7.2-3.

Show that the running time of QUICKSORT is $\Theta(n^2)$ when the array A contains distinct elements and is sorted in decreasing order.

Answer.

Given an array $A[p..r]$ contains distinct elements in decreasing order, PARTITION divide it into two subarrays with 0 elements in $A[p..i]$ and $n-1$ elements in $A[i+1..r-1]$, reducing the size of the problem by 1 in $\Theta(n)$ time. We then obtain the recurrence

$$T(n) = T(n-1) + \Theta(n)$$

on the running time of quicksort. Figure 1 shows the recursion tree for this recurrence. Using substitution method, one can easily prove that the recurrence $T(n) = T(n-1) + \Theta(n)$ has the solution $T(n) = \Theta(n^2)$.

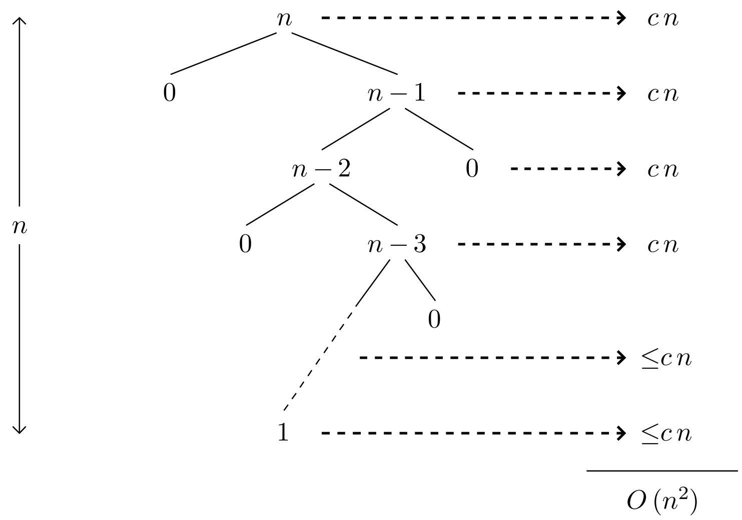

Figure 1. A recursion tree for QUICKSORT in which the array array $A[p..r]$ contains distinct elements in decreasing order, yielding a running time of $O(n^2)$. Node show subproblem sizes, with pre-level costs one the right. The pre-level costs include the constant c implicit in the $\Theta(n)$ term.

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