6.1-3.

Show that in any subtree of a max-heap, the root of the subtree contains the largest value occurring anywhere in that subtree.

Proof.

- If the subtree S consists only one node, then the root contains the only value of S, thus the largest one.

- If the subtree S contains more than one node, we denote the root of S by $A[x]$. The property of complete binary tree suggests that the two children of $A[x]$ are $A[2\,x]$ and $A[2\,x+1]$, and they each are the root of a subtree of S. Suppose $A[2\,x]$ and $A[2\,x+1]$ each contains the largest value in one of the subtrees of S. According to the max-heap property, which states that for every node i other than the root, $A[\textsc{Parent}(i)] \geq A[i]$, thus we have $A[x] \geq A[2\,x]$ and $A[x] \geq A[2\,x+1]$. So $A[x]$ is not smaller than any elements in the two substrees of S. Since $A[x]$ together with all the elements in the two subtrees of S makes the entire subtree S. Hence, the root $A[x]$ contains the largest value in the subtree S.

Therefore, we can show by mathematical induction that in any subtree of a max-heap, the root of the subtree contains the largest value occurring anywhere in that subtree. □

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