EXTREMUM VALUES

Step-1 : $f(x,y)$ is given.

Step-2 : $\frac{\partial f}{\partial x} = 0 \longrightarrow A$ | $\frac{\partial f}{\partial y} = 0 \longrightarrow B$

Solve A & B & find values of 'x' & 'y' & make pair.

Step-3 : $\frac{\partial^2 f}{\partial x^2} = r$ (let say) , $t = \frac{\partial^2 f}{\partial y^2}$; $s = \frac{\partial^2 f}{\partial x \partial y}$.

Step-4 : If, $rt - s^2 > 0$ and

i) $r < 0$, then $f(x,y)$ has max value

ii) $r > 0$, then $f(x,y)$ has min value

Step-5 : If $rt - s^2 < 0$, then $f(x,y)$ has no extremum values.

Step-6 : If $rt - s^2 = 0$, then need further discussion / doubtful cases.

$\Rightarrow$ $f(x,y) = x^3 + y^3 - 3axy$, discuss extremum values :

Soln $\frac{\partial f}{\partial x} = 3x^2 - 3ay = 0$

$\Rightarrow y = \frac{x^2}{a}$

$\frac{\partial f}{\partial y} = 3y^2 - 3ax = 0$

$\Rightarrow x(x^3 - a^3) = 0$

$x = 0, a$

then, $y = 0, a$.

$\therefore$ pairs are $(0,0)$ & (a,a).

$r = \frac{\partial^2 f}{\partial x^2} = 6x$

$t = \frac{\partial^2 f}{\partial y^2} = 6y$

$s = \frac{\partial^2 f}{\partial x \partial y} = -3a$

$rt - s^2 = 36xy - 9a^2$

at $(0,0) \longrightarrow -9a^2 < 0 \longrightarrow$ no extremum values.

at $(a,a) \longrightarrow 27a^2 > 0$

$r = 6a > 0 \longrightarrow$ Min^m value.

$\Rightarrow$ Show that the f^n, $f(x,y) = x^3 + y^3 - 63x - 63y + 12xy$ is max^m at $(-7,-7)$ & min at $(3,3)$.

Sol^n) $\dfrac{\partial f}{\partial x} = 3x^2 - 63 + 12y = 0$. | $\dfrac{\partial f}{\partial y} = 3y^2 - 63 + 12x = 0$.

$3y^2 + 12x = 63$

$\Rightarrow$ $3x^2 + 12y = 63$ ~~$x=y,\ x+y=4$~~.

$\therefore 3x^2 + 12y = 3y^2 + 12x$

$3(x^2 - y^2) = 12(x-y)$

$3(x+y)(x-y) = 12(x-y)$.

$\therefore 3(x-y)[(x+y) - (4)] = 0$.

$\therefore x = y, \quad x + y = 4$

$\therefore r = 6x, \quad s = 12, \quad t = 6y$.

if $(-7,-7)$ is max^m, then, $rt - s^2 > 0$ & $r < 0$.

$\therefore 36xy - 144$ | $6(-7) < 0$

$\Rightarrow 36(49) - 144 > 0$

Hence, $(-7,-7)$ is max^m.

if $(3,3)$ is min^m, then $rt - s^2 > 0$ & $r > 0$

$36(9) - 144 > 0$ | $6(3) > 0$

Hence $(3,3)$ is min^m

⇒ A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimensions of box requiring least material for its construction.

Sol^n)

$V = 32$ cc.

$lbh = 32$

$b = \dfrac{32}{lh}$

$S = 2(l+b)h + lb.$

$S = 2\left(l + \dfrac{32}{lh}\right)h + l\left(\dfrac{32}{lh}\right)$

$S = 2lh + \dfrac{64}{l} + \dfrac{32}{h}.$

$\dfrac{\partial S}{\partial l} = 2h - \dfrac{64}{l^2} = 0$

$h = \dfrac{32}{l^2}$

$\dfrac{\partial S}{\partial h} = 2l - \dfrac{32}{h^2} = 0.$

$l = \dfrac{\cancel{32}\,16}{h^2}$

$l = \dfrac{16\,(l^4)}{(32)^2}$

$l^3 = \dfrac{32 \times \cancel{32}^{\,2}}{\cancel{16}} = 64$

$l = \cancel{8}\,4$, $h = \cancel{8}\,2$, $b = 4$

$\therefore \dfrac{\partial^2 S}{\partial l^2} = \dfrac{\cancel{64}\,128}{l^3} = \dfrac{128}{64} = 2.$

$\dfrac{\partial^2 S}{\partial l\,\partial h} = 2$, $\dfrac{\partial^2 S}{\partial h^2} = 8.$

Vimp

$\therefore rt - s^2 \Rightarrow \cancel{4-2>0}$

$\Rightarrow 16 - 4 > 0$

$\therefore 2 > 0$

$\therefore S$ is min^m at $l = 4$, $h = 2$, $b = 4$

learn absolute maxima and minima, it is in syllabus