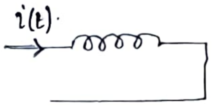
MAGNETIC CIRCUITS

* EM - It deals with analysis of electric or magnetic fields

* Magnetic coupling :

i) Self Inductance :

Current forms the flux and flux builds opp. emf



Acc to faradays law of EMI, EMF induced in circuit is proportional to no. of turns and rate of change of flux.

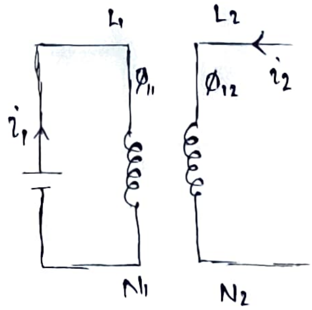
$$e = N \cdot \frac{d\phi}{dt}$$

$$\phi \propto I.$$

$$\therefore e = N \cdot \frac{d\phi}{di} \cdot \frac{di}{dt}$$

$$\Rightarrow e = L \frac{di}{dt}$$

ii) Mutual Inductance :



Case 1 : (Assume $i_2 = 0$)

Total flux $(\phi_1) = \phi_{11} + \phi_{12}$

$$\therefore V_1 = L_1 \cdot \frac{di_1}{dt} = N_1 \frac{d\phi_1}{di_1} \cdot \frac{di_1}{dt}$$

$$V_2 = M_{21} \frac{di_1}{dt} = N_2 \frac{d\phi_{12}}{di_1} \cdot \frac{di_1}{dt}$$

Case : 2 : (Assume $i_1 = 0$).

$$V_1 = M_{12} \cdot \frac{di_2}{dt}, \quad V_2 = L_2 \frac{di_2}{dt}$$

coeff of magnetic coupling $(K) = \frac{\phi_{12}}{\phi_{11}}$ (wrt prim. wiring).

" " " " $= \frac{\phi_{21}}{\phi_{22}}$ (wrt sec. winding).

$\therefore K \longrightarrow$ fraction of total flux that is produced by coil 1 linking coil 2.

* Coil 1 : $K = \frac{d\phi_{12}}{\phi_1} \Rightarrow d\phi_{12} = K\phi_1$

* Coil 2 : $K = \frac{d\phi_{21}}{\phi_2} \Rightarrow d\phi_{12} = K\phi_2$

$$\therefore M_{21} = N_2 \frac{d\phi_{12}}{di_1}, \quad M_{12} = N_1 \cdot \frac{d\phi_{21}}{di_2}$$

if $M_{21} = M_{12} = M$.

$$M^2 = K^2 \left(\frac{N_2\phi_1}{di_1}\right)\left(\frac{N_1\phi_2}{di_2}\right)$$

$$M^2 = K^2 L_1 L_2 \Rightarrow M = K\sqrt{L_1 L_2}$$

$$\therefore K = \frac{M}{\sqrt{L_1 L_2}}$$

* Note : For iron core : $K > 0.99$

Air core : $K \longrightarrow 0.4 - 0.8$.

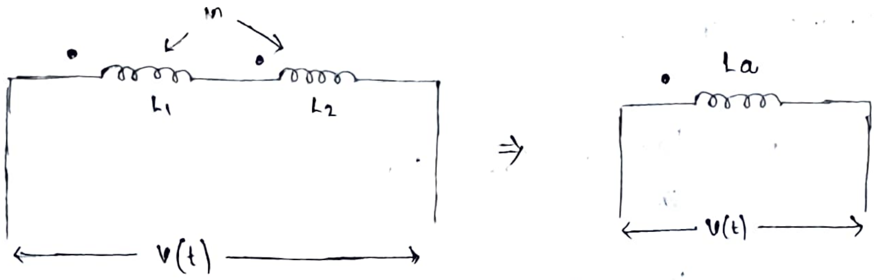
* if K_{max}, i.e, $K = 1$, then $M = \sqrt{L_1 L_2}$

→ Combination of conductively connected mutual coupled coil.

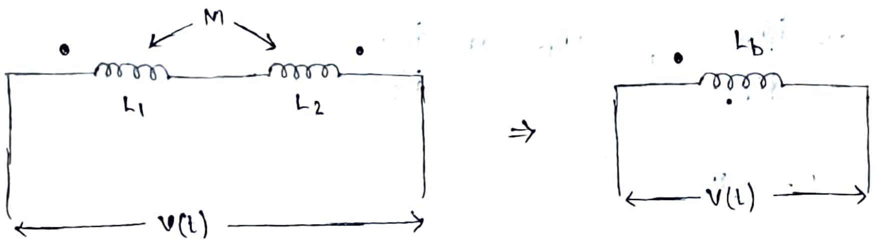
Series connection — series aiding / cumulative
— series opposition / differential.

Parallel connection — parallel aiding / "
— parallel opposition / "

* Series Aiding :



* Series opposition :



→ Aiding :

The current is entering both the coils through dotted terminal.

We can write :

$$L_1 \frac{di}{dt} + M\frac{di}{dt} + L_2 \frac{di}{dt} + M\frac{di}{dt} = v(t).$$

$$\Rightarrow \frac{di}{dt}\left(L_1 + M + L_2 + M\right) = v(t)$$

$$\Rightarrow v(t) = \frac{di}{dt}(L_a) \quad , \quad \text{where } L_a = L_1 + L_2 + 2M.$$

→ Series opposition :

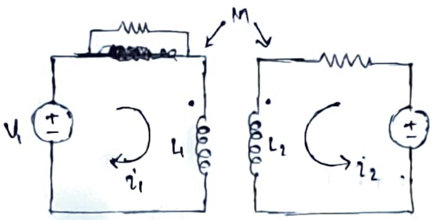
$$L_1 \frac{di}{dt} - M\frac{di}{dt} + L_2 \frac{di}{dt} - M\frac{di}{dt} = v(t).$$

$$\Rightarrow \frac{di}{dt}\left[L_1 + L_2 - 2M\right] = v(t).$$

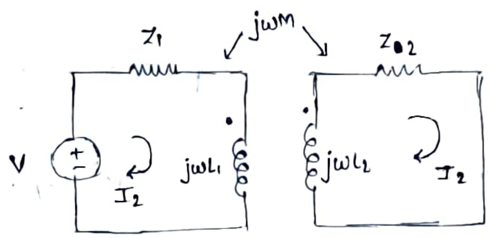
$$\Rightarrow v(t) = \frac{di}{dt}\left[L_b\right] \quad , \quad L_b = L_1 + L_2 - 2M.$$

→ Analysis of coupled coils :

Time domain :



Frequency domain.



→ Note :

Self Inductance :

$$\epsilon = L\frac{di}{dt} , \quad \text{where} \quad L = N\frac{d\phi}{di}$$

while solving eqn for 'K'

we have used : $L = \dfrac{N\phi}{di \text{ (or) } i}$

bcs $L \cdot \dfrac{di}{dt} = N \cdot \dfrac{d\phi}{dt}$.

Using MBS :

$$Li = N\phi$$

$$L = \frac{N\phi}{i} \Rightarrow N \cdot \frac{d\phi}{di}$$

⇒ Obtain the max^m possible mutual inductance b/w two coils of inductance 16H and 4H

Sol^n) $M = K\sqrt{L_1L_2}$.

$K_{max} = 1$

$\therefore M = \sqrt{L_1L_2} = 8H$.

⇒ Two inductively coupled coils have self inductance $L_1 = 50$ mH and $L_2 = 200$ mH. If the coeff. of coupling is 0.5

i) Find the value of M

ii) What is maxm possible M.

Sol^n) i) $M = K\sqrt{L_1L_2} = (0.5)\sqrt{(50)(200)} = 50$ mH.

ii) $M_{max} = 100$ mH.

⇒ Two identical coils A and B of 1000 turns each lie in parallel planes such that 80% flux produced by one coil links with another coil. A current of 5A flowing in coil A produces a flux of 0.05 mWb in it. If the current in coil A changes from +12A to −12A in 0.02 sec. Calculate:

i) M ii) E in coil B.

Sol^n) $N_1 = N_2 = 1000$ turns.

$\phi_{12} = 0.8 \times 0.05 \times 10^{-3} = 0.04 \times 10^{-3}$ Wb.

i) $M = \dfrac{N_2\phi_{12}}{i_1} = \dfrac{1000 \times 0.04 \times 10^{-3}}{5} = 8\times10^{-3}$ H.

ii) $E_2 = M\dfrac{di_1}{dt}$, $di_1 = 12 - (-12) = 24A$

$dt = 0.02$ sec

$$e_2 = \frac{8\times10^{-3} \times 24}{0.02} = \underline{\underline{9.6\ V}}$$

⇒ The no. of turns into coupled coils in 500 turns, 1500 turns resp. When 5A current flows in coil 1, the total flux in this coils is 0.6×10^{-3} Wb and the flux linking the second coil is 0.3×10^{-3} Wb. Determine L_1, L_2, M, K.

Sol^n $N_1 = 500$, $N_2 = 1500$, & $i_1 = 5A$, $\phi_{12} = 0.3\times10^{-3}$ Wb.

$$\therefore\ \phi_1 = \phi_{12} + \phi_{11}$$

$$\phi_1 = 0.6\times10^{-3} + 0.3\times10^{-3} = 0.9\times10^{-3}\ \text{Wb}.$$

$$L_1 = \frac{N_1\phi_1}{i_1} = 0.09\ H. \qquad M = \frac{N_2\phi_{12}}{i_1} = 0.09\ H.$$

$$K = \frac{\phi_{12}}{\phi_1} = 0.333$$

$$M = K\sqrt{L_1L_2} \Rightarrow L_2 = \frac{M^2}{K^2L_1} = 0.811\ H.$$

⇒ Two coils connected in series have an equivalent inductance of 0.8H when connected in aiding and an equivalent inductance 0.4H when connected in opposition. Calculate M.

Sol^n

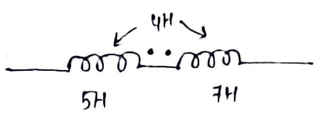
$$L_1 + L_2 + 2M = 0.8H$$

$$L_1 + L_2 - 2M = 0.4H$$

$$\therefore\ 4M = 0.4\ H$$

$$\underline{\underline{M = 0.1H}}$$

⇒ Determine equivalent inductance of series combination :



Sol^n

$$L_{eq} = L_1 + L_2 - 2M$$

$$= \underline{\underline{4H}}$$

⇒ Two coupled coils of self inductance $L_1 = 2H$, $L_2 = 4H$ are coupled in (i) series aiding ii) series opp iii) parallel aiding iv) parallel opp. If the mutual ind. is 0.5H, find the equivalent ind. in each case.

Sol^n^ i) $L_1 + L_2 + 2M = 7H$

imp.

ii) $L_1 + L_2 - 2M = 5H$

iii) $L_{eq} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = 1.55H$

iv) $L_{eq} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = 1.1071\ H$

⇒ Two identical coupled coils in series has an equivalent inductance value of 0.08 H & 0.0354 H, find the values of ind., M and K

Sol^n^ $L_{eq_1} = 0.08$, $L_{eq_2} = 0.0354$.

~~$L_{eq} = L_1 + L_2 \pm 2M$~~

$\therefore \quad L_{eq_1} = L_1 + L_2 + 2M \quad , \quad L_{eq_2} = L_1 + L_2 - 2M.$

$\therefore \quad 4M = L_{eq_1} - L_{eq_2}$

$M = \dfrac{0.08 - 0.0354}{4} = 0.0115\ H$

$\therefore \quad L_1 + L_2 = 0.08 - 2M = 0.0617$

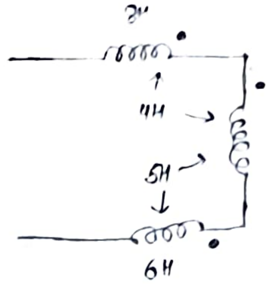
~~$L_1 + L_2 = 0.0354 + 2M =$~~

Since coils are identical : $L_1 = L_2 = L$

$L = 0.02885\ H$

$K = \dfrac{M}{\sqrt{L_1 L_2}} = 0.3614$

⇒ Calculate eff. inductance of circuit shown in fig :



Sol^n

$$V = 8\frac{di}{dt} - 4\frac{di}{dt} + 10\frac{di}{dt} - 4\frac{di}{dt} + 5\frac{di}{dt}$$

$$+ 6\frac{di}{dt} + 5\frac{di}{dt}$$

$$V = 26\frac{di}{dt}$$

$$\therefore V = L\frac{di}{dt} \qquad \therefore \underline{L_{eq} = 26\,H}$$

⇒ A coil has self ind. of 10H. If a current of 200 mA is reduced to zero in a time of 1 ms, find the avg. value of induced emf across the terminals of coils.

Sol^n

$L = 10\,H$

$dI = 200 \times 10^{-3}$ A.

$dt = 10^{-3}$ sec.

$$\therefore V = L\frac{di}{dt} = (10)\frac{(200\times10^{-3})}{10^{-3}} = \underline{2000\,V}$$

⇒ Calculate the emf induced in a coil of 200 turns when the flux linking with it changes from 1 mm to 3 mm in 0.1 sec

Sol^n

$$V = N\frac{d\phi}{dt} = (200)\frac{(2\times10^{-3})}{10^{-1}} = \underline{4V}$$

⇒ A coil has self ind. of 30mH. Calculate the emf induced in the coil when the current in the coil.

i) (↑) at rate of 300 A/sec

ii) Raises from 0 to 10A in 0.06 sec.

Sol^n:

i) $V = L\frac{di}{dt} = (30\times10^{-3})(300) = \underline{\underline{9V}}$.

ii) $V = (30\times10^{-3})\left(\frac{10}{6\times10^{-2}}\right) = \underline{\underline{5V}}$

⇒ When two coils are connected in series, their eff. inductance is found to be 10H. When the connections of one coil is reversed the eff. ind. is 6H. If the coeff. of coupling is 0.6, calculate L of each coil & M.

Sol^n:

$L_{eq_1} = 10, \quad L_{eq_2} = 6H, \quad K = 0.6.$

$\therefore \left.\begin{aligned} L_1+L_2+2M &= 10 \\ L_1+L_2-2M &= 6 \end{aligned}\right\} \rightarrow 4M = 4$

$\underline{\underline{M = 1H}}$.

$\therefore L_1+L_2 = 10-2M \Rightarrow L_1+L_2 = 8H.$

$\Rightarrow L_1 = 8-L_2$

$\therefore K = \frac{M}{\sqrt{L_1L_2}} \Rightarrow L_1L_2 = \frac{M^2}{K^2}$

$\Rightarrow (8-L_2)(L_2) = \frac{1}{0.36}$

$\Rightarrow L_2^2 - 8L_2 + 2.78 = 0.$

$\therefore L_2 = \frac{8\pm\sqrt{64-4(1)(2.78)}}{2} = \frac{8+7.27}{2}, \frac{8-7.27}{2}$

$\therefore L_2 = 7.635H$ or $0.365H$

$L_1 = 0.365$ or $7.635H$

MAGNETIC CIRCUITS

FORMULAS :

* $B = \dfrac{\phi}{A}$ wb/m² (or) Tesla.

$\phi \rightarrow$ flux

$A \rightarrow$ Area.

$B \rightarrow$ flux density

* mmf $= N \times I$ AT $\rightarrow$ ampere turns.

mmf $\rightarrow$ Magnetomotive force

N $\rightarrow$ no.of turns. of coil

* $H = \dfrac{mmf}{l} = \dfrac{N \times I}{l} = \dfrac{B}{\mu_0 \mu_r}$ AT/m

H $\rightarrow$ Magnetic field strength (or) Magnetic field intensity (or) Magnetising force.

* $\mu_0 = 4\pi \times 10^{-7}$

$\mu_r = 1$ (for air)

* $S \,(R_m) = \dfrac{l}{\mu_0 \mu_r A}$ AT/wb $= \dfrac{Ni}{\phi}$

S $\rightarrow$ Reluctance (opp. to flow of flux).

* $A = \dfrac{\pi d^2}{4}$ m²

d $\rightarrow$ inner diameter of ring

D $\rightarrow$ Mean diameter.

* $l = \pi D$ m

l $\rightarrow$ mean circumference (or) mean length (or), mean magnetic path.

* ϕ (flux) $\rightarrow$ wb. $= \dfrac{mmf}{reluctance}$

Toroidal core :

It is a circular core

Applications : Transformers, inductors to power electronic circuits.

* Permeability : (μ)

A property of magnetic material which indicates the ability of magnetic circuit to carry magnetic flux

$\mu = \dfrac{B}{H}$, $\mu_r = \dfrac{\mu}{\mu_0}$

Hopkinson's law :

MMF = flux × reluctance

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It is known as Ohm's law of magnetic circuit.

⇒ A magnetizing force of 8000 A/m is applied to circular magnetic circuit of mean diameter of mean diameter 30cm by passing a current through a coil wound on circuit. If the coil is uniformly wound around the circuit and has 750 turns, find i.

Sol^n)

$H = 8000$ A/m

$D = 30 \times 10^{-2}$ m.

$N = 750$

$l = \pi D = 30\pi \times 10^{-2}$ m.

$\therefore H = \frac{N \times i}{l}$

$\Rightarrow i = \frac{H \times l}{N} = \frac{8000 \times \cancel{30}\pi \times 10^{-2}}{\cancel{750}\ 25}$

$\Rightarrow i = 10.05$ A

⇒ Determine mmf req. to generate total flux of 100 μWb in air air gap coil 0.2 cm long. The cross sectional area of air gap is 25 cm².

Sol^n)

$\phi = 100 \times 10^{-6}$ Wb

$l = 0.2 \times 10^{-2}$ m.

$A = 25 \times 10^{-4}$ m²

$\cancel{mmf = \frac{l}{\mu_0 \mu_r A}}$

$\cancel{mmf = \frac{0.2 \times 10^{-2}}{4\pi \times 10^{-7} \times 25 \times 10^{-4}}}$

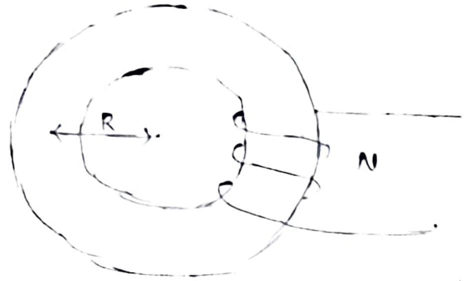
$B = \frac{\phi}{A} = \frac{100 \times 10^{-6}}{25 \times 10^{-4}} = 4 \times 10^{-2}$ Wb/m²

$H = \frac{B}{\mu_0 \mu_r} = \frac{4 \times 10^{-2}}{4\pi \times 10^{-7}} = 3.18 \times 10^{4}$ AT/m

$mmf = H \times l = 3.18 \times 10^{4} \times 0.2 \times 10^{-2} = 63.7$ AT

B in Toroidal core :

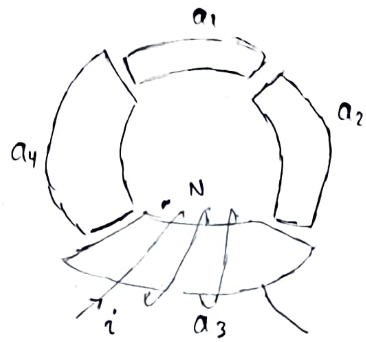
$$B = \frac{\mu N i}{2\pi R}$$



Series Magnetic Circuits :

Total reluctance $\Rightarrow S = \sum_{i=1}^{n} S_i$

where $S_i = \dfrac{l_i}{a_i \mu_0 \mu_{ri}}$



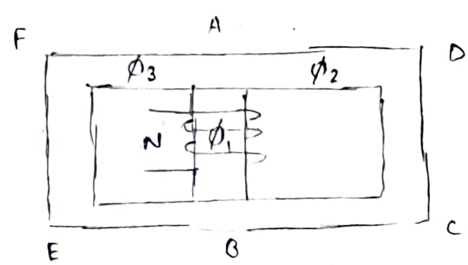
Total mmf $= \phi S$

$= \phi \left[\sum_{i=1}^{n} S_i \right]$

$= \sum \dfrac{B_i l_i}{\mu_0 \mu_{ri}} = \sum_{i=1}^{n} H_i l_i$

Parallel Magnetic Circuits :

$\phi_1 = \phi_2 + \phi_3$



S_1 = reluctance of BA $= \dfrac{l_1}{a_1 \mu_0 \mu_{r1}}$

S_2 = reluctance of AB,CD $= \dfrac{l_2}{a_2 \mu_0 \mu_{r2}}$

S_3 = reluctance of AFEB $= \dfrac{l_3}{a_3 \mu_0 \mu_{r3}}$

Total mmf or AT $= \phi_1 S_1 + \phi_2 S_2 = \phi_1 S_1 + \phi_3 S_3$

like
$i_1 = i_2 + i_3$
in current dc
we have
$\phi_1 = \phi_2 + \phi_3$
in magnetic circuit

Leakage flux :

$\phi = \phi_u + \phi_L$

$\phi_u \rightarrow$ usefull flux

$\phi_L \rightarrow$ leakage flux.

Leakage co-efficient or leakage factor :

$$\lambda = \frac{\phi}{\phi_u}$$

⇒ An iron ring of 400 cm mean circumference is made from iron of cross section 20cm². Its permeability is 500. If it is wound with 400 turns, what current would be required to produce flux of 0.01 wb ?

Soln

$l = 400 \times 10^{-2}$ m.

$A = 20 \times 10^{-4}$ m².

$\mu = 500$.

$N = 400$.

$\phi = 0.01$ wb.

$$H = \frac{Ni}{l} \Rightarrow i = \frac{Hl}{N}.$$

⇒ ~~A three~~ :

$$\mu = \frac{B}{H} \Rightarrow H = \frac{B}{\mu} = \frac{\phi/A}{\mu} = \frac{0.01}{20 \times 10^{-4} \times 500} = 10^{-2}.$$

$$i = \frac{(10^{-2})(400 \times 10^{-2})}{400} = \underline{\underline{10^{-4}\ A}}.$$

⇒ A three layer Toroidal core has flux density of core cross-section is 3T in the highest cross sectional area. The core is made of material having a relative permeability of 100. The first layer length is 3cm and area is $2cm^2$, 5cm & $1.5\ cm^2$, 8cm & $1cm^2$. The no. of turns of coil is 200. Find the reluctance of coil and determine current in coil.

Soln)

$B = 3T$.

$\mu_H = 100$

$l_1 = 3\times10^{-2}$, $a_1 = 2\times10^{-4}$.

$l_2 = 5\times10^{-2}$, $a_2 = 1.5\times10^{-4}$

$l_3 = 8\times10^{-2}$, $a_3 = 10^{-4}$

$N = 200$.

$$\Rightarrow S = S_1 + S_2 + S_3.$$

$$\Rightarrow \mu_0 S = \frac{3\times10^{-2}}{2\times10^{-4}\times100} + \frac{5\times10^{-2}}{1.5\times10^{-4}\times10^2} + \frac{8\times10^{-2}}{10^{-4}\times10^2}$$

$$\Rightarrow \mu_0 S = \frac{3}{2} + \frac{10}{3} + 8$$

$$\Rightarrow \mu_0 S = \frac{9+20+48}{6} = \frac{77}{6}$$

$$\Rightarrow S = \frac{77}{6\mu_0}$$

$\cancel{B = \frac{\phi}{A}}$

$$B = \frac{\phi}{A}$$

$\therefore$ &

$\Rightarrow \cancel{\phi = BA = 3\times}$

$$\Rightarrow \phi = BA = (3)($$

⇒ A circular iron ring of mean diameter 25 cm & cross sectional area 9 cm² is bound with a coil of 100 turns and carries a current of 1.5 A. The relative permeablity of iron is 2000. Calc. amt of flux produced in ring.

Sol^n) $i = 1.5\ A$

The mean length of flux path $(l) = \pi D$

$= (3.14)(25 \times 10^{-2})$

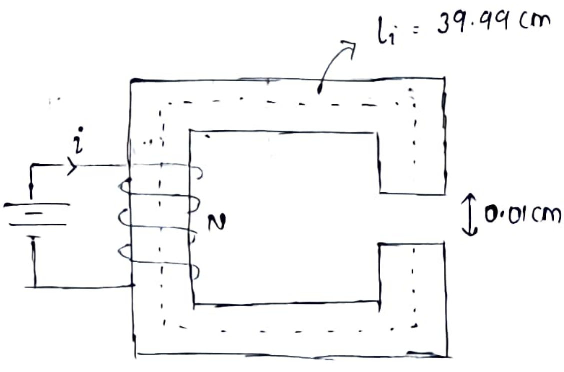
$= \underline{0.785\ m}$

$$\text{flux } (\phi) = \frac{\text{MMF}}{\text{reluctance}} = \frac{Ni}{\frac{l}{\mu_0 \mu_r A}} = \frac{100 \times 1.5}{0.785} \times 4\pi \times 10^{-7} \times 200 \times 9 \times 10^{-4}$$

$= \underline{432\ \mu Wb}$

⇒ A rect shape iron core has an air gap of 0.01 cm. The mean length of flux part through iron is 39.99 cm. The relative permeability of iron is 2000. The coil has 1000 turns. Cross sectional area is 9 cm². Calc. current read to produce flux of 1 mWb.

Sol^n)



The total reluctance of flux path

↳ reluctance of iron path + that of air gap.

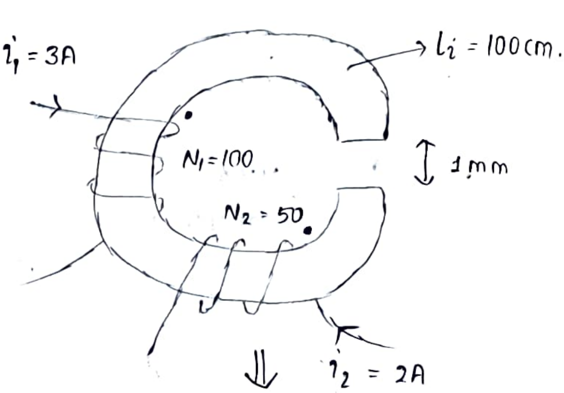
$$S = \frac{l_i}{\mu_0 \mu_r A} + \frac{l_g}{\mu_0 A} = 265.214 \times 10^3$$

$$\phi = \frac{Ni}{S} \Rightarrow i = 265.214 \times 10^{-3}$$

$\Rightarrow i = 0.265$

⇒ An iron ring of mean length of an iron core is 100 cm having uniform CA of 10 cm². It is bound by a mag. coil as shown in fig. The dirxn of current flowing through a coil are such that they produce ϕ in opp. dirn. The permeable of iron is taken as 2000. There is a cut in ring creating an air gap of 1 mm. Calc. flux in air gap.

Sol^n: if flux is opp, the i_1 & i_2 are opp.

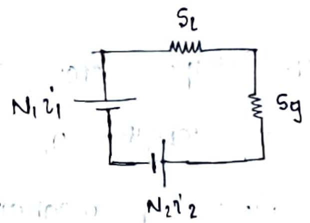


$$\boxed{\cancel{\text{Total mmf}} = \cancel{N_1 i_1 - N_2 i_2} = \cancel{200}}$$

$\therefore$ Total mmf $= N_1 i_1 - N_2 i_2$

$= (100)(3) - (50)(2)$

$= 200$



$$S = \frac{l_i}{a \mu_0 \mu_r} + \frac{l_g}{a \mu_0 \mu_r}$$

$$S = \frac{100 \times 10^{-2}}{(10 \times 10^{-4})(\mu_0)(2000)} + \frac{1 \times 10^{-3}}{(10 \times 10^{-4})(\mu_0)}$$

$$S = \frac{1}{\mu_0 (2)} + \frac{1}{\mu_0} = \frac{1 + 2}{2\mu_0} = \frac{3}{2\mu_0}$$

$$S = \frac{3}{2(4\pi \times 10^{-7})}$$

$\therefore$

$$\phi = \frac{\text{mmf}}{\text{reluctance}} = \frac{200\,(2)\,(4\pi \times 10^{-7})}{3}$$

$$= \underline{\underline{1.67 \times 10^{-4} \text{ Wb}}}$$