* $f(x) = \frac{a}{x} + bx$, $f(2) = 1$, has extreme value at $x = 2$

find a & b.

Sol^n $f'(x) = -\frac{a}{x^2} + b = 0$ $\qquad\qquad -ax^{-2}$

$b = \frac{a}{x^2}$ $\qquad\qquad -a(-2x^{-3})$

$x^2 = \frac{a}{b} \Rightarrow x = \pm\sqrt{\frac{a}{b}}$ $\qquad\qquad \frac{2a}{x}$

$\cancel{f''(x) = \frac{2a}{x^3} = \frac{2a}{\left(\frac{a}{b}\right)^{3/2}}}$ $\longrightarrow$ $\underline{a = 4b} \leftarrow x = 2.$

$f(2) = 1$

$\frac{a}{2} + 2b = 1$

$a + 4b - 2 = 0.$

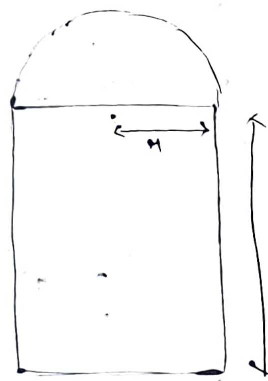
$8b = 2$

$b = 1/4$, $a = 1$

$\Rightarrow$ A window has the form of rectangle summounted by a semi-circle. If the perimeter is 40ft, find its dimensions so that the greatest amount of light may be admitted.

Sol^n. Perimeter $= \pi r + (2l + 2r) = 40$

$\pi r + 2l + 2r = 40.$



Area $= \frac{\pi r^2}{2} + l(2r) + \frac{\pi r^2}{2} - \frac{\pi r^2}{2}$

$= \pi r^2 + 2rl - \frac{\pi r^2}{2}$

$= r(\pi r + 2l) - \frac{\pi r^2}{2}$

$= r(40 - 2r) - \frac{\pi r^2}{2}$

$\frac{dA}{dr}$ $= 40r - 2r^2 - \frac{\pi r^2}{2}$ $\qquad$ diff & solve

AREA BETWEEN THE CURVES :

→ Area bounded by the curve $y = f(x)$, $x = a$, $x = b$.

$$\int_a^b f(x)\,dx$$

→ Area bounded ⇒ $x = f(y)$, $y = a$, $y = b$.

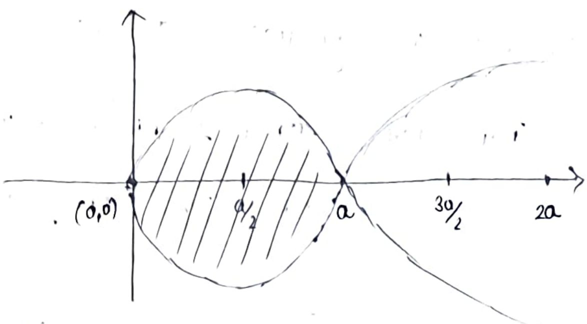
$$\int_a^b f(y)\,dy$$

⇒ $y^2 = \frac{x}{3a}(x-a)^2$

Soln $y = \pm \frac{\sqrt{x}(x-a)}{\sqrt{3a}} = f(x)$

x	0	$a/2$	a	$3a/2$	$2a$
y	0	$\frac{-\frac{a}{2}\sqrt{a}}{\sqrt{3a}}$	0	$\frac{\frac{a}{2}\frac{\sqrt{3a}}{2}}{\sqrt{3a}}$	$\frac{\sqrt{2a}\,a}{\sqrt{3a}}$

∴ Area $= \int_0^a \frac{\sqrt{x}(x-a)}{\sqrt{3a}}\,dx$



$A = \frac{1}{\sqrt{3a}} \int_0^a x^{3/2} - a x^{1/2}\,dx$

$A = \frac{1}{\sqrt{3a}} \left[\frac{2x^{5/2}}{5} - \frac{2a x^{3/2}}{3} \right]_0^a$

$A = \frac{1}{\sqrt{3a}} \left[\frac{2}{5}(a)^{5/2} - \frac{2a}{3}(a^{3/2}) \right]$

$A = \frac{1}{\sqrt{3a}} \left[\frac{2}{5}a^{5/2} - \frac{2}{3}a^{5/2} \right]$

$A = \frac{a^{5/2}}{\sqrt{3}\,a^{1/2}} \left[\frac{6-10}{15} \right]$ ⇒ $A = \frac{+4a^2}{15\sqrt{3}}$

for both sides of x

∴ Ans $= 2\left(\frac{4a^2}{15\sqrt{3}}\right)$

⇒ Find area b/w $a^2x^2 = y^3(2a-y)$

Sol^n) $a^2x^2 = 2ay^3 - y^4$

$$x = \pm\sqrt{\frac{2ay^3 - y^4}{a^2}} = \frac{(y^{3/2})(\sqrt{2a-y})}{a}$$

Let $y = 2a\sin^2\theta$.

$$x = \frac{(2a\sin^2\theta)^{3/2}\sqrt{2a - 2a\sin^2\theta}}{a}$$

$$x = 2a^{1/2}\sin^2\theta\sqrt{2a(1-\sin^2\theta)}$$

$$x = 2a^{1/2}\sin^2\theta(\sqrt{2a})(\cos^1\theta)$$

$4\sin^2\theta\cos^2\theta$

$\sin^2 2\theta$

$$x = (2a^{1/2})(2^{1/2}\cdot a^{1/2})\sin^2\theta\cos^2\theta$$

$$x = (2^{3/2}a)\frac{\sin^2 2\theta}{4}$$

$$x = 2^{-1/2}a\sin^2 2\theta$$

$$x = 2^{-1/2}a$$