4.3-9.

Solve the recurrence $T(n) = 3\,T(\sqrt{n}) + \log n$ by making a change of variables. Your solution should be asymptotically tight. Do not worry about whether values are integral.

Answer.

Renaming $m = \log n$ yields

$$T(2^m) = 3\,T\left(2^{m/2}\right) + m$$

We can now rename $S(m) = T(2^m)$ to produce the new recurrence

$$S(m) = 3\,S(m/2) + m$$

This new recurrence has the solution: $S(m) = \Theta(m^{\lg 3})$. Changing back from $S(m)$ to $T(n)$, we obtain

$$T(n) = T(2^m) = S(m) = \Theta(m^{\lg 3}) = \Theta\left((\log n)^{\lg 3}\right)$$