

## A.C. CIRCUITS

→ Alternating current or Voltage :

It is the current in which the magnitude and dir<sup>n</sup> vary periodically. The nature of AC is that the current first ( $\uparrow$ ) to max<sup>m</sup> & then falls back to the min<sup>m</sup> to repeat the process in the opp. dir<sup>n</sup> in sinusoidal func<sup>n</sup>.

→ Cycle: One complete set of ~~change in~~ +ve & -ve values of an alternating quantity constitutes one cycle.

→ Time Period: Time taken to complete one cycle by a AC.

→ Frequency: The no. of cycles completed per second.

$$f = \frac{1}{T}$$

→ Instantaneous value :

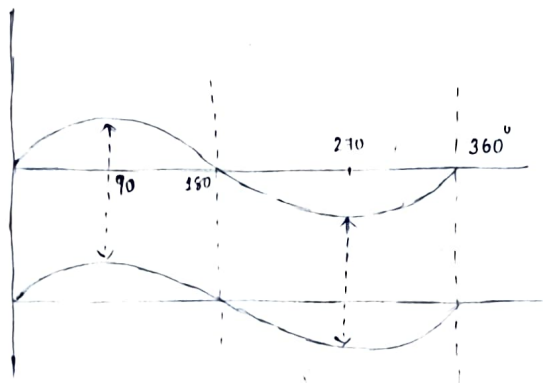
The value of AC at particular instant in a cycle.

→ Peak Value (or) Magnitude (or) Amplitude (or) Max<sup>m</sup> value.

The max<sup>m</sup> value of AC (current/voltage) during a cycle.

→ Inphase quantities :

When two alternating quantities reach their max<sup>m</sup> & min<sup>m</sup> values simultaneously.



→ Out of phase

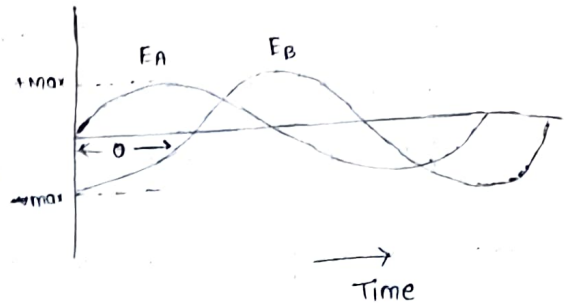
→ Out of phase (Phase diff).

When two alternating quantities do not attain max<sup>m</sup> and min<sup>m</sup> values simultaneously.

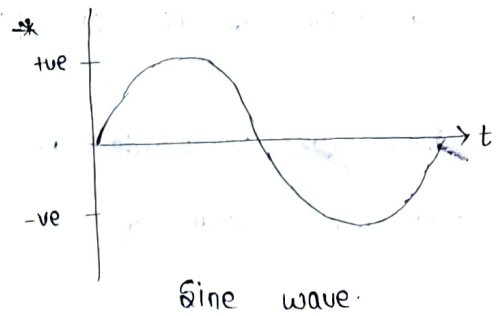
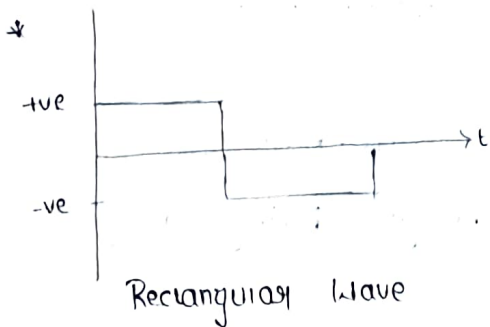
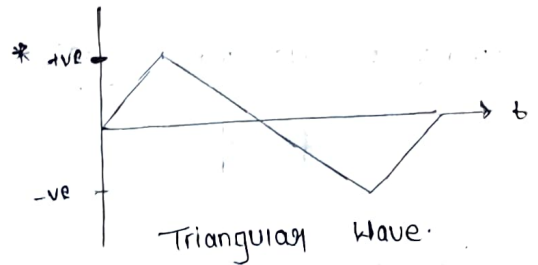
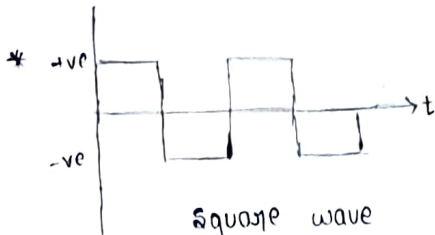
→ Phase Angle:

It is an angular displ<sup>n</sup> b/w two alternating quantities.

$\theta$  → phase angle.



→ Diff types of AC, waves:



→ Positive temp co-efficient of resistance:

The resistance of metal ( $\uparrow$ ) with rise in temp.

$\therefore R \propto T$ , ex: Cu, Al, etc.

→ Negative temp. co-efficient of resistance:

$\therefore R \propto \frac{1}{T}$ , ex: electrolytes, insulators.

# Freq

# Frequency Or A.C. Wave :

The frequency of the alternating voltages generated by an alternator depends upon the no. of poles of the alternator & the speed at which it rotates.

$$f = \frac{PN}{120}$$

where  $P \rightarrow$  Total no. of poles of generator  
 $N \rightarrow$  speed of generator in rpm.

$f \rightarrow$  frequency.

\* Alternator : It converts mechanical energy into AC energy.

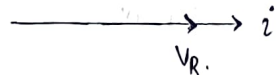
\* Generator : Converts mechanical energy to either DC / AC.

# A.C. Through Pure Resistor ?

\*  $i$  &  $V_R$  are in same phase.

$$* i = \frac{V}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} (\sin \omega t)$$

$$i = i_m \sin \omega t \rightarrow \text{same phase}$$



$$\cos \theta = \frac{R}{Z}$$
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$\rightarrow$  Power :

$$P = Vi$$

$$= (V_m \sin \omega t) (i_m \sin \omega t) = V_m i_m \sin^2 \omega t$$
$$= \frac{V_m i_m}{2} (1 - \cos 2\omega t)$$

$$\therefore P = \underbrace{\frac{V_m i_m}{2}}_{\text{Constant Part}} - \underbrace{\frac{V_m i_m \cos 2\omega t}{2}}_{\text{Fluctuating Part}}$$

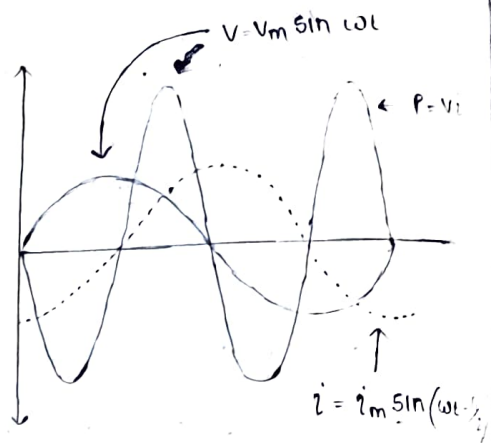
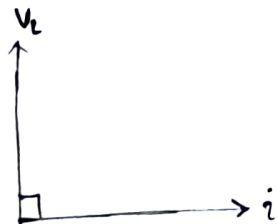
But ~~the~~ avg power consumed is zero in complete cycle.

$$\therefore \text{Power consumed} \Rightarrow P = \frac{V_m}{\sqrt{2}} \cdot \frac{i_m}{\sqrt{2}} = \underline{\underline{V_{rms} i_{rms}}}$$

→ AC Through pure inductor :

\* The AC current flowing in the inductor will set up a magnetic field that is alternating in nature but its magnitude will change at every instant.

\* This will give rise to an ~~on AC~~ alternating induced EMF, often called the EMF of self ~~induction~~ induction, the magnitude of which depends on :  
 i) rate of change of current  
 ii) inductance of coil.



$$* \quad V = -L \frac{di}{dt} \quad \text{---}$$

$$* \quad V = V_m \sin \omega t$$

$$L \frac{di}{dt} = V_m \sin \omega t$$

$$di = \frac{V_m}{L} \sin \omega t \, dt$$

$$\Rightarrow i = \frac{V_m}{\omega L} (-\cos \omega t)$$

$$\Rightarrow i = \frac{V_m}{\omega L} (\sin (\omega t - \frac{\pi}{2}))$$

$$\Rightarrow i = \underline{i_m (\sin (\omega t - \frac{\pi}{2}))}$$

\* Power :

$$P = Vi$$

$$P = V_m \sin \omega t \cdot I_m \sin (\omega t - \frac{\pi}{2})$$

$$P = -V_m I_m \sin \omega t \cos \omega t$$

$$P = -V_m I_m \frac{\sin 2\omega t}{2}$$

$$P = \frac{-V_m I_m}{2} (\sin 2\omega t)$$

\* Avg Power for one complete cycle :

$$P_{avg} = \int_0^{2\pi} \frac{-V_m I_m}{2} \sin 2\omega t \, dt = \underline{\underline{0}}$$

→ Inductive reactance : ( $X_L$ )

$$* \quad X_L = \omega L = 2\pi f L$$

\* The opposition offered by an inductor to AC .

→ AC Through pure Capacitive circuit :

\* When an alternating voltage is applied to the plates of capacitor, it is charged first in one direction & then in the opp. dir<sup>n</sup>.

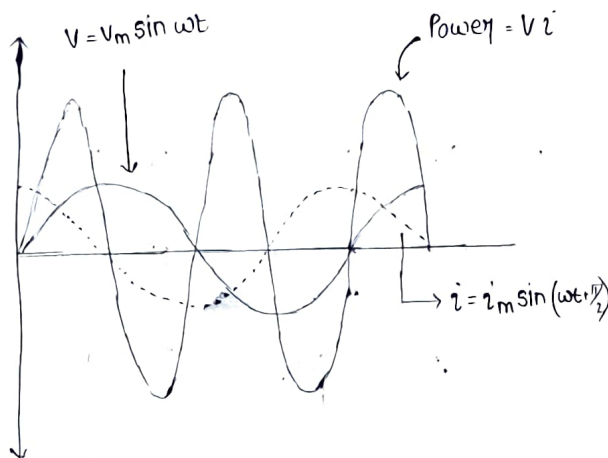
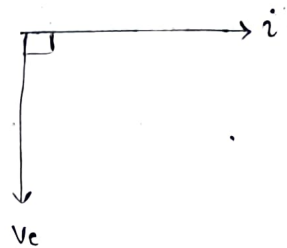
\*  $q = CV$ ,  $i = \frac{dq}{dt}$ ,  $V = V_m \sin \omega t$

$$\Rightarrow i = \frac{d}{dt} (C \cdot V_m \sin \omega t)$$

$$\Rightarrow i = \omega \cdot C \cdot V_m \cos \omega t$$

$$\Rightarrow i = \frac{V_m}{\frac{1}{\omega C}} \cos \omega t$$

$$\Rightarrow i = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin \left(\omega t + \frac{\pi}{2}\right)$$



\* Power :

$$P = Vi$$

$$P = (V_m \sin \omega t) (i_m \sin (\omega t + \frac{\pi}{2}))$$

$$P = V_m i_m \sin \omega t \cdot \cos \omega t$$

$$P = \frac{V_m i_m}{2} \sin 2\omega t$$

\* Capacitive reactance : ( $X_c$ )

$$* X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

\* Avg Power for one cycle :

$$P_{avg} = \int_0^{2\pi} \frac{V_m i_m}{2} \sin 2\omega t \, dt = 0$$

remember.  $\phi = \frac{\pi}{2}$

\*

$$\langle P_{avg} \rangle = C_{rms} i_{rms} \cos \phi$$

⇒ An AC circuit consists of pure resistor of  $10\Omega$  and is connected across AC supply of  $230V$ ,  $50\text{ Hz}$ . Calculate:

i) Current      ii) Power      iii) Eq<sup>n</sup> for voltage & current

Sol<sup>n</sup> i)  $i = \frac{V}{R} = \frac{230}{10} = 23\text{ A}$

ii)  $P = Vi = (23)(230) = \underline{\underline{5290\text{ W}}}$  (rms value)

iii)  $V_m = \sqrt{2} V_{rms} = 325.27\text{ V}$

$i_m = \sqrt{2} i_{rms} = 32.52\text{ A}$

$\therefore \omega = 2\pi f = 2\pi(50) = 314\text{ rad/s}$

$\therefore \begin{aligned} V &= (325.27) \sin(314)t \\ i &= (32.52) \sin(314)t \end{aligned} \quad \left. \vphantom{\begin{aligned} V &= (325.27) \sin(314)t \\ i &= (32.52) \sin(314)t \end{aligned}} \right\} \text{same phase}$

⇒ In pure resistive circuit, the instantaneous voltage &  $i$  is:

$V = 250 \sin 314t$ ,  $i = 10 \sin 314t$

i) Peak power      ii) Avg Power

Sol<sup>n</sup> i) Power (Peak) =  $V_m i_m = \underline{\underline{2500\text{ W}}}$  <sup>imp</sup>

ii)  $P_{avg} = \frac{V_m i_m}{2} = \underline{\underline{1250\text{ W}}}$

⇒ Two capacitors of  $80\mu\text{F}$  &  $50\mu\text{F}$  resp. connected in series.

i) Current      ii) Max<sup>m</sup> energy stored in the circuit when  $200V$  at  $50\text{ Hz}$  are applied across the series circuit

Sol<sup>n</sup>  $C = \frac{C_1 C_2}{C_1 + C_2} = \frac{400}{13} \mu\text{F}$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \underline{\underline{103.4 \Omega}}$$

$$\therefore i = \frac{V_{rms}}{X_c} = \frac{200}{103.4} = 1.93 \text{ A}$$

$$\text{ii) Max energy} = \frac{1}{2} C V_m^2 = \frac{1}{2} \left( \frac{400}{13} \times 10^{-6} \right) (283)^2$$

$\uparrow$   
 Given in capacitor chapter.

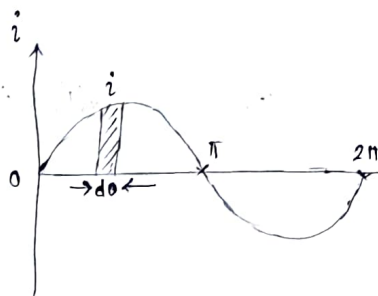
$$= \underline{\underline{1.23 \text{ J}}}$$

$V_m = \sqrt{2} (V_{rms})$

→ Average value of sinusoidal current :

$$i = i_m \sin \theta$$

$$\begin{aligned} \text{Area of half cycle : } \int_0^\pi i \, d\theta &= \int_0^\pi i_m \sin \theta \, d\theta \\ &= i_m [-\cos \theta]_0^\pi \\ &= -i_m [\cos \pi - \cos 0] \\ &= \underline{\underline{2 i_m}} \end{aligned}$$



Avg value :  $I_{av} = \frac{\text{Area of half cycle}}{\text{Base length of half cycle}} = \frac{2 I_m}{\pi}$

$$\therefore I_{av} = 0.637 I_m$$

$$\therefore \underline{\underline{V_{av} = 0.637 V_m}}$$

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 for  $V_{avg}$  also



⇒ A voltage of  $100 \sin \omega t$  is applied to  $100 \Omega$ . Find the

$i_{\text{inst}}$ ,  $i_{\text{avg}}$ ,  $i_{\text{rms}}$  &  $P_{\text{avg}}$ .

Sol<sup>n</sup>

$$V = 100 \sin \omega t$$

$$R = 100 \Omega$$

$$i) i_{\text{inst}} = i(t) = \frac{V(t)}{R} = \frac{100 \sin \omega t}{100} = \sin \omega t \text{ A}$$

$$ii) i_{\text{avg}} = \frac{\Delta I_m}{\pi} = \frac{\Delta \left( \frac{V_m}{R} \right)}{\pi} = \frac{\Delta (1)}{\pi} = 0.697 \text{ A}$$

$$iii) i_{\text{rms}} = \frac{I_m}{\sqrt{2}} = 0.707 \text{ A}$$

$$\begin{aligned} iv) \text{ Power} &= V(t) \times i(t) = 100 \sin^2 \omega t \\ &= 100 \left( \frac{1 - \cos 2\omega t}{2} \right) \\ &= \underline{\underline{50 (1 - \cos 2\omega t)}} \end{aligned}$$

⇒ An inductive coil takes  $10 \text{ A}$  & dissipates  $1000 \text{ W}$  when connected to a supply of  $125 \text{ V}$ ,  $50 \text{ Hz}$ . Calculate the impedance/reactance, inductance & power factor.

Sol<sup>n</sup> .  $P = Vi \cos \theta = i^2 R$

$$R = \frac{P}{i^2} = \frac{1000}{10^2} = \underline{\underline{10 \Omega}} \rightarrow \text{reactance.}$$

$$Z = \frac{V}{i} = \frac{125}{10} = \underline{\underline{12.5 \Omega}} \rightarrow \text{impedance.}$$

$$\cos \phi = \frac{R}{Z} = \frac{10}{12.5} = 0.8$$

$$X_L = \sqrt{Z^2 - R^2} = \underline{\underline{7.5 \Omega}} \rightarrow \text{inductance}$$

$$L = \frac{7.5}{2\pi(50)} = 23.8 \text{ mH} \rightarrow \text{inductance.}$$



⇒ A coil of ~~10 Ω~~  $R = 10 \Omega$ , and inductance is  $0.1 \text{ H}$  is connected in series with capacitor of  $150 \mu\text{F}$  across  $200 \text{ V}$ ,  $50 \text{ Hz}$  supply. Calculate inductive reactance, capacitive reactance, current, power factor,  $V$  across resistor, inductor & capacitor.

Sol<sup>y</sup>  $R = 10 \Omega$ ,  $L = 0.1 \text{ H}$ ,  $C = 150 \mu\text{F}$ ,  $V = 200 \text{ V}$ ,  $f = 50 \text{ Hz}$ .

$$X_L = \omega L = 2\pi fL = 2\pi(50)(0.1) = 31.42 \Omega \rightarrow (1)$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(50)(150 \mu\text{F})} = 21.22 \Omega \rightarrow (2)$$

~~$$i = \frac{200}{10} = 20 \text{ A}$$~~

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 14.28 \Omega$$

$$i = \frac{200}{14.28} = 14 \text{ A} \rightarrow (3)$$

$$\cos \phi = \frac{R}{Z} = \frac{10}{14.28} = 0.7 \rightarrow (4)$$

(lag)  
↳  $\therefore X_L > X_C$

$$V_R = iR = (14)(10) = 140 \text{ V} \rightarrow (5)$$

$$V_L = iX_L = (14)(31.42) = 439.88 \text{ V} \rightarrow (6)$$

$$V_C = iX_C = (14)(21.22) = 297.08 \text{ V} \rightarrow (7)$$

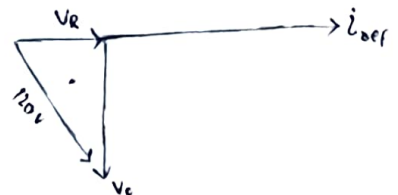
⇒ A capacitance of  $20 \mu\text{F}$ ,  $R = 100 \Omega$  are connected in series across  $120 \text{ V}$ ,  $60 \text{ Hz}$  main. Determine the avg power observed by the circuit and also draw the phasor diagram.

Sol<sup>y</sup>  $X_C = \frac{1}{\omega C} = \frac{1}{2\pi(60)(20 \times 10^{-6})} = 132.6 \Omega$

$$Z = \sqrt{R^2 + X_C^2} = 166.1 \Omega$$

$$i = \frac{120}{166.1} = 0.72 \text{ A}$$

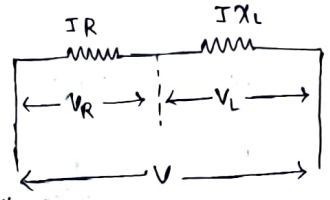
$$\cos \phi = \frac{R}{Z} = 0.601, \quad P = 43.2 \text{ W}$$



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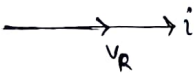
## # RESISTANCE - INDUCTANCE (R-L) series circuit :

- \* In series, same amount of  $i$  flows through all elements  
so, current should be taken as reference.

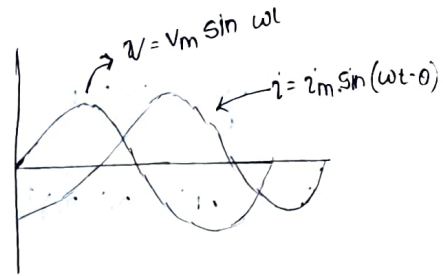
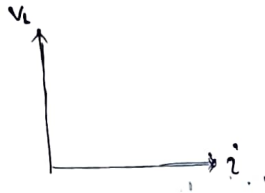


- \*  $V_R = iR$ ,  $V_L = iX_L$

- \* In resistor :



- In Inductor :



- \*  $V^2 = V_R^2 + V_L^2$

$$\Rightarrow V = \sqrt{V_R^2 + V_L^2} = \sqrt{(iR)^2 + (iX_L)^2} = i \sqrt{R^2 + X_L^2}$$

$$\Rightarrow i = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z}$$

- \*  $\tan \theta = \frac{V_L}{V_R} = \frac{X_L}{R}$

- \*  $X_L = \omega L = 2\pi fL$

- \* Power factor  $\Rightarrow \cos \theta = \frac{R}{Z}$

- \* Instantaneous Power :  $P = V \times i$

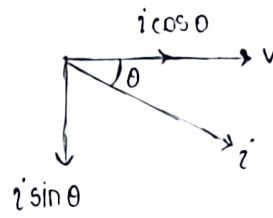
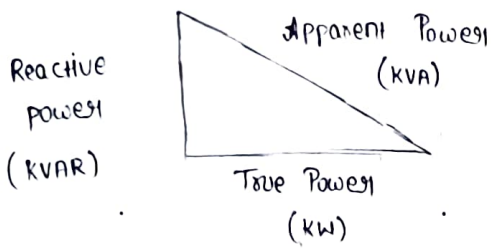
$$\Rightarrow P = V_m \sin \omega t \cdot i_m \sin (\omega t - \theta)$$

$$\Rightarrow P = \frac{V_m}{\sqrt{2}} \cdot \frac{i_m}{\sqrt{2}} \cos \theta - \frac{1}{2} V_m i_m \cos (2\omega t - \theta)$$

(fluctuating power is zero for one cycle)

- \* Avg Power  $\Rightarrow P_{avg} = \frac{V_{rms} i_{rms}}{2} \cos \theta$

→ Apparent Power, True Power, Reactive Power & Power factor:



\* Apparent Power :  $S = V_i = \frac{V_i}{1000} \times \frac{I_i \text{ ampere}}{1000} \text{ KVA}$

\* True Power (or) Real Power (or) Active Power  $\Rightarrow P = V_i \cos \theta = \frac{V_i \cos \theta}{1000} \text{ (kW)}$  <sup>Watts</sup>

\* Reactive Power  $\Rightarrow Q = V_i \sin \theta = \frac{V_i \sin \theta}{1000} \text{ KVAR}$   $\rightarrow$  reactive

\*  $\text{KVA} = \sqrt{(\text{kW})^2 + (\text{KVAR})^2}$

\* Real Power : (P)  $\rightarrow$  The actual power consumed in an A.C circuit

\* Reactive Power : The power absorbed by pure reactance

\* Apparent Power (or) Total Power (S) : It is given by 'product of  $V_i$

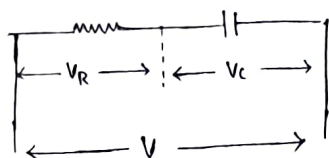
\* Power factor :  $\cos \theta = \frac{R}{Z} = \frac{V_i \cos \theta \rightarrow \text{real power}}{V_i \rightarrow \text{app. power}}$

\* The max value of P.F. is 1  
min value of P.F. is 0

## # R-c Series circuit :

A circuit containing a resistance in series with a capacitance.

$$* V = \sqrt{V_R^2 + V_C^2} = i \sqrt{R^2 + X_C^2}$$

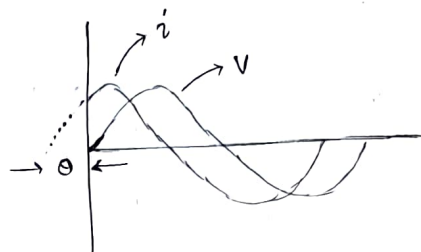


$$* \tan \theta = \frac{V_C}{V_R} = \frac{X_C}{R}$$

\* Instantaneous voltage  $\Rightarrow V = V_{\max} \sin \omega t$

ii current  $\Rightarrow i = i_m \sin (\omega t + \theta)$

$$\therefore P = V i \cos \theta$$



## # R-L-C Series circuit :

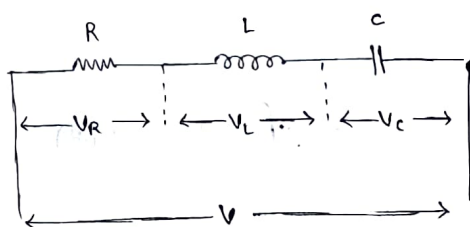
~~Case :~~

→ Conditions :

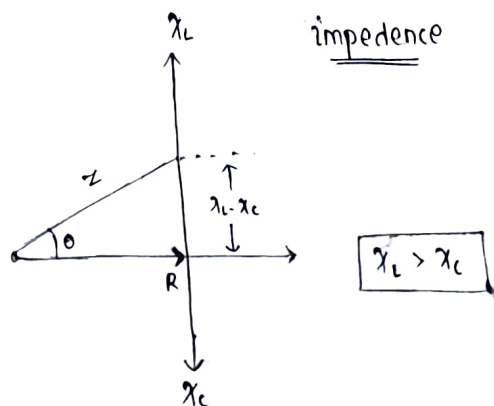
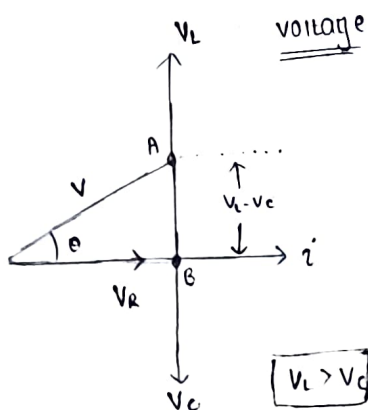
i)  $X_L > X_C \rightarrow$  inductive

ii)  $X_C > X_L \rightarrow$  capacitive

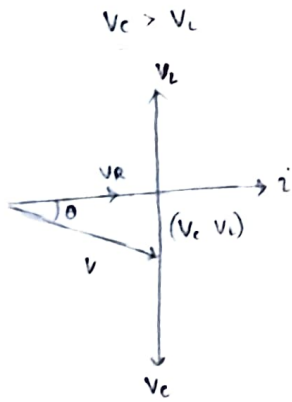
iii)  $X_L = X_C \rightarrow$  Resistive.



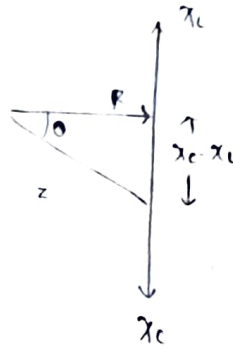
→ Case - 1 :



→ Case - 2 :

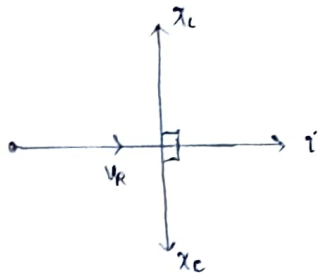


$\chi_c > \chi_L$



→ Case - 3 :

$\chi_L = \chi_c$



$(\chi_L - \chi_c) = 0$

\*  $\chi_L > \chi_c$  → Nature of circuit is inductive, so  $i$  lags the voltage  
 $\theta > 0$  ( $\theta \rightarrow$  phase angle)

\*  $\chi_c > \chi_L$  → Nature of circuit is capacitive, so  $i$  leads the voltage  
 $\theta < 0$

\*  $\chi_c = \chi_L$  → Nature of circuit is resistive,  $i$  &  $v$  are in same phase  
 $\theta = 0$

## # CONVERTING POLAR FORM INTO RECTANGULAR FORM & VICE-VERSA :

\* Rectangular form :  $A + jB$

\* Polar form :  $A \angle B$

\* Note : To perform addition or subtraction, rect form is easy

To perform multiplication or division, polar form is easy.

⇒ Convert  $6 + j8$  to polar form :

Sol<sup>n</sup>) Magnitude :  $\sqrt{6^2 + 8^2} = 10$

$$\tan \theta = \frac{8}{6} \Rightarrow \theta = \tan^{-1} \left( \frac{8}{6} \right) = \underline{\underline{53.1^\circ}}$$

Polar form :  $10 \angle 53.1^\circ$

Exponential form :  $10e^{j53.1}$

I quad  $\rightarrow \theta$

II  $\rightarrow \pi - \theta$

III  $\rightarrow \theta - \pi$

IV  $\rightarrow -\theta$

⇒ Convert  $10 \angle 53.1^\circ$  to rect form :

Sol<sup>n</sup>) polar form :  $10 \angle 53.1^\circ$

$$10 (\cos 53.1^\circ + j \sin 53.1^\circ)$$

$$10 (0.6 + j 0.799)$$

$$6 + 7.99j$$

$$\underline{\underline{6 + 8j}}$$

→ Addition & Subtraction of vector quantities :

$$\vec{v}_1 = a_1 + jb_1 \quad \& \quad \vec{v}_2 = a_2 + jb_2$$

$$\text{Addition : } \vec{v} = \vec{v}_1 + \vec{v}_2 = \underline{\underline{(a_1 + a_2) + j(b_1 + b_2)}}$$

$$|\vec{v}| = \sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}, \quad \theta = \tan^{-1} \left( \frac{b_1 + b_2}{a_1 + a_2} \right)$$

→ Multiplication & division of vector quantities.

$$\vec{V}_1 = a_1 + jb_1, \quad \vec{V}_2 = a_2 + jb_2$$

↓

$$\vec{V}_1 = a_1 \angle b_1$$

↓

$$\vec{V}_2 = a_2 \angle b_2$$

$$\begin{aligned} * \text{ Multiplication } \rightarrow \vec{V}_1 \times \vec{V}_2 &= a_1 \angle b_1 \times a_2 \angle b_2 \\ &= a_1 a_2 \angle b_1 + b_2 \end{aligned}$$

$$\begin{aligned} * \text{ Division } \rightarrow \frac{\vec{V}_1}{\vec{V}_2} &= \frac{a_1 \angle b_1}{a_2 \angle b_2} = \frac{a_1}{a_2} \angle b_1 - b_2 \end{aligned}$$

$$\Rightarrow 10 \angle 30^\circ + 16 \angle -30^\circ$$

$$\begin{aligned} \text{Soln} \rightarrow 10 (\cos 30^\circ + j \sin 30^\circ) &= 8.66 + j5 \\ 16 (\cos 30^\circ - j \sin 30^\circ) &= 13.86 - j8 \end{aligned} \quad \left. \vphantom{\begin{aligned} 10 (\cos 30^\circ + j \sin 30^\circ) \\ 16 (\cos 30^\circ - j \sin 30^\circ) \end{aligned}} \right\} \Rightarrow \underline{\underline{22.52 + j3}}$$

$$\begin{aligned} \Rightarrow \frac{\vec{A} \cdot \vec{B}}{\vec{C}} \quad \vec{A} &= 10 + j10 \\ \vec{B} &= 15 \angle -120^\circ, \quad \vec{C} = 5 + j0 \end{aligned}$$

$$\text{Soln} \rightarrow \vec{A} = 14.14 \angle 45^\circ, \quad \vec{B} = 15 \angle -120^\circ, \quad \vec{C} = 5 \angle 0^\circ$$

$$\frac{\vec{A} \cdot \vec{B}}{\vec{C}} = \frac{(14.14 \times 15)}{5} \angle 45^\circ - 120^\circ - 0 = \underline{\underline{42.42 \angle -75^\circ}}$$

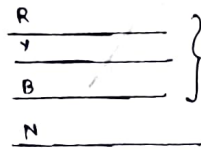


# THREE PHASE SYSTEM

→ SINGLE PHASE :



→ THREE PHASE : (Types)



- \* Balanced system
- \* Unbalanced system

\* Balanced system → Same  $i$  through (R, Y, B)

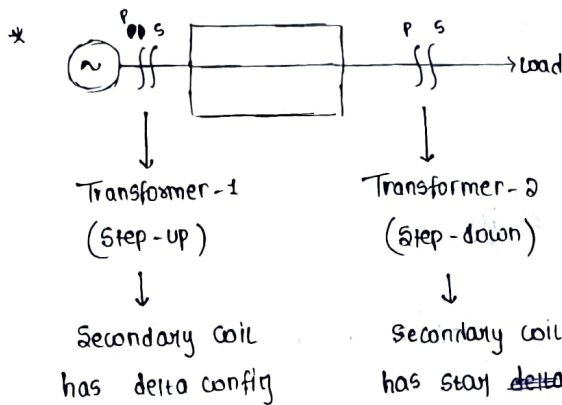
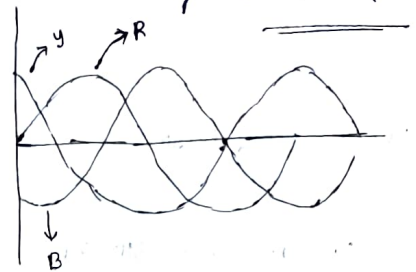
\* Unbalanced system → Not same  $i$  through (R, Y, B)

$$\begin{aligned} R &\rightarrow E_m \sin \omega t \\ Y &\rightarrow E_m \sin (\omega t + \theta) \\ B &\rightarrow E_m \sin (\omega t - \theta) \end{aligned}$$

\* Phase Order : (~~Phase sequence~~)

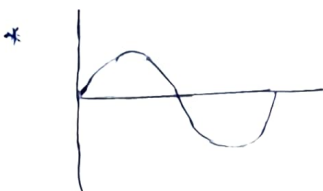
RYB → Positive sequence

BYR → Negative sequence



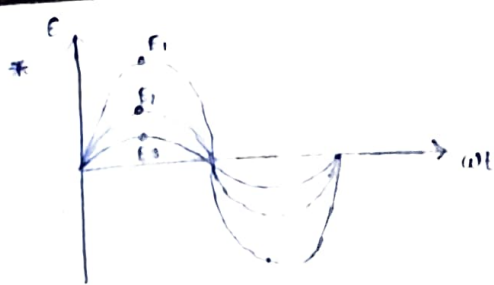
} → one line diagram.

~~\*\*\*~~ \* 3 phase is used for continuous supply of current in a circuit

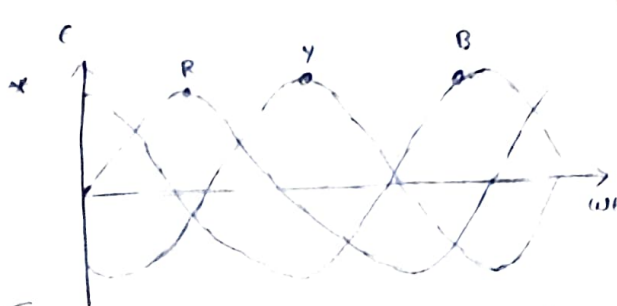


→ In single phase, value goes to zero three times, which may affect the appliance, so we use 3 phase.

\* we don't use 4 & 5 phase because the phases may cancel each other



diff values of  $E$   
at one  $t$



diff  $t$  for same  $E$  at  
diff time,

$$R = \sin(\omega t)$$

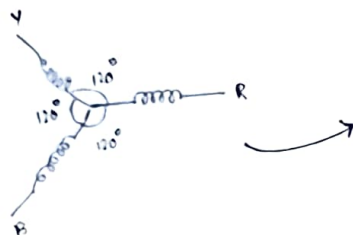
$$Y = \sin(\omega t - 120^\circ)$$

$$B = \sin(\omega t - 240^\circ)$$

+ve phase seq : RYB

-ve phase seq : BYR

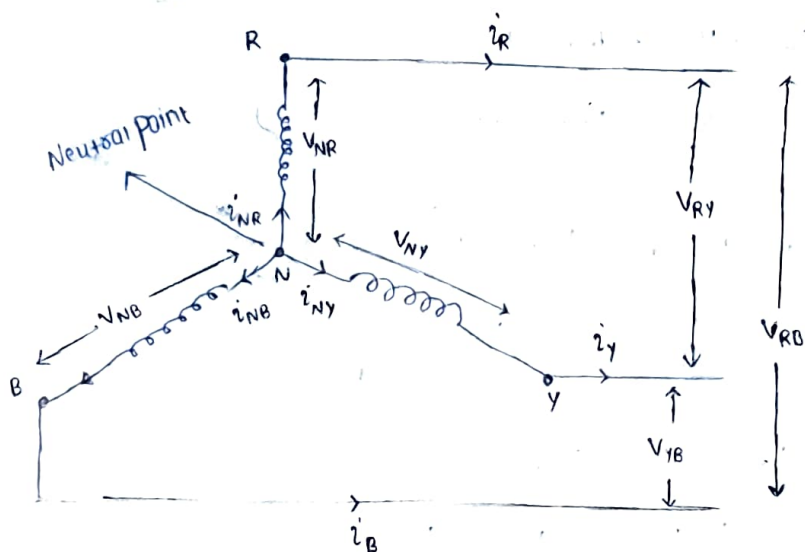
The phase diff. is  $120^\circ$  most times.



# Connection of 3 phase AC  $\rightarrow$  Star  
 $\rightarrow$  delta.

$\rightarrow$  STAR CONNECTION : (Balanced circuits)

It is a type of connection, where one terminal from each moving winding is connected to a common point and the remaining 3 terminals are connected to circuit.



Line quantities :

\* Line voltages :  $V_{RY} = V_{YB} = V_{BR} \} \rightarrow V_L$

\* Line currents :  $I_R = I_Y = I_B \} \rightarrow I_L$

It is balanced  
because phase  
is  $120^\circ$  each

Phase quantities

\* Phase voltages :  $V_{NR} = V_{NY} = V_{NB} \} \rightarrow V_{Ph}$

\* Phase currents :  $i_{NR} = i_{NY} = i_{NB} \} \rightarrow i_{Ph}$

→ For BALANCED CONNECTION :

\* Relation b/w line and phase currents :

$$\begin{array}{ccc} i_{NR} = i_R & i_{NY} = i_Y & i_{NB} = i_B \\ & \downarrow & \swarrow \\ \therefore i_{Ph} = i_L \end{array}$$

\* Relation b/w line & phase voltages :

Applying KVL in mesh : (in pre fig.)

Loop (NRYN)  $\rightarrow V_{NR} + V_{RY} - V_{NY} = 0$

$$V_{RY} = V_{NY} - V_{NR}$$

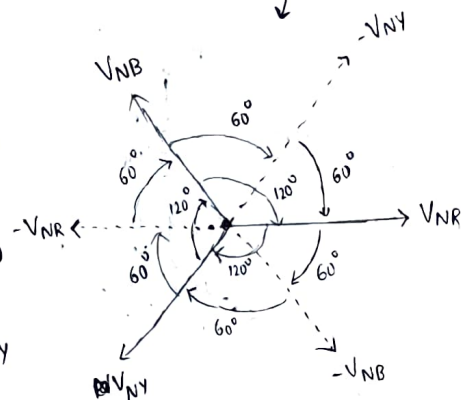
Loop (NYBN)  $\rightarrow V_{NY} + V_{YB} - V_{NB} = 0$

$$\therefore V_{YB} = V_{NB} - V_{NY}$$

Loop (NRBN)  $\rightarrow V_{NR} - V_{BR} - V_{NB} = 0$

$$\therefore V_{BR} = V_{NR} - V_{NB}$$

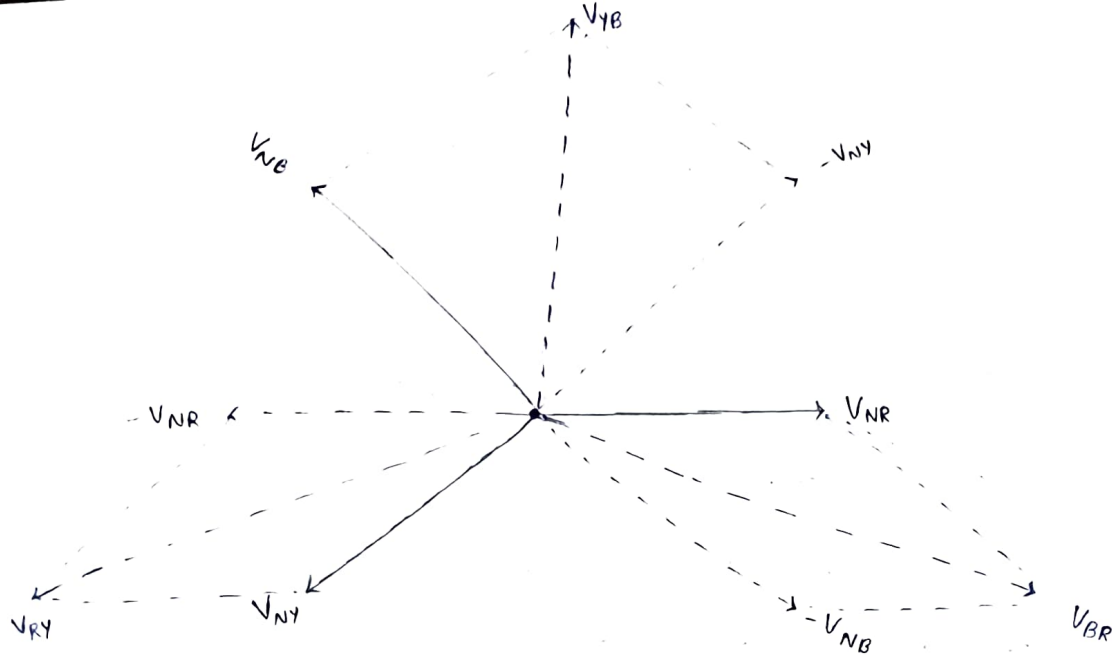
sequence of drawing  
is  $R \rightarrow Y \rightarrow B$



$$\therefore |V_{RY}| = \sqrt{V_{NY}^2 + V_{NR}^2 + 2V_{NY}V_{NR}\cos\phi}$$

$$|V_L| = \sqrt{V_{Ph}^2 + V_{Ph}^2 + 2V_{Ph}^2\cos 60^\circ} \rightarrow |V_L| = \sqrt{3} V_{Ph}$$

from phasor above.

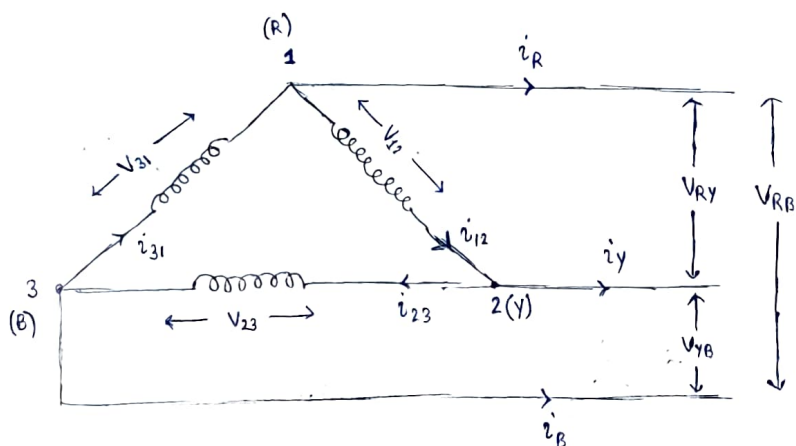


Phasor diagram

Draw phase voltages phases

### → DELTA CONNECTION : (Balanced)

\* It is a type of connection of 3 phase windings where all the coils are connected to back to back arrangement.



### \* line quantities :

\* line voltage :  $V_{RY} = V_{YB} = V_{BR} = V_L$

\* line currents :  $i_R = i_Y = i_B = i_L$

### Phase quantities :

\* Phase voltage :  $V_{12} = V_{23} = V_{31} = V_{Ph}$

\* Phase current :  $i_{12} = i_{23} = i_{31} = i_{Ph}$

\* Relation b/w line and phase voltages :

$$V_{RY} = V_{12} \quad , \quad V_{YB} = V_{23} \quad , \quad V_{RB} = V_{31}$$

$$\therefore \boxed{V_{\text{line}} = V_{\text{phase}}}$$

\* Relation b/w line and phase currents :

By KCL :

$$i_{31} = i_{12} + i_R \Rightarrow i_R = i_{31} - i_{12}$$

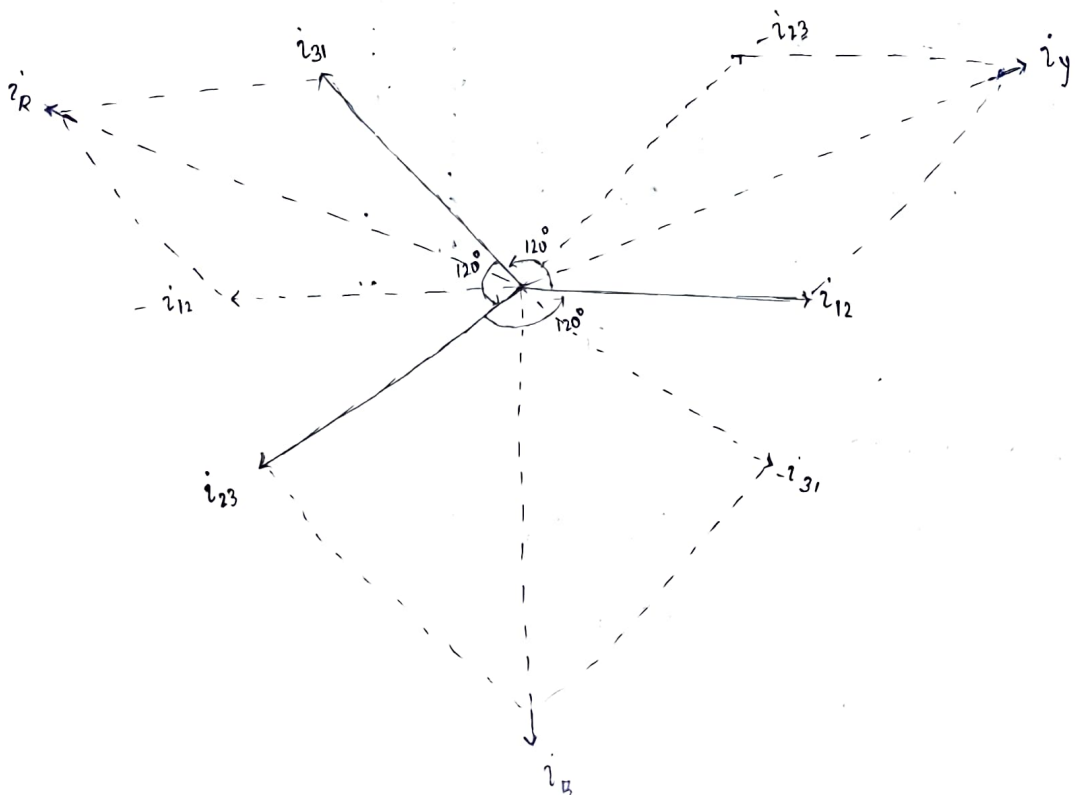
$$i_{12} = i_{23} + i_Y \Rightarrow i_Y = i_{12} - i_{23}$$

$$i_{23} = i_{31} + i_B \Rightarrow i_B = i_{23} - i_{31}$$

$$\therefore |i_R| = \sqrt{i_{31}^2 + i_{12}^2 + 2i_{31}i_{12} \cos \phi}$$

$$i_L = \sqrt{i_{ph}^2 + i_{ph}^2 + 2i_{ph}^2 \cos 60^\circ} = \underline{\underline{\sqrt{3} i_{ph}}}$$

$$\therefore \underline{\underline{i_L = \sqrt{3} i_{ph}}}$$

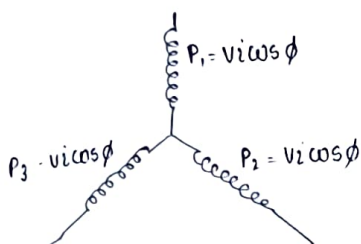


## # Power In $3\phi$ AC : (Balanced)

- \* Active Power : (P) :  $V_{rms} i_{rms} \cos \phi$
- \* Reactive Power (Q) :  $V_{rms} i_{rms} \sin \phi$
- \* Apparent Power (S) :  $V_{rms} i_{rms}$

In  $3\phi$  AC

The power across any load is in line form except impedance which is in phase form



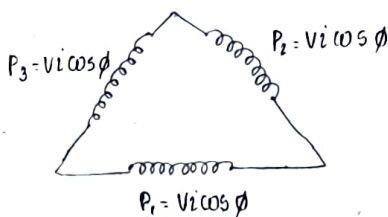
$$\therefore P_T = P_1 + P_2 + P_3$$

$$P_T = 3 V_i \cos \phi$$

$$\therefore V_1 = V_2 = V_3 = V_{ph}$$

$$i_1 = i_2 = i_3 = i_{ph}$$

$$\therefore P_{ph} = 3 V_{ph} i_{ph} \cos \phi$$



$$\therefore P_T = P_1 + P_2 + P_3$$

$$\therefore V_1 = V_2 = V_3 = V_{ph}$$

$$i_1 = i_2 = i_3 = i_{ph}$$

$$P_{ph} = 3 V_{ph} i_{ph} \cos \phi$$

$\therefore$  Power won't change in phase values irrespective of type of connection.

$$I_L = I_{ph} \text{ \& \; } V_L = \sqrt{3} V_{ph}$$

$$V_L = V_{ph} \text{ \& \; } I_L = \sqrt{3} I_{ph}$$

$$P_L = \sqrt{3} V_L I_L \cos \phi$$

$$P_L = \sqrt{3} V_L I_L \cos \phi$$

Power won't change in line values irrespective of type of connection.

$\therefore$  we take power mostly in terms of line because.

the load is connected to line currents or voltages

ex:



we connect load through A, B, C &  $i$  flowing through A, B, C are line currents and line voltages

\* Power in  $3\phi$  AC : In terms of phase In terms of line

Active Power (P) :  $P = 3 V_{ph} i_{ph} \cos \phi$  (watts)  $\sqrt{3} V_L i_L \cos \phi$  (watts)

Reactive Power (Q) :  $Q = 3 V_{ph} i_{ph} \sin \phi$  (VAR)  $\sqrt{3} V_L i_L \sin \phi$  (VAR)

Apparent Power (S) :  $S = 3 V_{ph} i_{ph}$  (VA)  $\sqrt{3} V_L i_L$  (VA)

\* in  $3\phi$  AC, we express anything by default in line form except given in que.

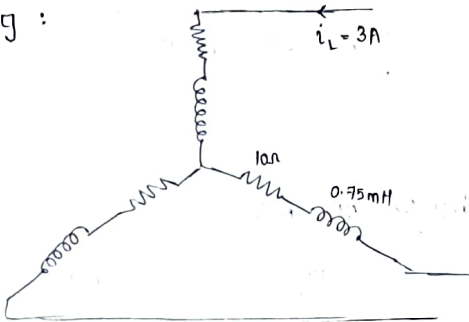
## # NUMERICALS :

$\Rightarrow$  In  $3\phi$  balanced star system is having a coil of resistance  $10\Omega$  & inductance  $0.75$  mH at supply of  $3\phi$ . Calculate :

a) Total impedance b) Total supply voltage

c) Net power d) Power factor

Soln) fig :



$$\therefore X_L = 2\pi f L = 2\pi (50) (0.75 \times 10^{-3})$$

$$X_L = 0.235 \Omega$$

a)  $Z_{ph} = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + (0.235)^2} = \underline{10.002 \Omega}$

b)  $V_L$  is asked

$$\therefore V_{ph} = i_{ph} \times Z_{ph} = i_L \times Z_{ph} \quad (\because \text{in star } i_{ph} = i_L)$$

$$= (3) (10.002)$$

$$V_{ph} = 30.006 \text{ V}$$

$$\therefore V_L = \sqrt{3} V_{ph} = \underline{51.971 \text{ V}}$$

$Z \rightarrow$  always in phase

so, if in que given

$V_L$  &  $Z$  and asked  $i_L$

then,  $i_{ph} = \frac{V_{ph}}{Z}$

then  $i_{ph} \rightarrow i_L$

we cannot do,  $i_L = \frac{V_L}{Z}$

cause  $Z$  in phase

If nothing mention then assume  $V$  &  $i$  as line



$$c) P = \sqrt{3} V_L I_L \cos \phi \quad , \quad \cos \phi = \frac{R}{Z} = \frac{10}{10.002} = 0.999$$

$$P = (\sqrt{3}) (51.971) (3) (0.999) = \underline{\underline{269.779 \text{ W}}}$$

d) we need  $V_m, I_m, \phi$  for phasor:

$$V_m = \sqrt{2} V_{rms} = \sqrt{2} V_L = 73.498 \text{ V}$$

$$I_m = \sqrt{2} I_{rms} = \sqrt{2} I_L = 4.242 \text{ V}$$

$$\phi = \cos^{-1} \left( \frac{R}{Z} \right) = 2.562^\circ$$

~~$I_L$~~   
 $I_L$  &  $I_{ph}$  are by default rms values

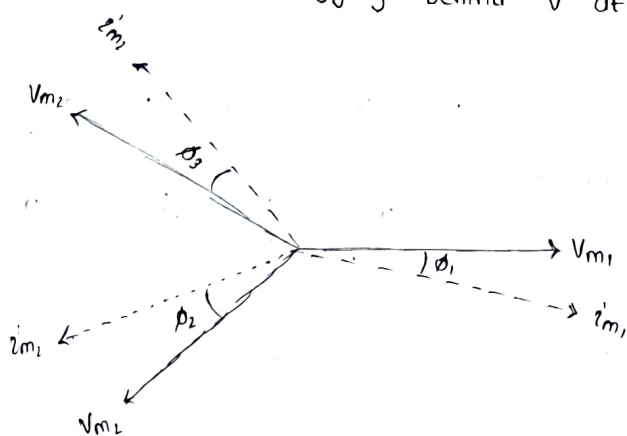
for balanced,  $V_{m1} = V_{m2} = V_{m3} = V_m$

$$I_{m1} = I_{m2} = I_{m3} = I_m$$

$$\phi_1 = \phi_2 = \phi_3 = \phi$$

first draw,  $V$ ,  $120^\circ$  apart ( $\because$  it is balanced).

then draw,  $I$ , lagging behind  $V$  at  $\phi^\circ$  ( $\because$  inductor present)



(Ans)

$\Rightarrow$  If a load of  $Z = 3 - j7\Omega$  in series with an inductor of  $5\Omega$  is connected in balanced  $3\phi$  delta across  $440\text{V}, 50\text{Hz}$  supply. Calculate

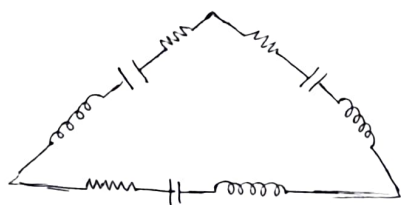
a) Total impedance

b) Net supply current

c) If same load at same current in a  $3\phi$  balanced star formation then find  $k\text{VAR}$  rating. , d) phasor

Soln)  $Z = 3 - j7\Omega \longrightarrow R = 3\Omega, X_C = 7\Omega$

$X_L = 5\Omega, V_L = 440\text{V}, f = 50\text{Hz}$



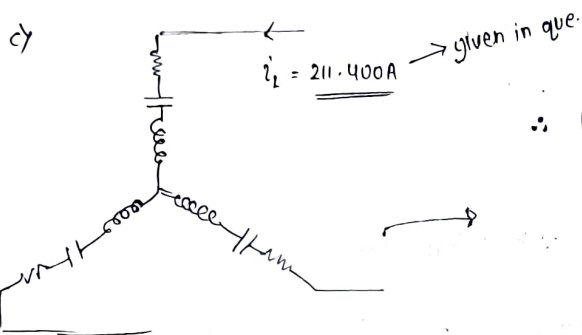
$$\text{a) } Z = \sqrt{R^2 + (X_L - X_C)^2} = Z_{ph}$$



$$Z_{ph} = \underline{\underline{3.605 \Omega}}$$

$$\text{b) } i_{ph} = \frac{V_{ph}}{Z} = \frac{V_L}{Z} = \frac{440}{3.605} = \underline{\underline{122.052 \text{ A}}} \quad (\because V_L = V_{ph} \text{ in 'delta'})$$

$$\therefore i_L = \sqrt{3} i_{ph} = \underline{\underline{211.400 \text{ A}}}$$



$$\therefore \text{KVAR} \Rightarrow Q = \sqrt{3} V_L i_L \sin \phi \text{ VAR}$$

$$V_{ph} = i_{ph} \times Z = (122.052) \times (3.605)$$

$$V_{ph} = 440.09$$

$$V_L = \sqrt{3} V_{ph} = \underline{\underline{1319.99 \text{ V}}}$$

$$Q = (\sqrt{3}) (1319.99) (211.4) (\sin \phi)$$

$$Q = 268133.234 \text{ VAR}$$

$$Q = \underline{\underline{268.133 \text{ KVAR}}} \quad (\text{Ans})$$

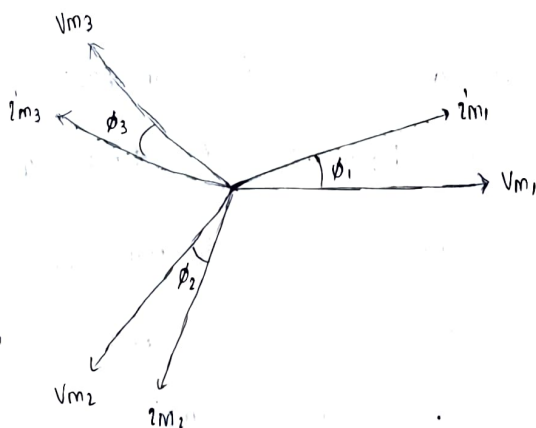
$$\cos \phi = \frac{R}{Z} = \frac{3}{3.605} = 0.822$$

$$\phi = 33.695^\circ$$

$$\text{d) } V_m = 682.253 \text{ V}$$

$$i_m = 298.964 \text{ V}$$

$$\phi = 33.695^\circ$$



\* wattmeter consists of 2 coils namely current & pressure coil.  
It is used to find power in 3 $\phi$

\* we use two wattmeter method to find power in 3 $\phi$ , i.e., wattmeter present on 2 wires.



$$P_{\text{Total}} = W_1 + W_2$$

⇒ Two wattmeters  $P_{out} = 150 \text{ kW}$ ,  $440 \text{ V}$ , 3 phase  $\Delta$  pping induction motor running at full load, the wattmeter reading are  $115 \text{ kW}$ ,  $50 \text{ kW}$ .

i) The input to the motor

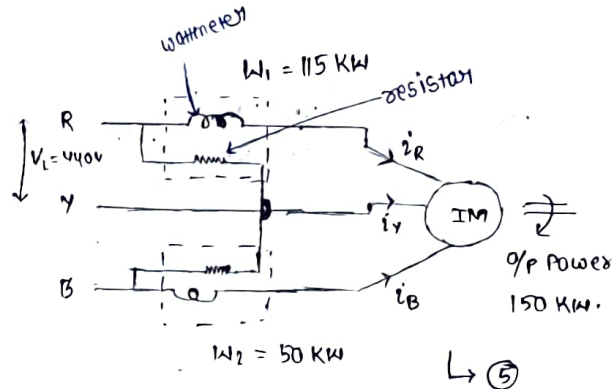
ii) Power factor of the motor

iii) Line current drawn by the motor.

iv)  $\eta$ .

v) Diagram.

Soln) Given :  $150 \text{ kW} \rightarrow \text{o/p Power}$   
 $440 \text{ V} \rightarrow \text{Voltage}$   
 3 phase i.m  
 $W_1 = 115 \text{ kW}$ ,  $W_2 = 50 \text{ kW}$



Power i/p to the motor  $= W_1 + W_2 = 165 \text{ kW} \rightarrow \textcircled{1}$

$$\tan \theta = \sqrt{3} \left[ \frac{W_1 - W_2}{W_1 + W_2} \right] = \sqrt{3} \left[ \frac{65}{165} \right] = 0.682$$

$$\tan \theta = 0.682$$

$$\theta = 34.3^\circ$$

$$\cos 34.3^\circ = 0.826 \rightarrow \textcircled{2}$$

Power i/p to the motor :  $\sqrt{3} V_L I_L \cos \theta$

$$\Rightarrow 165 \text{ kW} = (\sqrt{3}) (440) (I_L) (0.826)$$

$$\Rightarrow I_L = 262.1 \text{ A} \rightarrow \textcircled{3}$$

$$\left. \begin{array}{l} \text{o/p} = 150 \text{ W} \\ \text{i/p} = 165 \text{ W} \end{array} \right\} \eta = \frac{\text{o/p}}{\text{i/p}} \times 100 = 90.9\% \rightarrow \textcircled{4}$$

⇒ 20 wattmeters connected to measure the power input to 3 phase circuit which indicates 15 kW, 1.5 kW resp. The latest reading being reversing the current coil connection.

Q Calculate Power and Power factor.

Soln

$$W_1 = 15 \text{ kW}$$

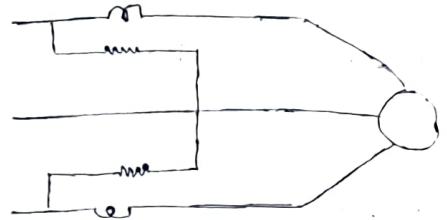
$$W_2 = -1.5 \text{ kW} \leftarrow \text{reversed}$$

$$\text{Total i/p power} : \underline{\underline{13.5 \text{ kW}}}$$

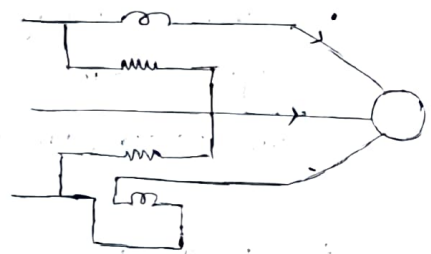
$$\tan \theta = \sqrt{3} \left[ \frac{W_1 - W_2}{W_1 + W_2} \right], \theta = 64.7^\circ$$

$$\therefore \cos \theta = \underline{\underline{0.43}}$$

→ The cosine angle b/w  $i$  &  $V$  of an L-circuit



⇓ reverse



⇒  $P_{\text{out}}$  to a 3 phase, 400V, 37.3 kW inductor motor  $\eta = 88\%$

$\cos \theta = 0.82$  is to be by a wattmeter method. Find

Reading of  $W_1, W_2$  &  $I_L$ .

$$\text{Soln } \eta = 88\%$$

$$\cos \theta = 0.82$$

$$V_L = 400 \text{ V}$$

$$\text{Rating} = 37.3 \text{ kW}$$

$$\eta = \frac{\%P}{i/P} \times 100$$

$$\therefore i/P = \frac{\%P}{\eta} \times 100 = \frac{37.3 \text{ kW}}{88} \times 100 = \underline{\underline{42.386 \text{ kW}}}$$

$$P_T = W_1 + W_2 = 42.386 \text{ kW}$$

$$\cos \theta = 0.82$$

$$\theta = 34.91^\circ$$

$$\tan \theta = \sqrt{3} \left[ \frac{W_1 - W_2}{W_1 + W_2} \right]$$

$$W_1 - W_2 = 17.081 \text{ kW}$$

$$\therefore W_1 = 29.734 \text{ kW}$$

$$W_2 = 12.65 \text{ kW}$$

$$P = \sqrt{3} V_L \cdot I_L \cos \theta$$

$$\text{Bco } I_L = \underline{\underline{74.8 \text{ A}}}$$