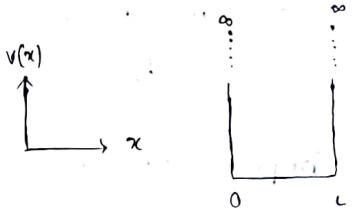
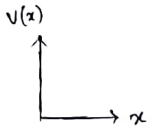
TUNNEL EFFECT :

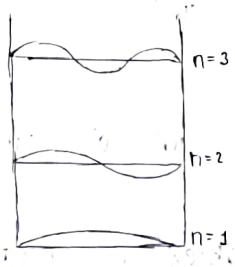
* We knew from particle in 1D box :

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), \quad n = 1, 2, 3, \ldots \quad \text{(eigenfunction)}$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$



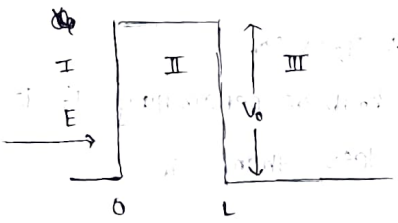
* 

$\Rightarrow$ 

$$\Rightarrow \int_{-\infty}^{\infty} |\psi|^2 \, dx = 1$$

$$\Rightarrow \int_{-\infty}^{0} |\psi|^2 dx + \int_{0}^{L} |\psi|^2 dx + \int_{L}^{\infty} |\psi|^2 dx = 1$$

$\Rightarrow$

* 

* Classically the particle would not exist in region III. But quantum mechanically we can find the prob. of finding the particle in reg III

$$T \simeq \exp(-2K_2 L)$$

where $K_2 = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$.

$\Rightarrow$ Compute the dim. of K_2.

Soln) $\exp(-2K_2 L) \quad \therefore K_2 = 1/L = L^{-1}$ ($\because$ exp is dimensionless).

$\therefore M^0 L^{-1} T^0$

⇒ The atoms in solid posses a certain min^m zero-point energy even at 0K, while no such restriction holds for molecules in ideal gas. Use uncertainity principle to explain.

Soln In Atom:

The particles are fixed to lattice due to which at 0K, the particles has Δx very less, so Δp is very large This shows that they have min^m zero point energy associated with their non-zero momentum, which keeps them vibrating even at zero temp.

In Molecule:

In ideal gas, molecules are far apart, hence here Δx is large, so, Δp is small. This means that in ideal gas, the molecules can have their KE reduced to very close to zero at 0K.

In summary, the principle prevents atoms in solid from coming to complete stop at 0K, leading to min^m zero point energy due to Δp, On other hand, in ideal gas, the relatively larger ~~large~~ Δp Δx, allows molecules to have KE close to zero at 0K.

Δx & KE

far > 10 M

far 5-M : For solid, there is restriction for position of each atoms and so, we can't set Δx as infinity, means Δp is finite so there should be energy even if temp is 0K. But for molecule there is no restriction, so it can be zero at 0K.