

**10.1-6.**

Show how to implement a queue using two stacks. Analyze the running time of the queue operations.

**Answer.**

Using two stacks  $S_1$  and  $S_2$ , we can ENQUEUE an element to the queue  $Q$  by PUSHing it into  $S_1$  and DEQUEUE the tail element by POPing from  $S_2$ . Whenever  $S_1$  becomes full or  $S_2$  becomes empty, we transmit as much elements in  $S_1$  to  $S_2$  as possible using TRANSMIT. A queue is said to be empty if and only if both of its stacks  $S_1$  and  $S_2$  are empty. Likewise, the queue becomes full when  $S_1$  and  $S_2$  are both full.

QUEUE-EMPTY( $Q$ )

```

1  if STACK-EMPTY( $S_1$ ) and STACK-EMPTY( $S_2$ )
2      return TRUE
3  else return FALSE

```

QUEUE-FULL( $Q$ )

```

1  if STACK-FULL( $S_1$ ) and STACK-FULL( $S_2$ )
2      return TRUE
3  else return FALSE

```

ENQUEUE( $Q, x$ )

```

1  if QUEUE-FULL( $Q$ )
2      error "overflow"
3  else
4      if STACK-FULL( $S_1$ )
5          TRANSMIT( $S_1, S_2$ )
6      PUSH( $S_1, x$ )
7       $Q.head = S_1.top$ 
8       $Q.tail = S_2.top + 1$ 

```

DEQUEUE( $Q$ )

```

1  if QUEUE-EMPTY( $Q$ )
2      error "underflow"
3  else
4      if STACK-EMPTY( $S_2$ )
5          TRANSMIT( $S_1, S_2$ )
6      POP( $S_2$ )
7       $Q.tail = S_2.top + 1$ 
8       $Q.head = S_1.top$ 

```

TRANSMIT( $S_1, S_2$ )

```

1  if STACK-EMPTY( $S_2$ )
2      POUR( $S_1, S_2$ )
3  else
4       $T = \text{CREAT-STACK}()$ 
5      POUR( $S_2, T$ )
6      POUR( $S_1, T$ )
7      POUR( $T, S_2$ )

```

POUR( $S_1, S_2$ )

```

1  while not STACK-FULL( $S_2$ )

```

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2       $\text{PUSH}(S_2, \text{POP}(S_1))$