MODULE - 3

TAYLOR'S SERIES OF TWO VARIABLES :

* If $f(x,y)$ and all its partial derivatives upto the n^{th} order are finite & continuous for all points (x,y), where

$$a \leq x < a+h, \quad b < y < b+K$$

Then $$f(a+h, b+K) = f(a,b) + \left[h\frac{\partial}{\partial x} + K\frac{\partial}{\partial y}\right]f + \frac{1}{2!}\left[h\frac{\partial}{\partial x} + K\frac{\partial}{\partial y}\right]^2 f + \ldots\ldots$$

* In exam, we get upto 3^{rd} order :

⇒ Expand $e^x \cdot \sin y$ in powers of x & y, $x=0, y=0$ as far as terms of 3^{rd} degree.

Sol^n $$f(x,y) = f(0,0) + \left[x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right]f(0,0) + \frac{1}{2!}\left[x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right]^2 + \frac{1}{3!}\left[x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right]^3$$

~~f(x,y) = f(0,0) =~~ $f(x,y) = e^x \sin y = 0$ (at $x=0, y=0$)

$f_x(x,y) = e^x \sin y = 0$

$f_y(x,y) = e^x \cos y = 1$

$f_{xx}(x,y) = e^x \sin y = 0$

$f_{xy}(x,y) = e^x \cos y = 1$

$f_{yy}(x,y) = -e^x \sin y = 0$

$f_{xxx}(x,y) = e^x \sin y = 0$

$f_{xxy}(x,y) = e^x \cos y = 1$

$f_{xyy}(x,y) = -e^x \sin y = 0$

$f_{yyy}(x,y) = -e^x \cos y = -1$

$$f(x,y) = f(0,0) + x f_x(0,0) + y f_y(0,0) + \frac{x^2}{2!} f_{xx}(0,0) + \frac{2xy}{2!} f_{xy}(0,0) + \frac{y^2}{2!} f_{yy}(0,0)$$

$$+ \frac{1}{3!} x^3 f_{xxx}(0,0) + \frac{3x^2y}{3!} f_{xxy}(0,0) + \frac{3}{3!} xy^2 f_{xyy}(0,0) + \frac{1}{3!} y^3 f_{yyy}(0,0)$$

$$f(x,y) = 0 + x(0) + y(1) + \frac{x^2}{2}(0) + xy(1) + \frac{y^2}{2}(0) + \frac{x^3}{6}(0) + \frac{3x^2y}{6}(1)$$

$$+ \frac{3xy^2}{6}(0) + \frac{y^3}{6}(-1) + \dots$$

$$f(x,y) = y + xy + \frac{x^2y}{2} - \frac{y^3}{6} + \dots$$

Example Ex :- 82.
Ex - 1.17 → 4.

~~⇒ Expand $e^x \cos y$ near the point $(1, \pi/4)$ by Taylor's Thm.~~

~~Soln~~

⇒ Expand $\sin(xy)$ in powers of $(x-1)$ & $(y-\pi/2)$ as far as terms of second degree.

Soln) $f(x,y) = \sin(xy)$

$f(a+h, b+k) = f(x,y)$

$\therefore a+h = x$	$b+k = y$.
given $h = x-1$	given $k = y-\pi/2$
$a + x - 1 = x$	$b + y - \pi/2 = y$
$a = 1$	$b = \pi/2$.

$$\therefore f(x,y) = f(1,\pi/2) + \frac{1}{1!}\left[h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right] f(1,\pi/2) + \frac{1}{2!}\left[h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right]^2$$

$$f(x,y) = 1 + \left[(x-1) f_x(1,\pi/2) + (y-\pi/2) f_y(1,\pi/2)\right] + \frac{1}{2!}\Big[(x-1)^2 f_{xx}(1,\pi/2)$$

$$+ 2(x-1)(y-\pi/2) f_{xy}(1,\pi/2) + (y-\pi/2)^2 f_{yy}(1,\pi/2)\Big].$$

$$\therefore f(x,y) = 1 + (x-1)(0) + (y-\pi/2)(0) + \frac{1}{2!}\left[(x-1)^2\left[\frac{-\pi^2}{4}\right] + 2(x-1)(y-\pi/2)\right.$$

$$\left. + (y-\pi/2)^2(-1)\right]$$

$$f(x,y) = \sin(xy) = 1 - \frac{\pi^2}{8}(x-1)^2 - \frac{\pi}{2}(x-1)(y-\pi/2) - \frac{1}{2}(y-\pi/2)^2$$

⇒ Expand $e^x \cos y$ near the point $(1, \pi/4)$ by Taylor's Th^m.

Sol^n) ~~$h = x-1$, | $K = y - \pi/4$~~

~~→ Here, we are assuming x & y~~

~~so, $f(x+h, y+K)$~~

~~(But in pre que, x & y are given in que~~

~~so, we assumed a & b)~~

$\therefore$ $\cancel{f(1+(x-1), y+(y-\pi/4))}$

$\cancel{f(x+h, y+K)}$ →

$$f(a+h, b+K) = f(x,y)$$

$a+h = x$	$b+K = y$.
$h = x-1$	$K = y-\pi/4$
$a+x-1 = x$	$b+y-\pi/4 = y$
$a = 1$	$b = \pi/4$.

$$\therefore f(x,y) = f(a,b) + \left[h\frac{\partial}{\partial x} + K\frac{\partial}{\partial y}\right]f(a,b) + \frac{1}{2!}\left[h\frac{\partial}{\partial x} + K\frac{\partial}{\partial y}\right]^2 f(a,b).$$

$$f(x,y) = f(1,\pi/4) + (x-1)f_x(1,\pi/4) + (y-\pi/4)f_y(1,\pi/4) + \frac{1}{2!}\left[(x-1)^2 f_{xx}(1,\pi/4)\right.$$

$$\left. + 2(x-1)(y-\pi/4)f_{xy}(1,\pi/4) + (y-\pi/4)^2 f_{yy}(1,\pi/4)\right].$$

$\Rightarrow f(x,y) = \frac{e}{\sqrt{2}}\left[1+(x-1) - \left(y-\frac{\pi}{4}\right) + \frac{(x-1)^2}{2} - (x-1)\left(y-\frac{\pi}{4}\right) - \left(y-\frac{\pi}{4}\right)^2 + \cdots\right]$

$\Rightarrow$ Expand $\frac{(x+h)(y+k)}{x+h+y+k}$ in powers of h, k upto & inclusive of the second degree terms

$\frac{xy}{x+y} = f(x,y)$

$(x+h)(y+k)$

Sol^n^ ~~f(x+h)~~

$$f(x+h, y+k) = \frac{(x+h)(y+k)}{x+h+y+k}$$

$$f(x,y) = \frac{xy}{x+y}, \quad \frac{\partial f}{\partial x} = \frac{y^2}{(x+y)^2}, \quad \frac{\partial f}{\partial y} = \frac{x^2}{(x+y)^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{-2y^2}{(x+y)^3}, \quad \frac{\partial^2 f}{\partial y^2} = \frac{-2x^2}{(x+y)^3}, \quad \frac{\partial^2 f}{\partial x \partial y} = \frac{2xy}{(x+y)^3}$$

$$\therefore f(x+h, y+k) = f(x,y) + \left[h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right] + \frac{1}{2!}\left[\text{——}\right]^2$$

$$\Rightarrow \frac{xy}{x+y} + \frac{hy^2}{(x+y)^2} + \frac{kx^2}{(x+y)^2} - \frac{h^2y^2}{(x+y)^3} + \frac{2hkxy}{(x+y)^3} - \frac{k^2x^2}{(x+y)^3} + \cdots$$

$\Rightarrow$ Expand $(x^2y + \sin y + e^x)$ in power of ~~2nd order a=1, b=π~~ $(x-1)$ & $(y-\pi)$ with 2^nd^ order $a=1,\ b=\pi$

$\Rightarrow$ If $x = a\cosh\xi \cos\eta$, $y = a\sinh\xi \sin\eta$, show that

$$\frac{\partial(x,y)}{\partial(\xi,\eta)} = \frac{a^2}{2}(\cosh 2\xi - \cos 2\eta)$$

assume this h in sin and ξ as angle

Sol^n^

$$\begin{vmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} \end{vmatrix} = \begin{vmatrix} a\sinh\xi\cos\eta & -a\cosh\xi\cos\eta \\ a\cosh\xi\sin\eta & a\sinh\xi\cos\eta \end{vmatrix} = a^2\left[\sinh^2\xi\cos^2\eta + \cosh^2\xi\sin^2\eta\right]$$

$x = 1$, $(x-1) = 0$

$$= \frac{a^2}{2}\left[\cosh 2\xi - \cos 2\eta\right]$$

EXTREMUM VALUES

Step-1 : $f(x,y)$ is given.

Step-2 : $\frac{\partial f}{\partial x} = 0 \rightarrow A$ | $\frac{\partial f}{\partial y} = 0 \rightarrow B$

Solve A & B & find values of 'x' & 'y' & make pair.

Step-3 : $\frac{\partial^2 f}{\partial x^2} = r$ (let say), $t = \frac{\partial^2 f}{\partial y^2}$; $s = \frac{\partial^2 f}{\partial x \partial y}$.

Step-4 : If, $rt - s^2 > 0$ and

i) $r<0$, then $f(x,y)$ has max value

ii) $r>0$, then $f(x,y)$ has min value

Step-5 : If $rt - s^2 < 0$, then $f(x,y)$ has no extremum values.

Step-6 : If $rt - s^2 = 0$, then need further discussion / doubtful cases.

$\Rightarrow f(x,y) = x^3 + y^3 - 3axy$, discuss extremum values :

Sol^n

$\frac{\partial f}{\partial x} = 3x^2 - 3ay = 0$

$\Rightarrow y = \frac{x^2}{a}$

$\frac{\partial f}{\partial y} = 3y^2 - 3ax = 0$

$\Rightarrow x(x^3 - a^3) = 0$

$x = 0, a$

then, $y = 0, a$.

$\therefore$ pairs are $(0,0)$ & (a,a).

$r = \frac{\partial^2 f}{\partial x^2} = 6x$

$t = \frac{\partial^2 f}{\partial y^2} = 6y$

$s = \frac{\partial^2 f}{\partial x \partial y} = -3a$

$rt - s^2 = 36xy - 9a^2$

at $(0,0) \longrightarrow -9a^2 < 0 \longrightarrow$ no extremum values.

at $(a,a) \longrightarrow 27a^2 > 0$

$r = 6a > 0 \longrightarrow$ Min^m value.

⇒ Show that the f^n, $f(x,y) = x^3 + y^3 - 63x - 63y + 12xy$ is max^m at $(-7,-7)$ & min at $(3,3)$.

Sol^n) $\frac{\partial f}{\partial x} = 3x^2 - 63 + 12y = 0$ | $\frac{\partial f}{\partial y} = 3y^2 - 63 + 12x = 0$

$3y^2 + 12x = 63$

⇒ $3x^2 + 12y = 63$

$\therefore 3x^2 + 12y = 3y^2 + 12x$

$3(x^2 - y^2) = 12(x - y)$

$3(x+y)(x-y) = 12(x-y)$.

$3(x-y)[(x+y) - (4)] = 0$

$\therefore x = y, \quad x + y = 4$

$\therefore r = 6x, \quad s = 12, \quad t = 6y$.

if $(-7,-7)$ is max^m, then, $rt - s^2 > 0$ & $r < 0$.

$\therefore 36xy - 144$ | $6(-7) < 0$

⇒ $36(49) - 144 > 0$

Hence, $(-7,-7)$ is max^m.

if $(3,3)$ is min^m, then $rt - s^2 > 0$ & $r > 0$

$36(9) - 144 > 0$ | $6(3) > 0$

Hence $(3,3)$ is min^m

⇒ A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimensions of box requiring least material for its construction.

Sol^n

$V = 32$ cc.

$lbh = 32$

$b = \frac{32}{lh}$

$S = 2(l+b)h + lb$.

$S = 2\left(l + \frac{32}{lh}\right)h + l\left(\frac{32}{lh}\right)$

$S = 2lh + \frac{64}{l} + \frac{32}{h}$

$\frac{\partial S}{\partial l} = 2h - \frac{64}{l^2} = 0$

$h = \frac{32}{l^2}$

$\frac{\partial S}{\partial h} = 2l - \frac{32}{h^2} = 0$.

$l = \frac{\cancel{32}\,16}{h^2}$

$l = \frac{16\,(l^4)}{(32)^2}$

$l^3 = \frac{32 \times \cancel{32}^{\,2}}{\cancel{16}} = 64$

$l = 4$, $h = 2$, $b = 4$

$\therefore \frac{\partial^2 S}{\partial l^2} = \frac{128}{l^3} = \frac{128}{64} = 2$.

$\frac{\partial^2 S}{\partial l\,\partial h} = 2$, $\frac{\partial^2 S}{\partial h^2} = 8$.

Vimp

$\therefore rt - s^2 \Rightarrow$ ~~4 − 64 < 0~~

$\Rightarrow 16 - 4 > 0$

$\therefore 2 > 0$

$\therefore S$ is min^m at $l = 4$, $h = 2$, $b = 4$

Learn absolute maxima and minima, it is in syllabus

⇒ Find the extremum values of $u = x^2y^2 - 5x^2 - 8xy - 5y^2$.

Sol^n) $\frac{\partial u}{\partial x} = 2xy^2 - 10x - 8y = 0$ $\frac{\partial u}{\partial y} = 2x^2y - 8x - 10y = 0$.

$\cancel{2xy^2 - 10x - 8y = 2x^2y - 8x - 10y}$

$\cancel{2y(xy-8) - 10x = 0}$

$2x(y^2 - 5) - 8y = 0$

$x = \frac{4y}{y^2-5}$

$2\left(\frac{4y}{y^2-5}\right)^2 y - 8\left(\frac{4y}{y^2-5}\right) - 10y = 0$

$2y\left[\frac{(16y^2)}{(y^2-5)^2} - 4\left(\frac{4}{y^2-5}\right) - 5\right] = 0$

$y = 0$

$16y^2 - 16(y^2-5) - 5(y^2-5)^2 = 0$.

$16y^2 - 16y^2 + 80 - 5(y^2-5)^2 = 0$

$(y^2-5)^2 = 16$.

$y^2 - 5 = \pm 4$

$y^2 = 9$ & $y^2 = 1$

$y = \pm 3$, $y = \pm 1$

∴ pairs : $(-1, 1)$, $(1, -1)$, $(3, 3)$, $(-3, -3)$, $(0, 0)$

∴ $r = \frac{\partial^2 u}{\partial x^2} = 2y^2 - 10$

$s = \frac{\partial^2 u}{\partial x \partial y} = 4x - 8$

$t = \frac{\partial^2 u}{\partial y^2} = 2x^2 - 10$

∴ $rt - s^2$

$(2y^2-10)(2x^2-10) - (4x-8)^2$

↓

put above 4 values.

↓

we get that

only at $(0,0)$, function is max^m.

LAGRANGIES METHOD :

Step-1 : Given : $f(x,y,z) =$ ______ $\rightarrow$ ①

$\phi(x,y,z) =$ ______ $= 0 \longrightarrow$ ②

Step-2 : let , $F(x,y,z) = f(x,y,z) + \lambda\, \phi(x,y,z)$

'λ' be the undetermined coefficient.

Step-3 :

$\dfrac{\partial F}{\partial x} = 0$	$\dfrac{\partial F}{\partial y} = 0$	$\dfrac{\partial F}{\partial z} = 0$
$x =$ ______	$y =$ ______	$z =$ ______

(x,y,z) have value including 'λ'

Step-4 : Remove 'λ' from 'x', 'y', 'z' by substituting in eq^n ②

then $\lambda =$ ______

Step-5 : Substitute 'λ' value in ③, ④, ⑤

$x =$ ______ , $y =$ ______ , $z =$ ______

Step-6 : put the values of 'x', 'y', 'z' in eq^n ①.

then we have , $f(x,y,z) =$ ______.

$\Rightarrow$ Find the max value of $v = x^p y^q z^r$, subjected to

$ax + by + cz = p+q+r$.

Sol^n) $v = x^p y^q z^r$, $\phi(x,y,z) = ax + by + cz - p - q - r = 0$

$\log v = p\log x + q\log y + r\log z$ $\longrightarrow$ we can put $\log v$ directly because if v is max then $\log v$ is also max^m

$F(x,y,z) = p\log x + q\log y + r\log z + \lambda\left[ax + by + cz - p - q - r\right]$

$\dfrac{\partial F}{\partial x} = 0$	$\dfrac{\partial F}{\partial y} = 0$	$\dfrac{\partial F}{\partial z} = 0$
$x = \dfrac{-p}{\lambda a}$	$y = \dfrac{-q}{\lambda b}$	$z = \dfrac{-r}{\lambda c}$

~~$\lambda = 1$~~

put x, y, z in ϕ to get $\lambda = -1$.

$\therefore x = \frac{p}{a}, \quad y = \frac{q}{b}, \quad z = \frac{r}{c}$

$\therefore f(x,y,z) = \left(\frac{p}{a}\right)^p \left(\frac{q}{b}\right)^q \left(\frac{r}{c}\right)^r$.

It is finding max or min for 3 variable eq^n

$\Rightarrow$ Find the minimum & max^m dist^n of the point $(3,4,12)$ from sphere, $x^2+y^2+z^2=1$

Sol^n

$\phi = x^2+y^2+z^2-1 = 0$

$D = \sqrt{(x-3)^2+(y-4)^2+(z-12)^2}$

$\therefore f = (x-3)^2+(y-4)^2+(z-12)^2$ $\rightarrow$ ~~we can take common magnitude and make a function same done in pre que i.e, log V.~~

$\therefore \frac{\partial f}{\partial x} + \lambda \frac{\partial \phi}{\partial x} \Rightarrow 2(x-3) + \lambda(2x) = 0 \rightarrow (1)$

$\Rightarrow 2(y-4) + \lambda(2y) = 0 \rightarrow (2)$

$\Rightarrow 2(z-12) + \lambda(2z) = 0 \rightarrow (3)$

$\therefore$ Multiply (1) $\times x$, (2) $\times y$, (3) $\times z$

$\therefore (x^2+y^2+z^2) - 3x - 4y - 12z + \lambda(x^2+y^2+z^2) = 0$

$1 - 3x - 4y - 12z + \lambda = 0 \rightarrow (4)$

from 1,2,3 $\rightarrow$ $x = \frac{3}{1+\lambda}, \quad y = \frac{4}{1+\lambda}, \quad z = \frac{12}{1+\lambda}$

put in (4) $\rightarrow$ $1+\lambda - \frac{9}{1+\lambda} - \frac{16}{1+\lambda} - \frac{144}{1+\lambda} = 0 \Rightarrow 1+\lambda = \pm 13$

putting value of $1+\lambda$ in x,y,z $\Rightarrow \left(\frac{3}{13}, \frac{4}{13}, \frac{12}{13}\right) \left(\frac{-3}{13}, \frac{-4}{13}, \frac{-12}{13}\right)$

$\therefore$ Min^m dist^n $= \sqrt{\left(3-\frac{3}{13}\right)^2 + \text{———}} = 12$

Max^m dist^n $= \sqrt{\left(3+\frac{3}{13}\right)^2 + \text{———}} = 14$

In langrange we cannot determine whether point is min or max, just put value of x,y,z to find if its min/max

$\Rightarrow$ If, $u = ax^2 + by^2 + cz^2$, where $x^2 + y^2 + z^2 = 1$ and $lx + my + nz = 0$

P.T. stationary value of 'u' satisfies the eq^n.

$$\frac{l^2}{a-u} + \frac{m^2}{b-u} + \frac{n^2}{c-u} = 0$$

Sol^n

$u = ax^2 + by^2 + cz^2$

$\phi = x^2 + y^2 + z^2 - 1 = 0$

$\psi = lx + my + nz \longrightarrow (4)$

$\frac{\partial u}{\partial x} = 2ax$, $\frac{\partial u}{\partial y} = 2by$, $\frac{\partial u}{\partial z} = 2cz$

$\frac{\partial \phi}{\partial x} = 2x$, $\frac{\partial \phi}{\partial y} = 2y$, $\frac{\partial \phi}{\partial z} = 2z$

$\frac{\partial \psi}{\partial x} = l$, $\frac{\partial \psi}{\partial y} = m$, $\frac{\partial \psi}{\partial z} = n$

$\frac{\partial u}{\partial x} + \lambda_1 \frac{\partial \phi}{\partial x} + \lambda_2 \frac{\partial \psi}{\partial x} = 0 \Rightarrow 2ax + 2x\lambda_1 + \lambda_2 l = 0 \longrightarrow (1)$

$\frac{\partial u}{\partial y} + \lambda_1 \frac{\partial \phi}{\partial y} + \lambda_2 \frac{\partial \psi}{\partial y} = 0 \Rightarrow 2by + 2y\lambda_1 + \lambda_2 m = 0 \longrightarrow (2)$

$\Rightarrow 2cz + 2z\lambda_1 + \lambda_2 n = 0 \longrightarrow (3)$

Multiply (1), (2), (3) with x, y, z resp & add:

$$(2ax^2 + 2by^2 + 2cz^2) + (2x^2 + 2y^2 + 2z^2)\lambda_1 + (lx + my + nz)\lambda_2 = 0$$

$\Rightarrow 2u + 2\lambda_1 = 0$, $\lambda_1 = -u$

putting λ_1 in (1), (2), (3), we get:

$x = \frac{-\lambda_2 l}{2(a-u)}$, $y = \frac{-\lambda_2 m}{2(b-u)}$, $z = \frac{-\lambda_2 n}{2(c-u)}$

put in eq^n in eq (4), we get:

$$\frac{-\lambda_2 l^2}{2(a-u)} + \frac{-\lambda_2 m^2}{2(b-u)} + \frac{-\lambda_2 n^2}{2(c-u)} = 0$$

$$\Rightarrow \frac{l^2}{(a-u)} + \frac{m^2}{(b-u)} + \frac{n^2}{(c-u)} = 0 \text{ (Ans)}$$

⇒ Find the volume of the largest rect. llpiped that can be inscribed in the ellipsoid, $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Sol^n/ $\phi = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0$

Volume of llpiped ⇒ $8xyz$ ($\because$ Length = $2x$, bredth = $2y$, height = $2z$)

$\therefore \frac{\partial v}{\partial x} = 8yz$, $\frac{\partial v}{\partial y} = 8xz$, $\frac{\partial v}{\partial z} = 8xy$.

$\frac{\partial \phi}{\partial x} = \frac{2x}{a^2}$, $\frac{\partial \phi}{\partial y} = \frac{2y}{b^2}$, $\frac{\partial \phi}{\partial z} = \frac{2z}{c^2}$

$\therefore \frac{\partial v}{\partial x} + \lambda \frac{\partial \phi}{\partial x} = 0 \Rightarrow 8yz + \frac{2x\lambda}{a^2} = 0 \longrightarrow (1)$

$\Rightarrow 8xz + \frac{2y\lambda}{b^2} = 0 \longrightarrow (2)$

$\Rightarrow 8xy + \frac{2yz\lambda}{c^2} = 0 \longrightarrow (3)$

Multiply (1)×x, (2)×y, (3)×z

$\therefore 24xyz + 2\lambda\left[\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right] = 0$

$\Rightarrow \lambda = -12xyz$

$\therefore x = \frac{a}{\sqrt{3}}$, $y = \frac{b}{\sqrt{3}}$, $z = \frac{c}{\sqrt{3}}$

$\therefore$ Volume ⇒ $8xyz = 8\left(\frac{a}{\sqrt{3}}\right)\left(\frac{b}{\sqrt{3}}\right)\left(\frac{c}{\sqrt{3}}\right)$

imp.

* Ex - 98 in HK Dass

* Volume of pyramid = $\frac{1}{3}$ (Area of base) (height)

* Surface area of pyramid = $\frac{1}{2}$ (perimeter) (slant height).

* Volume of cone = $\frac{1}{3}\pi r^2 h$.