MODULE - 2

→ Gradient : For a scalar (function)

$$\boxed{\text{grad}} \xrightarrow{f} \boxed{\text{vector fields}}$$

Notation : grad or $\vec{\nabla}$
(nabla).

∴ grad $f(x,y,z)$ (or) $\vec{\nabla} f(x,y,z)$

$$\Rightarrow \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k}$$

* $f_1(x,y,z) = (x+y+z)$

$$\text{grad}(f_1) = \frac{\partial (f_1)}{dx} \hat{i} + \frac{\partial (f_1)}{\partial y} \hat{j} + \frac{\partial (f_1)}{\partial z} (\hat{k})$$

$$= 1\hat{i} + 1\hat{j} + 1\hat{k}$$

This is a constant vector.

* $f_2(x,y,z) = x^2yz$

$$\text{grad}(f_2) = 2xyz\,\hat{i} + x^2z\,\hat{j} + x^2y\,\hat{k}$$

(imp)

↳ this is 'a' ~~constant~~ vector.
↳ one in many

it has infinite points so, we call it as vector fields.

grad (f_2) at particular point → then it is the vector
↳ only one

$$\underset{(x,y,z)}{\nabla \overrightarrow{\Delta T}} = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}$$

det T is a scalar for $T(x,y,z)$

$$dT = \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz$$

$$dT = \vec{\nabla}T \cdot \vec{dl}$$

where $\vec{l} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector

$$\vec{dl} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$dT = \vec{\nabla}T \cdot \vec{dl} = |\vec{\nabla}T|\,|\vec{dl}|\cos\theta$$ [dot ~~cross~~ product]

* if we fix the magnitude $|\vec{dl}|$ and search around various directions (i.e, we vary θ), the max^m change the func^n (i.e T) evidently occurs when θ is zero (or $\cos\theta = 1$)

* Which more precisely to say; for a fixed distant ($|\vec{dl}|$ = const), dT (change of the scalar) is max^m where you move in the same dir^n of grad (T)

$$dT = \vec{\nabla}T \cdot \vec{dl} = |\vec{\nabla}T|\,|\vec{dl}|\cos\theta$$

* The gradient ($\vec{\nabla}T$) points in the max^m increment of the func^n.

* The magnitude (i.e, $|\vec{\nabla}T|$) gives the slope (rate of increment) along this maximal dir^n.

⇒ The height of a certain hill.

$H(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$

where y is the distⁿ in north (some unit length).

x is the distⁿ in east (some unit length).

i) Where is the top of the hill located ?

ii) How high is the hill ?

Solⁿ) Using gradient

i) $\frac{\partial H}{\partial x} = 10[2y - 6x - 18] = 0$

$\Rightarrow y = 3x + 9 \longrightarrow (1)$

$\frac{\partial H}{\partial y} = 10[2x - 8y + 28] = 0$

$\Rightarrow x = 4y - 14 \longrightarrow (2)$

$\therefore y = 3(4y - 14) + 9 \quad \Rightarrow y = 12y - 42 + 9.$

$\Rightarrow 11y = 33$

$\Rightarrow y = 3$

$\& \; x = -2.$

∴ Top of hill is located at 2 miles west & 3 miles north.

ii) Height $= \sqrt{(-2-0)^2 + (3-0)^2} = \sqrt{4+9} = \sqrt{13}$

⇒ Prob

⇒ Complete the following gradient ?

$\vec{\nabla} r$ where $\vec{r}$ is the position vector

Sol^n ATQ : $\vec{r} = (x, y, z)$

$|\vec{r}| = \sqrt{x^2+y^2+z^2}$

grad is taken for magnitudes or scalar

$$\vec{\nabla} r = \frac{\partial r}{\partial x}\hat{i} + \frac{\partial r}{\partial y}\hat{j} + \frac{\partial r}{\partial z}\hat{k}$$

$$= \frac{\partial\sqrt{x^2+y^2+z^2}}{\partial x}\hat{i} + \frac{\partial\sqrt{x^2+y^2+z^2}}{\partial y}\hat{j} + \frac{\partial\sqrt{x^2+y^2+z^2}}{\partial z}\hat{k}$$

$$= \frac{x\hat{i}+y\hat{j}+z\hat{k}}{\sqrt{x^2+y^2+z^2}} = \frac{\vec{r}}{|\vec{r}|} = \hat{r} \text{ (unit vector)}$$

$\therefore$ $\vec{\nabla} r = \hat{r}$

$$\boxed{\frac{\partial\sqrt{x^2+y^2+z^2}}{\partial x} = \frac{1}{2}(2x) = x}$$

DIVERGENCE

* Notation $\longrightarrow$ $\text{div}(\vec{v})$ or $\vec{\nabla}\cdot\vec{v}$

$$\boxed{\text{div}} \xrightarrow{\text{vector field } (\vec{v})} \boxed{\begin{matrix}\text{scalar} \\ \text{function}\end{matrix}}$$

$$\vec{\nabla}\cdot\vec{v} = \left(\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial x} + \frac{\partial v_3}{\partial z}\right) \quad \text{where } \vec{v} = (v_1, v_2, v_3)$$

$$v_1 = v_1(x,y,z)$$
$$v_2 = v_2(x,y,z) \quad \Rightarrow v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$$
$$v_3 = v_3(x,y,z)$$

$\Rightarrow \vec{v_1} = x\hat{i} + y\hat{j}$

Soln $\text{div}(\vec{v_1}) = 1 + 1 = 2$ (const).

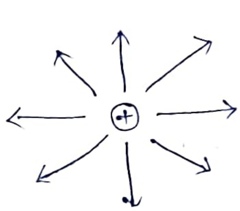
$\Rightarrow \vec{v_2} = x^2\hat{i} + y^2\hat{j} \Rightarrow \text{div}(\vec{v_2}) = 2(x+y)$ (not const).

But if we choose a point, then it is const.

$\therefore \text{div}(v_2)\big|_P = 2(a_1 + a_2) = \text{const}$

$\therefore$ $\text{div}(\vec{v})|_P > 0 \longrightarrow$ Fountain
$\text{div}(\vec{v})|_P < 0 \longrightarrow$ Sink
$\text{div}(\vec{v})|_P = 0 \longrightarrow$ Streamline

*



electric field

$\vec{E} = (x, y)$

$\therefore \text{div}(\vec{E})|_P > 0$ (Fountain)

* $\vec{\nabla}\cdot\vec{A}$, looks like dot product, but it is not

Because $\vec{\nabla}\cdot\vec{A} \neq \vec{A}\cdot\vec{\nabla}$

CURL

* Notation : curl $(\vec{A})$ or $\vec{\nabla}\times\vec{A}$

$$\boxed{\text{Curl}} \xrightarrow{\text{vector field}} \boxed{\text{vector field}}$$

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

where

$\vec{A} = (A_1, A_2, A_3)$

$A_n = A_n(x, y, z)$

$n = 1, 2, 3.$

$$\Downarrow$$

$$\hat{i}\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) + \hat{j}\left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x}\right) + \hat{k}\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)$$

$\Rightarrow \vec{V_1} = (-y, x)$, curl $(\vec{V_1})$

Sol^n ~~curl $(\vec{V_1})$~~ $\vec{V_1} = (-y, x, 0)$ using this

curl $(\vec{V_1}) = 2\hat{k}$

$\Rightarrow \vec{V_2} = (0, x, 0) \Rightarrow$ curl $(\vec{V_2}) = \hat{k}$

Sol^n
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & x & 0 \end{vmatrix} \Rightarrow \hat{i}\left(0 - \frac{\partial x}{\partial z}\right) - \hat{j}\left(0 - 0\right) + \hat{k}\left(\frac{\partial x}{\partial x} - 0\right)$$

$\Rightarrow \hat{k}$ (Ans)

30-Aug-2023

Volume / Surface Integral :

* Line Integral : $\int \vec{A} \cdot \vec{dl} \Rightarrow \int f(x)\, dx$

* Surface Integral : $\iint f(x,y)\, dx\, dy$

(or)

$$\iint \vec{A} \cdot \vec{ds} \quad \text{or} \quad \int_{surf} \vec{A} \cdot \vec{ds}$$

* Volume Integral : $\iiint f(x,y,z)\, dx\, dy\, dz$

$$\Rightarrow \int_{vol} T(x,y,z)\, d\underbrace{\tau}_{\text{volume}}$$

Three Theorems :

* $\int_{\vec{a}}^{\vec{b}} (\text{grad } T) \cdot \vec{dl} = T(\vec{b}) - T(\vec{a})$.

* $\int_{vol} (\text{div } \vec{v})\, d\tau = \int_{surf.} \vec{v} \cdot d\vec{s}$ (divergence theorem).

* $\int_{surf} (\text{curl } \vec{v}) \cdot d\vec{s} = \int_{line} \vec{v} \cdot \vec{dl}$ (Strokes theorem).

Maxwell's Eq^N in Differential Format :

* Gauss law of electrostatics : $\vec{\nabla} \cdot \vec{E} = \dfrac{\rho_{en}}{\epsilon_0}$

$$\iint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0} \Rightarrow \int (\vec{\nabla} \cdot \vec{E})\, d\cancel{\tau} = \frac{\iiint \rho\, d\cancel{\tau}}{\epsilon_0}$$

using divergence theorem.

* Gauss law of magnetostatics : $\vec{\nabla} \cdot \vec{B} = 0$ (no magnetic monopoles)

* Faraday's law : $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt}$$

$$\Rightarrow \iint_{surf} (\vec{\nabla} \times \vec{E}) \cdot \vec{ds} = -\frac{d}{dt} \iint_{surf} \vec{B} \cdot \vec{ds}$$

$\phi = \int \vec{B} \cdot \vec{ds}$

* Ampere's law (modified) : $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$, where $\vec{J}$ is current density.

MAXWELL'S EQ^N IN FREE SPACE : $(\rho = 0, \vec{J} = 0)$.

* $\vec{\nabla} \cdot \vec{E} = 0$
* $\vec{\nabla} \cdot \vec{B} = 0$
* $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$
* $\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

MAXWELL'S WAVE EQ^N IN FREE SPACE : (Derivation).

* We need to use a vector identity.

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$$

(or)

$$\text{curl}\,(\text{curl}\ \vec{A}) = \text{grad}\,(\text{div}\ \vec{A}) - \text{Lap}\,(\vec{A})$$

→ Case - 1 : For electric field $(\vec{E})$

LHS : $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(- \frac{\partial \vec{B}}{\partial t} \right)$

$= - \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B})$

$= - \frac{\partial}{\partial t} \left(\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) = - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$

RHS : $\vec{\nabla} (\cancel{\vec{\nabla} \cdot \vec{E}})^{0} - \nabla^2 \vec{E} = - \nabla^2 \vec{E}$ ← doubt

$\therefore - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = - \nabla^2 \vec{E}$

$\Rightarrow \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \nabla^2 \vec{E}$

$\Rightarrow \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \vec{E} = 0$

$\square \vec{E} = 0$

→ Case - 2 : For magnetic field : $(\vec{B})$.

$\square \vec{B} = 0$

$\square \begin{Bmatrix} \vec{E} \\ \vec{B} \end{Bmatrix} = 0$, in free space.

* Important :

i) curl (grad f) $= \nabla \times \nabla f = 0$

ii) div (grad f) $= \nabla \cdot \nabla f = \nabla^2 f$

iii) div (curl f) $= \nabla \cdot (\nabla \times f) = 0$

imp (proof after 2 pgs)

Trick: CG, DC = 0

DG = ∇^2

(or)

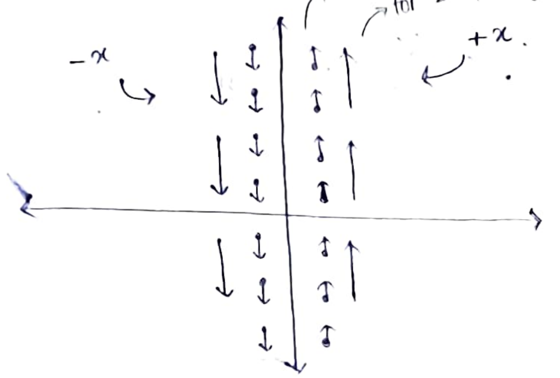
DCCG = 0

31-08-23

PROBLEMS ON MODULE-2 :

$\Rightarrow \vec{V_1} = x\hat{j}$, Plot this vector field in the (x,y) plane.

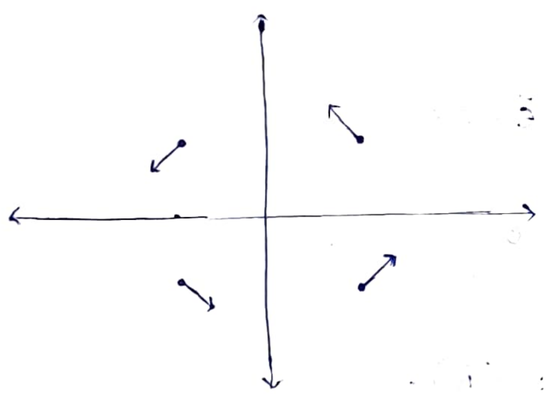
Sol^n)



$\text{curl } \vec{V_1} = \hat{k}$

$\Rightarrow \vec{V_2}(x,y) = (-y, x) = -y\hat{i} + x\hat{j}$

Sol^n)



$V_2(1,1) = -\hat{i}+\hat{j}$

$V_2(-1,1) = -\hat{i}-\hat{j}$

$V_2(-1,-1) = \hat{i}-\hat{j}$

$V_2(1,-1) = +\hat{i}+\hat{j}$

$\therefore \text{curl}(\vec{V_2}) = 2\hat{k}$

$\Rightarrow$ Compute grad (r^n).

Sol^n) $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$r = \sqrt{x^2+y^2+z^2}$

$r^n = (x^2+y^2+z^2)^{n/2}$ partial diff.

$\Rightarrow \frac{n}{2}\left[x^2+y^2+z^2\right]^{\frac{n-2}{2}} [2x+2y+2z]$

$\Rightarrow n r^{n-2} \vec{r}$

$\vec{\nabla} r^n = n r^{n-2} \vec{r}$

$\vec{\nabla} r = \frac{\vec{r}}{r} = \hat{r} \longrightarrow$ from pre que.

$\Rightarrow$ Compute $\nabla^2\left(\frac{1}{r}\right)$.

Sol^n/ $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$$\therefore \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(x^2+y^2+z^2)^{-1/2}\right) \Rightarrow \frac{\partial}{\partial x}\left(\frac{-1}{2}(x^2+y^2+z^2)^{-3/2}(2x)\right)$$

$$\Rightarrow \frac{\partial}{\partial x}\left(-x\,(x^2+y^2+z^2)^{-3/2}\right)$$

$$\Rightarrow \frac{\partial}{\partial x}\left(\frac{-x}{(x^2+y^2+z^2)^{3/2}}\right)$$

$$\Rightarrow \frac{(x^2+y^2+z^2)^{3/2}(-1) - (-x)\left(\frac{3}{2}(x^2+y^2+z^2)^{1/2}(2x)\right)}{(x^2+y^2+z^2)^3}$$

$$\Rightarrow \frac{-(x^2+y^2+z^2)^{3/2} + 3x^2(x^2+y^2+z^2)^{1/2}}{(x^2+y^2+z^2)^3}$$

$$\Rightarrow \frac{3x^2(x^2+y^2+z^2)^{1/2} - (x^2+y^2+z^2)^{1/2}(x^2+y^2+z^2)}{(x^2+y^2+z^2)^3}$$

$$\Rightarrow \frac{(x^2+y^2+z^2)^{1/2}\left[3x^2 - (x^2+y^2+z^2)\right]}{(\text{—})^3}$$

$$\Rightarrow \frac{2x^2-y^2-z^2}{(\text{—})^{5/2}}$$

$\therefore$ Similarly : for y : $\frac{2y^2-x^2-z^2}{(\text{—})^{5/2}}$

z : $\frac{2z^2-x^2-y^2}{(\text{—})^{5/2}}$

$\therefore$ Add we get zero

Hence : $\nabla^2\left(\frac{1}{r}\right) = 0$

$\Rightarrow$ curl (grad f) = ? , $f = f(x,y,z)$

$\Rightarrow$ div (curl $\vec{A}$) = ? , $\vec{A} = (f_1(x,y,z), f_2(x,y,z), f_3(x,y,z))$

Solⁿ 1) curl (grad f)

$$\text{grad } f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$\text{curl (grad } f) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix}$$

$$\Rightarrow \hat{i}\left(\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y}\right) - \hat{j}\left(\frac{\partial^2 f}{\partial x \partial z} - \frac{\partial^2 f}{\partial z \partial x}\right) + \hat{k}\left(\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x}\right)$$

$$= 0\hat{i} + 0\hat{j} + 0\hat{k}$$

Solⁿ 2) div (curl $\vec{A}$).

$$\text{curl } \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$\Rightarrow \hat{i}\left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right) - \hat{j}\left(\frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z}\right) + \hat{k}\left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)$$

$$\text{div (curl } \vec{A}) = \frac{\partial}{\partial x}\left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)$$

$$= \frac{\partial^2 f_3}{\partial x \partial y} - \frac{\partial^2 f_2}{\partial x \partial z} - \frac{\partial^2 f_3}{\partial y \partial x} + \frac{\partial^2 f_1}{\partial y \partial z} + \frac{\partial^2 f_2}{\partial z \partial x} - \frac{\partial^2 f_1}{\partial z \partial y}$$

$$= \underline{\underline{0}} \quad \text{(Ans)}$$

04-09-2023

DISPLACEMENT CURRENT :

(Qualitatively and Quantitatively)

→ Maxwell's eq^n before modification.

* $\vec{\nabla}\cdot\vec{E} = \dfrac{\rho}{\epsilon_0}$

* $\vec{\nabla}\cdot\vec{B} = 0$

* $\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$

* $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ ~~$\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$~~

* Theorem : div (curl $\vec{A}$) = 0

Apply the th^m in eq^n (3).

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$0 \qquad\qquad -\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{B}) = 0$$

Apply the th^m in eq^n (4).

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\cdot(\mu_0\vec{J})$$

$$0 \qquad\qquad \neq 0$$

* $\vec{\nabla}\cdot\vec{J} = -\dfrac{\partial\rho}{\partial t}$ → continuity eq^n in electrodynamics.

$$\therefore \vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\mu_0\vec{J}) + \vec{\nabla}\left(\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\mu_0\vec{J}) + \mu_0\left(\epsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E})\right)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\mu_0\vec{J}) + \mu_0\epsilon_0\frac{\partial\rho}{\partial t}$$

$\vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot(\#)$

↑ to get zero we have to add some value

$$\therefore \vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\because \left(\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}\right) \rightarrow = -\frac{\partial}{\partial t}(\epsilon_0\vec{\nabla}\cdot\vec{E})$$

$$= -\epsilon_0\left(\vec{\nabla}\frac{\partial E}{\partial t}\right)$$

$$\therefore \vec{\nabla}\cdot\left[\vec{J} + \epsilon_0\frac{\partial E}{\partial t}\right]$$

Hence, Maxwells eq^n after modification :

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial E}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\vec{J}_d$$

↳ disp^n current

$$\therefore \vec{J}_d = \epsilon_0\frac{\partial E}{\partial t}$$

⇒ Let $\vec{v}$ be a vector value func^n :

$$\vec{v} = (-4x - 3y + 4z\ ;\ -3x + 3y + 5z\ ,\ 4x + 5y + 3z)$$

Find a scalar func^n $f(x,y,z)$ such that $\vec{v} = grad(f)$

Sol^n/

$$V = \frac{\partial(f)}{\partial x}\hat{i} + \frac{\partial(f)}{\partial y}\hat{j} + \frac{\partial(f)}{\partial z}\hat{k}$$

$\therefore$ ~~$\frac{\partial(f)}{\partial x} = -4$~~

But to get f, we need to do partial integration.

$\therefore$ ~~$\vec{f} = (-2x^2 - 3xy + 4xz), (-\frac{3}{2}x^2 + 3xy + 5xz)$~~

$$f = (-2x^2 - 3xy + 4xz)\ ,\ (-3xy + \frac{3}{2}y^2 + 5zy)\ ,\ (4xz + 5yz + \frac{3}{2}z^2)$$

$$\therefore f(x,y,z) = -2x^2 + \frac{3}{2}y^2 + \frac{3}{2}z^2 \underbrace{- 3xy + 4xz + 5zy}$$

only repeat once
∵ when diff wrt x, y, z, same
value gives two values.

DEFINITIONS : (IMP)

→ GRADIENT :

* It is a vector quantity but it is applied on scalar quantity.

* It explains the variations of function in x, y, z dir^n.

* ex : if we apply gradient to function of temp., then from gradient we can understant rate of change of temp in x, y & z dir^n.

→ DIVERGENCE :

* It is scalar quantity but applied on vector quantity.

* ~~it explains overall variation of function in x,y & z dirn.~~

* it explains how fast the area of span is changing.

* ex : If 4 frnds float down the river, each making corner square. If square is getting bigger, then river has +ve div, if it shrinks then -ve div.

→ CURL :

* It is a vector quantity & applied on vector quantity.

* It describes the circulation of vector field in 3-D space

* ex : At surface of river turbine rotate fast but inside the river it rotates slowly.

→ Explanation of 4 Maxwells eqn :

i) Electric field diverges from electric charge.

ii) There are no isolated magnetic poles

iii) Electric field are produced by changing magnetic fields

iv) circulating magnetic field are produced by changing electric fields

→ Displacement Current :

The change in electric field gives rise to a current. As a result, magnetic field is induced.

→ Assumption of string vibration :

* Strings are assumed to be inextensible.

* The vibratory dispⁿ are small.

* The θ is very small.