Divergence

* Notation $\longrightarrow$ $\mathrm{div}(\vec{v})$ or $\vec{\nabla}\cdot\vec{v}$

$$\boxed{\mathrm{div}} \xrightarrow{\text{vector field } (\vec{v})} \boxed{\text{scalar function}}$$

$$\vec{\nabla}\cdot\vec{v} = \left(\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial x} + \frac{\partial v_3}{\partial z}\right)$$ where $\vec{v} = (v_1, v_2, v_3)$

$v_1 = v_1(x, y, z)$

$v_2 = v_2(x, y, z)$ $\Rightarrow$ $v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$

$v_3 = v_3(x, y, z)$

$\Rightarrow \vec{v_1} = x\hat{i} + y\hat{j}$

Sol^n $\mathrm{div}(\vec{v_1}) = 1 + 1 = 2$ (const)

$\Rightarrow \vec{v_2} = x^2\hat{i} + y^2\hat{j}$ $\Rightarrow$ $\mathrm{div}(\vec{v_2}) = 2(x+y)$ (not const)

But if we choose a point, then it is const :

$\therefore \mathrm{div}(v_2)\big|_P = 2(a_1 + a_2) = \text{const}$

$\therefore$ $\mathrm{div}(\vec{v})\big|_P > 0 \longrightarrow$ Fountain

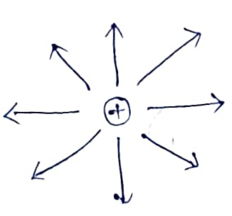
$\mathrm{div}(\vec{v})\big|_P < 0 \longrightarrow$ Sink

$\mathrm{div}(\vec{v})\big|_P = 0 \longrightarrow$ Streamline

*

electric field

$\vec{E} = (x, y)$



$\therefore \mathrm{div}(\vec{E})\big|_P > 0$ (Fountain)

04-09-2023

DISPLACEMENT CURRENT :

(Qualitatively and Quantitatively)

→ Maxwell's eqn before modification.

* $\vec{\nabla}\cdot\vec{E} = \dfrac{\rho}{\epsilon_0}$
* $\vec{\nabla}\cdot\vec{B} = 0$
* $\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$
* $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ ~~$+\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$~~
* Theorem : div (curl $\vec{A}$) = 0

Apply the thm in eqn (3).

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$0 \qquad\qquad -\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{B}) = 0$$

Apply the thm in eqn (4).

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\cdot(\mu_0\vec{J})$$

$$0 \qquad \neq \qquad 0$$

* $\vec{\nabla}\cdot\vec{J} = -\dfrac{\partial\rho}{\partial t}$ → continuity eqn in electrodynamics.

$$\therefore \vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\mu_0\vec{J}) + \vec{\nabla}\left(\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\mu_0\vec{J}) + \mu_0\left(\epsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E})\right)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\mu_0\vec{J}) + \mu_0\epsilon_0\frac{\partial\rho}{\partial t}$$

$$\vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot(\#)$$

↑ to get zero we have to add some value

$$\therefore \vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\because \left(\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}\right) \rightarrow -\frac{\partial}{\partial t}(\epsilon_0\vec{\nabla}\cdot\vec{E})$$

$$= -\epsilon_0\left(\vec{\nabla}\cdot\frac{\partial\vec{E}}{\partial t}\right)$$

$$\therefore \vec{\nabla}\cdot\left[\vec{J} + \epsilon_0\frac{\partial\vec{E}}{\partial t}\right]$$

Hence, Maxwell's eqn after modification :

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\vec{J}_d$$

↳ dispn current

$$\therefore \vec{J}_d = \epsilon_0\frac{\partial\vec{E}}{\partial t}$$