6.2-6.

Show that the worst-case running time of MAX-HEAPIFY on a heap of size n is $\Omega(\lg n)$. (*Hint:* For a heap with n nodes, give node values that cause MAX-HEAPIFY to be called recursively at every node on a simple path from the root down to a leaf.)

Answer.

Consider a heap of n nodes with the smallest node value resides at its root. If called at the root, MAX-HEAPIFY will have to exchange the node of the smallest value with its child all the way down to a leaf. For the node's value is always smaller than its children's, thus violating the max-heap property. The procedure terminates when the node eventually "floats down" to a leaf, where it has no child. The amount of swapping operation in this process is exactly the height of the tree, which is $\lg n$. So the worst-case running time of MAX-HEAPIFY on a heap of size n is $\Omega(\lg n)$.

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