7.2-1.

Use the substitution method to prove that the recurrence $T(n) = T(n-1) + \Theta(n)$ has the solution $T(n) = \Theta(n^2)$, as claimed at the beginning of Section 7.2.

Proof.

One must show that both $T(n) = O(n^2)$ and $T(n) = \Omega(n^2)$ hold to converge on the correct, asymptotically tight bound $T(n) = \Theta(n^2)$.

We first set out to prove that $T(n) = O(n^2)$. The substitution method requires us to show that $T(n) \le c\,n^2$ for an appropriate choice of the constant $c > 0$. We start by assuming that this bound holds for all positive $m < n$, in particular for $m = n-1$, yielding $T(n-1) \le c(n-1)^2$. Substituting into the recurrence yields

$$\begin{aligned} T(n) &\le c(n-1)^2 + \Theta(n) \\ &\le c(n-1)^2 + d\,n \\ &= c\,n^2 - 2\,c\,n + c + d\,n \\ &\le c\,n^2 \end{aligned}$$

where the last step holds as long as $0 \le d \le c$ and $n \ge 1$. So it turns out that $T(n) = O(n^2)$.

Similarly we can prove that $T(n) = \Omega(n^2)$. Therefore the recurrence $T(n) = T(n-1) + \Theta(n)$ has the solution $T(n) = \Theta(n^2)$. □

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