7.4-4.

Show that RANDOMIZED-QUICKSORT's expected running time is $\Omega(n \lg n)$.

Proof.

The derivation of the lower bound on RANDOMIZED-QUICKSORT's expected running time is almost identical to its upper bound, except that a few adjustment is required to handle $E[x]$ on the final step. We still use a change of variable $(k = j - 1)$ but bound it on a different direction which also employs the harmonic series in equation (A.7):

$$\begin{aligned} E[x] &= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{2}{j-i+1} \\ &= \sum_{i=1}^{n-1} \sum_{k=1}^{n-i} \frac{2}{k+1} \qquad\qquad (k \geq 1) \\ &\geq \sum_{i=1}^{n-1} \sum_{k=1}^{n-i} \frac{2}{2k} \\ &= \sum_{i=1}^{n-1} \Omega(\lg n) \\ &= \Omega(n \lg n) \end{aligned}$$

Therefore, the expected running time of RANDOMIZED-QUICKSORT is $\Omega(n \lg n)$. □

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