$$\iint curl(\vec{F}) \cdot \hat{n} \, ds$$

$$ds = \frac{dx \, dy}{1/\sqrt{14}}$$

$$\therefore \iint \frac{7}{\sqrt{14}} \frac{dx\,dy}{1/\sqrt{14}} = 7 \iint dx\,dy = 7\left[\frac{1}{2} \times 2 \times 3\right] = \underline{\underline{21}}$$

$\uparrow$

Area of ABC

Verified

GAUSS - DIVERGENCE THM

(Relation b/w surface to volume integral)

$$* \iint_S \vec{F}\cdot\hat{n}\,ds = \iiint_V \text{div}\,\vec{F}\,dv \quad , \quad dv = dx\,dy\,dz$$

⇒ State Gauss's DIVERGENCE THM $\iint_S \vec{F}\cdot\hat{n}\,ds = \iiint \text{div}\,\vec{F}\,dv$ where S is surface of sphere $x^2+y^2+z^2=16$ and $\vec{F} = 3x\hat{i}+4y\hat{j}+5z\hat{k}$

Sol^n) $\nabla\cdot\vec{F} = 3+4+5 = 12$

$$\iiint_V 14\,dv \Rightarrow 12V$$

$$= 12\left(\frac{4}{3}\pi(4)^3\right) = \frac{3584\pi}{3}$$

⇒ Evaluate $\iint_S \vec{F}\cdot\hat{n}\,ds$; where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$

Sol^n) $\vec{\nabla}\cdot\vec{F} = 4z - 2y + y = 4z - y$

$$\therefore \int_0^1\int_0^1\int_0^1 (4z-y)\,dx\,dy\,dz = 3/2.$$

$\frac{xyz}{dv}$ $\frac{y}{dx\,dy/dz}$

⇒ Evaluate $\iint_S (y^2z^2\hat{i} + z^2x^2\hat{j} + z^2y^2\hat{k})\,\hat{n}\,ds$, where S is the part of sphere $x^2+y^2+z^2=1$, above xy-plane and bounded by this plane

Sol^n) $\iint_S \vec{F}\cdot\hat{n}\,ds = \iiint_V \vec{\nabla}\cdot\vec{F}\,dv$

$$\therefore \vec{\nabla}\cdot\vec{F} = 2zy^2$$

$$\therefore \iiint 2zy^2\,dx\,dy\,dz$$

$x = r \sin\phi \cos\theta$

$y = r \sin\phi \sin\theta$

$z = r \cos\phi$

$\therefore dx\,dy\,dz = r^2 \sin\theta \, dr\, d\theta\, d\phi$

$$\therefore \int_0^{\pi/2}\int_0^{2\pi}\int_0^{1} 2\,(r\cos\phi)\,(r^2\sin^2\phi\,\sin^2\theta)\,(r^2\sin\theta\,dr\,d\theta\,d\phi)$$

$\Rightarrow \pi/12$ (Ans).

More examples from this topic.

$\Rightarrow$ Find $\iint \vec{F}\cdot\hat{n}\,ds$, $\vec{F} = (2x+3z)\hat{i} - (xz+y)\hat{j} + (y^2+2z)\hat{k}$ and S is surface of sphere having center $(3,-1,2)$ radius 3.

Sol^n now eq^n of sphere : $(x-3)^2 + (y+1)^2 + (z-2)^2 = 9$.

But $\hat{n}$ is diff. to find.

So use ; $\iint \vec{F}\cdot\hat{n}\,ds = \iiint \operatorname{div}\vec{F}\,dv$.

$\therefore \operatorname{div}\vec{F} = 3$

$\therefore \iiint 3\,dv \Rightarrow 3\iiint dv$

$= 3\left[\frac{4}{3}\pi r^3\right] = 108\pi$.