6.1-1.

What are the minimum and maximum numbers of elements in a heap of height h?

Answer.

A heap of height h consists of the minimum number of elements when the tree is completely filled on all the first $h-1$ levels, while the hth level contains merely one element on the leftmost branch

$$\begin{aligned} N_{\min} &= 1+2+2^2+\cdots+2^{h-1}+1 \\ &= \frac{1\,(1-2^h)}{1-2}+1 \\ &= 2^h \end{aligned}$$

Likewise, when the heap corresponds to a complete binary tree of the same height h, it has the maximum number of elements

$$\begin{aligned} N_{\max} &= 1+2+2^2+\cdots+2^h \\ &= \frac{1\,(1-2^{h+1})}{1-2} \\ &= 2^{h+1}-1 \end{aligned}$$

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