7.2-1.

Use the substitution method to prove that the recurrence $T(n) = T(n-1) + \Theta(n)$ has the solution $T(n) = \Theta(n^2)$, as claimed at the beginning of Section 7.2.

Proof.

The substitution method requires us to show that $T(n) \leq c\, n^2$ for an appropriate choice of the constant $c > 0$. We start by assuming that this bound holds for all positive $m < n$, in particular for $m = n-1$, yielding $T(n-1) \leq c\,(n-1)^2$. Substituting into the recurrence yields

$$\begin{array}{rcl} T(n) & \leq & c\,(n-1)^2 + \Theta(n) \\ & \leq & c\,(n-1)^2 + d\,n \\ & = & c\,n^2 - 2\,c\,n + c + d\,n \\ & \leq & c\,n^2 \end{array}$$

where the last step holds as long as $0 \leq d \leq c$ and $n \geq 1$. Therefore the recurrence $T(n) = T(n-1) + \Theta(n)$ has the solution $T(n) = \Theta(n^2)$. □

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