to the corresponding algorithm, respectively. $\tau_t = 10$.

LPSDM, DNSGA-II-A, DNSGA-II-B, MOEA/D, PPS and SGEA in the seven test problems. And it can be clearly seen that, compared with the other algorithms, LPSDM responds to changes more stably and recovers faster in most of the test problems, thereby obtaining higher convergence performance. Finally, we can make the following observations from Fig. 1.

First, when $t \geq 10$, LPSDM considerably outperforms the other strategies for almost all the test instances in terms of the IGD values. In addition, when $t \geq 10$, the IGD values of solutions obtained by LPSDM slightly fluctuate at a low level, indicating that the proposed method is able to track the environmental change efficiently and effectively.

Second, for FDA3, although the performance of LPSDM outperforms the other strategies among $t \leq 40$ and $t \geq 20$, the fluctuation of the LPSDM curve is very big, implying that the performance of LPSDM is unstable. In other words, the great improvement of LPSDM is needed to deal with FDA3. For dMOP2 and dMOP3, the performance of LPSDM is the best and most stable across the whole process of evolution. However, the IGD trend on LPSDM is similar to the IGD trend on SGEA when $t \leq 10$ (i.e., the value of IGD on LPSDM is similar to the value of IGD on SGEA when $t \leq 10$.).

The results indicate that the proposed LPSDM achieves consistently better results than the compared state-of-the-art algorithms in solving most selected DMOPs. In summary, the proposed LPSDM is highly competitive when tackling both simple and complex problems.

III. Appendix C

A. Influence of Severity of Change

In this section, to examine the influence of the severity of change on the algorithm's performance, experiments are carried out on FDA1, dMOP2, JY1, JY2, JY3, JY5, JY6 and JY9 problems with τ_t fixed to 10, and n_t set to 5, 10 and 30, which represent severe, moderate, and slight environmental changes, respectively. Experimental results of five algorithms on the HVD and IGD metrics are given in Table V. Analysis of the IGD and HVD metrics indicates that the proposed algorithm is capable of significantly improving the dynamic optimization performance.

It can be observed from Table V that all the algorithms are very sensitive to the severity of change, as can be seen from the improvement of the metrics when increasing the value of n_t. For different severity levels, LPSDM is able to produce impressive performance and wins on the majority of instances, and this algorithm is mainly exceeded by DNSGA-II-A on only one problem, i.e., JY3. One possible explanation is that on JY3 with time-varying non-monotonic dependencies between any two decision variables, the degree of diversity loss is roughly the same for different severity levels, but for different severity levels, LPSDM reacts to changes differently.

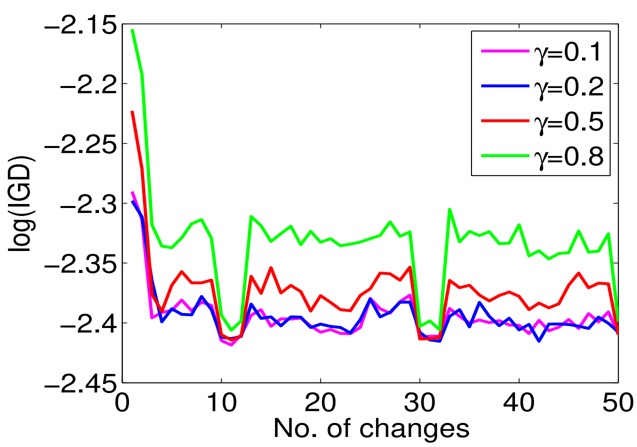


Fig. 2: The average IGD values of LPSDM with different γ values on FDA1.

B. Influence of γ Values

To study whether the proposed algorithm is sensitive to the value of γ, we test LPSDM with varying $\gamma \in \{0.1, 0.2, 0.5, 0.8\}$, fixed $\tau_t = 20$ and $n_t = 10$. The experimental result for FDA1 is shown in Fig. III-B. It shows γ is a key parameter affecting the performance of LPSDM. LPSDM has significant performance degradation for either too big or too small values of γ as the IGD value fluctuates widely for extreme cases. The reasons are as follows. A very small value of γ means a population has a small searching space,

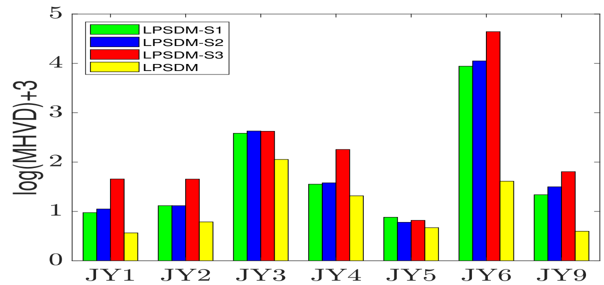


Fig. 3: MHVD values obtained by different components.

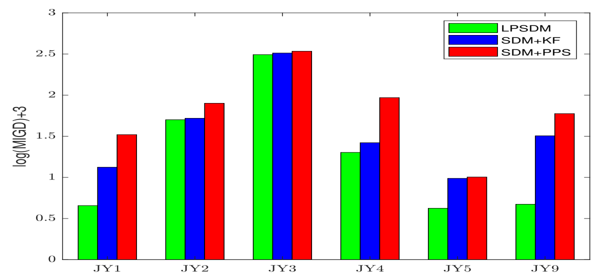


Fig. 4: Performance comparison between LPSDM and other prediction models for selected problems on MIGD with $n_t = 10$ and $\tau_t = 20$.

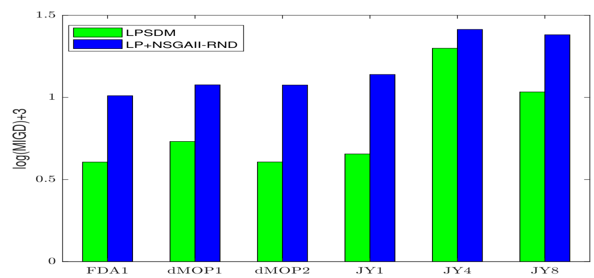


Fig. 5: Performance comparison between LPSDM and other diversity strategy for selected problems on MIGD with $n_t = 10$ and $\tau_t = 20$.

leading to reduced population diversity. On the contrary, a large value of γ has a big searching space; in this case, it may exist dramatically changed, leading to reduced population convergence.

Therefore, it is desirable to choose γ in between the extremes. The algorithm performance is good when $\gamma = 0.1$ or $\gamma = 0.2$ than $\gamma = 0.5$ or $\gamma = 0.8$, which is demonstrated in Fig. 2 where $\gamma = 0.2$ appears to be the best choice. Meanwhile, considering the balance between the convergence and the diversity. We thus suggest γ is reasonably near 0.2.

References

[1] R. Rambabu, P. Vadakkepat, K. C. Tan, and M. Jiang, "A mixture-of-experts prediction framework for evolutionary dynamic multiobjective optimization," *IEEE Transactions on Cybernetics*, pp. 1–14, 2019.

[2] F. Wilcoxon, *Individual Comparisons by Ranking Methods*. Springer New York, 1992.

TABLE VI: Mean and standard deviation values of MGD obtained by seven algorithms

Problems	(n_t, τ_t)	LPSDM	DNSGA-II-A	DNSGA-II-B	SGEA	MOEA/D	PPS	MoE
FDA1	(10, 10)	**1.66e-3(7.63e-5)**	2.08e-2(3.61e-3)‡	1.13e-1(1.17e-2)‡	9.14e-3(3.54e-4)‡	1.83e-2(2.02e-3)‡	1.30e-1(1.20e-1)‡	1.19e-1(2.66e-3)‡
	(10, 20)	**1.56e-3(1.00e-5)**	6.14e-3(6.15e-4)‡	1.58e-2(4.41e-4)‡	4.58e-3(1.78e-4)‡	6.64e-3(2.31e-4)‡	8.05e-2(4.46e-2)‡	1.07e-1(1.98e-3)‡
	(10, 30)	**1.50e-3(9.57e-6)**	3.69e-3(5.15e-5)‡	7.07e-3(1.63e-4)‡	3.22e-3(5.56e-5)‡	3.89e-3(3.36e-5)‡	6.34e-3(8.78e-4)‡	1.03e-1(1.00e-3)‡
FDA4	(10, 10)	**8.29e-3(2.31e-4)**	1.99e-1(1.41e-2)‡	5.66e-1(1.95e-2)‡	7.55e-2(4.42e-3)‡	4.48e-2(2.88e-3)‡	1.03e-1(9.79e-3)‡	2.58e-2(3.29e-3)‡
	(10, 20)	**9.09e-3(3.12e-4)**	1.00e-1(5.65e-3)‡	1.99e-1(9.43e-3)‡	3.79e-2(2.40e-3)‡	1.59e-2(5.00e-5)‡	6.85e-2(5.33e-3)‡	2.23e-2(6.76e-4)‡
	(10, 30)	**9.22e-3(2.05e-4)**	6.46e-2(6.57e-3)‡	7.84e-2(4.19e-3)‡	2.56e-2(7.15e-4)‡	1.04e-2(5.77e-5)‡	5.64e-2(2.96e-3)‡	1.80e-2(3.32e-4)‡
JY1	(10, 10)	**1.18e-3(7.50e-5)**	9.16e+0(1.52e+0)‡	1.52e-1(1.09e-2)‡	1.13e-2(1.16e-3)‡	1.02e-2(4.24e-4)‡	6.24e-2(7.42e-3)‡	4.53e-2(8.39e-3)‡
	(10, 20)	**1.11e-3(3.77e-5)**	7.21e+0(4.46e-1)‡	1.50e-2(5.74e-4)‡	4.46e-3(1.26e-4)‡	3.51e-3(2.56e-4)‡	6.15e-2(4.49e-2)‡	9.03e-3(2.85e-3)‡
	(10, 30)	**1.05e-3(2.98e-5)**	6.10e+0(5.30e-1)‡	5.35e-3(2.35e-4)‡	2.85e-3(1.09e-4)‡	1.82e-3(4.50e-5)‡	6.82e-3(3.22e-3)‡	3.26e-3(6.71e-4)‡
JY2	(10, 10)	**5.04e-2(1.00e-4)**	1.88e-1(5.33e-2)‡	1.69e-1(1.76e-2)‡	5.05e-2(1.35e-3)†	5.34e-2(1.14e-3)†	1.49e-1(1.16e-1)‡	1.87e-1(3.30e-3)‡
	(10, 20)	5.03e-2(8.16e-5)	1.06e-1(3.02e-2)‡	8.85e-2(4.70e-4)‡	**4.96e-2(3.39e-4)**	4.98e-2(2.06e-4)†	7.08e-2(3.28e-2)‡	1.75e-1(1.08e-3)‡
	(10, 30)	5.04e-2(1.15e-4)	1.94e-1(5.60e-2)‡	7.15e-2(1.87e-4)‡	4.99e-2(5.60e-4)†	**4.94e-2(5.00e-5)**	5.91e-2(9.80e-3)‡	1.73e-1(6.97e-4)‡
JY3	(10, 10)	2.16e-1(9.07e-2)	1.98e-1(7.87e-2)	1.83e-1(1.04e-2)‡	**1.36e-1(1.21e-2)**	1.72e-1(1.15e-2)	4.41e-1(1.07e-1)‡	9.61e-1(1.87e-2)‡
	(10, 20)	**1.01e-1(9.77e-2)**	2.76e-1(8.62e-2)‡	2.14e-1(6.06e-3)‡	1.51e-1(4.86e-2)‡	1.68e-1(1.60e-2)‡	2.70e-1(2.19e-2)‡	7.04e-1(4.48e-2)‡
	(10, 30)	**4.39e-2(8.79e-3)**	3.76e-1(4.21e-2)‡	2.26e-1(7.48e-3)‡	1.62e-1(5.44e-2)‡	1.66e-1(2.84e-2)‡	2.40e-1(1.23e-2)‡	1.13e-1(6.93e-2)‡
JY4	(10, 10)	**2.23e-3(6.65e-5)**	7.24e+0(4.87e-1)‡	1.95e-1(1.96e-2)‡	1.10e-2(4.32e-4)‡	7.25e-3(5.14e-4)‡	1.50e-1(5.42e-2)‡	1.55e-1(3.50e-3)‡
	(10, 20)	**1.86e-3(1.14e-4)**	9.12e+0(1.96e-1)‡	2.42e-2(7.73e-4)‡	4.37e-3(1.25e-4)‡	2.84e-3(1.50e-5)‡	9.92e-2(2.99e-2)‡	1.29e-1(1.06e-3)‡
	(10, 30)	**1.53e-3(4.19e-5)**	9.83e+0(1.58e-1)‡	7.16e-3(2.87e-4)‡	2.60e-3(1.87e-5)‡	2.04e-3(8.69e-5)‡	6.67e-2(5.06e-2)‡	1.22e-1(4.68e-4)‡
JY5	(10, 10)	**9.48e-4(9.43e-6)**	6.27e-1(1.44e-1)‡	6.26e-3(9.78e-4)‡	1.89e-3(4.54e-4)‡	1.15e-3(2.88e-5)‡	1.07e-2(1.06e-3)‡	1.53e-2(3.37e-3)‡
	(10, 20)	9.77e-4(1.47e-5)	8.04e-1(9.04e-2)‡	1.47e-3(1.19e-4)‡	1.10e-3(9.14e-5)‡	**9.44e-4(2.15e-5)**	2.92e-3(2.06e-4)‡	7.57e-3(1.51e-3)‡
	(10, 30)	9.94e-4(1.27e-5)	8.49e-1(1.24e-1)‡	1.06e-3(6.06e-5)‡	1.04e-3(5.30e-5)‡	**8.56e-4(5.19e-6)**	2.36e-3(2.76e-4)‡	4.39e-3(8.40e-4)‡
JY6	(10, 10)	**5.23e-2(9.15e-3)**	4.16e+0(8.97e-2)‡	6.84e+0(2.97e-1)‡	9.49e-1(1.15e-1)‡	5.79e-1(5.94e-2)‡	7.37e+0(1.84e-1)‡	3.57e+0(2.07e-1)‡
	(10, 20)	**2.36e-2(2.77e-2)**	1.72e+0(7.81e-2)‡	1.74e+0(1.10e-1)‡	2.08e-1(1.71e-2)‡	1.54e-1(2.70e-2)‡	5.77e+0(1.89e-1)‡	1.90e+0(1.79e-1)‡
	(10, 30)	**2.24e-2(1.90e-2)**	8.70e-1(2.09e-1)‡	4.91e-1(2.90e-2)‡	7.39e-2(9.35e-3)‡	7.26e-2(2.26e-3)‡	5.11e+0(2.26e-1)‡	9.76e-1(8.30e-2)‡
JY7	(10, 10)	**1.15e+0(5.85e-1)**	7.76e+0(6.52e-1)‡	2.07e+1(1.01e+0)‡	2.01e+0(6.80e-1)‡	1.96e+0(2.41e-1)‡	4.83e+1(2.18e+0)‡	5.72e+0(9.62e-1)‡
	(10, 20)	7.49e-1(4.68e-1)	1.49e+0(5.27e-1)‡	2.83e+0(5.04e-1)‡	**7.25e-1(3.71e-1)**	1.00e+0(7.19e-1)‡	3.42e+1(5.32e+0)‡	3.83e+0(1.30e+0)‡
	(10, 30)	1.43e+0(5.10e-1)	**2.76e-1(5.10e-2)**	4.85e-1(2.48e-1)‡	4.77e-1(3.52e-1)‡	1.33e+0(2.07e-1)‡	1.11e+1(1.18e+1)‡	3.49e+0(1.00e+0)‡
JY8	(10, 10)	**4.06e-3(3.36e-5)**	1.60e-1(6.53e-2)‡	8.89e-3(1.42e-3)‡	3.02e+0(9.83e-3)‡	6.27e-3(2.35e-4)‡	8.25e-3(2.04e-3)‡	2.73e-2(3.54e-3)‡
	(10, 20)	**4.07e-3(8.84e-5)**	1.41e-1(6.92e-2)‡	4.76e-3(1.59e-4)‡	3.01e+0(0.00e+0)‡	6.26e-3(3.50e-4)‡	4.91e-3(6.01e-4)‡	1.42e-2(1.83e-3)‡
	(10, 30)	**4.14e-3(7.11e-5)**	1.31e-1(5.91e-2)‡	4.47e-3(7.81e-5)‡	3.01e+0(0.00e+0)‡	6.24e-3(1.42e-4)‡	4.61e-3(1.76e-4)‡	1.18e-2(2.05e-3)‡
JY9	(10, 10)	**6.50e-3(5.54e-3)**	4.30e+1(8.52e+0)‡	1.11e-1(1.24e-2)‡	5.30e-1(4.87e-2)‡	1.45e-2(1.74e-3)‡	5.35e-1(4.31e-1)‡	9.58e-2(1.09e-2)‡
	(10, 20)	**1.43e-3(1.41e-4)**	4.66e+1(3.99e+0)‡	2.04e-2(1.96e-3)‡	3.83e-2(6.28e-3)‡	4.25e-3(1.03e-4)‡	3.51e-2(8.67e-3)‡	9.70e-3(3.40e-3)‡
	(10, 30)	**1.32e-3(1.74e-4)**	3.86e+1(1.83e+0)‡	6.87e-3(3.86e-4)‡	2.69e-2(8.16e-5)‡	2.37e-3(2.07e-4)	1.12e-2(2.41e-3)‡	3.25e-3(2.45e-4)‡

‡ and † indicate LPSDM performs significantly better than and equivalently to the corresponding algorithm, respectively.

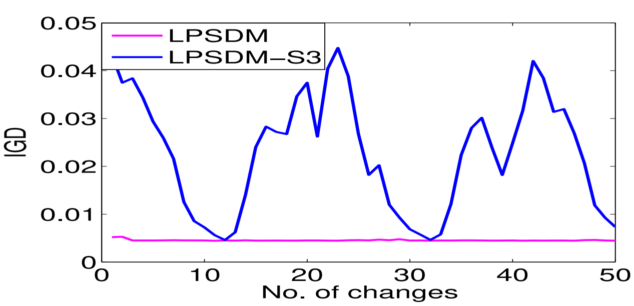


Fig. 6: The average IGD values of LPSDM and LPSDM-S3 with $n_t = 10$ and $\tau_t = 20$ on JY1.

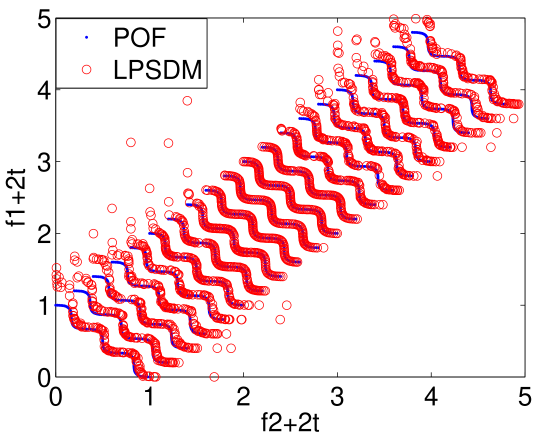
TABLE VII: Mean and standard deviation values of MSP obtained by seven algorithms

Problems	(n_t, τ_t)	LPSDM	DNSGA-II-A	DNSGA-II-B	SGEA	MOEA/D	PPS	MoE
FDA1	$(10, 10)$	**3.31e-3(2.44e-5)**	1.20e-2(3.60e-3)‡	1.09e-2(5.16e-4)‡	7.19e-3(3.93e-4)‡	7.77e-3(8.21e-4)‡	1.52e-2(4.31e-3)‡	1.29e-2(1.46e-4)‡
	$(10, 20)$	**3.27e-3(2.87e-5)**	5.98e-3(1.07e-3)‡	6.68e-3(1.72e-4)‡	4.21e-3(1.27e-4)‡	7.37e-3(1.81e-4)‡	8.20e-3(2.16e-3)‡	9.54e-3(9.00e-5)‡
	$(10, 30)$	**3.24e-3(3.20e-5)**	5.15e-3(5.18e-5)‡	5.49e-3(1.78e-4)‡	3.57e-3(8.64e-5)‡	6.50e-3(1.23e-4)‡	3.61e-3(1.13e-4)‡	7.15e-3(7.32e-5)‡
FDA4	$(10, 10)$	**3.48e-2(2.63e-4)**	8.52e-2(7.71e-3)‡	7.50e-2(1.79e-3)‡	4.10e-2(3.39e-3)‡	8.49e-2(2.56e-3)‡	4.62e-2(2.74e-3)‡	5.74e-2(4.44e-3)‡
	$(10, 20)$	**3.47e-2(1.89e-4)**	5.76e-2(2.82e-3)‡	5.21e-2(1.59e-3)‡	2.74e-2(1.37e-3)‡	6.06e-2(2.73e-3)‡	3.97e-2(1.87e-3)‡	5.43e-2(3.05e-3)‡
	$(10, 30)$	3.47e-2(6.60e-4)	4.81e-2(1.18e-3)‡	4.39e-2(8.41e-4)‡	**2.49e-2(8.38e-4)**	5.55e-2(6.27e-4)‡	3.89e-2(3.77e-4)‡	5.77e-3(2.16e-4)‡
JY1	$(10, 10)$	**3.89e-3(5.25e-5)**	1.29e-1(2.08e-2)‡	1.47e-2(7.92e-4)‡	1.63e-2(1.57e-3)‡	1.98e-2(1.89e-3)‡	1.53e-2(5.50e-4)‡	3.98e-2(7.76e-3)‡
	$(10, 20)$	**3.91e-3(4.61e-5)**	1.63e-1(5.05e-3)‡	8.13e-3(2.63e-4)‡	8.56e-3(4.87e-4)‡	1.25e-2(5.67e-4)‡	1.05e-2(2.89e-3)‡	1.79e-2(1.79e-3)‡
	$(10, 30)$	**3.87e-3(4.85e-5)**	2.03e-1(2.44e-2)‡	7.71e-3(1.79e-4)‡	6.39e-3(2.14e-4)‡	1.18e-2(9.57e-5)	5.55e-3(6.48e-4)‡	1.35e-2(6.58e-4)‡
JY2	$(10, 10)$	**3.43e-3(5.05e-5)**	2.13e-2(8.97e-3)‡	1.43e-2(7.22e-4)‡	1.30e-2(1.08e-3)‡	1.97e-2(2.02e-3)‡	2.05e-2(7.63e-3)‡	2.45e-2(7.34e-3)‡
	$(10, 20)$	**3.38e-3(5.00e-6)**	1.14e-2(3.59e-3)‡	7.42e-3(3.43e-4)‡	6.91e-3(2.92e-4)‡	1.03e-2(1.64e-3)‡	7.21e-3(3.69e-3)‡	1.10e-2(1.87e-3)‡
	$(10, 30)$	**3.39e-3(1.00e-5)**	1.28e-2(4.62e-3)‡	6.81e-3(3.23e-4)‡	4.98e-3(1.57e-4)‡	7.69e-3(2.31e-4)‡	5.99e-3(2.10e-3)‡	9.32e-3(1.87e-3)‡
JY3	$(10, 10)$	6.62e-2(6.67e-2)	3.33e-2(8.55e-3)	1.60e-2(2.72e-3)	1.79e-2(4.02e-3)	**1.06e-2(6.38e-4)**	1.24e-1(1.13e-1)‡	6.83e-2(3.92e-2)‡
	$(10, 20)$	5.71e-2(8.60e-2)	3.23e-2(7.75e-3)	1.14e-2(1.74e-3)‡	1.45e-2(7.56e-3)	**1.15e-2(5.37e-4)**	2.63e-2(1.15e-2)	1.35e-2(6.55e-3)
	$(10, 30)$	**5.63e-3(2.03e-3)**	2.70e-2(8.02e-3)‡	8.77e-3(1.34e-3)‡	1.02e-2(1.41e-3)‡	1.11e-2(5.50e-4)‡	1.18e-2(1.06e-3)‡	1.18e-2(3.40e-3)‡
JY4	$(10, 10)$	7.33e-3(5.39e-4)	1.61e-1(4.50e-3)‡	1.34e-2(7.84e-4)‡	2.65e-2(6.02e-4)‡	**3.43e-3(4.73e-4)**	1.80e-2(3.22e-3)‡	9.27e-3(7.65e-4)‡
	$(10, 20)$	5.87e-3(2.29e-4)	1.54e-1(1.30e-2)‡	9.34e-3(3.48e-4)‡	1.83e-2(2.53e-4)‡	**3.00e-3(1.51e-4)**	1.22e-2(7.55e-4)‡	8.33e-3(3.19e-4)‡
	$(10, 30)$	5.80e-3(5.19e-4)	1.55e-1(5.78e-3)‡	7.76e-3(2.38e-4)‡	1.61e-2(2.34e-4)‡	**2.47e-3(2.60e-4)**	7.56e-3(1.89e-3)‡	7.37e-3(2.00e-4)‡
JY5	$(10, 10)$	**3.67e-3(3.00e-5)**	3.06e-2(4.39e-3)‡	6.74e-3(5.97e-4)‡	4.22e-3(3.09e-4)‡	8.83e-3(3.07e-4)‡	7.59e-3(1.10e-3)‡	4.32e-2(1.69e-2)‡
	$(10, 20)$	3.73e-3(2.87e-5)	3.51e-2(3.09e-3)‡	5.83e-3(9.91e-5)‡	**3.47e-3(2.43e-4)**	8.81e-3(2.71e-4)‡	3.93e-3(5.35e-4)‡	4.30e-2(4.72e-3)‡
	$(10, 30)$	3.65e-3(2.44e-5)	3.69e-2(4.02e-3)‡	5.73e-3(2.11e-4)‡	**3.48e-3(3.37e-4)**	8.52e-3(3.40e-4)‡	3.53e-3(1.84e-4)	1.91e-2(4.24e-3)‡
JY6	$(10, 10)$	**1.31e-2(1.79e-3)**	1.40e-1(6.48e-3)‡	1.74e-1(8.18e-3)‡	2.59e-1(3.14e-2)‡	1.25e-1(1.44e-1)‡	3.68e-1(4.06e-2)‡	3.18e-1(2.88e-2)‡
	$(10, 20)$	**8.80e-3(5.34e-3)**	6.48e-2(2.04e-3)‡	5.92e-2(2.25e-3)‡	7.12e-2(5.16e-3)‡	6.89e-2(7.57e-2)‡	2.83e-1(2.49e-2)‡	1.88e-1(2.33e-2)‡
	$(10, 30)$	**7.07e-3(6.75e-4)**	3.90e-2(5.53e-3)‡	2.62e-2(2.04e-3)‡	3.67e-2(2.90e-3)‡	5.93e-2(2.03e-2)‡	2.18e-1(1.20e-2)‡	8.11e-2(1.55e-2)‡
JY7	$(10, 10)$	**2.08e-2(7.49e-3)**	3.79e-1(2.02e-2)‡	6.07e-1(2.89e-2)‡	4.41e-1(7.07e-2)‡	1.17e-1(1.27e-1)‡	2.27e+0(1.40e-1)‡	8.28e-1(2.85e-1)‡
	$(10, 20)$	**3.08e-2(1.98e-2)**	9.51e-2(1.63e-2)‡	1.11e-1(2.07e-2)‡	1.06e-1(2.28e-2)‡	7.17e-2(4.70e-2)‡	1.68e+0(1.60e-1)‡	3.83e-1(1.21e-1)‡
	$(10, 30)$	**2.91e-2(1.59e-3)**	4.68e-2(1.17e-2)‡	3.66e-2(1.60e-2)‡	5.47e-2(1.27e-2)‡	4.63e-2(8.58e-3)‡	9.05e-1(7.72e-1)‡	1.88e-1(4.41e-2)‡
JY8	$(10, 10)$	**4.66e-3(1.02e-4)**	4.00e-2(1.36e-2)‡	7.45e-3(3.68e-4)‡	1.12e-2(1.09e-2)‡	1.06e-2(4.19e-4)‡	7.38e-3(9.27e-4)‡	2.49e-2(5.73e-3)‡
	$(10, 20)$	**4.39e-3(6.13e-5)**	2.27e-2(1.22e-2)‡	6.76e-3(3.79e-4)‡	1.48e-3(8.23e-4)‡	1.16e-2(7.39e-4)‡	4.89e-3(3.96e-4)‡	1.91e-2(2.64e-3)‡
	$(10, 30)$	**4.43e-3(1.26e-4)**	2.18e-2(1.88e-2)‡	6.67e-3(2.55e-4)‡	3.06e-3(3.11e-3)‡	1.15e-2(2.06e-4)‡	4.57e-3(5.83e-5)‡	1.74e-2(1.22e-3)‡
JY9	$(10, 10)$	**4.71e-3(7.73e-4)**	4.55e-1(1.23e-1)‡	1.58e-2(5.24e-3)‡	1.09e-1(2.50e-2)‡	2.10e-2(4.42e-3)‡	5.93e-2(2.71e-2)‡	3.87e-2(8.93e-3)‡
	$(10, 20)$	**5.12e-3(1.15e-3)**	6.82e-1(7.65e-2)‡	8.69e-3(5.55e-4)‡	1.15e-2(1.10e-3)‡	1.26e-2(2.63e-4)‡	1.05e-2(1.31e-3)‡	2.13e-2(5.03e-3)‡
	$(10, 30)$	**4.41e-3(4.77e-4)**	8.21e-1(4.18e-2)‡	8.18e-3(1.90e-4)‡	4.90e-3(2.71e-4)‡	1.16e-2(1.50e-4)‡	6.18e-3(1.22e-4)‡	1.45e-2(1.56e-3)‡

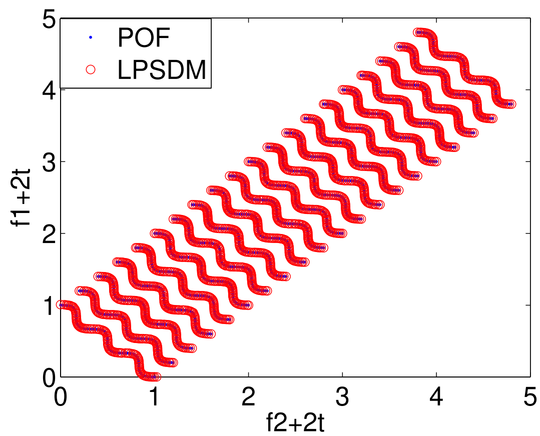
‡ and † indicate LPSDM performs significantly better than and equivalently to the corresponding algorithm, respectively.

TABLE VIII: P-values calculated for Wilcoxon rank-sum test with fixed $\tau_t = 20$, and $n_t = 10$, which NA occurs for the best performing algorithm itself.

Problems	(n_t, τ_t)	LPSDM	DNSGA-II-A	DNSGA-II-B	SGEA	MOEA/D	PPS	MoE
FDA1	$(10, 20)$	NA	0.0357	0.0307	0.0357	0.0201	0.0033	0.0070
FDA4	$(10, 20)$	NA	0.0644	0.0661	0.0542	0.0320	0.0233	0.0121
JY1	$(10, 20)$	NA	0.0034	0.0066	0.0034	0.0032	0.00233	0.0012
JY2	$(10, 20)$	NA	0.0002	0.0002	0.0001	0.0002	0.0003	0.0002
JY3	$(10, 20)$	NA	0.0476	0.0114	0.0714	0.2000	0.0357	0.6923
JY4	$(10, 20)$	NA	0.0002	0.0232	0.0451	0.0462	0.0004	0.0001
JY5	$(10, 20)$	NA	0.0032	0.0467	0.0574	0.0462	0.0013	0.0002
JY6	$(10, 20)$	NA	0.0001	0.0002	0.0001	0.0002	0.0003	0.0001
JY7	$(10, 20)$	NA	0.0361	0.0714	0.7857	0.7000	0.4000	0.0140
JY8	$(10, 20)$	NA	0.0413	0.0814	0.8871	0.5123	0.3202	0.0231
JY9	$(10, 20)$	NA	0.0015	0.0814	0.8871	0.5123	0.3202	0.0231

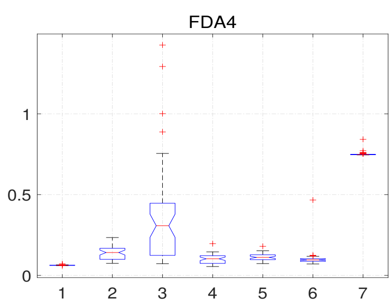


(a) LPSDM-S3

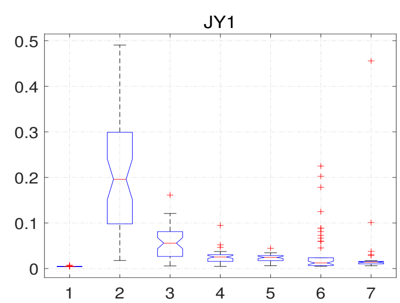


(b) LPSDM

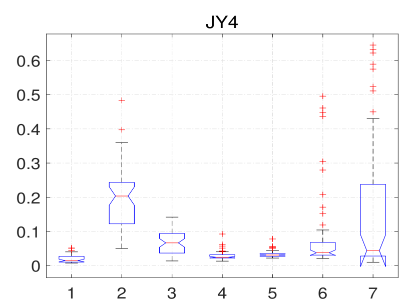
Fig. 7: Obtained POFs for JY1 with n_t =10 and τ_t=20.



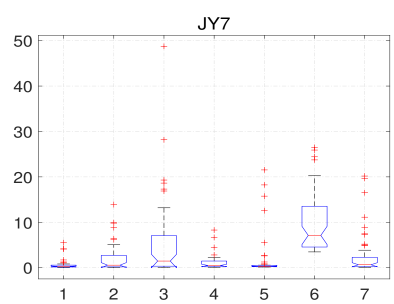
(a) FDA4



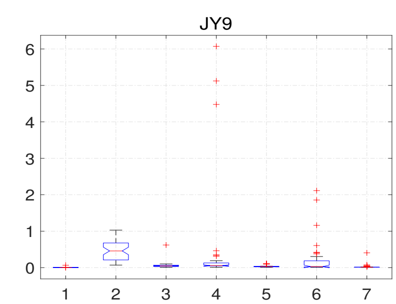
(b) JY1



(c) JY4

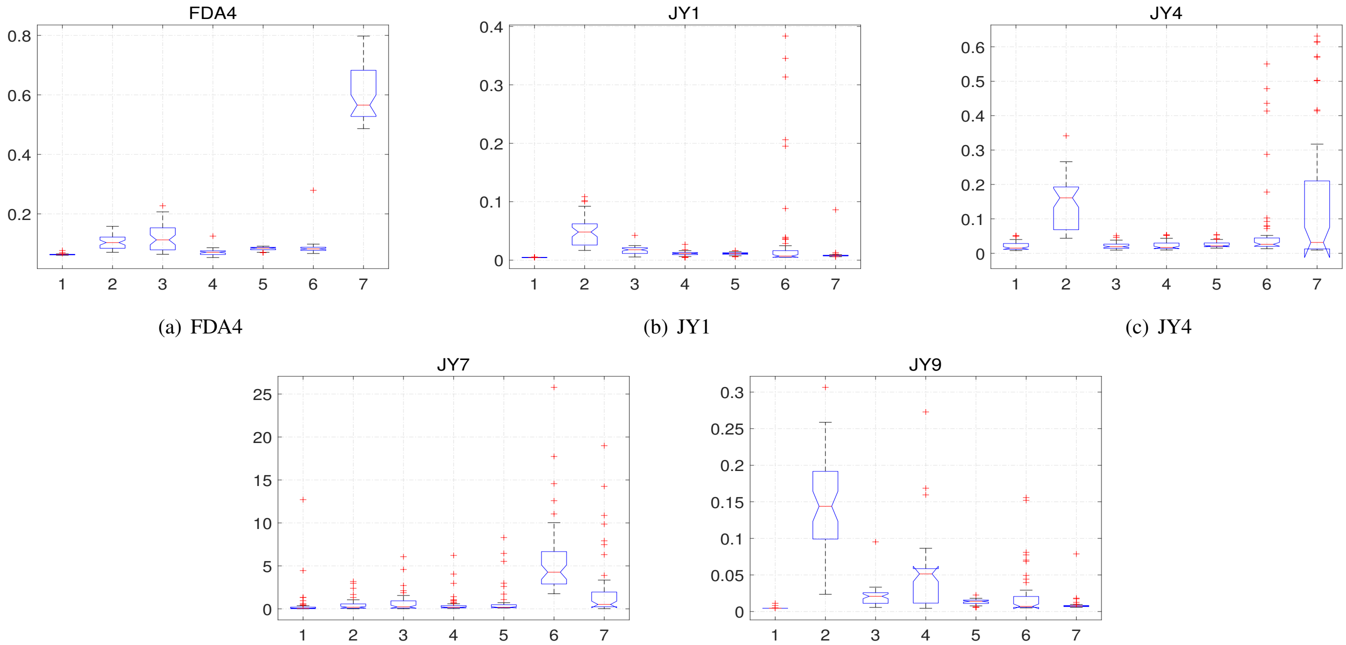


(d) JY7



(e) JY9

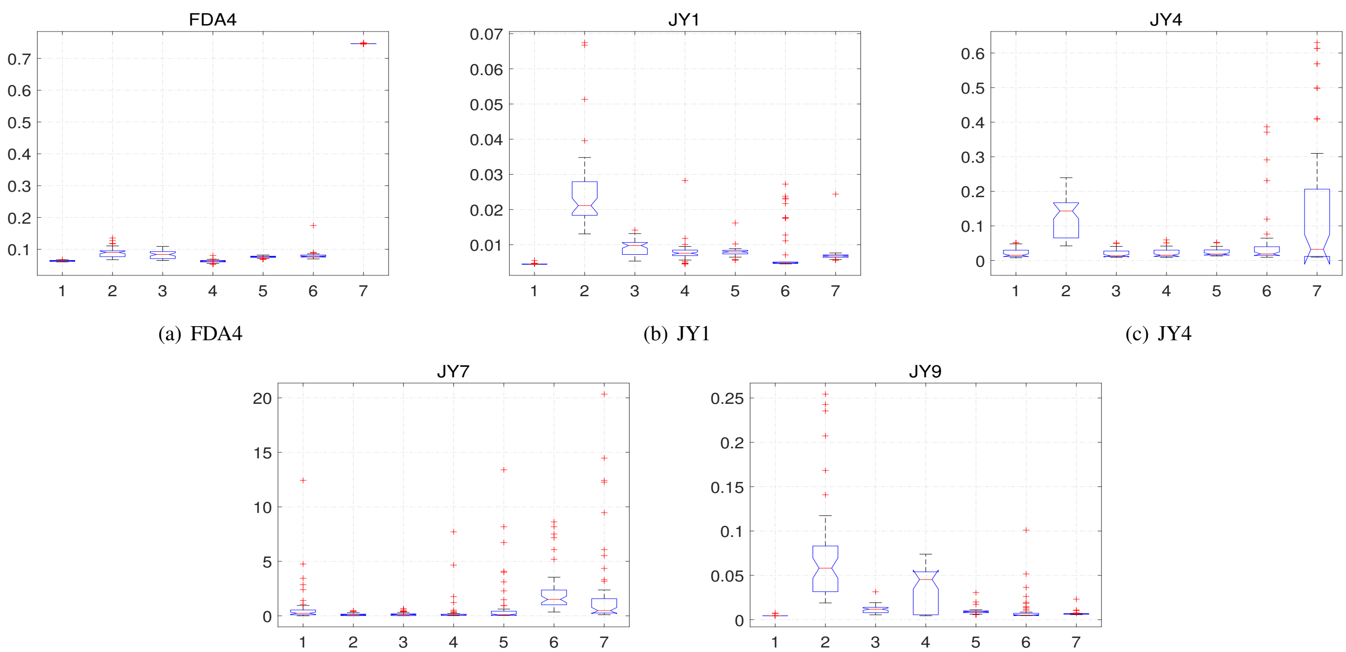
Fig. 8: Boxplot of average IGD values over 20 times of seven algorithms on seven problems with n_t =10 and τ_t=10. (1:LPSDM, 2: DNSGA-II-A, 3: DNSGA-II-B, 4: SGEA, 5: MOEA/D, 6: PPS, 7: MoE.)



(a) FDA4 (b) JY1 (c) JY4

(d) JY7 (e) JY9

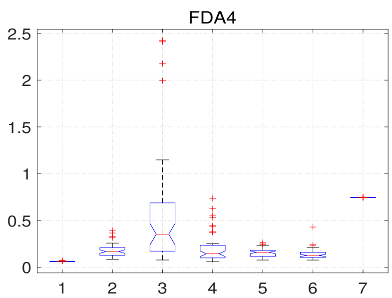
Fig. 9: Boxplot of average IGD values over 20 times of seven algorithms on seven problems with n_t =10 and τ_t=20. (1:LPSDM, 2: DNSGA-II-A, 3: DNSGA-II-B, 4: SGEA, 5: MOEA/D, 6: PPS, 7: MoE.)



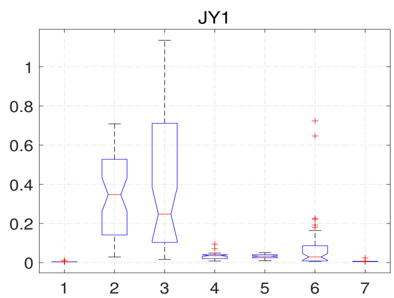
(a) FDA4 (b) JY1 (c) JY4

(d) JY7 (e) JY9

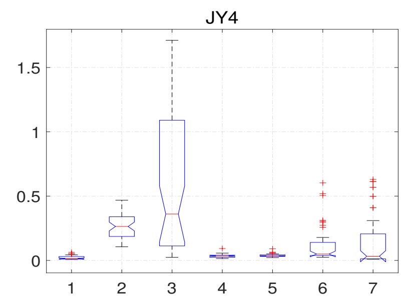
Fig. 10: Boxplot of average IGD values over 20 times of seven algorithms on seven problems with n_t =10 and τ_t=30. (1:LPSDM, 2: DNSGA-II-A, 3: DNSGA-II-B, 4: SGEA, 5: MOEA/D, 6: PPS, 7: MoE.)



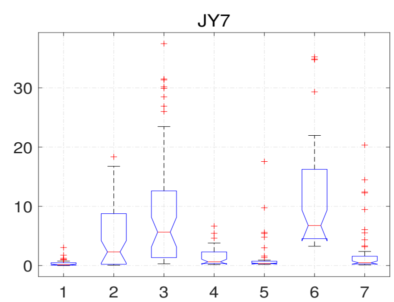
(a) FDA4



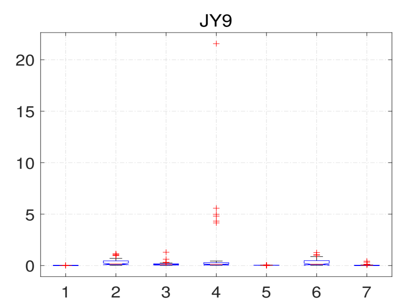
(b) JY1



(c) JY4



(d) JY7



(e) JY9

Fig. 11: Boxplot of average IGD values over 20 times of seven algorithms on seven problems with n_t =5 and τ_t=10. (1:LPSDM, 2: DNSGA-II-A, 3: DNSGA-II-B, 4: SGEA, 5: MOEA/D, 6: PPS, 7: MoE.)