7.3-2.

When Randomized-Quicksort runs, how many calls are made to the random-number generator Random in the worst case? How about in the best case? Give your answer in terms of Θ-notation.

Answer.

$\Theta(n)$ calls to Random in both cases. The number of calls made to Random is equal to that of Randomized-Partition, since Random is called once every time Randomized-Partition runs. So we can draw our interest on the partitioning operation to unreveal this problem.

The worst-case behavior occurs when the partitioning routine produces one subproblem with $n-1$ elements and one with 0 elements in each recursive call. Since the recursive call on an array of size 0 just return, $S(0)=\Theta(1)$, the recurrence for the number of calls made to Randomized-Partition is

$$\begin{aligned} S(n) &= S(n-1)+S(0)+\Theta(1) \\ &= S(n-1)+\Theta(1) \end{aligned}$$

Using substitution method, we can show that the recurrence $S(n)=S(n-1)+\Theta(1)$ has the solution $S(n)=\Theta(n)$.

In the best case, Randomized-Partition divides the problem into two subproblem with size $\lfloor n/2 \rfloor$ and $\lceil n/2 \rceil - 1$. The recurrence for the number of calls made to Randomized-Partition is

$$S(n) = 2\,S(n/2)+\Theta(1)$$

By case 1 of the master theorem, this recurrence has the solution $T(n)=\Theta(n)$.

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