16 Aug - 2023
Lec - 2

WAVE EQN IN THE STRING :

GOAL : To derive the 1D wave eqⁿ in a string.

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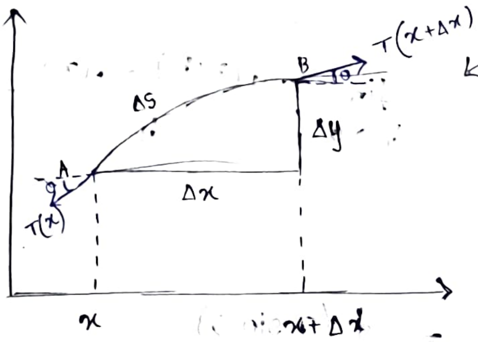
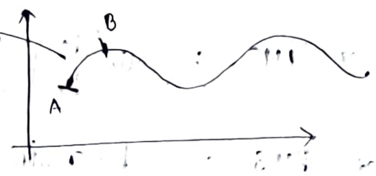


arc



$$\therefore \Delta S^2 = \Delta x^2 + \Delta y^2$$ ⟵ Pythogorus Theorem

for small value ΔS is st. line.

$$\Rightarrow \frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

* The magnitude of the tension point x & $x + \Delta x$ are resp. $T(x)$ & $T(x+\Delta x)$

* The net horizontal line is given as

$$[T(x+\Delta x) \cos\theta - T(x)\cos\theta]\,\hat{i}$$

* The net vertical line is given as :

$$[T(x+\Delta x)\sin\theta - T(x)\sin\theta]\,\hat{j}$$

* Let $\mu \longrightarrow$ mass per unit length.

* Because we are interested in the vertical ~~vertical~~ dispⁿ so, we are going to neglect horizontal motion.

* The notation of the transverse dispⁿ is denoted by y as a funcⁿ of x & t $[y(x,t)]$

* Using Newton's second law $[F = ma]$

$$\therefore [T(x+\Delta x)\sin\theta - T(x)\sin\theta] = (\mu \Delta S)\frac{d^2y}{dt^2}$$

now ~~multip~~ divide Δx both sides :

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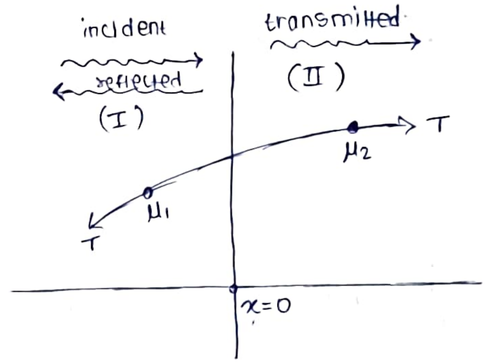
TRANSMISSION AND REFLETION OF WAVES :



$$V = \sqrt{\frac{T}{\mu}}$$

as $\mu_1 \neq \mu_2$

so, $V_1 \neq V_2$ & $K_1 \neq K_2$ $\left(K = \frac{\omega}{V}\right)$

→ GOAL :

To study the relation b/w the amplitude of incident wave, reflected wave & transmitted wave.

Region-1 $\begin{cases} f_I = Ae^{i(\omega t - K_1 x)} \\ f_R = Be^{i(\omega t + K_1 x)} \end{cases}$

Region-2 $\{ f_t = Ce^{i(\omega t - K_2 x)}$

* A geometric condition that the dispn is immediately same to the left & right of x=0 !

$$(f_I + f_R)\Big|_{x=0} = f_T\Big|_{x=0}.$$

* f & its derivatives must be continuous across the boundary.

$$\frac{d}{dx}(f_I + f_R)\Big|_{x=0} = \frac{df_T}{dx}\Big|_{x=0} \longrightarrow (1)$$

* If the above condition is not true, then the accn would be infinity.