7.4-3.

Show that the expression $q^2+(n-q-1)^2$ achieves a maximum over $q=0,1,\ldots,n-1$ when $q=0$ or $q=n-1$.

Proof.

Consider the function $f(x)=x^2+(n-x-1)^2$ on the range $0\le x\le n-1$. The first derivate of this expression with respect to x is $f'(x)=2[2x-(n-1)]$, which equals to 0 iff $2x-(n-1)=0$, that is, when $x=\frac{n-1}{2}$. Further more, we can conclude the sign of $f'(x)$ on the interval $[0,n-1]$, as Table 1 shows. The sign of $f'(x)$ indicates that $f(x)$ monotonically decreases on $\left[0,\frac{n-1}{2}\right)$ until drop to

x	$\left[0,\frac{n-1}{2}\right)$	$\frac{n-1}{2}$	$\left(\frac{n-1}{2},n-1\right]$
$f'(x)$	$f'(x)<0$	$f'(x)=0$	$f'(x)>0$

Table 1. The sign of expression $f'(x)=2[2x-(n-1)]$ on interval $[0,n-1]$.

the bottom at $\frac{n-1}{2}$, then the function grows steadly all the way on $\left(\frac{n-1}{2},n-1\right]$. The course of $f(x)$ reveals that the maximum value locates at either of its endpoint. As $f(0)=f(n-1)=(n-1)^2$, $x^2+(n-x-1)^2$ reaches its maximum when $x=0$ or $x=n-1$. Therefore, the expression $q^2+(n-q-1)^2$ achieves a maximum over $q=0,1,\ldots,n-1$ when $q=0$ or $q=n-1$. □

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