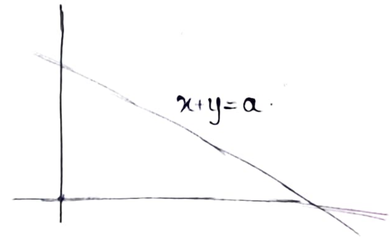
$\Rightarrow$ $\iiint_R (x^2+y^2+z^2)\, dx\, dy\, dz$. where R denotes region bound by

$x=0,\ y=0,\ z=0$ & $x+y+z=a$.

Sol^n $x+y+z=a$



in x-y plane $\rightarrow$ $x+y=a$

$\Rightarrow y=a-x$

for x-axis $\longrightarrow$ $x=a$

imp (must).

$$\int_0^a dx \int_0^{a-x} dy \int_0^{a-x-y} (x^2+y^2+z^2)\, dz$$

$\hookrightarrow$ solve

Whenever one eq^n comes then solve like this

INTEGRATION BY CHANGING INTO ~~POLAR~~ SPERICAL COORDINATES :

$x = r\sin\theta\cos\phi$

$y = r\ \cancel{\cos\theta}\ \sin\theta\ \sin\phi$

$z = r\cos\theta$.

$dx\, dy\, dz = |J|\, dr\, d\theta\, d\phi = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$\phi \rightarrow$ for x-y plane

$\theta \rightarrow$ for z plane

* It is used if the expression $x^2+y^2+z^2$ is involved in the problem

imp

* In a sphere $x^2+y^2+z^2=a^2$ the limits of r are 0 and a and limits of θ are $0, \pi$ and that of ϕ are 0 and 2π.

$\Rightarrow$ $\iiint_R x^2+y^2+z^2\, dx\, dy\, dz$, taken over the volume enclosed by sphere $x^2+y^2+z^2=1$

$\boxed{\phi \rightarrow 0 \text{ to } \pi}$ $\rightarrow$ i.e from +z to -z in most que unless said in que like next que.

imp

Sol^n

$$\int_0^{2\pi}\int_0^{2\pi}\int_0^1 r^2 (r^2 \sin\theta\, dr\, d\theta\, d\phi\, \cancel{dr})$$

$$\int_0^{2\pi} d\phi \int_0^{2\pi} \sin\theta\, d\theta \int_0^1 r^4\, dr = \frac{4\pi}{5}$$

⇒ $\iiint (x^2+y^2+z^2)\,dx\,dy\,dz$, over first octant of sphere

$x^2+y^2+z^2=a^2$

Soln: $\displaystyle\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{a} r^2\,(r^2 \sin\theta)\,dr\,d\theta\,d\phi$

first octant is

$\theta \rightarrow 0$ to $\pi/2$ &

$\phi \rightarrow 0$ to $\pi/2$

↳ solve

SOLVING WHEN TWO OR MORE EQN ARE GIVEN :

⇒ Find the volume of cylinder $x^2+y^2=4$ & $y+z=4$ & $z=0$

Soln: $z = 4-y$, $z=0$.

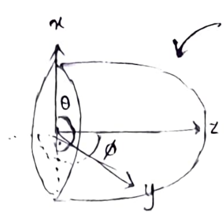
$\displaystyle\int_{-2}^{+2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{0}^{4-y} (\quad)$

↳ any eqⁿ given.

CONTINUING SPHERICAL COORDINATES :

⇒ $\displaystyle\iiint_E (x^2+y^2)\,dv$, E is region portion of $x^2+y^2+z^2=4$, with $y\geq 0$

Soln: from fig we can say :



Draw this in every que to understand que.

$\theta = 0 \rightarrow \pi$

$\phi = 0 \rightarrow \pi$.

$\therefore \displaystyle\int_0^{\pi}\int_0^{\pi}\int_0^{2} r^2 \sin^2\theta\; \cancel{d\theta\, d\phi\, dr}\,.\,(r^2 \sin\theta\; d\theta\, d\phi\, dr)$

$= \dfrac{128\pi}{15}$

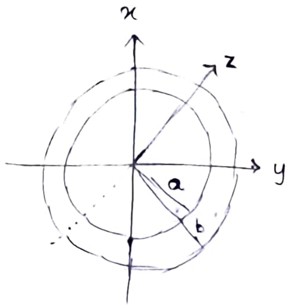
⇒ Find the volume of 2 concentric spheres $r_1 = a$ & $r_2 = b$

Sol^n here $r \Rightarrow a \rightarrow b$

around z-axis $\theta \Rightarrow 0 \rightarrow 2\pi$

$\phi \Rightarrow 0 \rightarrow \pi$

from +z to -z



$$\therefore \int_0^{2\pi}\int_0^{\pi}\int_a^{b} r^2 \sin\theta \, d\theta \, d\phi \, dr$$

↳ when nothing given in que, take $dv = r^2 \sin\theta \, d\theta \, d\phi \, dr$

(not in syllabus)

⇒ Find the volume of region above the cone $z^2 = x^2 + y^2$ and inside the sphere $x^2+y^2+z^2 = 2az$ $(a>0)$.

Sol^n $x^2+y^2+z^2 - 2az = 0$

$x^2+y^2+(z-a)^2 = a^2$

$\therefore$ centre $(0,0,a)$.

$r = a$.