7.4-1.

Show that in the recurrence

$$T(n) = \max_{0 \le q \le n-1} (T(q) + T(n-q-1)) + \Theta(n)$$

$T(n) = \Omega(n^2)$.

Proof.

We begin the argument with a guess that $T(n) \ge c\,n^2$ for some constant c. Substituting this guess into recurrence (7.1) above, we have

$$\begin{aligned} T(n) &\ge \max_{0 \le q \le n-1} (c\,q^2 + c\,(n-q-1)^2) + \Theta(n) \\ &= c \cdot \max_{0 \le q \le n-1} (q^2 + (n-q-1)^2) + \Theta(n) \end{aligned}$$

The expression $q^2 + (n-q-1)^2$ achieves a maximum over that parameter's range $0 \le q \le n-1$ at either endpoint. This indicates that $\max_{0 \le q \le n-1} (q^2 + (n-q-1)^2) \le (n-1)^2 = n - 2\,n + 1$. Going back to the bounding of $T(n)$, we obtain

$$\begin{aligned} T(n) &\ge c\,n^2 - c\,(2\,n-1) + \Theta(n) \\ &\ge c\,n^2 \end{aligned}$$

since we can pick the constant large enough so that the $c\,(2\,n-1)$ term dominates the $\Theta(n)$ term. Therefore, $T(n) = \Omega(n^2)$. □

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