7.2-5.

Suppose that the splits at every level of quicksort are in the proportion $1-\alpha$ to α, where $0<\alpha<1/2$ is a constant. Show that the minimum depth of a leaf in the recursion tree is approximately $-\lg n/\lg\alpha$ and the maximum depth is approximately $-\lg n/\lg(1-\alpha)$. (Don't worry about integer round-off.)

Answer.

Figure 1 shows the recursion tree for this split on every level of quicksort. We can see that the the minimum depth of a leaf in the recursion tree is approximately $\log_{\frac{1}{\alpha}} n = -\lg n/\lg\alpha$, and the maximum depth is approximately $\log_{\frac{1}{1-\alpha}} n = -\lg n/\lg(1-\alpha)$.

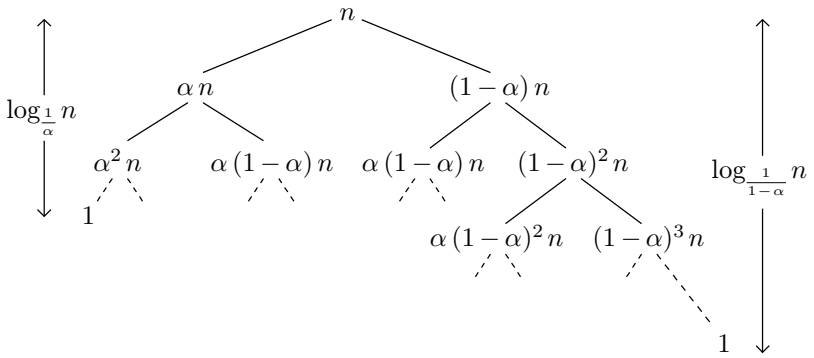

Figure 1. A recursion tree for QUICKSORT in which PARTITION always produces a split in the proportion $1-\alpha$ to α, where $0<\alpha<1/2$ is a constant.

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