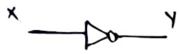
~~RESIDENTIAL WIRING~~ [30]

DIGITAL CIRCUITS / LOGIC CIRCUITS

LOGIC GATES :

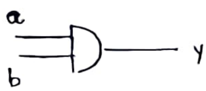
→ Not Gates : 

Truth Table

X	Y
0	1
1	0

Boolean eq^n

$y = \bar{a}$ or $\bar{x}$

→ AND Gate : 

↓

2^n tables

where $n \to$ no. of inputs

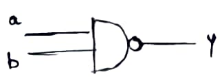
a	b	y
0	0	0
0	1	0
1	0	0
1	1	1

$y = a \cdot b$.

→ OR Gate : 

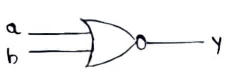
a	b	y
0	0	0
0	1	1
1	0	1
1	1	1

$y = a + b$.

→ NAND Gate : 

a	b	y
0	0	1
0	1	1
1	0	1
1	1	0

$y = \overline{a \cdot b}$

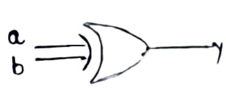
→ NOR Gate : 

a	b	y
0	0	1
0	1	0
1	0	0
1	1	0

$y = \overline{a + b}$

$ab + \bar{a}\bar{b}$

$a\bar{b} + \bar{a}b$

→ XOR Gate : 

a	b	y	y'
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

$y = A \oplus B$

$A \oplus B = \bar{A}B + A\bar{B}$

$\overline{A \oplus B} = AB + \bar{A}\bar{B}$

→ XNOR Gate :

a	b	y
0	0	1
0	1	0
1	0	0
1	1	1

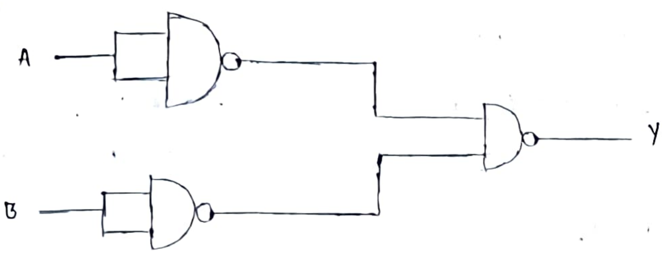
$Y = \overline{A \oplus B}$

* NOT, AND, OR } Basic Gates

NAND, NOR, ~~XOR~~, ~~XNOR~~ } Derived Gates (or) Universal Gates.

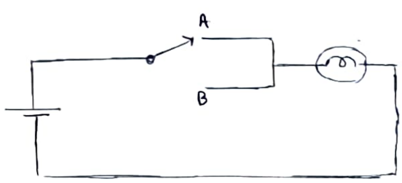
⇒ Generate OR operation using 3 NAND Gates ?

Sol^n^



$\therefore Y = \overline{\bar{A} \cdot \bar{B}} = A + B$

⇒



Sol^n^ for lamp on → 1
for lamp off → 0.

∴

A	B	y	
0	0	0	→ not possible
1	0	1	
0	1	1	
1	1	0	→ not possible.

XOR circuit. ←

→ Convert (37) decimal to binary :

* $2 \overline{)37}$ = 18, 36, 1 → 1 LSD
↳ least significant digit

* $2 \overline{)18}$ = 9, 18, 0 LSD

* $2 \overline{)9}$ = 4, 8, 1 LSD

* $2 \overline{)4}$ = 2, 4, 0 LSD

* $2 \overline{)2}$ = 1, 2, 0 LSD

* $2 \overline{)1}$ = 0, 0, 1 MSD.
↳ Max significa digit

∴ ~~10100~~ $(100101)_2$ → start from down.

⇒ Convert 0.95 to its binary equivalent ?

Sol^n^

Fraction	Radix	Result	Recorded Carriers.
0.95	2	$(0.95 \times 2) = 1.9 \rightarrow 1 + 0.9$	1
0.9	2	$1.8 \rightarrow 1 + 0.8$	1
0.8	2	$1.6 \rightarrow 1 + 0.6$	1
0.6	2	$1.2 \rightarrow 1 + 0.2$	1
0.2	2	$0.4 \rightarrow 0 + 0.4$	0
0.4	2	$0.8 \rightarrow 0 + 0.8$	0
0.8	2	$1.6 \rightarrow 1 + 0.6$	1 → once repetition reached stop.

$\therefore (0.1111001)_2$

⇒ 24.6 (decimal) ⟶ binary.

Sol^n^

$2 \overline{)24}$ = 12, 24, 0 ; $2 \overline{)12}$ = 6, 12, 0 ; $2 \overline{)6}$ = 3, 6, 0 ; $2 \overline{)3}$ = 1, 2, 1 ; $2 \overline{)1}$ = 0, 0, 1

$0.6 \times 2 = 1.2 = 1 + 0.2 = 1$

$0.2 \times 2 = 0.4 = 0 + 0.4 = 0$

$0.4 \times 2 = 0.8 = 0 + 0.8 = 0$

$0.8 \times 2 = 1.6 = 1 + 0.6 = 1$

$0.6 \times 2 = 1.2 = 1 + 0.2 = 1$

$\therefore$ Ans ⇒ $(11000.10011)_2$

⇒ Find 1's complement of $(1101)_2$ ⇒ 0010 (Ans)

⇒ Find 1's complement of $(1011001)_2$ ⇒ 0100110 (Ans)

⇒ Find 2's complement of $(1001)_2$ ⇒

Sol^n^

```
  1 0 0 1
  0 1 1 0   → 1's
+       1
---------
  0 1 1 1   → 2's complement.
```

⇒ Find 2's complement of $(10100011)_2$

Sol^n) 01011100 → 1's

+ 1

01011101 → 2's

~~⇒ Subtract $(101011)_2$ from $(111001)_2$ using 1's complement method~~

~~Sol^n) 111001~~

~~010100 → 1's of $(101011)_2$~~

⇒ Trick to find 2's complement directly.

Sol^n) (1001) 1 → write the same values till we encounter 1st one (1) from right

2 → then after that change the values

∴ 1001 ←

↓

0111 (Ans)

ii) 10100011 → 01011101 (Ans).

⇒ Addition of binary :

i) 110101 + 011010

11

110101

011010

1001111

we must carry 1 if we get 1+1

Rules for addition

A	B	Sum	Carry
0 +	0	0	0
0 +	1	1	0
1 +	0	1	0
1 +	1	0	1

⇒ Subtraction of binary :

i) when we get carry.

(110101) − (100101)

⇓ 1s

110101 + 011010

11

110101

011010

1001111

ignore carry and add 1

+ 1

010000 (Ans)

ii) when we don't get carry.

101011 − 111001

⇓ 1s

101011 + 000110

111

101011

000110

110001

⇓ 1s

001110 (Ans)

Rules for subtraction in binary.

A		B	diff. ~~sum~~	carry
0	-	0	0	0
0	-	1	1	1
1	-	0	1	0
1	-	1	0	0

Laws of Boolean Algebra.

i) Cumulative → $A+B = B+A$

ii) Associative → $AB = BA$

iii) Distributive → $A+(B+C) = (A+B)+C$

$(AB)C = A(BC)$

$\boxed{A(B+C) = AB + AC}$

Addition rules in Boolean Algebra :

* $A + AB = A$
* $A(A+B) = A$
* $A + \bar{A}B = A+B$
* $(A+B)(A+C) = A+BC$

$A + \bar{A}B = A + B.$

$A + \bar{A}B = A + AB + \bar{A}B.$

$= A + B(A+\bar{A})$

$= A + B$

$A + AB = A$

$\Rightarrow A(1+B) \Rightarrow A(1) = A$

$(A+B)(A+C) = AA + AC + BA + BC$

$= \cancel{A(A+B) + C(A+B)}$

$= \cancel{(A+B)(A+C)}$

$= A + AC + BA + BC \quad (\because AA = A)$

$= A(1+B+C) + BC$

$= A(1+D) + BC$

$= A(1) + BC = A+BC$

$A(A+B) = AA + AB$

$= A + A - A$

$= A$

Theorem's in Boolean Algebra :

→ Demorgan's Theorem : $(\overline{AB} = \bar{A}+\bar{B})$ $(\overline{A+B} = \bar{A}\bar{B})$

→ Consensus Theorem : $(AB + \bar{A}C + BC = AB + \bar{A}C)$

Laws Of Boolean Algebra :

And	OR
$A \cdot 0 = 0$	$A + 0 = A$
$A \cdot A = A$	$A + A = A$
$A \cdot 1 = A$	$A + 1 = 1$
$A\bar{A} = 0$	$A + \bar{A} = 1$

Imp

* $\boxed{A + (\bar{A} \cdot B) = A+B}$

⇒ Solve using Consensus Theorem, $\bar{A}B + AC + B\bar{C} + \bar{B}C + AB$.

Sol^n

$\underbrace{\bar{A}B + AB}_{1} + AC + B\bar{C} + \bar{B}C$

$\therefore \; 1 + AC + B\bar{C} + \bar{B}C$

⇒ Reduce expression, $\overline{\overline{AB} + \bar{A} + AB}$

$(\because \overline{AB} + AB = 1)$

Sol^n

$\overline{\bar{A} + \bar{B} + \bar{A} + AB}$ | $\overline{\overline{AB} + \bar{A} + AB}$

$\overline{\bar{A} + \bar{B} + AB}$ | $\overline{1 + \bar{A}} = \bar{1} = 0$

$\overline{\bar{A} + \bar{B} + A}$

$\overline{1 + \bar{B}} = \bar{1} = 0$ (Ans)

$1 - B + AB$

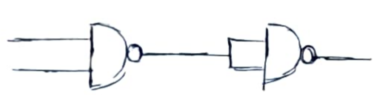
$1 - B(1 - A)$

OR, AND, NOT Gates using universal gates (NAND, NOR)

i) NOT :

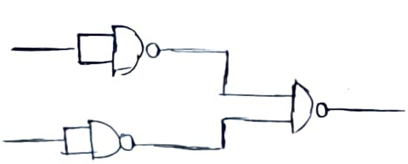
using NAND.

ii) AND :



using 2NANDS.

iii) OR :

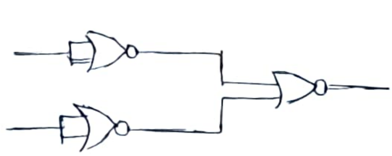


using 3 NANDS.

i) NOT :

using NOR

ii) AND :



using 3 NOR.

iii) OR :

using 2 NOR.

⇒ using demorgan find $A + B(\overline{C + \overline{DE}})$

Sol^n

$A + B(\bar{C} \cdot \overline{\overline{DE}})$ (or) $A + B(\overline{\bar{C} + \bar{D} + \bar{E}})$

$A + B\bar{C}DE$ | $A + B(\bar{C} \cdot D \cdot E)$

$A + B\bar{C}DE$

BINARY → OCTA :

decimal → 10
Octal → 8 → 2^3
binary → 2 → 2^1
~~decimal → 4 → 2~~
hexa decimal → 16 → 2^4

000 → 0
001 → 1
010 → 2
011 →
100
101
110
111 → 7

⇒ $(0110100)_2$ → $(\ )_8$

Sol^n^ → 0 110 100

means (000)

0 ↓ 0, 110 ↓ 6, 100 ↓ 4

∴ $(064)_8$

⇒ $(1110110)_2$

Sol^n^ 001 110 110

↓ 1, ↓ 6, ↓ 6

∴ $(166)_8$

Decimal to hexadecimal.

Hexadecimal

0	8
1	9
2	A → 10
3	B → 11
4	C → 12
5	D — 13
6	E — 14
7	F — 15

⇒ $(14)_{10}$

Sol^n^ 16 | 14, 00, 14

∴ $(E)_{16}$ (Ans)

⇒ $(9216)_{10}$

Sol^n^

remainders

16	9216	
16	576	— 0
16	36	— 0
16	2	— 4
	0	— 2

$(2400)_{16}$

Hexadecimal to decimal

⇒ $(A2)_{16}$

Sol^n^ A = 10 , 2

↓ 16^1, ↓ 16^0

$(10\times16) + (2\times1) = (162)_{10}$

⇒ $(12B7)_{16}$

Sol^n^ 1 2 B 7

↓ 16^3 × 1, ↓ 16^2 × 2, ↓ 16^1 × 11, ↓ 16^0 × 1

4096 + 512 + 176 + 7

= $(4791)_{10}$

⇒ Multiply 1011 by 1101

Sol^n^

```
        1011
      × 1101
    --------
     1 1
        1011
       0000×
      1011××
     1011×××
    --------
    10001111
```

} Here add of 3 1's occurs.

⇒ Divide 11101 ÷ 1100

Sol^n^

```
           10
      ---------
 1100 | 11101
        1100
        ----
          101
          ~~100~~0
          ----
          101
```

∴ Quotient $= (10)_2 = (2)_{10}$

Remainder $= (101)_2 = (5)_{10}$

⇒ Divide 110101.110 with 10

Sol^n^

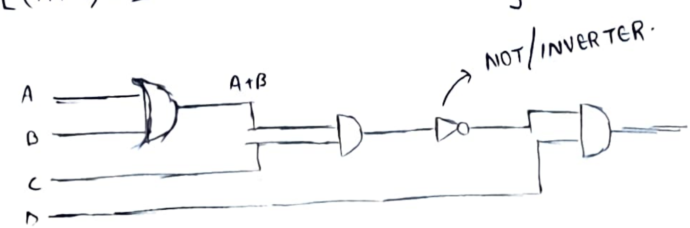
```
        11010.111
     ------------
  10 | 110101.110
       10
       --
        10
        10
        --
         1
         0
        --
         10
         10
         --
          01
          0
          --
           11
           10
           --
            11
            10
            --
             10
             10
             --
              0
```

∴ Quotein → 11010.111

Rem → 0

⇒ $[\overline{(A+B)C}]D$ → Draw fig.

Sol^n^



(or)

Boolean Func^N Representation

* SOP / S SOP $\longrightarrow$ Standard sum of products.
* POS / SPOS $\longrightarrow$ Standard product of sum

$\Rightarrow$ $F(A,B,C) = \bar{A}BC + AB\bar{C} \rightarrow$ (SOP)

$\Rightarrow$ $F(A,B,C) = (A+B+C)(\bar{A}+\bar{B}+\bar{C}) \longrightarrow$ (POS)

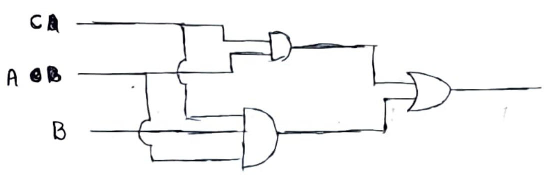
$\Rightarrow$ $f(A,B,C,D) = AB\bar{C}\bar{D} + \bar{A}\bar{B}CD + ABC\bar{D} \longrightarrow$ (SOP)

convert to POS.

$f(A,B,C,D) = (A+B+\bar{C}+\bar{D})(\bar{A}+\bar{B}+C+D)(A+B+C+\bar{D}) \longrightarrow$ (POS)

$\Rightarrow$ Draw ~~diagram~~ digital circuit for $AC + ABC$

Sol^n)



(or)

$AC(1+B) = AC$ $\Rightarrow$

$\longrightarrow$ SOP [Two input gate]

$\Rightarrow$ Given : TT

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

it is not AND/OR
it is just given in que.

Sol^n) complement $\longrightarrow$ 0
normal $\longrightarrow$ 1.

$\therefore$

A	B	Y	Product
0	0	0	$\bar{A}\bar{B}$
0	1	0	$\bar{A}B$
1	0	1	$A\bar{B}$
1	1	1	AB

$\therefore Y = AB + A\bar{B}$ $\longleftarrow$ only take values having $Y = 1$

→ POS (3 i/p gate) :

Sorry

Normal → 0

Complement → 1

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

Take complement..

~~ABC~~	Y_1
~~$AB\bar{C}$~~	1
	1
	1
	0
	1
	1
	0
	1

SOP → $\therefore Y = ABC + AB\bar{C} + A\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}$

POS → $Y = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)\cdot(\bar{A}+B+C)\cdot(\bar{A}+B+\bar{C}), (\bar{A}+\bar{B}+\bar{C})$.

* Complete Minterm, Maxterm, SOP → SSOP, $\overline{\text{POS}}$ → SPOS, before K-Map.

KARNAUGH MAP [K-Map] : (no. of cells → 2^n cells).

* Two variable K-Map :
↳ $2^2 = 4$

A \ B	0	1
0	0	1
1	2	3

* Three variable K-Map : $2^3 = 8$.

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

~~* Two Va~~

* Four variable K-Map : $2^4 = 16$

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Order to take binary :

00 → 01 → 11 → 10

Just change one no.
either left or right but not both

To rem. values, convert binary to decimal (or else rem.).

ex : $(1111)_2 \longrightarrow (15)_{10}$

$(1011)_2 \longrightarrow (11)_{10}$

→ Rules for creating group in K-Map :

* No zeros allowed
* Group can be vertical / horizontal but not diagonals.
* Overlapping allowed
* Group should be as large as possible
* Group must contain 2^n cells.

⇛ $F(A,B) = \Sigma(0,2,3)$

Step - 1 : Draw K-Map according to var.

Step - 2 : put "1" on cells given in bracket

Step - 3 : put binary above box

Step - 4 : Take pairs as large as possible

Step - 5 : Put values of cell, containing '1'

Step - 5 : Cancel values having 0 & 1 both

Step - 6 : $\bar{A} \to 0$
$A \to 1$

Sol^n A, B ⟶ Two variable

A \ B	0	1
0	1	
1	1	1

∴ P_1

A	B
~~0~~	0
~~1~~	0

P_2

A	B
1	~~0~~
1	~~1~~

∴ Put $0 \longrightarrow \bar{A}$
$1 \longrightarrow A$.

∴ $F(A,B) \longrightarrow B' + A$ (Ans)

⇛ $F(A,B,C) = \Sigma(0,1,2,3,5)$

Sol^n

A \ BC	00	01	11	10
0	1	1	1	1
1		1		

P_1

A	B	C
0	~~0~~	~~0~~
0	~~0~~	~~1~~
0	~~1~~	~~1~~
0	~~1~~	~~0~~

P_2

A	B	C
~~0~~	0	1
~~1~~	0	1

$F(A,B,C) = A' + B'C$ (Ans)

⇒ $F(A,B,C,D) = \Sigma(0,1,4,5,8,9,12,13)$

Sol^n^

AB \ CD	00	01	11	10
00	1	1		
01	1	1		
11	1	1		
10	1	1		

↳ we have to take grp as large as possible

AB \ CD	00	01	11	10
00	1	1		
01	1	1		
11	1	1		
10	1	1		

we can take a grp of 8 or 4 or 2 or 1

∴ E P₁

A	B	C	D
0	0	0	0
0	0	0	1
0	1	0	0
0	1	0	1
1	1	0	0
1	1	0	1
1	0	0	0
1	0	0	1

∴ C' (Ans).

MIN TERM & MAX TERM :

* Minterm ⟶ Each individual term in SSOP
* Maxterm ⟶ Each individual term in SPOS.

Variable		Minterm	Maxterm
A	B	SSOP	SPOS
0	0	$\bar{A}\bar{B} \to m_0$	$A+B \to M_0$
0	1	$\bar{A}B \to m_1$	$A+\bar{B} \to M_1$
1	0	$A\bar{B} \to m_2$	$\bar{A}+B \to M_2$
1	1	$AB \to m_3$	$\bar{A}+\bar{B} \to M_3$

for SOP ~~minterm~~ { 0 → complement, 1 → normal }

for POS { 0 → normal, 1 → complement }

* ex : $f(A,B) = AB + \bar{A}B$

Sol^n^ it is SOP ⟶ $\underset{3}{\overset{11}{AB}} + \underset{1}{\overset{01}{\bar{A}B}}$

∴ $\Sigma m(1,3)$

$\Rightarrow f(A,B) = (A+B)\,(\bar{A}+B)$

Sol^n $\overset{0\ 0}{(A+B)}\ \overset{1\ 0}{(\bar{A}+B)}$

$\quad\ \ \downarrow \qquad\quad \downarrow$

$\quad\ \ 0 \qquad\quad\ 2$

$\therefore$ ~~ΣM(0,2)~~

$\pi M(0,2)$

→ SOP & SSOP :

* SSOP → The function should contain all variables in every product

i.e, $AB'+A$ (✗ CSOP) (✓ SSOP)

$ABC'+A'B$ (✗ CSOP) (✓ SSOP)

$A'C+AB'+BC$ (✗ CSOP) (✓ SSOP).

** Converting SSOP to CSOP.

$\Rightarrow AB'+A$

Sol^n $AB'+A(1)$

$AB'+A(B+B')$

$AB'+AB+AB'$

$AB'+AB$ (SSOP)

$\Rightarrow ABC'+A'B$

Sol^n $ABC'+A'B(1)$

$ABC'+A'B(C+C')$

$ABC'+A'BC+A'BC'$

$\Rightarrow A'C+AB'+BC$

Sol^n $A'C(1)+AB'(1)+BC(1)$

$A'C(B+B')+AB'(C+C')+BC(A+A')$

$A'BC+A'CB'+AB'C+AB'C'+\ldots$

* → SPOS to CSPOS : (cannonical)

$\Rightarrow (A'+B)\,B$

Sol^n $(A'+B)(B+0)$

$(A'+B)(B+A\cdot A')$

$(A'+B)(B+A)(B+A')$

$\Rightarrow (A'+B)(A+B)$

$\Rightarrow (A+B)(B+C')(A'+C)$

Sol^n $(A+B+0)(B+C'+0)(A'+C+0)$

$(A+B+CC')(B+C'+AA')(A'+C+BB')$

$(A+B+C)(A+B+C')(B+C'+A)(B+C'+A')(A'+C+B)(A'+C+B')$

$(A+B+C)(A+B+C')(B+C'+A')(A'+C+B)(A'+C+B')$

⇒ Draw K-Map for

$$f = A'BC'D + ABC'D + A'BCD + ABCD + A'B'C'D'$$

Sol^n

CD \ AB	A'B'	A'B	AB	AB'
C'D'	1			
C'D		1	1	
CD		1	1	
CD'				

P

A	B	C	D
0	1	0	1
1	1	0	1
0	1	1	1
1	1	1	1

B

$\therefore f = A'B'C'D' + BD$ (Ans)

→ K-Map for POS :

⇒ $f = (A'+B)(A+B)$

Sol^n

B \ A	A 0	A' 1
B 0	0	0
B' 1		

$\therefore$

A	B
0	0
1	0

$\therefore$ B (Ans)

⇒ $F = (A+B+\bar{C})(A+B'+C')(A+B+C)$

Sol^n

A \ BC	BC 00	BC' 01	B'C' 11	B'C 10
A 0	0	0	0	
A' 1				

P_1

A	B	C
0	0	0
0	0	1

A+B

P_2

A	B	C
0	0	1
0	1	1

A+C'

$\therefore$ ~~AB + AC~~ $f = (A+B)(A+C')$ (Ans)

⇒ $F = \Pi(6, 7, 15)$

Sol^n

AB \ CD	00	01	11	10
00	0			
01			0	0
11			0	
10				

P_1

A	B	C	D
0	1	1	1
0	1	1	0

A + B' + C'

P_2

A	B	C	D
0	1	1	1
1	1	1	1

B' + C' + D'

$\therefore F = (A+B'+C')(B'+C'+D')$

KMap with don't care condition.

⇒ $f(A,B,C,D) = \sum m(0,2,3,6,7,12,13,14) + \underbrace{\sum d(1,4,11,15)}_{\text{don't care}}$

Sol^n^

AB \ CD	00	01	11	10
00	1	x	1	1
01	x		1	1
11	1	1	x	1
10			x	

x → can be 1 or 0 depending on pairing range.

only include 1 'x' in grp.

∴ P_1

A	B	C	D
0	0	1	1
0	0	1	0
0	1	1	1
0	1	1	0

$\bar{A}C$

P_2

A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	1
0	0	1	0

$\bar{A}\bar{B}$

P_3

A	B	C	D
1	1	0	0
1	1	0	1
1	1	1	1
1	1	1	0

AB

∴ $F = A'B' + A'C + AB$

⇒ $f(A,B,C,D) = \pi M(5,8,9,10)\ \pi D(1,4,11,15)$

Sol^n^

AB \ CD	00	01	11	10
00		X		
01	X	0		
11			X	
10	0	0	X	0

P_1

A	B	C	D
0	1	0	0
0	1	0	1

$AB'C$

P_2

A	B	C	D
0	0	0	1
0	1	0	1

ACD'

P_3

A	B	C	D
1	0	0	0
1	0	0	1
1	0	1	1
1	0	1	0

$A'B$

∴ $(A+B'+C)(A+C+D')(A'+B)$

CORNERING IN K-Map :

$\Rightarrow f(A,B,C,D) = \Sigma(0,5,7,8,9,10,11,14,15)$

Sol^n

AB \ CD	00	01	11	10
00	1			
01		1	1	
11			1	1
10	1	1	1	1

P_1

A B C D
0 0 0 0
1 0 0 0
$B'C'D'$

P_2

A B C D
0 1 0 1
0 1 1 1
$A'BD$

P_3

A B C D
1 1 1 1
1 1 1 0
1 0 1 1
1 0 1 0
AC

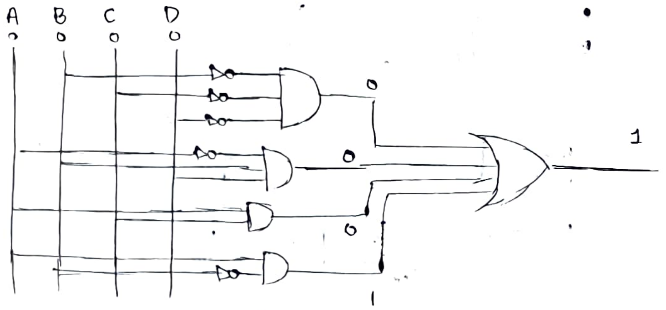
P_4

A B C D
1 0 0 0
1 0 0 1
1 0 1 1
1 0 1 0
AB'

$\therefore \; B'C'D' + A'BD + AC + AB' = f$

Always verify K-Map : (To get marks draw fig.) Imp do in every K-map que.

~~A B C D~~



Verified

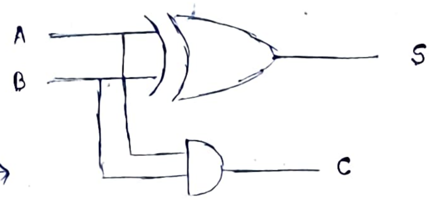
* Mux & De-Mux ~~con~~ will come in FAT.

HALF ADDER :

A	B	Sum	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$\therefore \; S = A'B + AB' = A \oplus B$

$C = AB$



ex :
$$\begin{array}{r} {}^{1}\;\; \\ 01 \\ +\,11 \\ \hline 0 \end{array}$$

$\therefore$ carry = 1

Sum = 0

in half adder we cannot add carry.

K-Map (Sum)

A \ B	0	1
0		1
1	1	

$\therefore Y = A\bar{B} + \bar{A}B$

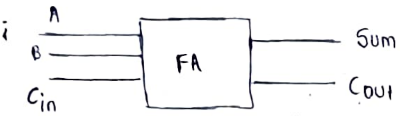
$Y = A \oplus B$

K-Map (Carry)

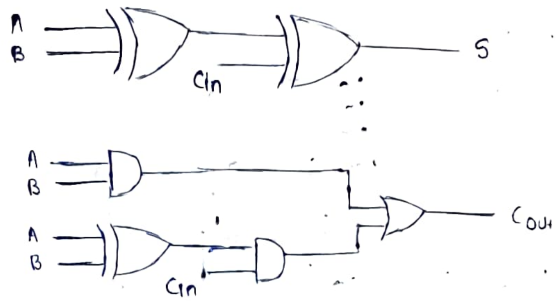
A \ B	0	1
0		
1		1

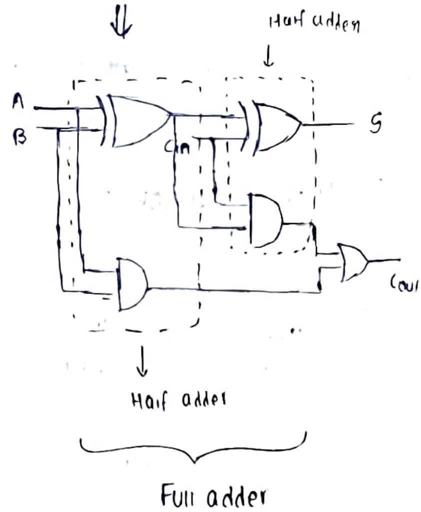
$Y = AB$

Full Adder :



A	B	C_{in}	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1





$S = \bar{A}\bar{B}C_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}\bar{C}_{in} + ABC_{in}$

$C_{out} = \bar{A}(\bar{B}C_{in} + B\bar{C}_{in}) + A(\bar{B}\bar{C}_{in} + BC_{in})$

$= \bar{A}(B \oplus C_{in}) + A(\overline{B \oplus C_{in}})$

$= \bar{A}X + A\bar{X}$

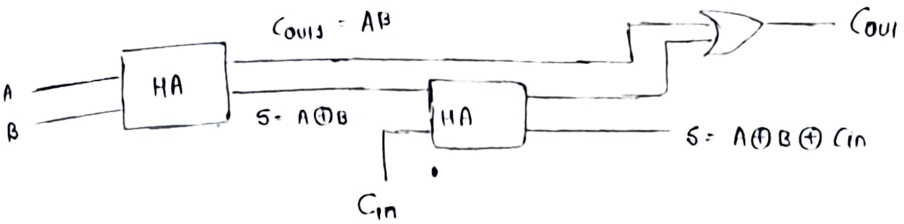
$= A \oplus X = A \oplus B \oplus C_{in}$

$C_{out} = \bar{A}BC_{in} + A\bar{B}C_{in} + AB\bar{C}_{in} + ABC$

A \ BC	00	01	11	10
0			1	
1		1	1	1

$\therefore$ ~~Cout = A~~

$C_{out} = AB(\bar{C}_{in} + C_{in}) + C_{in}(\bar{A}B + A\bar{B}) = AB + C_{in}(A \oplus B)$



Full adder using half adders

KMap for sum :

A \ BC	00	01	11	10
0		1		1
1	1		1	

$S = A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + ABC$

$S = A \oplus B \oplus C$ solve.

K-Map for carry :

A \ BC	00	01	11	10
0			1	
1		1	1	1

P1

A	B	C
1	0	1
1	1	1
AC		

P2

A	B	C
0	1	1
1	1	1
BC		

P3

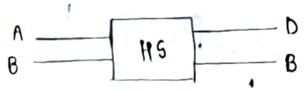
A	B	C
1	1	1
1	1	0
AB		

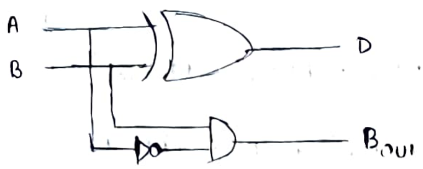
$\therefore C = AC + BC + AB$

$\therefore C_{out} = AC + BC + AB = AB + C_{in}(A \oplus B)$

$S = A \oplus B \oplus C$

HALF SUBTRACTOR :





A	B	D	B_{out}
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

ex :

$$\begin{array}{r} {}^{1}\ \ \\ 10 \\ -\,01 \\ \hline 1 \end{array}$$

$\therefore D = 1$

$B_{out} = 1$.

$D = A\bar{B} + B\bar{A} = A \oplus B$

$B = \bar{A}B$

in half subtractor we cannot subtract borrow.

K-Map (sum)

A \ B	0	1
0		1
1	1	

$S = AB' + A'B$

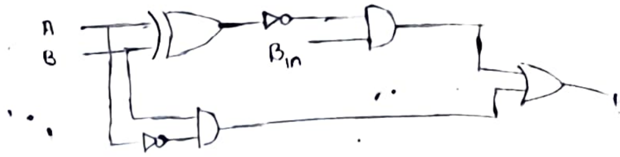
K-Map (Borrow)

A \ B	0	1
0		1
1		

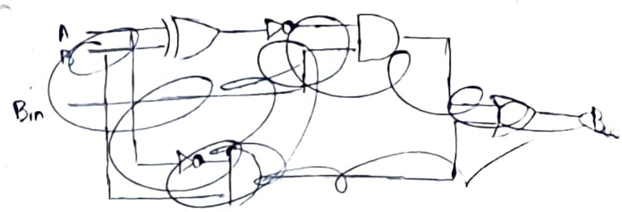
$B_{out} \ \bar{A}B$

Full Subtractor :

A	B	Bin	D	Bout
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1



⇓



$$D = \bar{A}\bar{B}B_{in} + \bar{A}B\overline{B_{in}} + A\bar{B}\overline{B_{in}} + ABB_{in}$$

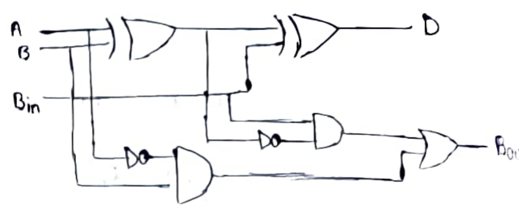
$$= \overline{B_{in}}(\bar{A}B + A\bar{B}) + B_{in}(\bar{A}\bar{B} + AB)$$

$$= \overline{B_{in}}(A \oplus B) + B_{in}(\overline{A \oplus B})$$

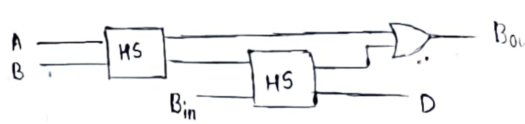
$$= \overline{B_{in}}\,X + B_{in}\,\bar{X}$$

$$= B_{in} \oplus X$$

$$= B_{in} \oplus A \oplus B$$



Full subtractor

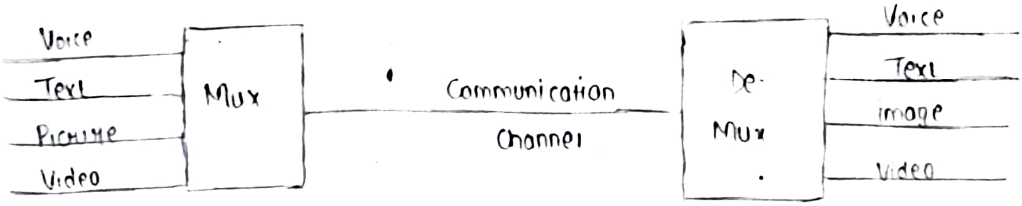


$$B_{out} = \bar{A}\bar{B}B_{in} + \bar{A}B\overline{B_{in}} + \bar{A}BB_{in} + ABB_{in}$$

$$= \bar{A}B(\overline{B_{in}} + B_{in}) + A\,B_{in}(\bar{A}\bar{B} + AB)$$

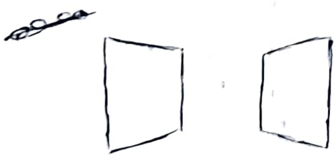
$$= \bar{A}B + B_{in}(\overline{A \oplus B})$$

Mux And Demux



Multiple input
to
single output

~~multiple ou~~
single input
to
multiple output



Mux
(Many to one)

De-Mux
(one to many)

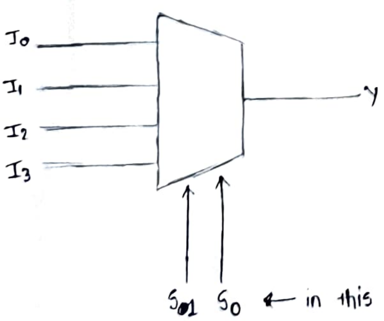
Mux $\longrightarrow$ $2^n - 1$

De-Mux $\longrightarrow$ $1 - 2^n$

$2^n - n$ $\longrightarrow$ Encoder

$n - 2^n$ $\longrightarrow$ Decoder

→ 4x1 Mux :



← in this order only.

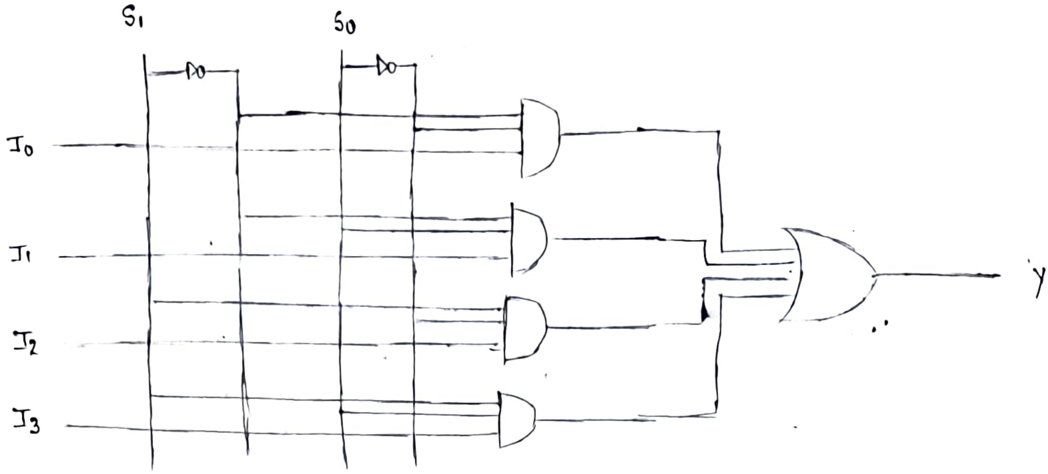
∴ 4 inputs,

so; $2^n = 4$

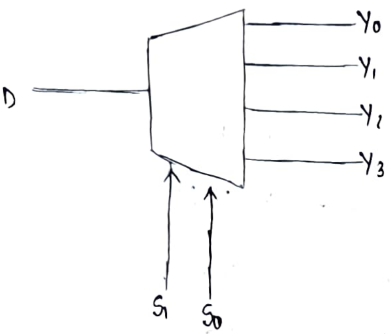
$n = 2$ $\longrightarrow$ i.e, S_0, S_1

S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

$$\therefore Y = \bar{S_1}\bar{S_0} I_0 + \bar{S_1} S_0 I_1 + S_1 \bar{S_0} I_2 + S_1 S_0 I_3$$

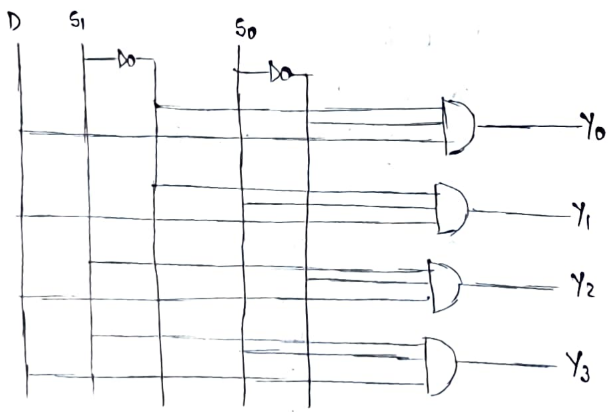


→ De - Mux : (1×4)



S_1	S_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	D
0	1	0	0	D	0
1	0	0	D	0	0
1	1	D	0	0	0

$$Y_0 = \overline{S_1}\,\overline{S_0}\,D$$
$$Y_1 = \overline{S_1}\,S_0\,D$$
$$Y_2 = S_1\,\overline{S_0}\,D$$
$$Y_3 = S_1\,S_0\,D$$

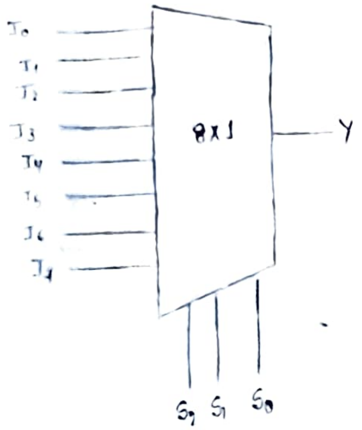


* $SOP = \overline{POS}$
* $POS = \overline{SOP}$

← In interchange eq^n

SOP ⟷ POS

→ Mux : (8x1)



S_2	S_1	S_0	Y
0	0	0	I_0
0	0	1	I_1
0	1	0	I_2
0	1	1	I_3
1	0	0	I_4
1	0	1	I_5
1	1	0	I_6
1	1	1	I_7.

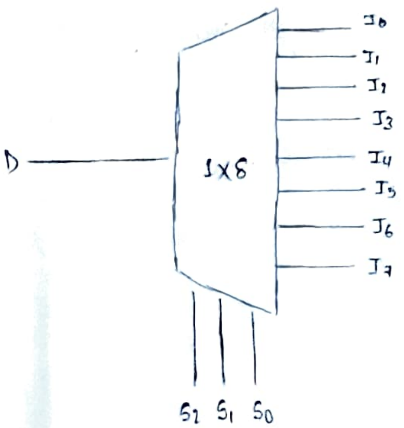
Draw diag.

$2^3 = 8$

$Y = \bar{S}_2 \bar{S}_1 \bar{S}_0 I_0 + \bar{S}_2 \bar{S}_1 S_0 I_1 + \dots$

$\dots + S_2 S_1 S_0 I_7$

→ De Mux : (1x8)



S_2	S_1	S_0	I_7	I_6	I_5	I_4	I_3	I_2	I_1	I_0
0	0	0	0	0	0	0	0	0	0	D
0	0	1	0	0	0	0	0	0	D	0
0	1	0	0	0	0	0	0	D	0	0
0	1	1	0	0	0	0	D	0	0	0
1	0	0	0	0	0	D	0	0	0	0
1	0	1	0	0	D	0	0	0	0	0
1	1	0	0	D	0	0	0	0	0	0
1	1	1	D	0	0	0	0	0	0	0.

$I_0 = \bar{S}_2 \bar{S}_1 \bar{S}_0 D$ . ... $I_6 = S_2 S_1 \bar{S}_0 D$

$I_1 = \bar{S}_2 \bar{S}_1 S_0 D$ $\quad$ $I_7 = S_2 S_1 S_0 D$

Draw diag:

* learn full wave and half wave rectifier .