$$\frac{\partial \phi}{\partial x} = \frac{r^n a_1 - (a_1 x + a_2 y + a_3 z)\, n r^{n-1} (\partial r/\partial x)}{r^{2n}}$$

$$r^2 = x^2 + y^2 + z^2 \Rightarrow 2r \frac{dr}{dr} = 2x \Rightarrow \frac{\partial r}{\partial x} = \frac{x}{r}.$$

$$\frac{\partial \phi}{\partial x} = \frac{a_1}{r^n} - \frac{n(a_1 x + a_2 y + a_3 z)x}{r^{n+2}}$$

$$\therefore \nabla \phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$$

$$= \frac{\vec{a}}{r^n} - \frac{n}{r^{n+2}}(\vec{a}\cdot\vec{r})\vec{r} \quad \text{(Ans)}.$$

⇒ Find the dirn derivative of div $(\vec{u})$ at point $(1,2,2)$ in dirn of outer normal of sphere $x^2+y^2+z^2 = 9$ for $\vec{U} = x^4\hat{i} + y^4\hat{j} + z^4\hat{k}$

Solny div $\vec{u} = \vec{\nabla}\cdot\vec{U}$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(x^4\hat{i} + y^4\hat{j} + z^4\hat{k}\right)$$

$$= 4x^3 + 4y^3 + 4z^3$$

$\therefore$ normal to sphere : $\nabla(x^2+y^2+z^2-9)$

$$= 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

at $(1,2,2)$ $= 2\hat{i} + 4\hat{j} + 4\hat{k}$

$$\therefore \nabla(\vec{\nabla}\cdot\vec{U}).$$

$$\nabla(4x^3 + 4y^3 + 4z^3) \cdot \frac{2\hat{i}+4\hat{j}+4\hat{k}}{\sqrt{36}}$$

$$= (12x^2\hat{i} + 12y^2\hat{j} + 12z^2\hat{k}) \cdot \frac{(2\hat{i}+4\hat{j}+4\hat{k})}{6}$$

$$= \underline{68}$$

⇒ if $\vec{V} = \dfrac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}}$, $\vec{\nabla}\times\vec{V}$.

Solⁿ

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \dfrac{x}{\sqrt{x^2+y^2+z^2}} & \dfrac{y}{\sqrt{x^2+y^2+z^2}} & \dfrac{z}{\sqrt{x^2+y^2+z^2}} \end{vmatrix} = \underline{\underline{0}}.$$

⇒ Determine a, b, such that curl of vector

$$\vec{A} = (2xy+3yz)\,\hat{i} + (x^2+axz-4z^2)\,\hat{j} - (3xy+byz)\,\hat{k} = 0$$

Solⁿ

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 2xy+3yz & x^2+axz-4z^2 & -(3xy+byz) \end{vmatrix} = 0$$

$$\hat{i}\left[-x(3+a)+z(8-b)\right] + 6y\,\hat{j} + z(-3+a)\,\hat{k} = 0.$$

$\therefore$ $a+3=0$, $a-3=0$, $8-b=0$

$\underline{\underline{a=-3,3}}$, $\underline{\underline{b=8}}$.

→ Scalar potential func^n (f) :

$\vec{F} = \nabla\phi$, find ϕ ?

$\Rightarrow d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz$

$\therefore \left(\hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}\right)\left(\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz\right)$

$\therefore d\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\phi\ (d\vec{r})$

$d\phi = \nabla\phi\, d\vec{r}$

$d\phi = F\, d\vec{r}$

$\phi = \int \vec{F}\cdot d\vec{r}$, let $\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$

$\therefore \boxed{\phi = \int F_1\,dx + \int F_2\,dy + \int F_3\,dz}$

$\Rightarrow \int y\,dz + z\,dy \longrightarrow d(yz) \longrightarrow \underline{\underline{yz + c}}$

~~⇒ when~~

⇒ If vector field is given by :

$\vec{F} = (x^2 - y^2 + x)\,\hat{i} - (2xy + y)\,\hat{j}$, Is $\vec{F}$ irrotational . If so, find scalar potential ?

Sol^n $\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 - y^2 + x & -(2xy + y) & 0 \end{vmatrix} = \underline{\underline{0}}$ (Hence irrotational)

$\vec{F} = \nabla\phi$

$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz$

$\phi = \int \vec{F}\cdot d\vec{r}$