6.3-3.

Show that there are at most $\lceil n/2^{h+1}\rceil$ nodes of height h in any n-element heap.

Proof.

We choose to prove this proposition by mathematical induction

Base case. Nodes of height 0 are leaves and there are at most $\lceil n/2\rceil = \lceil n/2^{0+1}\rceil$ leaves in an n-element heap (see Exercise 6.1-7). Thus, the proposition holds for the base case where $h=0$.

Inductive step. Suppose the proposition holds for the case of $h = k$, that is, there are at most $\lceil n/2^{k+1}\rceil$ nodes of height k in any n-element heap. As heap closely approximates to comple binary tree, nearly every two nodes on level of height k share a parent on level of height $k+1$. So there are at most $\lceil\lceil n/2^{k+1}\rceil/2\rceil = \lceil n/2^{(k+1)+1}\rceil$ nodes of height $k+1$, which indicates the proposition also holds for the case where $h = k+1$.

By the principle of mathematical induction, there are at most $\lceil n/2^{h+1}\rceil$ nodes of height h in any n-element heap. □

Email address: informlarry@gmail.com