MODULE - 2

→ Gradient : For a scalar (function)

$$\boxed{\text{grad}} \xrightarrow{f} \boxed{\text{vector fields}}$$

Notation : grad or $\vec{\nabla}$ (nabla).

$\therefore$ grad $f(x,y,z)$ (or) $\vec{\nabla} f(x,y,z)$

$$\Rightarrow \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

* $f_1(x,y,z) = (x+y+z)$

$$\text{grad}(f_1) = \frac{\partial(f_1)}{dx}\hat{i} + \frac{\partial(f_1)}{\partial y}\hat{j} + \frac{\partial(f_1)}{\partial z}(\hat{k})$$

$$= 1\hat{i} + 1\hat{j} + 1\hat{k}$$

→ this is 'a' constant vector.

* $f_2(x,y,z) = x^2yz$

it has infinite points so, we call it as vector fields.

$$\text{grad}(f_2) = 2xyz\,\hat{i} + x^2z\,\hat{j} + x^2y\,\hat{k}$$ (imp)

↳ this is 'a' ~~constant~~ vector.

↳ one in many

grad (f_2) at particular point → then it is the vector

↳ only one

$$\underset{(x,y,z)}{\nabla \vec{T}} = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}$$

Let T is a scalar for $T(x,y,z)$

$$dT = \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz$$

$$dT = \vec{\nabla}T \cdot d\vec{l}$$

where $\vec{l} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector

$$d\vec{l} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$dT = \vec{\nabla}T \cdot d\vec{l} = |\vec{\nabla}T|\,|d\vec{l}| \cos\theta$$ [dot ~~cross~~ product]

* if we fix the magnitude $|d\vec{l}|$ and search around various directions (i.e, we vary θ), the max^m change the func^n (i.e T) evidently occurs when θ is zero (or $\cos\theta = 1$)

* which more precisely to say; for a fixed distant ($|d\vec{l}|$ = const), dT (change of the scalar) is max^m where you move in the same dire^n of grad (T)

$$dT = \vec{\nabla}T \cdot d\vec{l} = |\vec{\nabla}T|\,|d\vec{l}| \cos\theta$$

* The gradient $(\vec{\nabla}T)$ points in the max^m increment of the func^n

* The magnitude (i.e. $|\vec{\nabla}T|$) gives the slope (rate of increment) along this maximal dir^n.

⇒ The height of a certain hill.

$$H(x,y) = 10\,(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$$

where y is the distⁿ in north (some unit length)

x is the distⁿ in east (some unit length).

i) Where is the top of the hill located ?

ii) How high is the hill ?

Solⁿ) Using gradient

i) $\dfrac{\partial H}{\partial x} = 10\left[2y - 6x - 18\right] = 0$

$$\Rightarrow y = 3x + 9 \longrightarrow (1)$$

$\dfrac{\partial H}{\partial y} = 10\left[2x - 8y + 28\right] = 0$

$$\Rightarrow x = 4y - 14 \longrightarrow (2)$$

$\therefore\ y = 3(4y - 14) + 9 \Rightarrow y = 12y - 42 + 9.$

$$\Rightarrow 11y = 33$$

$$\Rightarrow y = 3$$

$$\&\ x = -2$$

∴ Top of hill is located at 2 miles west & 3 miles north.

ii) Height $= \sqrt{(-2-0)^2 + (3-0)^2} = \sqrt{4+9} = \sqrt{13}$

~~Prob~~

⇒ Complete the following gradient :

$\vec{\nabla} r$ where $\vec{r}$ is the position vector

Soln) ATQ : $\vec{r} = (x, y, z)$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

grad is taken for magnitudes or scalar

$$\vec{\nabla} r = \frac{\partial r}{\partial x}\hat{i} + \frac{\partial r}{\partial y}\hat{j} + \frac{\partial r}{\partial z}\hat{k}$$

$$= \frac{\partial \sqrt{x^2+y^2+z^2}}{\partial x}\hat{i} + \frac{\partial \sqrt{x^2+y^2+z^2}}{\partial y}\hat{j} + \frac{\partial \sqrt{x^2+y^2+z^2}}{\partial z}\hat{k}$$

$$= \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}} = \frac{\vec{r}}{|\vec{r}|} = \hat{r} \text{ (unit vector)}$$

$$\therefore \vec{\nabla} r = \hat{r}$$

$$\boxed{\frac{\partial \sqrt{x^2+y^2+z^2}}{\partial x} = \frac{1}{2}(2x) = x}$$

DIVERGENCE

* Notation $\longrightarrow$ $\text{div}(\vec{v})$ or $\vec{\nabla} \cdot \vec{v}$

$$\boxed{\text{div}} \xrightarrow{\text{vector field } (\vec{v})} \boxed{\text{scalar function}}$$

$$\vec{\nabla} \cdot \vec{v} = \left(\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial x} + \frac{\partial v_3}{\partial z} \right)$$ where, $\vec{v} = (v_1, v_2, v_3)$

$v_1 = v_1(x, y, z)$
$v_2 = v_2(x, y, z)$ $\Rightarrow$ $v_1 \hat{i} + v_2 \hat{j} + v_3 \hat{k}$
$v_3 = v_3(x, y, z)$

$\Rightarrow$ $\vec{v_1} = x\hat{i} + y\hat{j}$

Sol^n $\text{div}(\vec{v_1}) = 1 + 1 = 2$ (const).

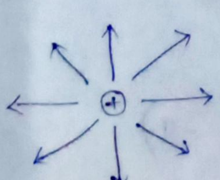
$\Rightarrow$ $\vec{v_2} = x^2 \hat{i} + y^2 \hat{j}$ $\Rightarrow$ $\text{div}(\vec{v_2}) = 2(x+y)$ (not const).

But if we choose a point, then it is const:

$\therefore$ $\text{div}(v_2)\big|_p = 2(a_1 + a_2) = \text{const}$

$\therefore$ $\text{div}(\vec{v})\big|_p > 0 \longrightarrow$ Fountain
$\text{div}(\vec{v})\big|_p < 0 \longrightarrow$ Sink
$\text{div}(\vec{v})\big|_p = 0 \longrightarrow$ Streamline

*



electric field
$\vec{E} = (x, y)$

$\text{div}(\vec{E})\big|_p > 0$ (Fountain)

* $\vec{\nabla}\cdot\vec{A}$, looks like dot product , but it is not

Because $\vec{\nabla}\cdot\vec{A} \neq \vec{A}\cdot\vec{\nabla}$

CURL

* Notation : curl $(\vec{A})$ or $\vec{\nabla}\times\vec{A}$

$\boxed{\text{Curl}} \xrightarrow{\text{vectorfield}} \boxed{\text{vector field}}$

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

where

$\vec{A} = (A_1, A_2, A_3)$

$A_n = A_n(x, y, z)$

$n = 1, 2, 3.$

$\Downarrow$

$$\hat{i}\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) + \hat{j}\left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x}\right) + \hat{k}\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)$$

$\Rightarrow \vec{v_1} = (-y, x)$, curl $(\vec{v_1})$

Soln ~~curl $(\vec{v_1})$~~ $\vec{v_1} = (-y, x, 0)$ using this

curl $(\vec{v_1}) = 2\hat{k}$.

$\Rightarrow \vec{v_2} = (0, x, 0) \Rightarrow$ curl $(\vec{v_2}) = \hat{k}$

Soln
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & x & 0 \end{vmatrix} \Rightarrow \hat{i}\left(0 - \frac{\partial x}{\partial z}\right) - \hat{j}(0 - 0) + \hat{k}\left(\frac{\partial x}{\partial x} - 0\right)$$

$\Rightarrow \hat{k}$ (Ans)

30-Aug-2023

Volume / Surface Integral :

* Line Integral : $\int \vec{A} \cdot \vec{dl} \Rightarrow \int f(x)\, dx$

* Surface Integral : $\iint f(x,y)\, dx\, dy$

(or)

$$\iint \vec{A} \cdot \vec{ds} \quad \text{or} \quad \int_{surf} \vec{A} \cdot \vec{ds}$$

* Volume Integral : $\iiint f(x,y,z)\, dx\, dy\, dz$

$$\Rightarrow \int_{vol} T(x,y,z)\, d\underset{\nwarrow \text{volume}}{\tau}$$

Three Theorems :

* $\int_{\vec{a}}^{\vec{b}} (\text{grad } T) \cdot \vec{dl} = T(\vec{b}) - T(\vec{a})$

* $\int_{vol} (\text{div } \vec{v})\, d\tau = \int_{surf} \vec{v} \cdot \vec{ds}$ (divergence theorem).

* $\int_{surf} (\text{curl } \vec{v}) \cdot \vec{ds} = \int_{line} \vec{v} \cdot \vec{dl}$ (Strokes theorem).

Maxwell's Eqn in Differential Format :

* Gauss law of electrostatics : $\vec{\nabla} \cdot \vec{E} = \frac{\rho_{en}}{\epsilon_0}$

$$\iint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0} \Rightarrow \int (\vec{\nabla} \cdot \vec{E})\, d\cancel{\tau} = \frac{\iiint \rho\, d\cancel{\tau}}{\epsilon_0}$$

using divergence theorem.

* Gauss law of magnetostatics : $\vec{\nabla} \cdot \vec{B} = 0$ (no magnetic monopoles)

* Faraday's law : $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt}$$

$$\Rightarrow \iint_{surf} (\vec{\nabla} \times \vec{E}) \cdot \vec{ds} = -\frac{d}{dt} \iint_{surf} \vec{B} \cdot \vec{ds}.$$

$\phi = \int \vec{B} \cdot \vec{ds}$

* Ampere's law : $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$, where
(modified)
$\vec{J}$ is current density.

MAXWELL'S EQ^N IN FREE SPACE : ($\rho = 0$, $\vec{J} = 0$).

* $\vec{\nabla} \cdot \vec{E} = 0$
* $\vec{\nabla} \cdot \vec{B} = 0$
* $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
* $\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$.

MAXWELL'S WAVE EQ^N IN FREE SPACE : (Derivation).

* We need to use a vector identity.

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}.$$

(or)

curl (curl $\vec{A}$) = grad (div $\vec{A}$) − Lap ($\vec{A}$)

→ Case - 1 : For electric field ($\vec{E}$)

LHS : $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t} \right)$

$= -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B})$

$= -\frac{\partial}{\partial t} \left(\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$

RHS : $\vec{\nabla} \cdot \cancel{(\vec{\nabla} \cdot \vec{E})}^{0} - \nabla^2 \vec{E} = -\nabla^2 \vec{E}$

$\therefore -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = -\nabla^2 \vec{E}$

$\Rightarrow \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \nabla^2 \vec{E}$

$\Rightarrow \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \vec{E} = 0$

$\Box \vec{E} = 0$

→ Case - 2 : For magnetic field : ($\vec{B}$).

$\Box \vec{B} = 0$

$\Box \begin{Bmatrix} \vec{E} \\ \vec{B} \end{Bmatrix} = 0$, in free space.

* Important :

i) curl (grad f) = $\nabla \times \nabla f = 0$

ii) div (grad f) = $\nabla \cdot \nabla f = \nabla^2 f$

iii) div (curl f) = $\nabla \cdot (\nabla \times f) = 0$

} imp (proof after 2 pgs)

Trick: CG, DC = 0

DG = ∇^2 }

(or)

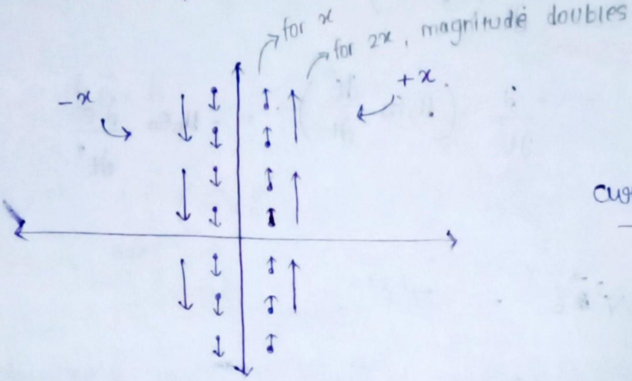
DCCG = 0

31-08-23

PROBLEMS ON MODULE-2 :

⇒ $\vec{V_1} = x\hat{j}$, Plot this vector field in the (x,y) plane.

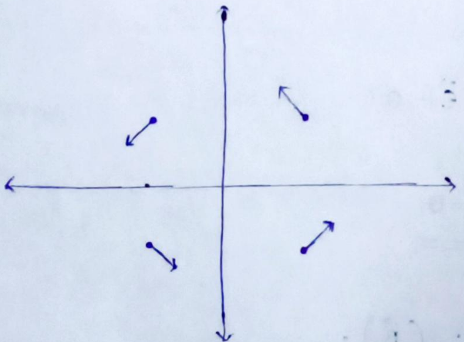
Sol^n)



$\text{curl } \vec{V_1} = \hat{k}$

⇒ $\vec{V_2}(x,y) = (-y, x) = -y\hat{i} + x\hat{j}$

Sol^n)



$V_2(1,1) = -\hat{i} + \hat{j}$

$V_2(-1,1) = -\hat{i} - \hat{j}$

$V_2(-1,-1) = \hat{i} - \hat{j}$

$V_1(1,-1) = +\hat{i} + \hat{j}$

$\therefore \text{curl}(\vec{V_2}) = 2\hat{k}$

⇒ Compute $\text{grad}(r^n)$

Sol^n) $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$r = \sqrt{x^2 + y^2 + z^2}$

$r^n = (x^2 + y^2 + z^2)^{n/2}$

partial diff.

$\Rightarrow \frac{n}{\cancel{2}}\left[x^2 + y^2 + z^2\right]^{\frac{n-2}{2}} \left[\cancel{2}x + \cancel{2}y + \cancel{2}z\right]$

$\Rightarrow n r^{n-2} \vec{r}$

$\vec{\nabla} r^n = n r^{n-2} \vec{r}$

$\vec{\nabla} r = \frac{\vec{r}}{r} = \hat{r}$ ⟶ from pre que.

$\Rightarrow$ Compute $\nabla^2\left(\frac{1}{r}\right)$.

Sol^n/ $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$\therefore \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(x^2+y^2+z^2)^{-1/2}\right) \Rightarrow \frac{\partial}{\partial x}\left(-\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2x)\right)$

$\Rightarrow \frac{\partial}{\partial x}\left(-x(x^2+y^2+z^2)^{-3/2}\right)$

$\Rightarrow \frac{\partial}{\partial x}\left(\frac{-x}{(x^2+y^2+z^2)^{3/2}}\right)$

$\Rightarrow \frac{(x^2+y^2+z^2)^{3/2}(-1) - (-x)\left(\frac{3}{2}(x^2+y^2+z^2)^{1/2}(2x)\right)}{(x^2+y^2+z^2)^3}$

$\Rightarrow \frac{-(x^2+y^2+z^2)^{3/2} + 3x^2(x^2+y^2+z^2)^{1/2}}{(x^2+y^2+z^2)^3}$

$\Rightarrow \frac{3x^2(x^2+y^2+z^2)^{1/2} - (x^2+y^2+z^2)^{1/2}(x^2+y^2+z^2)}{(x^2+y^2+z^2)^3}$

$\Rightarrow \frac{(x^2+y^2+z^2)^{1/2}\left[3x^2 - (x^2+y^2+z^2)\right]}{(\text{—})^3}$

$\Rightarrow \frac{2x^2-y^2-z^2}{(\text{—})^{5/2}}$

$\therefore$ similarly: for y: $\frac{2y^2-x^2-z^2}{(\text{—})^{5/2}}$

z: $\frac{2z^2-x^2-y^2}{(\text{—})^{5/2}}$

$\therefore$ Add we get zero

Hence: $\nabla^2\left(\frac{1}{r}\right) = 0$

$\Rightarrow$ curl (grad f) = ? , $f = f(x,y,z)$

$\Rightarrow$ div (curl $\vec{A}$) = ? , $\vec{A} = (f_1(x,y,z), f_2(x,y,z), f_3(x,y,z))$

Sol^n 1) curl (grad f)

$$\text{grad } f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$\text{curl}(\text{grad } f) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix}$$

$$\Rightarrow \hat{i}\left(\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y}\right) - \hat{j}\left(\frac{\partial^2 f}{\partial x \partial z} - \frac{\partial^2 f}{\partial z \partial x}\right) + \hat{k}\left(\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x}\right)$$

$$= 0\hat{i} + 0\hat{j} + 0\hat{k}$$

Sol^n 2) div (curl $\vec{A}$).

$$\text{curl } \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$\Rightarrow \hat{i}\left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right) - \hat{j}\left(\frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z}\right) + \hat{k}\left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)$$

$$\text{div}(\text{curl } \vec{A}) = \frac{\partial}{\partial x}\left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)$$

$$= \frac{\partial^2 f_3}{\partial x \partial y} - \frac{\partial^2 f_2}{\partial x \partial z} - \frac{\partial^2 f_3}{\partial y \partial x} + \frac{\partial^2 f_1}{\partial y \partial z} + \frac{\partial^2 f_2}{\partial z \partial x} - \frac{\partial^2 f_1}{\partial z \partial y}$$

$$= \underline{\underline{0}} \quad \text{(Ans)}$$

04-09-2023

DISPLACEMENT CURRENT :

(Qualitatively and Quantitatively)

→ Maxwell's eq^n before modification.

* $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$

* $\vec{\nabla}\cdot\vec{B} = 0$

* $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

* $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

* Theorem : div (curl $\vec{A}$) = 0

Apply the th^m in eq^n (3).

$$\underbrace{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{E})}_{0} = \vec{\nabla}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$-\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{B}) = 0$$

Apply the th^m in eq^n (4).

$$\underbrace{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B})}_{0} = \vec{\nabla}\cdot(\mu_0\vec{J})$$

$$\neq 0$$

* $\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$ → continuity eq^n in electrodynamics.

$$\therefore \vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\mu_0\vec{J}) + \vec{\nabla}\left(\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\mu_0\vec{J}) + \mu_0\left(\epsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E})\right)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\mu_0\vec{J}) + \mu_0\epsilon_0\frac{\partial\rho}{\partial t}$$

$\vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot(\#)$

↑ to get zero we have to add some value

$\therefore \vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$

$\because \left(\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}\right) \rightarrow -\frac{\partial}{\partial t}(\epsilon_0\vec{\nabla}\cdot\vec{E})$

$= -\epsilon_0\left(\vec{\nabla}\frac{\partial\vec{E}}{\partial t}\right)$

$\therefore \vec{\nabla}\cdot\left[\vec{J} + \epsilon_0\frac{\partial\vec{E}}{\partial t}\right]$

Hence, Maxwells eq^n after modification :

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\vec{J}_d$$

↳ disp^n current

$$\therefore \vec{J}_d = \epsilon_0\frac{\partial\vec{E}}{\partial t}$$

⇒ Let $\vec{V}$ be a vector value funcⁿ :

$$\vec{V} = (-4x - 3y + 4z , -3x + 3y + 5z , 4x + 5y + 3z)$$

Find a scalar funcⁿ $f(x,y,z)$ such that $\vec{V} = grad(f)$

Solⁿ)
$$V = \frac{\partial(f)}{\partial x}\hat{i} + \frac{\partial(f)}{\partial y}\hat{j} + \frac{\partial(f)}{\partial z}\hat{k}$$

$$\therefore \cancel{\frac{\partial(f)}{\partial x} = -4}$$

But to get f, we need to do partial integration.

$$\therefore \cancel{\vec{f} = \left(-2x^2 - 3xy + 4xz\right), \left(-\frac{3}{2}x^2 + 3xy + 5xz\right)}$$

$$f = \left(-2x^2 - 3xy + 4xz\right), \left(-3xy + \frac{3}{2}y^2 + 5zy\right), \left(4xz + 5yz + \frac{3}{2}z^2\right)$$

$$\therefore f(x,y,z) = -2x^2 + \frac{3}{2}y^2 + \frac{3}{2}z^2 \underbrace{- 3xy + 4xz + 5zy}$$

only repeat once
∵ when diff wrt x, y, z , same value gives two values.

DEFINITIONS : (IMP)

→ GRADIENT :

* It is a vector quantity but it is applied on scalar quantity.

* It explains the variations of function in x, y, z : dirⁿ

* ex: if we apply gradient to function of temp., then from gradient we can understant rate of change of temp in x, y & z dirⁿ.

→ DIVERGENCE :

* It is scalar quantity but applied on vector quantity.

* ~~it explains overall variation of function in x,y & z dir^n.~~

* it explains how fast the area of span is changing.

* ex : If 4 frnds float down the river, each making corner square. If square is getting bigger, then river has +ve div, if it shrinks then -ve div.

→ CURL :

* It is a vector quantity & applied on vector quantity.

* It describes the circulation of vector field in 3-D space.

* ex : At surface of river turbine rotate fast but ~~at~~ inside the river it rotates slowly.

→ Explanation of 4 Maxwells eq^n :

i) Electric field diverges from electric charge

ii) There are no isolated magnetic poles

iii) ~~E~~ Electric field are produced by changing magnetic fields

iv) circulating magnetic field are produced by changing electric fields

→ Displacement Current :

The change in electric field gives ~~e~~ rise to a current. As a result, magnetic field is induced.

→ Assumption of string vibration :

* Strings are assumed to be inextensible

* The vibratory displ^n are small

* The θ is very small