6.1-2.

Show that an n-element heap has height $\lfloor \lg n \rfloor$.

Answer.

A heap of height h has a number of elements n such that $2^h \leq n \leq 2^{h+1} - 1$ (see Exercise 6.1-1). So given a number of elements, the height of a heap stretches within the bound

$$\lg(n+1) - 1 \leq h \leq \lg n$$

which can be expanded a little bit into

$$\lg(n) - 1 < \lg(n+1) - 1 \leq h \leq \lg n$$

Hence, an n-element heap is bounded within the height $\lg(n) - 1 < h \leq \lg n$, which is in other word $h = \lfloor \lg n \rfloor$.

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