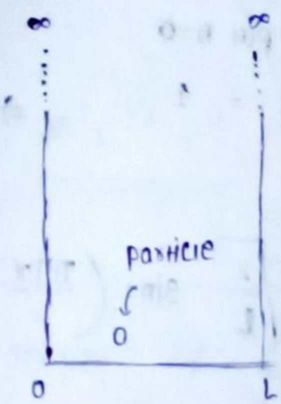
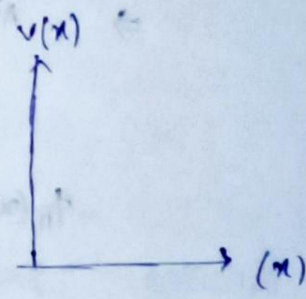
PARTICLE IN A BOX: (RIGID)

* STDE : $i\hbar \partial_t \Psi = \frac{-\hbar^2}{2m} \nabla^2 \Psi + V\Psi$

* STIE : $\nabla \Psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$

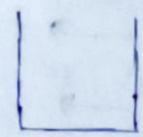
* GOAL : To find $\psi_n(x)$; E_n :





* $\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V(x))\psi = 0 \longrightarrow$ STIE.

$\therefore \psi = \psi(x)$

 $\Rightarrow V(x) = 0$

$V(x) = 0 \quad , \quad x \in (0, L)$

$V(x) = \infty$, otherwise.

 $\Rightarrow V(x) \neq 0$

$\downarrow$

slope of floor.

$$\left.\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E\Psi = 0 \right\} V = 0 \text{ (inside the wall)}$$

(or)

$\psi'' + K^2\psi = 0$, where $K^2 = \frac{2mE}{\hbar^2}$, $\boxed{' \longrightarrow \frac{d\alpha}{dx}}$

$\boxed{\text{from last 3rd page , } K^2 = \frac{P^2}{\hbar^2} = \frac{2mE}{\hbar^2}}$

$\psi(x) = A\sin(kx) + B\cos(kx)$

$\boxed{\psi(0) = 0 \quad \& \quad \psi(L) = 0}$ imp

↳ ψ is zero when $x \to \infty$ here ∞ are 0 & L

$\therefore$ $\psi(0) = 0 \Rightarrow B = 0$

$\Rightarrow \psi(x) = A\sin kx + 0$

$\Rightarrow \psi(L) = A\sin kL = 0$

it must be zero. so

$kL = n\pi$, $n = 1, 2, 3, 4.$

put $k^2 = \dfrac{2mE}{\hbar^2}$ in $\psi'' + k^2\psi = 0$

$\Rightarrow \psi'' + \dfrac{2mE}{\hbar^2}\psi = 0$

$k = \frac{n\pi}{L}$

$\Rightarrow \dfrac{n^2\pi^2}{L^2} = \dfrac{2mE_n}{\hbar^2}$

$\Rightarrow \boxed{E_n = \dfrac{n^2\pi^2\hbar^2}{2mL^2}}$

↳ imp derivation for FAT

$k = \frac{n\pi}{L}$

$\therefore$ $\psi(x) = A\sin\left(\dfrac{n\pi x}{L}\right)$

* $\displaystyle\int_{-\infty}^{+\infty} |\psi|^2\, dx = 1$ ← ~~non probability max~~ ~~of all possibilities~~ is 1

we should normalise equation (ψ)

$\Rightarrow \displaystyle\int_{-\infty}^{0} |\psi|^2 dx + \int_{0}^{L} |\psi|^2 dx + \int_{L}^{\infty} |\psi|^2 dx = 1$

put $n = 0$

$\Rightarrow A^2\left(\frac{L}{2}\right) = 1 \qquad \Rightarrow A = \sqrt{\dfrac{2}{L}}$

$\psi_n(x) = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{n\pi x}{L}\right)$, $n = 1, 2, 3, 4 \ldots\ldots$

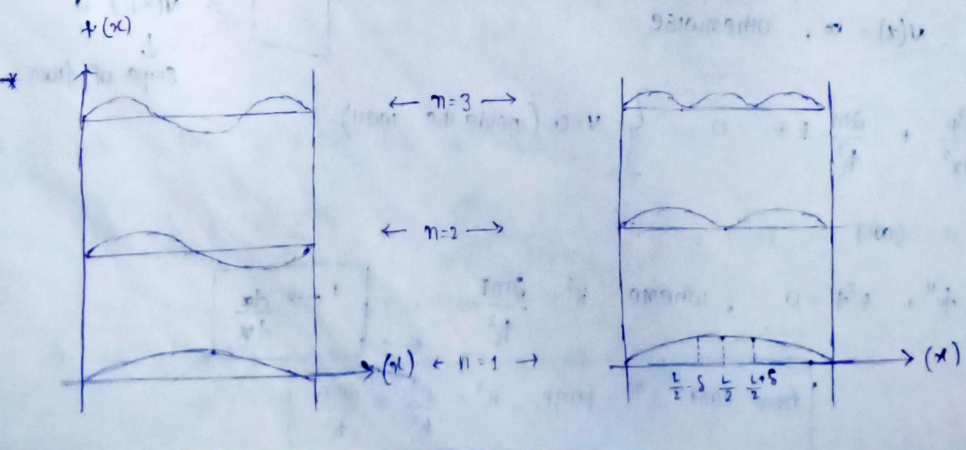
$E_n = \dfrac{n^2\pi^2\hbar^2}{2mL^2}$, $n = 1, 2, 3, \ldots\ldots$

if $n = 1 \longrightarrow$ (ground state)

$n = 2 \longrightarrow$ (1st excited state)

$n = 3 \longrightarrow$ (2nd excited state).

*



Probability $= \int_a^b |\psi_n|^2 dx = \int_{L/2-\delta}^{L/2+\delta} |\psi_1|^2 dx \rightarrow$ for pro diag.

$E_1 = \dfrac{\pi^2 \hbar^2}{2mL^2}$, $E_2 = 4E_1$, $E_n = n^2 E_1$ for $n \geq 2$.

⇒ Find the prob. that the particle is trapped in a box L (ie, b/w 0 to L) can be found b/w 0.45L to 0.55L for the ground state & first excited state ?

Sol^n) Probability : $\int_a^b \left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)\right)^2 dx$

⇒ Compute the dimension $\psi(x)$, $M^{\alpha} L^{\beta} T^{\gamma}$ for particle in 1D box ?

Sol^n) $\Psi(x) = \sqrt{\frac{2}{L}} \sin\left(\underbrace{\frac{n\pi x}{L}}_{\text{const}}\right)$ ∴ $M^0 L^{-1/2} T^0$

⇒ An X-ray photon of wavelength 10 pm is scattered through 110° by an e^-, what is KE of scattered e^-

Sol^n) $\lambda' - \lambda = \lambda_c(1-\cos\theta)$, $\lambda' \longrightarrow$ it is the wavelength of scattered photon, not of e^-

∴ $\lambda' = \lambda_c(1-\cos\theta) + \lambda$

$\lambda' = 13.26$ pm

∴ $KE_e = E - E'$

$= \dfrac{hc}{\lambda} - \dfrac{hc}{\lambda'} = 30.5$ keV

⇒ What about the time dependent wavefunction ?

Sol^n) $\Psi_n(x,t) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \exp\left(\frac{i n^2 \pi^2 \hbar}{2mL^2} t\right)$

$\Psi(x,t) = \psi e^{\frac{-iEt}{\hbar}} \longrightarrow$ put ψ & E.

⇒ A particle limited to x axis has the wave func^n $\psi(x) = ax$ b/w $x=0$ and $x=1$; $\psi(x)=0$ elsewhere.

i) Find prob. that the particle can be found b/w $x=0.45$ and $x=0.55$

ii) Find expectation value $\langle x \rangle$ of particle's position.

Sol^n) i) Prob $= \int_{0.45}^{0.55} (ax)^2 dx = a^2 \int_{0.45}^{0.55} x^2 dx = \frac{a^2}{3}\left[x^3\right]_{0.45}^{0.55}$

$= \frac{a^2}{3}[0.07525]$.

Now, $|\Psi|^2$ should be normalised so: see next 3rd que to understand.

$\boxed{\begin{array}{l} \psi(x) = ax \text{ for } x \in [0,1] \\ \psi(x) = 0 \text{ for } \cancel{\text{otherwise}} \text{ elsewhere} \end{array}}$

cause $L=1$

$\int_0^1 (\psi)^2 dx = 1 \longrightarrow$

$\Rightarrow a^2 \int_0^1 x^2 dx = 1 \Rightarrow \frac{a^2}{3}\left[x^3\right]_0^1 = 1 \Rightarrow \underline{a^2 = 3}$

$\therefore$ Prob $= \frac{3}{3}[0.07525] = \underline{0.07525}$

ii) $x\int_0^1 |\psi|^2 dx = a^2 \int_0^1 x^3 dx = \frac{a^2}{4}\left[x^4\right]_0^1 = \frac{a^2}{4} = \underline{\frac{3}{4}}$ (Ans).

imp

⇒ An eigenfunction of operator $\frac{d^2}{dx^2}$ is $f(x) = e^{4x}$. Find the correspondin eigenvalue

Sol^n) $\frac{df}{dx} = 4e^{4x}$

$\frac{d^2f}{dx^2} = 16e^{4x}$

$\therefore$ eigenvalue means $\boxed{Ax = \lambda x}$, $\lambda \to$ eigenvalue

$\therefore 16e^{4x} = \lambda e^{4x}$

$\therefore \underline{\lambda = 16}$ (Ans)

⇒ Find the prob. that a particle trapped in box L wide can be found b/w 0.45L & 0.55L for ground & first excited state.

Soln) $\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$

$$\therefore \text{Prob} = \int_{0.45L}^{0.55L} \left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)\right)^2 = \frac{2}{L}\int_{0.45L}^{0.55L} \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$$= \frac{2}{L} \int \frac{1-\cos\left(\frac{2n\pi x}{L}\right)}{2\left(\frac{2n\pi x}{L}\right)} dx$$

put $n = 1$ to get Prob^n at ground

put $n = 2$ to get prob at first excited.

⇒ A microscopic particle is represented by following wavefunc^n

$$\psi(x) = \frac{Ax}{a}, \quad x \in [0,a]$$

$$= \frac{A(b-x)}{(b-a)}, \quad x \in [a,b]$$

$$= 0, \quad \text{otherwise.}$$

Compute normalisation constant A.

Soln)
$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1 \Rightarrow \underbrace{\int_{-\infty}^{0} |\psi|^2 dx}_{0} + \int_{0}^{a} |\psi|^2 dx + \int_{a}^{b} |\psi|^2 dx + \underbrace{\int_{b}^{\infty} |\psi|^2 dx}_{0} = 1$$

$$\int_0^a \frac{A^2 x^2}{a^2} dx + \int_a^b \frac{A^2 (b-x)^2}{(b-a)^2} dx$$

solve

⇒ For a particle quantum system, certain microscopic particle has a wave funcⁿ of following form $\Psi(x,0) = c_1 \psi_1(x) + c_2 \psi_2(x)$

Here c_1, c_2 are constants and ψ_1 & ψ_2 are time independent wavefuncⁿ for above quantum system. What is wave funcⁿ $\Psi(x,t)$ at subsequent time? Probability density?

Imp
Must come in exam

Solⁿ:

~~The above eqⁿ is STIE~~

$$\Psi(x,t) = \psi(x)\, g(t) \longrightarrow \text{variable seperable.}$$

we know, $g(t) = e^{-iEt/\hbar}$

time dependent wave function

for our problem, $\Psi(x,t) = \underbrace{c_1 \psi_1(x)}_{\psi(x)}\, \underbrace{e^{-\frac{iE_1 t}{\hbar}}}_{g(t)} + c_2 \psi_2(x) e^{-\frac{iE_2 t}{\hbar}}$

∴ Prob density $= |\Psi|^2$

$$|\Psi|^2 = (\psi)(\psi)^* = |c_1|^2 \psi_1^2 + c_1 c_2^* \psi_1 \psi_2 e^{-\frac{i(E_1-E_2)t}{\hbar}} + c_2 c_1^* \psi_2 \psi_1 e^{\frac{i(E_1-E_2)t}{\hbar}} + |c_2|^2 \psi_2^2$$

giving star to c values to represent they are multiplied.

(Ans)

Scrodinger's Eqⁿ (STIE) from Eigenvalue (Eigenfunc perspective)

* Eigenvalue ⇒ If $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is given then you can find eigenvalues

* Eigenvalue eqⁿ : $\underset{\text{matrix}}{A}X = \overset{\text{eigenvalue}}{\lambda} \underset{\text{eigenvector}}{X}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \therefore X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

* $\hat{A} f = \lambda f$ where $\hat{A} \rightarrow$ operator

$f \rightarrow$ function (eigenvalue)

$\lambda \rightarrow$ eigenvalue (scale)

ex: $\hat{A} = \frac{d}{dx}$, $f_1 = \sin x$

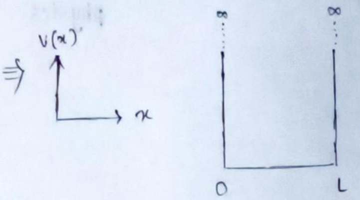
Soln) $\hat{A} f = \frac{d}{dx}(\sin x) = \cos x \neq \lambda f \rightarrow$ so cannot satisfy eigen

ex: $\hat{A} = \frac{d^2}{dx^2}$, $f = \sin x$

Soln) $\hat{A} f = \frac{d^2}{dx^2}(\sin x) = -\sin x = \lambda f \quad \therefore \underline{\lambda = -1}$

Hence $\frac{d^2}{dx^2}$ is eigen opera than d/dx

$\Rightarrow$



Can we write STIE as eigenvalue eqn

Soln) we know, $\frac{d^2\psi}{dx^2} + \frac{2m(E)}{\hbar^2}(\psi) = 0 \longrightarrow$ STIE

$\hookrightarrow$ when $V(x) = 0$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

Hence, here $\hat{A} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

$f = \psi, \quad \lambda = E$

$$\therefore \boxed{\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)\psi_n = E_n\psi_n} \leftarrow \text{eigen eq}^n \text{ of STIE}$$

imp

from STIE : $\Psi(x,t) = f(x)\, g(t)$

$g(t) \longrightarrow e^{-iEt/\hbar}$

$$\frac{d^2 f}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))f = 0$$

$$\therefore \Psi(x,t) = \exp\left[\frac{-i}{\hbar}(Et - px)\right] = \exp\left(-i\left(\frac{E}{\hbar}t - \frac{p}{\hbar}x\right)\right)$$

$$= \exp(-i(\omega t - kx))$$

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{-p^2}{\hbar^2}\Psi$$

$$\therefore \frac{-\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = \left(\frac{-\hbar^2}{2m}\right)\left(\frac{-p^2}{\hbar^2}\right)\Psi$$

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2 \psi}{\partial x^2} = \frac{p^2}{2m}\psi$$

$$\underbrace{\frac{-\hbar^2}{2m}\cdot\frac{\partial^2 \psi}{\partial x^2}}_{\text{This is known as KE operator.}} = E\psi$$

← KE ~~operator~~ eignen value

in phy we generally take KE as operator so we don't do

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} = \frac{2mE}{-\hbar^2}\psi}$$ → in maths we do like this, not in physics.

⇒ How can I recursse the eqⁿ of STIE when there is potential ? (i.e. $V(x) \neq 0$)

Sol^n

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))\psi = 0$$

$$\therefore -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

$$\therefore \hat{A} = \frac{-\hbar^2 \partial^2 \psi}{2m\,\partial x^2} + \hat{V}\psi$$

$$\lambda = E$$

$$f = \psi_n$$

∴ we can write above eqⁿ as :

$$\boxed{\hat{H} = \hat{K} + \hat{V}}$$

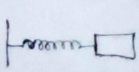
Hamilton $= -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$ V.

^ ⟶ operator

K ⟶ KE

V ⟶ potential

H ⟶ Hamiltonian operator.

∴ 

$$V(x) = \tfrac{1}{2}Kx^2$$

Here x acts as an operator

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}Kx^2$$

after deriving

$$\boxed{E_n = (n + \tfrac{1}{2})\hbar\omega}$$

imp

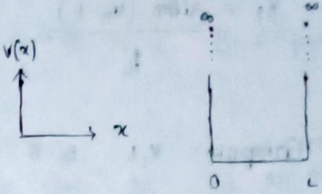
The KE operator of a wave funcⁿ gives energy as it eigenvalue

Tunnel Effect :

* We knew from particle in 1D box :

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \quad , n = 1,2,3,\ldots \quad \text{(eigenfunction)}$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$



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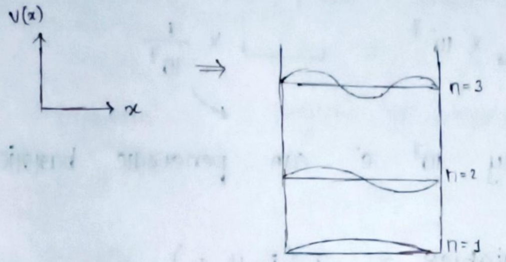
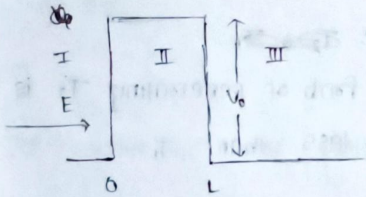


$$\Rightarrow \int_{-\infty}^{\infty} |\psi|^2 \, dx = 1$$

$$\Rightarrow \cancel{\int_{-\infty}^{0} |\psi|^2 dx} + \int_{0}^{L} |\psi|^2 dx + \cancel{\int_{L}^{\infty} |\psi|^2 dx} = 1$$

$\Rightarrow$

*



* Classically the particle would not exist in region III . But quantum mechanically we can find the prob. of finding the particle in reg III

$$T \simeq \exp(-2K_2 L)$$

$$\text{where} \quad K_2 = \frac{\sqrt{2m(V_0 - E)}}{\hbar}.$$

$\Rightarrow$ Compute the dim. of K_2.

Sol^n) $\exp(-2K_2 L)$ $\quad \therefore K_2 = \frac{1}{L} = L^{-1}$ $\quad$ ($\because$ exp is dimensionless).

$$\therefore \underline{M^0 L^{-1} T^0}$$

$\Rightarrow$ Electron with energy 1 eV is incident on barrier of 10 eV, 0.50 nm wide. Compute tunnelling prob.?

Soln) i) Compute K_2

convert eV to J then put in formula

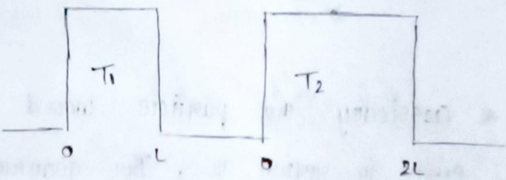
$$K_2 = \frac{\sqrt{2m(V_0 - E)}}{\hbar} =$$

ii) Compute $K_2 L \simeq 8$, $L \rightarrow$ width.

iii) $e^{-2K_2 L} = e^{-16} = \_\_\_\_ \times 10^{-7} = \_\_\_\_ \times \frac{1}{10^7}$

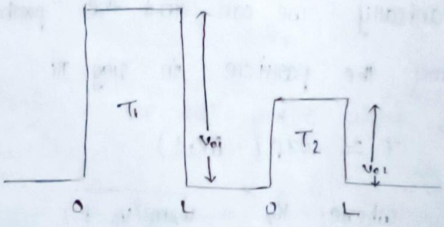
This means, 1 e^- in every 10^7 e^- can penetrate barrier.

$\therefore$ T depends on 3 variables : $T(E, V_0, L)$



$\therefore$ $\cancel{T_2 > T_1}$

Prob of penetrating T_2 is less than T_1.



Prob of penetrating T_2 is more than T_1, cause $V_0 (\downarrow)$

$\longrightarrow$ Applications :

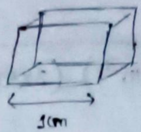
* Tunnel Diode .. $\longrightarrow$ CHATGPT
* Scanning Tunneling Microsope (STM)

} May come in FAT.

NANOPHYSICS

* It mean the realm of 10^{-9} m. to 100×10^{-9} m

* Confinement :

Degree of freedom (DOF)  → it is a 'Bulk' material.

in 3D str.

* If we confine 1D ↑, then structure will be 2D. ex: nano gold particle are dispersed in ruby glass.

when e^- are confined, the particle will have more oscillations this results in colour change

System	Degree of confinement	Degree of freedom.
Bulk	0	3
Q- Well	1	2
Q- wire	2	1
Q- dot	3	0.

imp FAT

Quantum

⇒ The wave func^n of particle is given as :

$$\psi = A \cos^2 x \quad , \quad x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$
$$= 0 \quad , \quad \text{otherwise}$$

Compute A

Sol^n/

$$\int_{-\pi/2}^{\pi/2} |\psi|^2 \, dx = 1$$

$$\int_{-\pi/2}^{\pi/2} A^2 \cos^4 x \, dx = 1 \Rightarrow \int_{-\pi/2}^{\pi/2} \left(\frac{\cos 2x + 1}{2}\right)^2 dx = \frac{1}{A^2}$$

$$\Rightarrow \int_{-\pi/2}^{\pi/2} \cos^2 2x + 1 + 2\cos 2x \, dx = \frac{4}{A^2}$$

$$\Rightarrow \int_{-\pi/2}^{\pi/2} \frac{\cos 4x + 1}{2} + \int_{-\pi/2}^{\pi/2} 1 + 2\cos 2x \, dx = \frac{4}{A^2}$$

$$\Rightarrow \frac{1}{2}\left(\frac{\sin 4x}{4} + x\right)_{-\pi/2}^{\pi/2} + \left(x + \sin 2x\right)_{-\pi/2}^{\pi/2} = \frac{4}{A^2}$$

$$\therefore A = \sqrt{\frac{8}{3\pi}}$$

⇒ Find the smallest possible uncertainity in the position of e^- moving with velocity 3×10^7 m/s ?

Sol^n)

$$\Delta p \cdot \Delta x \geq \frac{h}{4\pi}$$

we always represent at 1 decimal place

$$\therefore \Delta x \geq \frac{h}{4\pi (9.1 \times 10^{-31}) (3 \times 10^7)} \simeq \underline{1.92 \times 10^{-12}}$$

⇒ Compare the uncertainities of velocity of proton & electron confined in $20\mathring{A}$ box ?

Sol^n)

$$\Delta p \cdot \Delta x \geq \frac{h}{4\pi} \qquad \therefore \frac{\Delta V_p}{\Delta V_e} = \frac{m_e}{m_p} = 5.69 \times 10^{-4}$$

$$\therefore m \Delta V \cdot \Delta x = \frac{h}{4\pi}$$

$$\Delta V \propto \frac{1}{m}$$

⇒ An X-ray photon is found to have doubled its wavelength on being scattered by $90°$. Find energy and λ of incident photon.

Sol^n)

$$\lambda' - \lambda = (2.43 \times 10^{-12}) (1 - \cos 90°)$$

$$2\lambda - \lambda = 2.43 \times 10^{-12}$$

$$\lambda = 2.43 \times 10^{-12}$$

$$\lambda = \underline{0.0243\ \mathring{A}}$$

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.43 \times 10^{-12}} = \underline{0.513 \text{ MeV}}$$

⇒ Calculate the energy in eV corresponding to wavelength of $1\mathring{A}$ e^- & neutron

Sol^n $\cancel{\lambda = \frac{h}{\sqrt{2m\,KE}}}$ $\Rightarrow$ $KE = \frac{h^2}{2m\lambda^2}$

For $KE_e = \cancel{\frac{(6.6\times10^{-34}\ Js)^2}{2(9.1\times10^{-31})(10^{-10})^2}} = \frac{2.39\times10^{-17}}{1.6\times10^{-19}}\ eV = \underline{149.375}$

---

first check if, $(pc) > (m_0c^2)$ or $(pc) < E_0$.

$$pc = \frac{hc}{\lambda} = \frac{(6.6\times10^{-34})(3\times10^{8})}{10^{-10}} = 1980\times10^{-18} = 19.8\times10^{-16}$$

$$m_0c^2 = (9.1\times10^{-31})(3\times10^{8})^2 = 819\times10^{-16}$$

Here $pc < m_0c^2$ (not relativistic)

so, use $KE = \frac{1}{2}mv^2$ or $\lambda = \frac{h}{\sqrt{2mKE}}$

imp.

$\Rightarrow$ An e^- has de-Broglie λ 2 p.m. KE ? , V = ?

Sol^n $\cancel{\lambda = \frac{h}{\sqrt{2m\,KE}}}$ $\cancel{\Rightarrow KE = \frac{h^2}{2m\lambda^2} = 598.35\times10^{-16}\ J}$

$$\left.\begin{aligned} pc &= \frac{hc}{\lambda} = 990\times10^{-16} \\ &\quad \hookrightarrow 618750 \\ m_0c^2 &= 819\times10^{-16} \\ &\quad \hookrightarrow 511875 \end{aligned}\right\} \text{Hence relativistic}$$

$\therefore KE = E - E_0$

$KE = \sqrt{(pc)^2 + (m_0c^2)^2} - m_0c^2$ $\leftarrow$ use this to find KE

$= 804036 - 511875\ eV$

$= \underline{292\ keV}$

Here we can't use $KE = \frac{1}{2}mv^2$ ($\because$ it is relativistic)

$\therefore \left.\begin{aligned} E &= mc^2 \\ E_0 &= m_0c^2 \end{aligned}\right\}$ $E = \gamma E_0$ $\therefore$ $m = \gamma m_0$

$$\gamma = \frac{E}{E_0} = \frac{804}{511} = 1.573$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1.573 \Rightarrow 1-\frac{v^2}{c^2} = 0.404$$

$$\cancel{\frac{49}{16} = 1-\frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} =}$$

$$\Rightarrow \frac{v^2}{c^2} = 0.596$$

$$\Rightarrow v^2 = 0.596\,(3\times10^8)^2$$

$$\Rightarrow v = 231603108$$

$$\Rightarrow v = 2.31 \times 10^8$$

⇒ A H_2 atom is 5.3×10^{-11} m in radius. Estimate the min. energy an e^- can have in this atom?

Soln)

$$\cancel{r = 0.53\,\mathring{A} \longrightarrow n = 1}$$

$$\cancel{\Delta p\cdot\Delta x \geq \frac{h}{4\pi}} \qquad \cancel{r = 0.53\,\mathring{A} \longrightarrow n=1}$$

$$\cancel{E = -13.6\left[\frac{z^2}{n^2}\right] = -13.6}$$

$$\Delta x\cdot\Delta v \geq \frac{\hbar}{2m}$$

$$\Delta v \geq 1.1\times10^8$$

$$\therefore K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}\frac{(9.1\times10^{-31})(1.1\times10^8)^2}{(1.6\times10^{-19})}\,eV = 3.4\ eV$$

⇒ A photon and a particle have same wavelength. Can anything be said about how their linear momenta compare? About how the photon's energy compares with particle's total energy? About how the photon's energy compares with particle's kinetic energy?

Soln) * Linear momentum :-

Linear momentum of photon & particle with same wavelength is same because momentum is directly proportional to wavelength and wavelength is same, momenta will be equal.

* Photon's Energy v/s Particles Total Energy :

The energy of photon & ν, energy of particle depends on mass & velocity (i.e, KE & PE).

Hence with same wavelength, it's not possible to make direct comparison b/w these two.

(or)

p is same

$$\underbrace{\sqrt{(m_0c^2)^2 + (pc)^2}}_{\text{particle}} > \underbrace{pc}_{\text{photon}}$$

$$\underbrace{\sqrt{(m_0c^2)^2 + (pc)^2} - m_0c^2}_{\text{particle}} < \underbrace{pc}_{\text{photon}}$$

* Photon's Energy v/s Particle's KE.

Photon's energy depends on ν

Particle's KE depends on m & v

$\therefore$ not comparable.

⇒ Discuss the prohibition of E_0 for a particle trapped in box L wide in terms of uncertainity principle. How does the min^m momentum of such a particle compare with momentum uncertainity required by uncertainity principle if we take $\Delta x = L$

Sol^n $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$.

$\Delta p \geq \frac{\hbar}{2L}$ $(\Delta x = L)$.

Hence $\Delta p \neq 0$

if E_0 is prohibited, KE $= 0$ then $v = 0$ then $p = 0$

But according to $\Delta p \geq \frac{\hbar}{2\pi}$ $\therefore (\Delta p \neq 0)$

Hence to satisfy, there must be some momentum uncertainity means particle cannot have zero velocity.

⇒ The atoms in solid posses a certain min^m zero-point energy even at 0K, while no such restriction holds for molecules in ideal gas. Use uncertainity principle to explain.

Sol^n In Atom:

The particles are fixed to lattice due to which at 0K, the particles has Δx very less, so Δp is very large. This shows that they have min^m zero point energy associated with their non-zero momentum, which keeps them vibrating even at zero temp.

In ~~Mom~~ molecule:

In ideal gas, molecules are far apart, hence here Δx is large, so, Δp is small. This means that in ideal gas, the molecules can have their KE reduced to very close to zero at 0K.

In summary, the principle prevents atoms in solid from coming to complete stop at 0K, leading to min^m zero point energy due to Δp. On other hand, in ideal gas, the relatively larger ~~large~~ ~~Δp~~ Δx, allows molecules to have KE close to zero at 0K.

for ~~50~~ 10 M

$\boxed{\Delta x \; \& \; KE}$

for 5-M: For solid, there is restriction for position of each atoms and so, we can't set Δx as infinity, means Δp is finite so there should be energy even if temp is 0K. But for molecule there is no restriction, so it can be zero at 0K.

⇒ A particle moving in 1-D potential box of width 25 Å. Calculate prob. of finding the particle within an interval of 5 Å at the centers of box when it is in its state of least energy?

Sol^n) $L = 25\,\text{Å} = a$

$n = 1 \rightarrow$ least energy.

~~Prob at centre~~

At center of box : $x = a/2$.

$$\therefore\ |\psi(x)|^2 = \left[\sqrt{\frac{2}{a}}\ \sin\frac{\pi(a/2)}{a}\right]^2 = \frac{2}{a}.$$

$$\therefore\ P = |\psi(x)|^2\ \Delta x$$

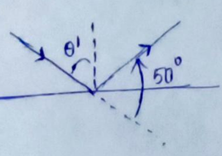
$$P = \frac{2}{a}(5\,\text{Å}) = \underline{\underline{0.4}}$$

⇒ A beam of 50 keV e^- is directed at crystal & diff. e^- are found at an angle of 50° relative to original beam. What is spacing of atomic planes of crystal?

Sol^n) $$\lambda = \frac{h}{\sqrt{2mKE}} = 5.4 \times 10^{-12}\ \text{m}$$

Let incident angle of beam be θ'.

ATQ :



$$\therefore\ 2\theta' + 50^\circ = 180^\circ$$

$$\theta' = 65^\circ$$

$$\therefore\ d = \frac{\lambda}{2\sin\theta'} = \underline{\underline{3\ \text{pm}}}$$

⇒ a) How much time is needed to measure the KE of e^- whose speed is 10 m/s with an uncertainity of no more than 0.1%. How far the e^- have travelled in this period of time.

b) Make the same calculation for 1g insect whose speed is same. What do these fig. indicate.

Sol^n a) $\frac{\Delta E}{E} \times 100 = 0.1$ & $\Delta E \cdot \Delta t = \frac{\hbar}{2}$

$\therefore \Delta t = \frac{\hbar}{mv^2 \times 10^{-3}} = 1.16 \times 10^{-3}$ s.

$\therefore e^-$ travelled 1.16×10^{-3} m.

b) $\Delta t = \frac{\hbar}{mv^2 \times 10^{-3}} = 1.06 \times 10^{-30}$ s

This show that time taken to find KE is less and precise for massive obj rather than light object.

⇒ An unstable elementary particle called eta meson has rest mass 549 MeV/c^2 & mean life time of 7×10^{-19} s. What is uncertainity in its rest mass?

Sol^n. $E_0 = m_0 c^2$

$E = mc^2$

$\therefore \Delta E = \Delta m c^2$

$\therefore \frac{\Delta E}{E} = \frac{\Delta m c^2}{mc^2} = \frac{\Delta m}{m}$

$\therefore \frac{\Delta m}{m} = \frac{\hbar}{2 \Delta t E} = 8.56 \times 10^{-7}$

⇛ Applications of STM : (will come in FAT).

Solⁿ

Soln * The STM shows the positions of atoms more precisely.

* STM's are versatile.

* STMs give the 3D profile of surface, which allows researchers to examine a multitude of characteristic, including roughness, surface defects & molecule size.

* STM is used in study of structure, growth, morphology, electronic str., thin films and nano str.. Lateral resolution of 0.1 nm to 0.01 nm of resolution in depth can be achieved.